

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD

DO

MATEMATIKA 2: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaci pribor, tablica osnovnih integrala, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

53

IME I PREZIME:

BROJ INDEKSA: 57097

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1. Riješiti integrale:

— (a)  $\int \frac{x^3}{x^2 + 3x + 5} dx$ , 8

— (b)  $\int \frac{\ln x}{x^2} dx$ , 10

— 2. Odrediti površinu između krivulja  $y = x + 2$  i  $y = 4 + x - x^2$ . 20

— 3. Odrediti ekstreme funkcije  $f(x, y) = 3x^2 + xy - y^3 + 2$ . 10

— 4. Riješiti diferencijalnu jednačinu:  $y'' + y' - 2y = 1$ . 5

— 5. Razviti funkciju  $f(x) = \cos x$  u Taylorov red po potencijama  $x - \frac{\pi}{2}$ . Izračunati barem prva 4 člana. ~~0~~

1. b)  $\int \frac{\ln x}{x^2} dx =$

$\ln x = u \quad dv = \frac{1}{x^2} dx$   
 $\frac{1}{x} dx = du \quad v = \int \frac{1}{x^2} dx$

$\int = u \cdot v - \int v du$

$\int = -\ln x \cdot \frac{1}{x} - \int -\frac{1}{x} \cdot \frac{1}{x} dx$

$\int = -\ln x \cdot \frac{1}{x} + \int \frac{1}{x^2} dx$

$\int = -\ln x \cdot \frac{1}{x} - \frac{1}{x}$

$\int = \frac{-\ln x}{x} - \frac{1}{x}$  ✓

10

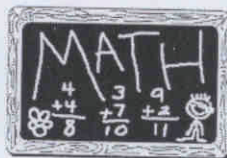
$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} = -\frac{1}{x}$

$v = \int x^{-2} dx$

$v = \frac{x^{-1}}{-1}$

$v = -\frac{1}{x}$

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



$$I_1 = 2 \int \frac{2x+3}{x^2+3x+5} dx = 2 \int \frac{dt}{t} = 2 \ln |t| = 2 \ln |x^2+3x+5|$$

$$I_2 = -6 \int \frac{1}{x^2+3x+5} dx = -6 \int$$

$$\begin{aligned} x^2+3x+5 &= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 5 \\ &= \left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{4}\right) + \frac{20}{4} \\ &= \left(x + \frac{3}{2}\right)^2 + \frac{-9+20}{4} \\ &= \left(x + \frac{3}{2}\right)^2 + \frac{11}{4} \end{aligned}$$

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$$x + \frac{3}{2} = t \quad k^2 = \frac{11}{4}$$

$$dx = dt \quad k = \frac{\sqrt{11}}{2}$$

$$I_2 = -6 \int \frac{1}{\left(x + \frac{3}{2}\right)^2 + \frac{11}{4}} dx = -6 \int \frac{1}{t^2 + \frac{11}{4}} dt$$

$$= -6 \cdot \frac{1}{\frac{\sqrt{11}}{2}} \arctan \frac{x + \frac{3}{2}}{\frac{\sqrt{11}}{2}}$$

$$\begin{aligned} &= \frac{-6}{\frac{\sqrt{11}}{2}} \arctan \frac{\frac{2x+3}{2}}{\frac{\sqrt{11}}{2}} = \frac{-18}{\sqrt{11}} \arctan \frac{2(2x+3)}{2\sqrt{11}} \\ &= \frac{-18}{\sqrt{11}} \arctan \frac{2x+3}{\sqrt{11}} // \end{aligned}$$

IME I PREZIME:

Lore Gjordanic

BROJ INDEKSA:

$$2. y = x + 2 \text{ i } y = 4 + x - x^2$$

$$y = x + 2$$

$$x_1 = 0$$

$$y_2 = 0$$

$$y = 0 + 2$$

$$0 = x + 2$$

$$y_2 = 2 //$$

$$-x = 2$$

$$x_2 = -2 //$$

$$y = 4 + x - x^2$$

$$y = -x^2 + x + 4$$

$$x_0 = \frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2}$$

$$y_0 = \frac{4ac - b^2}{4a} = \frac{4 \cdot (-1) \cdot 4 - 1^2}{-4} =$$

$$\frac{-16 - 1}{-4} = \frac{-17}{-4} = 4.25$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 4}}{-2}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 + 16}}{-2} = \frac{-1 \pm \sqrt{17}}{-2}$$

$$x_1 = \frac{-1 - \sqrt{17}}{-2} = \frac{-1 - 4.12}{-2} = 2.56 //$$

$$x_2 = \frac{-1 + 4.12}{-2} = -1.56 //$$

$$y = y$$

$$4 + x - x^2 = x + 2$$

$$4 + x - x^2 - x - 2 = 0$$

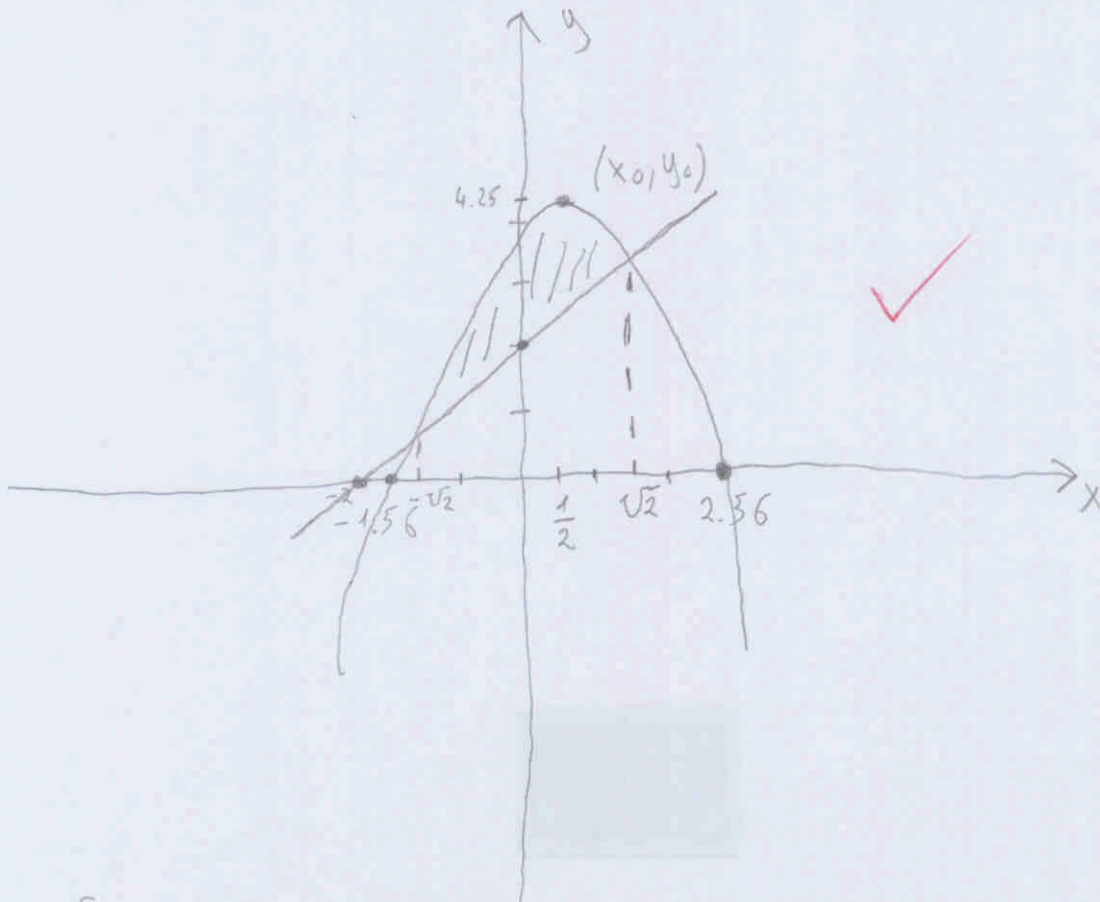
$$-x^2 + 2 = 0$$

$$-x^2 = -2$$

$$x^2 = 2$$

$$x = \pm \sqrt{2} \quad \checkmark$$

2.



$$P = \int_{-\sqrt{2}}^{\sqrt{2}} (4+x-x^2) dx - \int_{-\sqrt{2}}^{\sqrt{2}} (x+2) dx = \int_{-\sqrt{2}}^{\sqrt{2}} (4+x-x^2-x-2) dx = \int_{-\sqrt{2}}^{\sqrt{2}} (2-x^2) dx$$

JEDNOSTAVNIJE =

$$I_1 = \int (4+x-x^2) dx = \int 4 dx + \int x dx - \int x^2 dx = 4 \int dx + \frac{x^2}{2} - \frac{x^3}{3}$$

$$= \frac{4x}{1} + \frac{x^2}{2} - \frac{x^3}{3}$$

$$I_2 = \int (x+2) dx = \int x dx + \int 2 dx = \frac{x^2}{2} + 2x$$

$$I = \underbrace{\left( 4x + \frac{x^2}{2} - \frac{x^3}{3} \right)}_{I_1} \Big|_{-\sqrt{2}}^{\sqrt{2}} - \underbrace{\left( \frac{x^2}{2} + 2x \right)}_{I_2} \Big|_{-\sqrt{2}}^{\sqrt{2}}$$

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IME I PREZIME:

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BROJ INDEKSA:

$$2. \bar{I}_1 = \left( -4\sqrt{2} + \frac{(-\sqrt{2})^2}{2} - \frac{(-\sqrt{2})^3}{3} \right) - \left( 4\sqrt{2} + \frac{(\sqrt{2})^2}{2} - \frac{(\sqrt{2})^3}{3} \right)$$

$$\bar{I}_2 = \left( \frac{(-\sqrt{2})^2}{2} + 2 \cdot (-\sqrt{2}) \right) - \left( \frac{(\sqrt{2})^2}{2} + 2 \cdot \sqrt{2} \right)$$

$$3. f(x, y) = 3x^2 + xy - y^3 + 2$$

$$z_x = 6x + y \quad \checkmark A$$

$$z_y = x - 3y^2 \quad \checkmark$$

$$z_{xx} = 6 \quad (A)$$

$$z_{yy} = -6y \quad (C)$$

$$z_{xy} = 0 \quad (B)$$

$$z_{yx} = 0$$

$$z_{xy} = z_{yx} = 1$$

X

x = 0

$$x - 3y^2 = 0$$

$$x = 3 - \left( \frac{1}{18} \right)^2 = 0$$

$$x = 3 - \frac{1}{324} = 0$$

$$x = \frac{1}{108} = 0$$

$$x_2 = \frac{1}{108} //$$

$$6x + y = 0 \quad \checkmark$$

$$x - 3y^2 = 0 \quad \checkmark$$

$$x = 3y^2$$

$$y \sqrt{18(y+1)} = 0$$

$$3y^2 = x$$

$$y = 0 \quad x_1 = 0$$

$$18y + 1 = 0$$

$$18y = -1$$

$$y_2 = -\frac{1}{18} \quad \checkmark$$

$$T_1(0, 0) \quad T_2\left(\frac{1}{108}, -\frac{1}{18}\right) \quad \checkmark$$

$$\frac{3}{x} = \frac{1}{108} = 0$$

X

3.  $T_1(0,0)$

$A = 6$  ✓

$B = 0$  ✓

✗

$L = -6y$

$\Delta = A \cdot C - B^2 = 6 \cdot 0 - 0^2 = 0$

$C = -6 \cdot 0$

$C = 0$  ✓

$T_2\left(\frac{1}{108}, -\frac{1}{18}\right)$  ✓

$A = 6$

$B = 0$  ✓

✗

$\Delta = AC - B^2 = 6 \cdot \frac{1}{3} - 0^2$

$L = -6y$

$L = -6 \cdot -\frac{1}{18}$

$= \frac{6}{3} - 0$

$L = \frac{6}{18} = \frac{1}{3}$  ,,

$= 2$  ,,

$\Delta > 0$

$A = 6 > 0$

$m\left(\frac{1}{108}, -\frac{1}{18}, \frac{1}{3}\right)$

$z = 3x^2 + xy - y^3 + 2$

$z = 3 \cdot \left(\frac{1}{108}\right)^2 + \left(\frac{1}{108} \cdot -\frac{1}{18}\right) - \left(-\frac{1}{18}\right)^3 + 2$

$z_m = 3 \cdot \left(\frac{1}{11664}\right) + \left(\frac{-1}{1944}\right) - \left(\frac{-1}{5832}\right) + 2$

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$$4. \quad y'' + y' - 2y = 1$$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-2)}}{2} = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$\lambda_{1,2} = \frac{-1 \pm 3}{2}$$

$$\lambda_1 = \frac{-1+3}{2} = 1_{11} \quad \checkmark$$

$$\lambda_2 = \frac{-1-3}{2} = -2_{11} \quad \checkmark$$

$$y = C_1 e^{y_1 x} + C_2 e^{y_2 x} \quad \checkmark$$

$$y = C_1 e^x + C_2 e^{-2x} \quad \checkmark \quad \text{RJEŠENJE HOMOGENE}$$

KAKO DO RJEŠENJA POLAZNE JEDNAČEBE?



$$5. f(x) = \cos x \quad x - \frac{\sqrt{1}}{2} = 0$$

$$f\left(\frac{\sqrt{1}}{2}\right) = \cos \frac{\sqrt{1}}{2} \quad x = \frac{\sqrt{1}}{2} //$$

$$= 0,99 // \text{X}$$

$$\cos \frac{\pi}{2} = 0$$

$$f'(x) = \sin x \quad \checkmark$$

$$f'\left(\frac{\sqrt{1}}{2}\right) = \sin \frac{\sqrt{1}}{2}$$

$$= 0,27 // \text{X}$$

$$f = f(x_0) + (x-x_0) \cdot f'(x_0) + \frac{(x-x_0)^2}{2!} \cdot f''(x_0) + \dots + \frac{f^{(n)}(x_0)}{n!} \cdot (x-x_0)^n + \dots$$

$$f''(x) = -\cos x \quad \checkmark$$



$$f''\left(\frac{\sqrt{1}}{2}\right) = -\cos \frac{\sqrt{1}}{2}$$

$$= -0,99 // \text{X}$$

VIDI DURBIĆ, DUNATOV ✓

$$f'''(x) = -\sin x$$

$$f'''\left(\frac{\sqrt{1}}{2}\right) = -\sin \frac{\sqrt{1}}{2}$$

$$= -0,27 //$$

$$f^{(4)}(x) = \cos x$$

$$f^{(4)}\left(\frac{\sqrt{1}}{2}\right) = \cos \frac{\sqrt{1}}{2}$$

$$= 0,99 //$$