

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD 15:00 DO

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljšavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

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 Broj ↓
 bodova

IME I PREZIME: KREŠIMIR KERČ

BROJ INDEKSA: 0269023278

1. Ako su z_1 i z_2 rješenja kvadratne jednadžbe $z^2 + 2 = 0$, izračunati:

- (a) $\overline{\left(\frac{z_1 - z_2}{z_2 - 2}\right)}$;
- (b) $\overline{\left(\frac{z_2}{z_1}\right)}$.

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2. Gaussovom metodom riješiti sustav jednadžbi:

$$\begin{aligned} 5x_1 + x_2 + x_3 - x_4 &= 3 \\ x_1 + x_2 - x_3 + 2x_4 &= -10 \\ -2x_1 - x_2 + x_3 + x_4 &= -10 \\ x_2 + x_3 &= 4 \end{aligned}$$

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3. Ispitati tok funkcije: $f(x) = \sqrt{8+x} - \sqrt{8-x}$. Da li postoje lokalni ekstremi?

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4. Pronaći prvu i drugu derivaciju funkcije: $g(x) = (\arctan x)^2$.

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2) Gauss

$$\begin{aligned} 5x_1 + x_2 + x_3 - x_4 &= 3 \\ x_1 + x_2 - x_3 + 2x_4 &= -10 \\ -2x_1 - x_2 + x_3 + x_4 &= -10 \\ x_2 + x_3 &= 4 \end{aligned}$$

Augmented matrix row reduction steps:

$$\left[\begin{array}{cccc|c} 5 & 1 & 1 & -1 & 3 \\ 1 & 1 & -1 & 2 & -10 \\ -2 & -1 & 1 & 1 & -10 \\ 0 & 1 & 1 & 0 & 4 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 5 & 1 & 1 & -1 & 3 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 1 & 1 & 0 & 4 \end{array} \right] \xrightarrow{R_2 - 5R_1} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & -4 & 6 & -11 & 53 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 1 & 1 & 0 & 4 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & -4 & 6 & -11 & 53 \\ 0 & 1 & 1 & 0 & 4 \end{array} \right] \xrightarrow{R_3 + 4R_2} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & 10 & -11 & 71 \\ 0 & 1 & 1 & 0 & 4 \end{array} \right] \xrightarrow{R_3 : 10} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1.1 & 7.1 \\ 0 & 1 & 1 & 0 & 4 \end{array} \right] \xrightarrow{R_3 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 0 & 1 & -1.1 & 7.1 \\ 0 & 1 & 1 & 0 & 4 \end{array} \right] \xrightarrow{R_2 \cdot (-1)} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 0 & -1 & 1.1 & -7.1 \\ 0 & 1 & 1 & 0 & 4 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & -1 & 1.1 & -7.1 \end{array} \right] \xrightarrow{R_2 - R_3} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 1 & 2 & -1.1 & 11.1 \\ 0 & 0 & -1 & 1.1 & -7.1 \end{array} \right] \xrightarrow{R_2 : 2} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 1 & 1 & -1.1 & 5.55 \\ 0 & 0 & -1 & 1.1 & -7.1 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 0 & -1 & 1.1 & -7.1 \\ 0 & 1 & 1 & -1.1 & 5.55 \end{array} \right] \xrightarrow{R_2 \cdot (-1)} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 0 & 1 & -1.1 & 7.1 \\ 0 & 1 & 1 & -1.1 & 5.55 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 1 & 1 & -1.1 & 5.55 \\ 0 & 0 & 1 & -1.1 & 7.1 \end{array} \right] \xrightarrow{R_2 - R_3} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 1 & 0 & 0 & -1.55 \\ 0 & 0 & 1 & -1.1 & 7.1 \end{array} \right] \xrightarrow{R_2 \cdot (-1)} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & -10 \\ 0 & 1 & 0 & 0 & 1.55 \\ 0 & 0 & 1 & -1.1 & 7.1 \end{array} \right] \xrightarrow{R_1 - R_2} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & -11.55 \\ 0 & 1 & 0 & 0 & 1.55 \\ 0 & 0 & 1 & -1.1 & 7.1 \end{array} \right] \xrightarrow{R_1 + R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0.9 & -4.45 \\ 0 & 1 & 0 & 0 & 1.55 \\ 0 & 0 & 1 & -1.1 & 7.1 \end{array} \right] \xrightarrow{R_1 : 10} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0.9 & -4.45 \\ 0 & 1 & 0 & 0 & 1.55 \\ 0 & 0 & 1 & -1.1 & 7.1 \end{array} \right] \xrightarrow{R_1 \cdot 10} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 9 & -44.5 \\ 0 & 1 & 0 & 0 & 1.55 \\ 0 & 0 & 1 & -1.1 & 7.1 \end{array} \right]$$

g) PRONAĆI PRVU I DRUGU DERIVACIJU FUNKCIJE

$$g(x) = (\arctan x)^2$$

$$f'(x) = 2(\arctan x)^1 \cdot \frac{1}{2\sqrt{1+(1-x)^2}} = \frac{2(\arctan x)}{2\sqrt{1+(1-x)^2}} = \frac{\arctan x}{(1+\sqrt{1-x})^2}$$

$$f''(x) = \frac{(\arctan x)' \cdot (1+\sqrt{1-x}) - \arctan x \cdot (1+\sqrt{1-x})'}{(1+\sqrt{1-x})^2}$$

$$f''(x) = \frac{\frac{1}{1+\sqrt{1-x}} + \sqrt{1-x} - \arctan x \cdot (-1)}{(1+\sqrt{1-x})^2} = \frac{\sqrt{1-x} - \arctan x}{(1+\sqrt{1-x})^2}$$

1) AKO SU z_1 I z_2 RJEŠENJA KVAADRATNE JED. $z^2 + 2 = 0$ IZRAČUNATI

$$a) \frac{z_1 - z_2}{z_2 - 2} = \frac{(1+i) - (-1-i)}{(-1-i) - 2} = \frac{1+i - (-1-i)}{-1-i}$$

$$b) \frac{z_2}{z_1} = \frac{-1-i}{1+i} = \frac{-2i}{-1-i}$$

$$z^2 - 27 + 2 = 0 \quad z = \frac{-2 \pm \sqrt{4 - 4 \cdot (-27)}}{2} = \frac{-2 \pm \sqrt{112}}{2} = -1 \pm \sqrt{28}$$

$$z = \frac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i \quad \boxed{\frac{z_1 - z_2}{z_2 - 2} = 1 - i}$$

$$z = \frac{2 + 2i}{2}$$

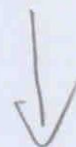
$$z = \frac{2(1+i)}{2}$$

$$z_1 = 1+i$$

$$z_2 = 1-i$$

$$b) \frac{z_2}{z_1} = \frac{1-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i-i-i^2}{1-i^2} = \frac{1-i^2-2i}{1-i^2} = \frac{1-i^2-2i}{1-i^2} = \frac{-2i}{2} = -i$$

$$\boxed{\frac{z_2}{z_1} = -i}$$



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$$1.) \frac{2-3i}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{6-9i-2i+12i^2}{3^2-(4i)^2} = \frac{6-11i-12}{9-16i^2} = \frac{6-11i}{9+16}$$

$$= \frac{6-11i}{25} = \frac{6}{25} - \frac{11i}{25}$$

$$\frac{2-3i}{i} \cdot \frac{-i}{-i} = \frac{-2i+3i^2}{-i^2} = \frac{-2i+3i^2}{1} = -3-2i$$

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