OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO .

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE <u>JEDNOSTRANO</u> NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. M3

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IME I PREZIME: SAŠA VAIŠTINA **BROJ INDEKSA:** 53437

1. X je zadan kao paralelogram s vrhovima $O(0,0)$, $A(1,0)$, $B(2,1)$ i $C(1,1)$. Skicirati taj paralelogram i izračunati dvostruki integral 20

$$\iint\limits_X y^2 \mathrm{d}x\mathrm{d}y$$

2. Neka je K kvadar $(0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3)$ i ∂K rub te kocke. Izračunati plošni integral

$$\iint\limits_{\widehat{\partial K}} x\,\mathrm{d}y\mathrm{d}z + zy\,\mathrm{d}x\mathrm{d}z + xy\,\mathrm{d}x\mathrm{d}y$$

3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=9$ za koji vrijedi $z \ge 2$.

4. Izračunati

$$\int\limits_{(3,2)}^{(5,5)} (x+y)\,(\mathrm{d}y + \mathrm{d}x)$$

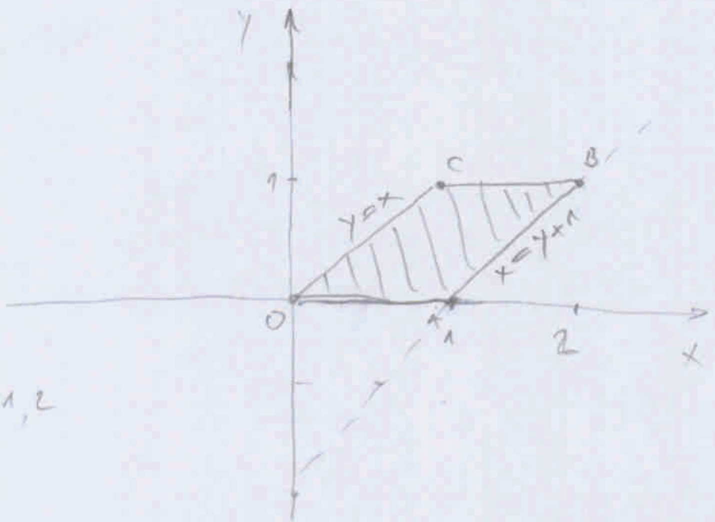
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) - 2y(t) = e^{-t}, \quad y(0) = y''(0) = 1,\ y'(0) = 2.$$

①

$y = x+1$

$x = y-1$



-1

$0, 1, 2$

$y = x+1$

$= 0+1$

$= 1$

$1+2$

$2+1$

$x = y-1$

$0-1$

$= -1$

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$$\int_0^1 dy \int_y^{y-1} y^2 dx =$$

$$\int_0^1 y^2 dy \int_y^{y+1} dx = \int_0^1 y^2 dy \cdot x \Big|_y^{y+1}$$

$$= \int_0^1 y^2 dy \cdot \left[(\cancel{y}+1) - \cancel{y}\right]$$

$$= \int_0^1 y^2 dy \cdot 1 = \frac{y^3}{3}\Big|_0^1$$

$$= \frac{1}{3} - 0 + c$$

$$= \frac{1}{3} + c$$

IME I PREZIME: SAFA VAISANA BROJ INDEKSA: 53437

(5) $y'''(t) - 2y(t) = e^{-t}$ $\quad y(0) = y''(0) = 1$

$y'(0) = 2$

$\mathcal{L}\left(y'''(t) - 2y(t)\right) = \mathcal{L}\, e^{-t}$

$\left(s^3 Y + s^2 y(0) + s^1 y'(0) + s^0 y''(0)\right) - 2\left(s^1 Y + s^0 y(0)\right) = \frac{1}{s+1}$

$\left(s^3 Y + s^2 \cdot 1 + s^1 \cdot 2 + 1\right) - 2\left(sY + 1\right) = \frac{1}{s+1}$

$s^3 Y + s^2 + 2s + 1 - 2sY - 2 = \frac{1}{s+1}$

$s^3 Y - 2sY = -s^2 - 2s + 1 + \frac{1}{s+1}$

$Y(s^3 - 2s) = -s^2 - 2s + 1 + \frac{1}{s+1}$

$Ys(s^2 - 2) = -s^2 - 2s + 1 + \frac{1}{s+1} \quad / : s(s^2-2)$

$Y = \frac{-\cancel{s}^{2}{}^{s}}{\cancel{s}(s^2-2)} - \frac{2\cancel{s}}{\cancel{s}(s^2-2)} + \frac{1}{s(s^2-2)} + \frac{1}{s(s^2-2)(s+1)}$

$= \frac{-s}{s^2-2} - \frac{2}{s^2-2} + \frac{1}{s(s^2-2)} + \frac{1}{s(s^2-2)(s+1)}$

$=$

IME I PREZIME: SARA VAISTINA　　　　　BROJ INDEKSA: 53437

$$= -\frac{s}{(s-\sqrt{2})(s+\sqrt{2}} = \frac{A}{s-\sqrt{2}} + \frac{B}{s+\sqrt{2}} = \frac{As + A\sqrt{2} + Bs - B\sqrt{2}}{(s-\sqrt{2})(s+\sqrt{2})}$$

$$= \frac{(A+B)s + A\sqrt{2} - B\sqrt{2}}{(s-\sqrt{2})(s+\sqrt{2})}$$

$1 = A+B$

$0 = A\sqrt{2} - B\sqrt{2} \Rightarrow A\sqrt{2} = B\sqrt{2}$

$A = B$

$A = \frac{1}{2} \quad B = \frac{1}{2}$

$$\boxed{= -\frac{1}{2}\frac{1}{s-\sqrt{2}} - \frac{1}{2}\frac{1}{s+\sqrt{2}}} - (1)$$

$$\frac{-2}{(s-\sqrt{2})(s+\sqrt{2})} = \frac{A}{(s-\sqrt{2})} + \frac{B}{(s+\sqrt{2})} = \frac{As + A\sqrt{2} + Bs - B\sqrt{2}}{(s-\sqrt{2})(s+\sqrt{2})}$$

$$= \frac{s(A+B) + A\sqrt{2} - B\sqrt{2}}{(s-\sqrt{2})(s+\sqrt{2})} =$$

$0 := -A + B$

$A = B$

$2A = A$

$2A - A = 1$

$A = 1$

$-2 = A\sqrt{2} - B\sqrt{2}$

$B\sqrt{2} = A\sqrt{2} + 2 \quad /:\sqrt{2}$

$B = \frac{A\cancel{\sqrt{2}} + 2}{\cancel{\sqrt{2}}} = 2A$

$B = 2 \cdot 1$

$= 2$

$$= \boxed{\frac{1}{s-\sqrt{2}} - \frac{1}{2}\frac{1}{s+\sqrt{2}}} \quad (2)$$

$$\frac{1}{s(s^2-2)} = \frac{1}{s[(s-\sqrt{2})(s+\sqrt{2})]} = \frac{A}{s} + \frac{B}{s-\sqrt{2}} + \frac{C}{s+\sqrt{2}} =$$

$$= \frac{A(s-\sqrt{2})(s+\sqrt{2}) + B(s(s+\sqrt{2})) + C(s(s-\sqrt{2})}{s[(s-\sqrt{2})(s+\sqrt{2})]} =$$

$$= \frac{As^2 - 2A + Bs^2 + Bs\sqrt{2} + Cs^2 - Cs\sqrt{2}}{s[(s-\sqrt{2})(s+\sqrt{2})]} = \frac{s^2(A+B+C) + s(B\sqrt{2} - C\sqrt{2}) - 2A}{s[(s-\sqrt{2})(s+\sqrt{2})]}$$

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$0 = A + B + C$ $0 = B\sqrt{2} - C\sqrt{2}$ $1 = -2A$

$\boxed{A = -B - C}$ $B\sqrt{2} = C\sqrt{2}$ $2A = -1 \;/ : 2$

$\boxed{B = C}$ $\boxed{A = -\frac{1}{2}}$

$-\frac{1}{2} = -B - C$ $\frac{1}{2} - C = C$

$\boxed{B = \frac{1}{2} - C}$ $-C - C = -\frac{1}{2}$

$-2C = -\frac{1}{2} \;/ : (-2)$

$B = \frac{1}{2} - \frac{1}{4}$ $\boxed{C = \frac{1}{4}}$

$B = \frac{}{4} = \boxed{\frac{3}{4}}$

$$= \boxed{-\frac{1}{2} \cdot \frac{1}{s} + \frac{3}{4} \frac{1}{s-\sqrt{2}} + \frac{1}{4} \cdot \frac{1}{s+\sqrt{2}}} \quad (3)$$

$$\frac{1}{[s(s-\sqrt{2})(s+\sqrt{2})](s+1)} = \frac{A}{s} + \frac{B}{s-\sqrt{2}} + \frac{C}{s+\sqrt{2}} + \frac{D}{s+1}$$

$$= \frac{[A(s^2-2)(s+1)] + B\,s(s+\sqrt{2})(s+1) + C\,s((s-\sqrt{2})(s+1) + D\,s(s^2-2)}{[s(s-\sqrt{2})(s+\sqrt{2})](s+1)}$$

$$= \frac{As^3 + As^2 - 2As - 2A + Bs^3 + Bs^2 + B\sqrt{2}s^2 + B\sqrt{2}s + Cs^3 + Cs^2 - C\sqrt{2}s^2 - C\sqrt{2}s + \ldots}{[s(s-\sqrt{2})(s+\sqrt{2})](s+1)}$$

$\ldots + Ds^3 - 2Ds$

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$$= \frac{s^3(A+B+C)+s^2(A+B+B\sqrt{2}+C-C\sqrt{2})+s(-2A+B\sqrt{2}-C\sqrt{2}-2D)}{[s(s-\sqrt{2})(s+\sqrt{2})](s+1)}$$

$$\frac{-2A}{}$$

$0^3 = A+B+C$ $0^2: \; A+B+B\sqrt{2}+C-C\sqrt{2}=0$

$-A = B+C$

$\boxed{A=-B-C}$

$1=-2A$

$2A=-1$

$\boxed{A=-\frac{1}{2}}$

$0^1 \; -2A+B\sqrt{2}-C\sqrt{2}-2D=0$

$-\frac{1}{2} = -B-C$

$\boxed{B=\frac{1}{2}-C}$

$-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}-\cancel{C}+\sqrt{2}(\frac{1}{2}-C)+\cancel{C}-C\sqrt{2}$

$\frac{\sqrt{2}}{2}-C\sqrt{2}-C\sqrt{2}=0$

$B=\frac{1}{2}-\frac{1}{4}$

$B=\frac{2-1}{4}=\boxed{\frac{1}{4}}$

$-2C\sqrt{2}=-\frac{\sqrt{2}}{2} \; /:(-2\sqrt{2})$

$C=-\frac{\cancel{\sqrt{2}}}{2(-2\cancel{\sqrt{2}})} \; -\frac{1}{-4} = \boxed{\frac{1}{4}}$

$-2D = 2A-B\sqrt{2}+C\sqrt{2}$

$-2D = \cancel{2}\cdot(-\frac{1}{\cancel{2}})-\cancel{\frac{\sqrt{2}}{4}}+\cancel{\frac{\sqrt{2}}{4}}$

$-2D=-1 \; /:(-2$

$\boxed{D=\frac{1}{2}}$

$-\frac{1}{2}\frac{1}{s}+\frac{1}{4}\frac{1}{s-\sqrt{2}}+\frac{1}{4}\frac{1}{s+\sqrt{2}}+\frac{1}{2}\frac{1}{s+1}$

(5)

IME I PREZIME: SARA VAISTINA BROJ INDEKSA: 53437

$$-\frac{1}{2}e^{\sqrt{2}t}-\frac{1}{2}e^{-\sqrt{2}t}+e^{\sqrt{2}t}-\frac{1}{2}e^{-\sqrt{2}t}-\frac{1}{2}+\frac{3}{4}e^{\sqrt{2}t}+\frac{1}{4}e^{-\sqrt{2}t}-$$

$$-\frac{1}{2}+\frac{1}{4}e^{\sqrt{2}t}+\frac{1}{4}e^{-\sqrt{2}t}+\frac{1}{2}e^{-t}$$ ✓

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