

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaci pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljšavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

xooo

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IME I PREZIME: **MARKO BUBIČIĆ**

BROJ INDEKSA: **54768-2007** Broj bodova ↓

1. Ispitati domenu i asimptote funkcije  $f(x) = \frac{\sqrt{x^2-3} - (3+x)}{x+2}$

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2. Zadane su točke  $A(1,2,3)$ ,  $B(3,1,2)$ ,  $C(2,3,1)$  i  $D(3,2,-1)$ . Odredi volumen paralelepipeda određenog usmjerenim dužinama  $\vec{AB}$ ,  $\vec{AC}$  i  $\vec{AD}$ .

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3. Ispitati tok funkcije:  $g(x) = (\arctan x)^2$  ako je zadano da druga derivacija ima jedine nultočke u  $x_1 \approx -0.8$  i  $x_2 \approx 0.8$ .

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4. Naći  $f'''(0)$  ako je  $f(x) = e^{-x} \sin x$ .

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1.

$$f(x) = \frac{\sqrt{x^2-3} - (3+x)}{x+2}$$

$$x+2 \neq 0$$

$$x \neq -2$$

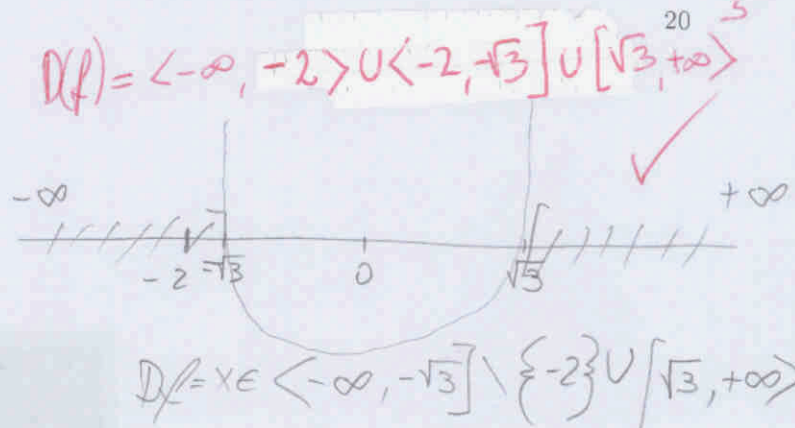
$$x^2-3 \geq 0$$

$$x^2 \geq 3$$

$$x^2 = \sqrt{3}$$

$$x_1 = -\sqrt{3}$$

$$x_2 = +\sqrt{3}$$



$$D_f = x \in \langle -\infty, -\sqrt{3} \rangle \cup \langle -\sqrt{3}, -2 \rangle \cup \langle -2, \sqrt{3} \rangle \cup \langle \sqrt{3}, +\infty \rangle$$

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V.A.

$$\lim_{x \rightarrow -\sqrt{3}} \frac{\sqrt{x^2-3} - (3+x)}{x+2} = \frac{\sqrt{(-\sqrt{3})^2-3} - (3+(-\sqrt{3}))}{-\sqrt{3}+2} = \frac{\sqrt{0} - 1,267}{-1,267+2} =$$

$$\frac{-1,267}{0,733} = -1,728$$

NEMA ✓

$$\lim_{x \rightarrow -2} \frac{\sqrt{x^2-3} - (3+x)}{x+2} = \frac{\sqrt{(-2)^2-3} - (3-2)}{-2+2} = \frac{1-1}{0} = \frac{0}{0} ?$$

NEMA X

$$\lim_{x \rightarrow \sqrt{3}} \frac{\sqrt{x^2-3} - (3+x)}{x+2} = \frac{\sqrt{(\sqrt{3})^2-3} - (3+\sqrt{3})}{\sqrt{3}+2} = \frac{-4,732}{3,732} =$$

H.A.

$$\lim_{x \rightarrow \pm\infty} \frac{\sqrt{x^2-3} - (3+x)}{x+2} \stackrel{H.A.}{=} \frac{\frac{\sqrt{x^2-3}}{x} - \frac{3}{x} - \frac{x}{x}}{\frac{x}{x} + \frac{2}{x}} = \frac{1 - 0 - 1}{1 + 0} = \frac{0}{1} = 0$$

IMA

$$\frac{\sqrt{\frac{x^2-3}{x^2}} - \left(\frac{3}{x} + \frac{x}{x}\right)}{\frac{x}{x} + \frac{2}{x}} = \frac{\sqrt{1-1} - 1}{1} = \frac{0}{1} = 0$$

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$$y = kx + l$$

$$\lim_{x \rightarrow \infty} k = \frac{f(x)}{x} = \frac{\frac{\sqrt{x^2-3} - (3+x)}{x+2}}{\frac{x}{1}} = \frac{\sqrt{x^2-3} - (3+x)}{x+2}$$

$$\lim_{x \rightarrow \infty} l = (f(x) - k \cdot x) \quad ?$$

VIDI LUKA SJAUŠ

VIDI IVAN MAMIĆ

4.  $f'''(0)$  ako je  $f(x) = e^{-x} \sin x$

$$f'(x) = e^{-x} \sin x + e^{-x} \cos x \quad \times$$

$$f'(x) = e^{-x} (\sin x + \cos x)$$

$$f''(x) = e^{-x} (\sin x + \cos x) + e^{-x} (-\cos x - \sin x)$$

$$f''(x) = e^{-x} (\sin x + \cos x + \cos x - \sin x)$$

$$f''(x) = e^{-x} \cdot 2 \cos x$$

$$f'''(x) = e^{-x} \cdot 2 \cos x + e^{-x} \cdot (-2 \sin x)$$

$$f'''(x) = e^{-x} (2 \cos x - 2 \sin x)$$

$$f'''(0) = e^{-0} (2 \cos 0 - 2 \sin 0)$$

$$f'''(0) = 1 \cdot (2 \cdot 1 - 2 \cdot 0)$$

$$f'''(0) = 1 \cdot (2 - 0)$$

$$f'''(0) = 2$$

$$\begin{aligned} f'(x) &= e^{-x} \cdot (-1) \cdot \sin x + e^{-x} \cos x \\ &= -e^{-x} \sin x + e^{-x} \cos x \\ &= e^{-x} (\cos x - \sin x) \end{aligned}$$

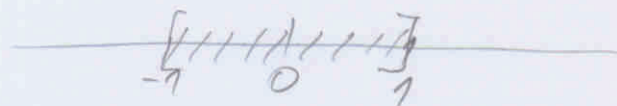
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3.  $g(x) = (\arctan x)^2$

$x_1 \approx -0,8$  i  $x_2 \approx 0,8$

$$-1 \leq x \leq 1 \quad \times$$

$$x \geq -1 \quad x \leq 1$$

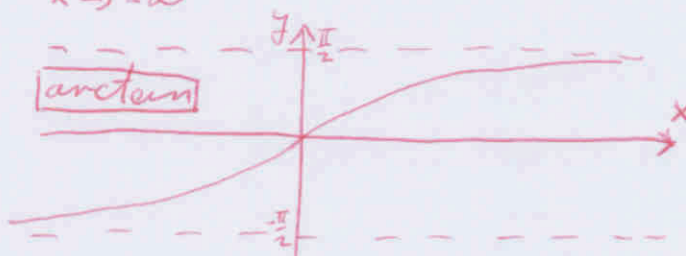


$$D_f = E_x [-1, 1] \quad \times$$

$$\mathcal{D}(g) = \mathbb{R}$$

$$\lim_{x \rightarrow +\infty} \arctan x = \frac{\pi}{2}$$

$$\lim_{x \rightarrow -\infty} \arctan x = -\frac{\pi}{2}$$



V.A.

$$\lim_{x \rightarrow -1} (\arctan x)^2 = (\arctan -1)^2 = \left(-\frac{1}{4}\pi\right)^2 = -\frac{\pi^2}{16} = -\frac{\pi}{16}$$

NEMA

$$\lim_{x \rightarrow 1} (\arctan x)^2 = (\arctan 1)^2 = \left(\frac{1}{4}\pi\right)^2 = \left(\frac{\pi}{4}\right)^2 = \frac{\pi^2}{16}$$

H.A.  $\lim_{x \rightarrow \pm\infty} (\arctan x)^2 = (\arctan \infty)^2 = 0$

$$f(x) = (\arctan x)^2$$

$$f'(x) = 2 \arctan x \cdot \frac{1}{1+x^2}$$

	$-\infty$	$-1$	$0$	$1$	$+\infty$
$f'(x)$	-	-	+	+	
$f(x)$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$	

$$f'(x) = \frac{2 \arctan x}{1+x^2}$$

$$f''(x) = \frac{2 \cdot \frac{1}{1+x^2} \cdot (1+x^2) - 2 \arctan x \cdot 2x}{(1+x^2)^2}$$

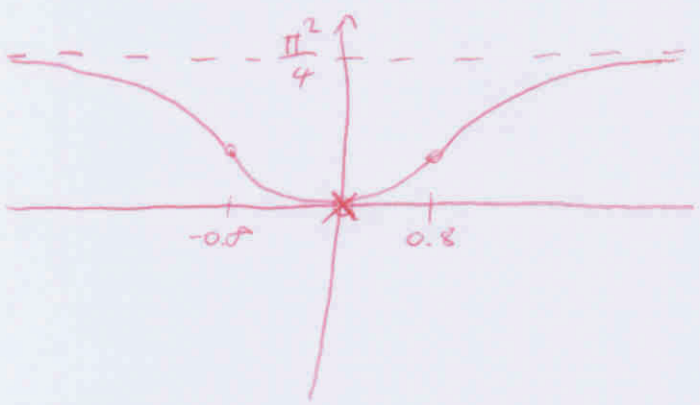
$m(0,0)$  ✓

$$f''(x) = \frac{2 \cdot \frac{1+x^2}{1+x^2} - 4x \arctan x}{(1+x^2)^2}$$

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$$f''(x) = \frac{2 - 4x \arctan x}{1+x^4}$$

H.A.  $\lim_{x \rightarrow \pm\infty} f(x) = \left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{4}$



	$-\infty$	$-0,8$	$+0,8$	$+\infty$
$f''(x)$	+	+	-	
$f(x)$	U	U	n	

	$-\infty$	$-0,8$	$+0,8$	$+\infty$
$f''(x)$	-	+	-	
$f(x)$	$\searrow$	$\searrow$	$\searrow$	