OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

OXXX

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IME I PREZIME: ANTE DUJEVIĆ

BROJ INDEKSA: 57641

Broj ↓ bodova

1. Odrediti (ako postoji) inverz matrice: 20 ∅

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & 1 & 3 & 3 \\ 1 & 3 & 3 & 0 \end{bmatrix}$$

2. Riješiti jednadžbu: $z^3 - \frac{(1-i)^3}{i^{333}} = 0$. 20 5

3. Ispitati tok funkcije: $g(x) = \dfrac{x^2 \ln x}{2}$. 40 ∅

4. Ispitati domenu, asimptote i prvu derivaciju i ekstreme funkcije: $f(x) = x + \sqrt{x^2 - x}$. 20 ∅

SARRUSOVO PRAVILO VRIJEDI SAMO ZA DETERMINANTE REDA 3

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & 1 & 3 & 3 \\ 1 & 3 & 3 & 0 \end{vmatrix} \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 3 \\ 1 & 3 & 3 \end{matrix} = 1\cdot1\cdot3\cdot0 + 1\cdot1\cdot3\cdot1 + 1\cdot3\cdot1\cdot3 + 1\cdot1\cdot1\cdot3 -$$

$$- 1\cdot1\cdot1\cdot1 - 3\cdot3\cdot3\cdot1 - 3\cdot3\cdot1\cdot1 - 0\cdot1\cdot1\cdot1$$

$$= 0 + 3 + 9 + 3 - 1 - 27 - 9 - 0 = -22$$

X

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & 1 & 3 & 3 \\ 1 & 3 & 3 & 0 \end{vmatrix}$$

$$M_{11} = 1 \times \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & 3 \\ 3 & 3 & 0 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 3 \\ 3 & 3 \end{matrix} = 1 \times (1\cdot3\cdot0 + 1\cdot3\cdot3 + 3\cdot1\cdot3 - 3\cdot3\cdot3 - 3\cdot3\cdot1 - 0\cdot1\cdot1)$$

$$= 1 \times (0 + 9 + 9 - 27 - 9 - 0) = -18$$

$$M_{12} = -1 \times \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & 3 \\ 1 & 3 & 0 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 3 \\ 1 & 3 \end{matrix} = -1 \times (0 + 3 + 9 - 9 - 9 - 0) = 6$$

$$M_{13} = 1 \times \begin{vmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 1 & 3 & 0 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 1 \\ 1 & 3 \end{matrix} = 1 \times (0 + 3 + 9 - 3 - 9 - 0) = 0$$

$$M_{14} = -1 \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 1 & 3 & 3 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 1 \\ 1 & 3 \end{matrix} = -1 \times (3 + 3 + 3 - 1 - 9 - 3) = 4$$

$$M_{21} = -1 \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 3 & 3 & 0 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 3 \\ 3 & 3 \end{matrix} = -1 \times (0 + 9 + 3 - 9 - 9 - 0) = 6$$

$$M_{22} = 1 \times \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & 3 \\ 1 & 3 & 0 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 3 \\ 1 & 3 \end{matrix} = 1 \times (0 + 3 + 9 - 9 - 9 - 0) = -6$$

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$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & 1 & 3 & 3 \\ 1 & 3 & 3 & 0 \end{vmatrix}$$

$$\Pi_{23} = -1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 3 & 0 & 1 & 3 \end{array}\right| = -1 \times (0+3+3-1-9-0) = \underline{\underline{4}}$$

$$\Pi_{24} = 1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 3 & 3 & 1 & 3 \end{array}\right| = 1 \times (\cancel{3}+3+3-1-9-\cancel{3}) = \underline{\underline{-4}}$$

$$\Pi_{31} = 1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 3 & 3 & 0 & 3 & 3 \end{array}\right| = 1 \times (0+\cancel{9}+\cancel{3}-\cancel{3}-\cancel{9}-0) = \underline{\underline{0}}$$

$$\Pi_{32} = -1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 3 & 0 & 1 & 3 \end{array}\right| = -1 \times (0+3+3-1-9-0) = \underline{\underline{4}}$$

$$\Pi_{33} = 1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 3 & 0 & 1 & 3 \end{array}\right| = 1 \times (0+3+3-1-9-0) = \underline{\underline{-4}}$$

$$\Pi_{34} = -1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 3 & 1 & 3 \end{array}\right| = -1 \times (\cancel{3}+1+\cancel{3}-1-\cancel{3}-\cancel{3}) = \underline{\underline{0}}$$

$$\Pi_{41} = -1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 3 & 3 & 1 & 3 \end{array}\right| = -1 \times (\cancel{3}+3+3-1-9-\cancel{3}) = \underline{\underline{4}}$$

$$\Pi_{42} = 1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 3 & 3 & 1 & 3 \end{array}\right| = 1 \times (3+3+3-1-9-3) = \underline{\underline{-4}}$$

$$\Pi_{43} = -1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \end{array}\right| = -1 \times (\cancel{3}+\cancel{3}+\cancel{1}-\cancel{1}-\cancel{3}-\cancel{3}) = \underline{\underline{0}}$$

$$\Pi_{44} = 1 \times \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \end{array}\right| = 1 \times (\cancel{3}+\cancel{1}+\cancel{1}-\cancel{1}-\cancel{1}-\cancel{3}) = \underline{\underline{0}}$$

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$$\begin{vmatrix} -18 & 6 & 0 & 4 \\ 6 & -6 & 4 & -4 \\ 0 & 4 & -4 & 0 \\ 4 & -4 & 0 & 0 \end{vmatrix} \cancel{x \cdot 22} = \begin{vmatrix} 396+132 & 0 & 88 \\ 132 & -132 & 88 & -88 \\ 0 & 88 & -88 & 0 \\ 88 & -88 & 0 & 0 \end{vmatrix}^T = \begin{vmatrix} 396 & 132 & 0 & 88 \\ 132 & -132 & 88 & -88 \\ 0 & 88 & -88 & 0 \\ 88 & -88 & 0 & 0 \end{vmatrix}$$

VIDI SLIČNO MIKULANDRA

∅

2. $z^3 - \frac{(1-i)^3}{i^{333}} = 0$

$i^{333} = i^1 = i$

$333 : 4 = 83$

$z^3 - \frac{(1-i)^3}{i} = 0$

$1^3 - 3 \cdot 1 \cdot i + i^3$

$1 - 3i + (-i)$

$1 - 3i - i$

$z^3 = \frac{(1-i)^3}{i}$

$= (1 - 1i - i1 + i^2)(1-i)$

$= 1 - 1i - 1i + ii^2 - 1i + 1i^2 + i^2 - i^3$

$z^3 = \frac{(1-i)\cdot(1-i)\cdot(1-i)}{i}$ ✓

$= 1 - \cancel{i} - i + i^2 - i + i^2 + \cancel{i^2} - i^3$

$z^3 = \frac{1-3i-i}{i} = \frac{1-4i}{i}$ ✗

$= 1 - i + i^2 - i + i^2 - i^3$

$= \cancel{1} - i - \cancel{1} - i - 1 - i$

$= -i - i - i - 1$ ✗

$z^3 = \frac{1}{i} - \frac{4i}{i}$

$|z| = \sqrt{1^2 + (-4)^2}$

$|z| = \sqrt{1+16}$

$|z| = \sqrt{17}$

$z^3 = \frac{1}{i} - 4$

$z^3 = \frac{1}{1} - 4$

$z = \sqrt[3]{\frac{1}{1} - 4}$

$\boxed{\frac{1}{i} = \frac{1}{i} \cdot \frac{i}{i} = \frac{i}{-1} = -i}$

$\text{tg}\,\varphi = \frac{y}{x} = \frac{4}{1} = 4$

$\varphi = 0{,}069$

$(1-i)^3 = (1-i)^2(1-i)$
$= (1 - i - i + i^2)(1-i)$
$= (\cancel{1} - 2i - \cancel{1})(1-i)$
$= -2i + 2i^2 = -2 - 2i$

4. $z^3 = (\sqrt{17})^3 (\cos 3\, 0{,}069 + i \sin 3\, 0{,}069)$

$k = 0, 1, 2$

∅

$\sqrt[n]{z} = \sqrt[n]{|z|} \cdot \left(\cos \frac{\varphi + 2k\pi}{n} + i \sin \frac{\varphi + 2k\pi}{n}\right)$

$k=0$

$z_1 = \sqrt[3]{\sqrt{17}} \cdot \left(\cos \frac{0{,}069 + 2 \cdot 0 \cdot \pi}{3} + i \sin \frac{0{,}069 + 2 \cdot 0 \pi}{3}\right)$

$= \sqrt[3]{\sqrt{17}} \cdot \left(\cos \frac{0{,}069}{3} + i \sin \frac{0{,}069}{3}\right)$ GDJE JE z_0, z_1, z_2?

IME I PREZIME: ANTE DUŠEVIĆ BROJ INDEKSA: 57641

4. $f(x) = x + \sqrt{x^2 - x}$

$f'(x) = x' + \left(\sqrt{x^2 - x}\right)'$

$= 1 + \left(\sqrt{2x - 1}\right)'$

$= 1 + \sqrt{2x - 1}$ X ∅

$= \infty + \sqrt{\infty} = \infty$

$\lim_{x \to \infty} x + \sqrt{x^2 - x} = \lim_{x \to \infty} x + \sqrt{x^2 - x} \cdot \frac{x - \sqrt{x^2 - x}}{x - \sqrt{x^2 - x}}$

$= \lim_{x \to \infty} \frac{x^2 - (\sqrt{x^2 - x})^2 / x^2}{x - \sqrt{x^2 - x} / x^2} = \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{x^2}{x^2} + \frac{x}{x^2}}{\frac{x}{x} - \sqrt{\frac{x^2}{x^2} - \frac{x}{x^2}}} =$

$= \frac{1 - 1 + \frac{1}{x}}{1 - \sqrt{1 - \frac{1}{x}}} = \frac{0}{1 - \sqrt{1}} = 0$ X

$\frac{0}{0}$ = NEODREĐENI OBLIK

VIDI KONČURAT

3. $g(x) = \frac{x^2 \ln x}{2}$

$g'(x) = \frac{(x^2 \ln x)' \cdot 2 - x^2 \ln x \cdot 2'}{2^2} = \frac{2x \ln 1 \cdot 2 - x^2 \ln x \cdot 0}{4}$

$= \frac{2x \ln 2}{4}$ X

2o) NASTAVAK

$k = 1$

$z_2 = \sqrt[3]{\sqrt{17}} \left(\cos \frac{0,069 + 2 \cdot 1\pi}{3} + i \sin \frac{0,069 + 2 \cdot 1\pi}{3}\right)$

$z_2 = \sqrt[3]{\sqrt{17}} \left(\cos \frac{0,069 + 2\pi}{3} + i \sin \frac{0,069 + 2\pi}{3}\right)$

$k = 2$

$z_3 = \sqrt[3]{\sqrt{17}} \left(\cos \frac{0,069 + 2 \cdot 2\pi}{3} + i \sin \frac{0,069 + 2 \cdot 2\pi}{3}\right)$

$= \sqrt[3]{\sqrt{17}} \left(\cos \frac{0,069 + 4\pi}{3} + i \sin \frac{0,069 + 4\pi}{3}\right)$

5