JOSIP MILETIĆ

Obavezno popuniti vrijeme rješavanja ispita: od   do

MATEMATIKA 2: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. Zadatke rješavate jednostrano na papire koje dobijete od nastavnika.

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Ime i prezime: JOSIP MILETIĆ   **Broj indeksa:**

1. Izračunati:

   (a) površinu omeđenu x-koordinatnom osi i grafom funkcije $f(x) = \sin x$ na intervalu $[\pi, 2\pi]$, Ø

   (b) $\int \frac{x^2+x+3}{x^2-1}\, dx$. Ø

2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. 20

3. Zadana je funkcija $f(x,y) = (x+y)\left(1 + \frac{1}{xy}\right)$. Odrediti ekstreme. 5

4. Riješiti diferencijalnu jednadžbu, a zatim rješenje uvrstiti u jednadžbu i provjeriti da je uistinu zadovoljava:

$$y'' + 2y' + y = e^x.$$

5. Zadano je $f(x) = \frac{x^2}{\sqrt{9+4x^2}}$. Približno izračunati integral $\int_0^2 f(x)\, dx$ nekom metodom numeričke integracije (Simpsonova ili trapezna formula). Ø

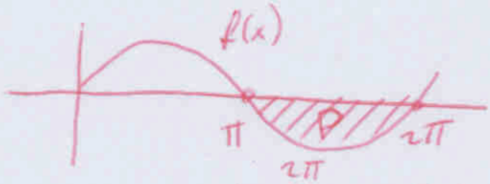
1.

a) $f(x) = \sin x \qquad [\pi, 2\pi]$ Ø

$\int_{\pi}^{2\pi} \sin x = -\cos x \Big|_{\pi}^{2\pi} = -\cos 2\pi - (-\cos \pi)$

X

$= -\cos 2\pi + \cos \pi = -2$?



$P = \int_{\pi}^{2\pi} [-f(x)]\, dx = -\int_{\pi}^{2\pi} \sin x\, dx$

b) $\int \frac{x^2+x+3}{x^2-1} dx =$

$\frac{x^2+x+3}{x^2-1} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2-1} \Big/ \cdot (x^2-1)$ Ø

X

$x^2+x+3 = A(x-1) - B + C$

$x^2+x+3 = Ax - A - B + C$

$x^2 = 0$

$x = Ax$

$1 = A$

VIDI BILKIĆ

IME I PREZIME: JOSIP MILETIC BROJ INDEKSA: $x^4 = 4x^3$

3. $f(x,y) = (x+y)\left(1+\frac{1}{xy}\right)$

$\frac{1}{x} = x^{-1} \overset{!'}{=} -1\, x^{-2} = -x^{-2} = -\frac{1}{x^2}$

$f(x,y) = x + \frac{\cancel{x}}{\cancel{x}y} + y + \frac{\cancel{y}}{x\cancel{y}}$

$f(x,y) = x + \frac{1}{y} + y + \frac{1}{x}$

$\frac{1}{y} = y^{-1} \overset{!'}{=} -y^{-2} = -\frac{1}{y^2}$

$\nabla z \begin{pmatrix} 1 - \frac{1}{x^2} \\ -\frac{1}{y^2} + 1 \end{pmatrix}$?

$-x^{-2} = 2x^{-3} = \frac{2}{x^3}$

$-y^{-2} = 2x^{-3} = \frac{2}{y^3}$

$1 - \frac{1}{x^2} = 0$ $\qquad -\frac{1}{y^2} + 1 = 0$

$-\frac{1}{x^2} = -1 \quad / \cdot x^2$ $\qquad -\frac{1}{y^2} = -1 \quad \cdot y^2$

$1 = x^2$ $\qquad -1 = -y^2$

$x_1 = \sqrt{1}$ $\qquad y^2 = 1$

$x = 1$ $\qquad x = \pm 1$ $\qquad y = \sqrt{1}$

$y = 1$ $\qquad y = \pm 1$

$T_1(1,1)$

$T_2(-1,1)$

$T_3(1,-1)$

$T_4(-1,1)$

? $\nabla z = \frac{dz}{dxx}\left(\frac{2}{x^3}\right)$

? $\nabla z = \frac{dz}{dxy}\left(\frac{2}{y^3}\right)$

$\nabla z = \frac{dz}{dyx}(0)$

$\nabla z \frac{dz}{dyx}(0)$

$\begin{vmatrix} \frac{2}{x^3} & 0 \\ 0 & \frac{2}{y^3} \end{vmatrix} = \frac{2}{x^3} \cdot \frac{2}{y^3} - 0 \cdot 0 = \frac{4}{x^3 y^3}$

5

$f(x,y) = \frac{4}{1^3 \cdot 1^3} = 4$? ZAKLJUČAK?

5. $f(x) = \frac{x^2}{\sqrt{9+4x^2}} = \int_0^2 \frac{x^2}{\sqrt{9+4x^2}} = \int_0^2 \frac{x^2}{\sqrt{9}+\sqrt{4x^2}} = \int_0^2 \frac{x^2}{3+2x}\,dx$

$= \left| \begin{matrix} 3+2x = u & x^2 = dv \\ 2dx = du & v = \frac{x^3}{3} \end{matrix} \right) = (3+2x)\cdot\frac{x^3}{3}\Big|_0^2 - \int_0^2 \frac{x^3}{3} - 2dx =$

0

$= (3+(2\cdot 2)-(2\cdot 0))\cdot\left(\frac{x^3}{3} - \frac{x^0}{3}\right) - \left(\left(\frac{1}{3}\frac{x^4}{4} - 2x\right)\right)\Big|_0^2$

$= 3+4\cdot\left(\frac{8}{3} - \frac{1}{3}\right) - \left(\frac{1}{3}\cdot\left(\frac{2^4}{4} - \frac{2^0}{4}\right) - 2\cdot(2-0)\right)$

$= 7\cdot\frac{7}{3} - \left(\frac{1}{3}\cdot\left(\frac{16}{4} - \frac{1}{4}\right) - 4\right)$

$= \frac{49}{3} - \left(\frac{1}{3}\cdot\frac{15}{4}\right) - 4 = \frac{49}{3} - \frac{15}{12} - 4 = \frac{196-15-84}{12} = \frac{97}{12}$

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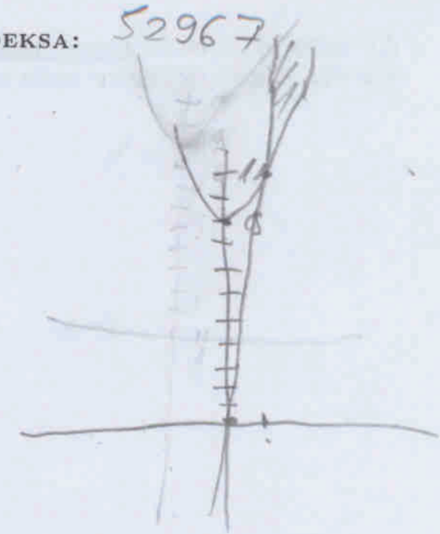

2. $y=2x^2+9 \quad y=9x$

x	y
0	9
1	11

x	y
0	0
1	9

$2x^2+9=9x$

$2x^2+9-9x=0$

$2x^2-9x+9=0$

$x_{(1,2)}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_{(1,2)}=\frac{9\pm\sqrt{9^2-4\cdot 2\cdot 9}}{2\cdot 2}=\frac{9\pm 3}{4}=$

$x_1=\frac{9-3}{4}=\frac{6}{4}=\frac{3}{2}$

$x_2=\frac{9+3}{4}=\frac{12}{4}=3$

$\int_{\frac{3}{2}}^{3}\int_{2x^2+9}^{9x} dy\,dx=\int_{\frac{3}{2}}^{3} 9x-(2x^2+9)=\int_{\frac{3}{2}}^{3}(9x-2x^2-9)$

$=9\frac{x^2}{2}\Big|_{\frac{3}{2}}^{3}-2\cdot\frac{x^3}{3}\Big|_{\frac{3}{2}}^{3}-9x\Big|_{\frac{3}{2}}^{3}=9\cdot\left(\frac{3^2}{2}-\frac{\left(\frac{3}{2}\right)^2}{2}\right)-2\cdot\left(\frac{3^3}{3}-\frac{\left(\frac{3}{2}\right)^3}{3}\right)$

$-9\cdot\left(3-\frac{3}{2}\right)=9\cdot\left(\frac{9}{2}-\frac{\frac{9}{4}}{2}\right)-2\cdot\left(\frac{27}{3}-\frac{\frac{27}{8}}{3}\right)-9\cdot\left(\frac{6-3}{2}\right)$

$=9\cdot\left(\frac{9}{2}-\frac{9}{8}\right)-2\cdot\left(\frac{27}{3}-\frac{27}{24}\right)-9\cdot\frac{3}{2}$

$=9\cdot\frac{36-9}{8}-\cancel{2}\cdot\frac{216-27}{\cancel{24}\,12}-\frac{27}{2}$

$=9\cdot\frac{27}{8}-\frac{189}{12}-\frac{27}{2}=\frac{243}{8}-\frac{189}{12}-\frac{27}{2}=\frac{729-378-324}{24}$

$=\frac{27}{24}=\frac{9}{8}$

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