

MATEMATIKA 2: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaci pribor, tablica osnovnih integrala, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita.

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ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

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1. Riješiti integrale:

(a) $\int \frac{x^3}{x^2+3x+5} dx$, 8

(b) $\int \frac{\ln x}{x^2} dx$, 10

2. Odrediti površinu između krivulja $y = x + 2$ i $y = 4 + x - x^2$. 20

3. Odrediti ekstreme funkcije $f(x, y) = 3x^2 + xy - y^3 + 2$. 5

4. Riješiti diferencijalnu jednadžbu: $y'' + y' - 2y = 1$.

5. Razviti funkciju $f(x) = \cos x$ u Taylorov red po potencijama $x - \frac{\pi}{2}$. Izračunati barem prva 4 člana. 20

1. a) $\int \frac{x^3}{x^2+3x+5} dx = \int x dx - 3 \int dx + \int \frac{4x+15}{x^2+3x+5} dx$

$\left\{ \begin{array}{l} 2x+3=1 \\ 2x+3=1 \end{array} \right.$

$x^3 : (x^2+3x+5) = x - 3 + \frac{4x+15}{x^2+3x+5}$
 $\begin{array}{r} x^3 : (x^2+3x+5) = x - 3 + \frac{4x+15}{x^2+3x+5} \\ \underline{+ x^3 + 3x^2 + 5x} \\ - 3x^2 - 9x + 15 \\ \underline{+ 3x^2 + 9x + 15} \\ 30x + 30 \end{array}$

$= \frac{x^2}{2} - 3x + \int \frac{4x}{x^2+3x+5} dx + 15 \int \frac{dx}{x^2+3x+5}$

$\left\{ \begin{array}{l} x^2+3x+5=t \\ (2x+3)dx=dt \end{array} \right.$

$= \frac{x^2}{2} - 3x + 2 \int \frac{2x+3-3}{x^2+3x+5} dx + 15 \int \frac{dx}{x^2+3x+5}$

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$= \frac{x^2}{2} - 3x + 2 \int \frac{dx}{x^2+3x+5} - 6 \int \frac{dx}{x^2+3x+5} + 15 \int \frac{dx}{x^2+3x+5}$

$= \frac{x^2}{2} - 3x + 2 \ln|x^2+3x+5| + 9 \int \frac{dx}{(x+\frac{3}{2})^2 - \frac{9}{4} + \frac{20}{4}}$

$\int \frac{dx}{(x+\frac{3}{2})^2 + \frac{11}{4}} = \int \frac{dx}{t^2+k^2} = \frac{1}{\sqrt{11}} \arctan \frac{x+\frac{3}{2}}{\frac{\sqrt{11}}{2}} + C = \frac{6}{\sqrt{11}} \arctan \frac{4x+6}{\sqrt{11}} + C$

$x+\frac{3}{2}=t$
 $dx=dt$
 $k^2 = \frac{11}{4}$
 $k = \frac{\sqrt{11}}{2}$

$= \frac{x^2}{2} - 3x + 2 \ln|x^2+3x+5| + \frac{36}{\sqrt{11}} \arctan \frac{4x+6}{\sqrt{11}} + C$

$$b) \int \frac{\ln x}{x^2} dx = u \cdot v - \int v \cdot du = \frac{\ln x}{x} - \int -\frac{1}{x} \cdot \frac{1}{x} dx = -\frac{\ln x}{x} + \int x^{-2} dx = -\frac{\ln x}{x} - \frac{1}{x} + C \checkmark$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$dv = x^{-2}$$

$$v = \frac{x^{-1}}{-1} = -\frac{1}{x} + C$$

$$= -\frac{\ln x + 1}{x} + C$$

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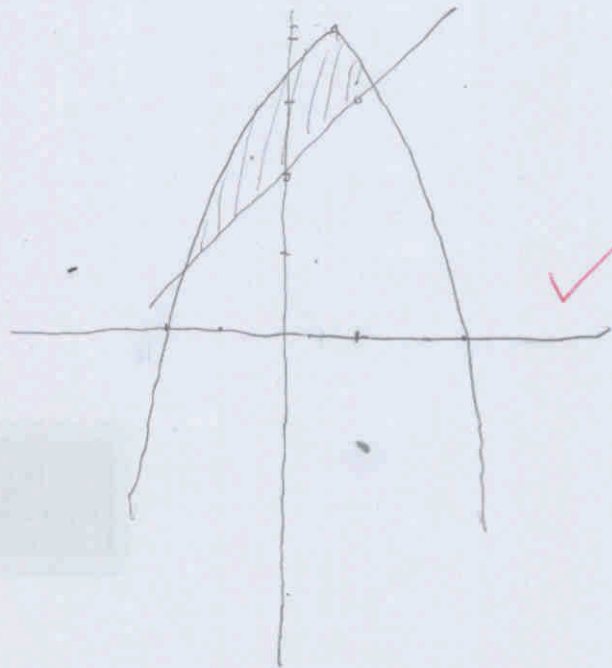
2. $y = -x^2 + x + 4$

$y = x + 2$

$a = -1 \Rightarrow \wedge$

$b = 1$
 $c = 4$

x	0	1	2
y	2	3	4



$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot (-1) \cdot 4}}{-2}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{17}}{-2}$$

$$x_1 = \frac{-1 + 4.12}{-2} = -1.56$$

$$x_2 = \frac{-1 - 4.12}{-2} = 2.56$$

$$T\left(-\frac{b}{2a}, -\frac{b^2 - 4ac}{4a}\right)$$

$$T\left(\frac{1}{2}, -\frac{1 - 4 \cdot (-1) \cdot 4}{-4}\right)$$

$$T\left(\frac{1}{2}, -\frac{17}{-4}\right)$$

$$T\left(\frac{1}{2}, 6.25\right)$$

$y = y$

$-x^2 + x + 4 = x + 2$

$-x^2 + x - x + 4 - 2 = 0$

$-x^2 + 2 = 0$

$x_{1,2} = \frac{\pm \sqrt{0^2 - 4 \cdot (-1) \cdot 2}}{-2}$

$x_1 = \frac{2 \cdot 0.2}{-2} = -1.41 \checkmark$

$x_2 = -\frac{2 \cdot 0.2}{-2} = 1.41 \checkmark$

$P = P_1 - P_2$

$$= \int_{-1.41}^{1.41} (-x^2 + x + 4) dx - \int_{-1.41}^{1.41} (x + 2) dx$$

$$= -\frac{x^3}{3} \Big|_{-1.41}^{1.41} + \frac{x^2}{2} \Big|_{-1.41}^{1.41} + 4x \Big|_{-1.41}^{1.41} - \frac{x^2}{2} \Big|_{-1.41}^{1.41} - 2x \Big|_{-1.41}^{1.41}$$

$$= -\frac{(1.41)^3 - (-1.41)^3}{3} + 2(1.41 + 1.41)$$

$= 15.64 \text{ kvadrati} \checkmark$

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4. $y'' + y' - 2y = 1$

$y = x \cdot e^{-x} \cdot \sqrt{x}$

$y' = e^{-x} C - 2y - 1$

$\frac{dy}{dx} = \dots$

\dots

\dots

\dots

\dots

\dots

\dots

VIDI DUMATOV

$u = 1 - 2y \quad v = x^2$
 $2 dy = -du \quad v \cdot d^2$

3. $f(x,y) = 3x^2 + 8y - y^3 + 2$

$\frac{dx}{dx} = 6x + y$

$\frac{dy}{dy} = x - 3y^2$

$6x + y = 0 \Rightarrow 18y^2 + y = 0 \Rightarrow y(18y + 1) = 0$

$x - 3y^2 = 0 \Rightarrow x = 3y^2$

$y = 0 \quad 18y + 1 = 0$

$x = 0 \quad 18y = -1$
 $y = -\frac{1}{18}$

$S(0,0) \quad S(\frac{1}{18}, -\frac{1}{18})$

$6x - \frac{1}{18} = 0$
 $6x = \frac{1}{18} \cdot 6$
 $x = \frac{1}{18}$

3. varstava

$$S(0,0) \checkmark$$

$$S\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \checkmark$$

$$r = \frac{d^2z}{dx^2} = 6 > \checkmark$$

$\frac{\partial^2 z}{\partial x^2} = 6 > 0$

$\frac{\partial^2 z}{\partial y^2} = 1 > 0$

$$s = \frac{d^2z}{dxdy} = 1 > \checkmark$$

$$t = \frac{d^2z}{dy^2} = -6y > \checkmark$$

$$r \cdot t - s^2 = -36y - 1$$

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5. $x - \frac{\pi}{2} = 0$

$$x = \frac{\pi}{2}$$

$$f(x) = \cos x$$

$$f'(x) = -\sin x$$

$$f''(x) = -\cos x$$

$$f'''(x) = \sin x$$

$$f^{(4)}(x) = \cos x$$

$$f\left(\frac{\pi}{2}\right) = 0$$

$$f'\left(\frac{\pi}{2}\right) = -1$$

$$f''\left(\frac{\pi}{2}\right) = 0$$

$$f'''\left(\frac{\pi}{2}\right) = 1$$

$$f^{(4)}\left(\frac{\pi}{2}\right) = 0$$

$$f'(x) = -\sin x$$

$$f''(x) = -\cos x$$

$$f'''(x) = \sin x$$

$$f^{(4)}(x) = \cos x$$

$$f'\left(\frac{\pi}{2}\right) = -1$$

$$f''\left(\frac{\pi}{2}\right) = 0$$

$$f'''\left(\frac{\pi}{2}\right) = 1$$

$$f^{(4)}\left(\frac{\pi}{2}\right) = 0$$

$$f(x) = 0 - \frac{1}{1!} \left(x - \frac{\pi}{2}\right) - \frac{0}{2!} \left(x - \frac{\pi}{2}\right)^2 + \frac{1}{3!} \left(x - \frac{\pi}{2}\right)^3 + \frac{0}{4!} \left(x - \frac{\pi}{2}\right)^4$$

$$= -\frac{1}{25!} \left(x - \frac{\pi}{2}\right)^5 - \frac{0}{6!} \left(x - \frac{\pi}{2}\right)^6 + \frac{1}{7!} \left(x - \frac{\pi}{2}\right)^7 + \frac{0}{8!} \left(x - \frac{\pi}{2}\right)^8$$

$$= x - \frac{\pi}{2} + \frac{1}{6} \left(x - \frac{\pi}{2}\right)^3 - \frac{1}{120} \left(x - \frac{\pi}{2}\right)^5 + \frac{1}{5040} \left(x - \frac{\pi}{2}\right)^7 \checkmark$$

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