OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD 15^{00} DO 16^{30}.

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

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IME I PREZIME: LUKA SJAUŠ

BROJ INDEKSA: 57680

Broj ↓ bodova

1. Ako su z_1 i z_2 rješenja kvadratne jednadžbe $z^2 + 2 = 0$, izračunati: 20

   (a) $\overline{\left(\frac{z_1 - z_2}{z_2 - 2}\right)}$;

   (b) $\overline{\left(\frac{z_2}{z_1}\right)}$.

VIDI HAMARIĆ

2. Gaussovom metodom riješiti sustav jednadžbi: 20 0

$$\begin{aligned} 5x_1 + x_2 + x_3 - x_4 &= 3 \\ x_1 + x_2 - x_3 + 2x_4 &= -10 \\ -2x_1 - x_2 + x_3 + x_4 &= -10 \\ x_2 + x_3 &= 4 \end{aligned}$$

3. Ispitati tok funkcije: $f(x) = \sqrt{8+x} - \sqrt{8-x}$. Da li postoje lokalni ekstremi? 40 10

4. Pronaći prvu i drugu derivaciju funkcije: $g(x) = (\arctan x)^2$. 20 20

2)

$$\begin{bmatrix} 5 & 1 & 1 & -1 & 3 \\ 1 & 1 & -1 & 2 & -10 \\ -2 & -1 & 1 & 1 & -10 \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \cdot 2 \sim \begin{bmatrix} 1 & -1 & 3 & 1 & -17 \\ 1 & 1 & -1 & 2 & -10 \\ -2 & -1 & 1 & 1 & -10 \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \cdot(-1) \cdot 2 \sim \begin{bmatrix} 1 & -1 & 3 & 1 & -17 \\ 0 & 2 & -4 & 1 & 7 \\ 0 & -3 & 7 & 3 & -44 \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \cdot 2$$

$$\sim \begin{bmatrix} 1 & -1 & 3 & 1 & -17 \\ 0 & 2 & -4 & 1 & 7 \\ 0 & 1 & -1 & 5 & -30 \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 3 & 1 & -17 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 1 & -1 & 5 & -30 \\ 0 & 2 & -3 & 1 & -7 \end{bmatrix} \cdot(-1) \cdot(-2) \sim \begin{bmatrix} 1 & -1 & 3 & 1 & -17 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & -2 & 5 & -34 \\ 0 & 0 & -6 & 1 & -15 \end{bmatrix} \cdot(-3)$$

$$\sim \begin{bmatrix} 1 & -1 & 3 & 1 & -17 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & -2 & 5 & -34 \\ 0 & 0 & 0 & -14 & 87 \end{bmatrix}$$

$-14 x_4 = 87$

$x_4 = 87 / (-14)$

$x_4 = \frac{87}{14}$

$$\begin{bmatrix} 5 & 1 & 1 & -1 & 3 \\ 1 & 1 & -1 & 2 & -10 \\ -2 & -1 & 1 & 1 & -10 \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \cdot(-1) \cdot(-1) \sim \begin{bmatrix} 5 & 0 & 0 & -1 & -1 \\ 1 & 0 & -2 & 2 & -14 \\ -2 & 0 & 2 & 1 & -6 \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \cdot(-5) \cdot 2 \sim$$

$$\sim \begin{bmatrix} 0 & 0 & 10 & -11 & 69 \\ 1 & 0 & -2 & 2 & -14 \\ 0 & 0 & -2 & 5 & -34 \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \cdot(-1) \cdot 5 \sim \begin{bmatrix} 0 & 0 & 0 & 14 & -101 \\ 1 & 0 & 0 & -3 & 20 \\ 0 & 0 & -2 & 5 & -34 \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 & 1 & -\frac{101}{14} \\ 1 & 0 & 0 & -3 & 20 \\ 0 & 0 & -2 & 5 & -34 \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \cdot 3 \cdot(-5)$$

$$\sim \begin{bmatrix} 0 & 0 & 0 & 1 & -\frac{101}{14} \\ 1 & 0 & 0 & 0 & -\frac{23}{14} \\ 0 & 0 & -2 & 0 & \frac{29}{14} \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 & 1 & -\frac{101}{14} \\ 1 & 0 & 0 & 0 & -\frac{23}{14} \\ 0 & 0 & 1 & 0 & -\frac{29}{28} \\ 0 & 1 & 1 & 0 & 4 \end{bmatrix} \cdot(-1) \sim \begin{bmatrix} 0 & 0 & 0 & 1 & -\frac{101}{14} \\ 1 & 0 & 0 & 0 & -\frac{23}{14} \\ 0 & 0 & 1 & 0 & -\frac{29}{28} \\ 0 & 1 & 0 & 0 & \frac{141}{28} \end{bmatrix}$$

$x_1 = -\frac{23}{14}$, $x_2 = \frac{141}{28}$, $x_3 = -\frac{29}{28}$, $x_4 = -\frac{101}{14}$

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4) $g(x) = (\arctan x)^2$

$g'(x) = 2\arctan x \cdot \frac{1}{1+x^2}$

$g'(x) = \frac{2\arctan x}{1+x^2}$ ✓

$g''(x) = \frac{2 \cdot \frac{1}{1+x^2} \cdot (1+x^2) - 2\arctan x \cdot 2x}{(1+x^2)^2}$ ✓

$g''(x) = \frac{\frac{2(1+x^2)}{1+x^2} - 2\arctan x \cdot 2x}{(1+x^2)^2}$?

$g''(x) = \frac{\frac{1+x^2}{1+x^2} - \frac{2\arctan x \cdot 2x}{1}}{(1+x^2)^2}$ ✗

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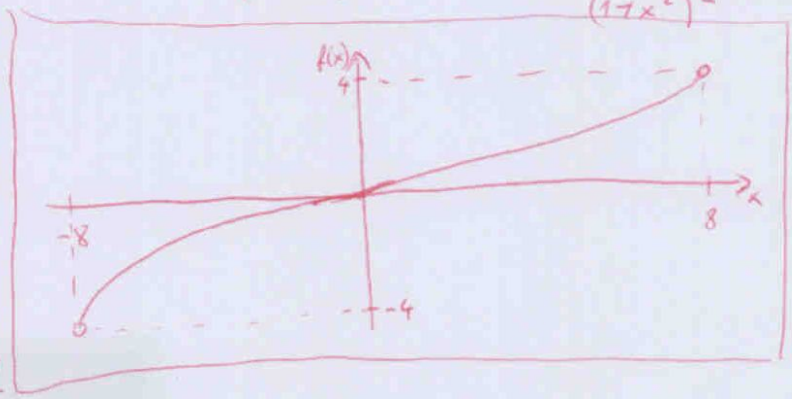
$g''(x) = \frac{1 - 2\arctan x \cdot 2x}{1+x^2}$ ✗

$g''(x) = \frac{1 - 2(\arctan x \cdot x)}{1+x^2}$ ✗

$g''(x) = \frac{2 \cdot \frac{1}{1+x^2} \cdot (1+x^2) - 2(\arctan x) \cdot 2x}{(1+x^2)^2}$

$= \frac{2 - 4x\arctan x}{(1+x^2)^2} = \frac{2(1 - 2x\arctan x)}{(1+x^2)^2}$



3) $f(x) = \sqrt{8+x} - \sqrt{8-x}$

$D = R \setminus \{-8, 8\}$ ✗

$8+x \geq 0$ $\quad 8-x \geq 0$

$8+x \overset{?}{=} 0$ $\quad 8-x \overset{?}{=} 0$

$x_1 = -8$ $\quad -x = -8$

$x_2 = 8$

$\mathcal{D} = ?$

$8-x \geq 0 \Leftrightarrow x \leq 8$

$8+x \geq 0 \Leftrightarrow x \geq -8$

$\Rightarrow \mathcal{D}(f) = [-8, 8]$

HORIZONTALNIH I KOSIH ASIMPTOTA NEMA JER NIJE DEFINIRANA U DIJELU PREMA $\pm\infty$.

$\lim_{x \to -8} f(x) = 0 - \sqrt{16} = -4$, $\lim_{x \to 8} f(x) = \sqrt{16} - 0 = 4$

$f'(x) = 0$ za $\underbrace{\sqrt{8-x}}_{\geq 0} + \underbrace{\sqrt{8+x}}_{\geq 0} = 0 \Leftrightarrow \underbrace{\sqrt{8-x}}_{\geq 0} = \underbrace{-\sqrt{8+x}}_{\leq 0}$ ↯

$f'(x) = \frac{\sqrt{8-x} + \sqrt{8+x}}{2\sqrt{8+x}\sqrt{8-x}} \geq 0 \Rightarrow f$ je rastuća

$f(-x) = \sqrt{8-x} - \sqrt{8+x}$ NEPARNA ✓

$f'(x) = (8+x)^{\frac{1}{2}} - (8-x)^{\frac{1}{2}}$ ✓

$f'(x) = \frac{1}{2}(8+x)^{-\frac{1}{2}} \cdot 1 - \left[\frac{1}{2}(8-x)^{-\frac{1}{2}} \cdot (-1)\right]$

$f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{8+x}} + \frac{1}{2} \cdot \frac{1}{\sqrt{8-x}}$ ✓

$f'(x) = \frac{1}{2\sqrt{8+x}} + \frac{1}{2\sqrt{8-x}} = \frac{\sqrt{8-x} + \sqrt{8+x}}{2(\sqrt{8+x} \cdot \sqrt{8-x})}$ ✓

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$f''(x) = \frac{-1}{4(8+x)^{\frac{3}{2}}} + \frac{1}{4(8-x)^{\frac{3}{2}}} = \frac{(8+x)^{\frac{3}{2}} - (8-x)^{\frac{3}{2}}}{4(8+x)^{\frac{3}{2}}(8-x)^{\frac{3}{2}}}$

$f''(x) = 0$ za $(8+x)^{\frac{3}{2}} - (8-x)^{\frac{3}{2}} = 0$

$\Leftrightarrow (8+x)^{\frac{3}{2}} = (8-x)^{\frac{3}{2}}$

$\Leftrightarrow 8+x = 8-x$

$\Leftrightarrow 2x = 0$

$\Leftrightarrow x = 0$

	-8		0		8
$f''(x)$		−		+	
$f(x)$		↷		↳	