MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ D	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: FILIP GORŠEK **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0369-2014 PROF. NIKICA UGLEŠIĆ	

1. Izračunati površinu lika omeđenog parabolom $y = x^2 - 4x - 5$ i pravcem $y = x + 1$. 20
2. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = x^2 - y^2$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme? 10
3. Pronaći:
   (a) koliko iznosi $\int_0^3 x^2 \ln x \, dx$, 10
   (b) partikularno rješenje diferencijalne jednadžbe $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. Na kraju provjeri rješenje. 10
4. Izračunati $\int\limits_0^3 \frac{dx}{2-\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 8% ili manje. 15
5. Nađi partikularno rješenje jednadžbe $9y'' - 6y' + y = 0$ uz uvjet $x = 1$, $y = 0$ i $y' = 0$. Ø

Ukupno: 65

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

FILIP GORŠEK

(1.) $y_1 = x^2 - 4x - 5$ $y_2 = x + 1$

$x^2 - 4x - 5 = x + 1$

$x^2 - 5x - 6 = 0$

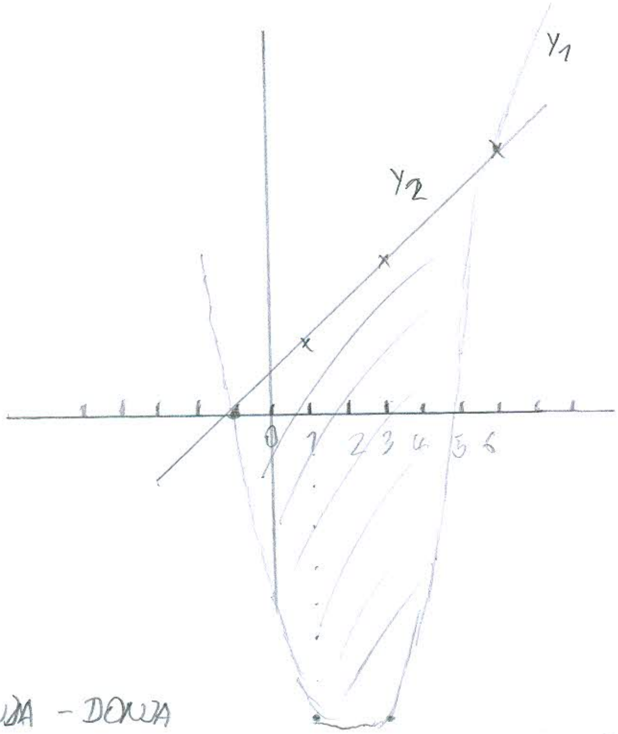
$x_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot (-6)}}{2}$

$x_1 = 6$

$x_2 = -1$

x	-1	1	3	6
y_1	0	-8	-8	7

x	-1	1	3	6
y_2	0	2	4	7



GORNJA - DONJA

$$P = \int_{-1}^{6} x + 1 - x^2 + 4x + 5 \, dx = \frac{5x^2}{2} - \frac{x^3}{3} + 6x \Big|_{-1}^{6} = 54 + \frac{19}{6} = \frac{343}{6}$$

$$\int 5x - x^2 + 6 \, dx = \int 5x \, dx - \int x^2 \, dx + 6 \int dx = \frac{5x^2}{2} - \frac{x^3}{3} + 6x + C$$

3. a) $\int_0^3 x^2 \ln x \, dx = \begin{bmatrix} u = \ln x & dv = x^2 dx \\ du = \frac{1}{x} dx & v = \frac{x^3}{3} \end{bmatrix} \approx 6.88751 \ldots = 6.88751$

$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \frac{dx}{x}$

$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^{\cancel{3}\,2} \frac{dx}{\cancel{x}}$

$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 dx$

$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \frac{x^3}{3} + c$

b) $x y y' = 1 - x^2 \,/ :x$

$y(1) = 1$

$2 \ln(1) - 1^2 + 2C = 1$

$-1 + 2C = 1$

$2C = 2 \,/ :2$

$\boxed{C = 1}$

$y y' = \frac{1-x^2}{x}$

$y \frac{dy}{dx} = \frac{1-x^2}{x} \,/ \cdot dx$

$\int y dy = \int \frac{1-x^2}{x} dx$

$\frac{y^2}{2} = \int \frac{1}{x} dx - \int x dx$

$\frac{y^2}{2} = \ln x - \frac{x^2}{2} + c \,/ \cdot 2$

$y^2 = 2 \ln x - x^2 + 2C \,/ \sqrt{}$

$y = \sqrt{2 \ln x - x^2 + 2C}$

$\boxed{y_p = \sqrt{2 \ln x - x^2 + 2}}$

$y' = \frac{1}{2} (2 \ln x - x^2 + 2) \cdot \left(\frac{2}{x} - 2x \right) / \cdot 2$

$y' = (2 \ln x - x^2 + 2) \cdot \left(\frac{2}{x} - 2x \right)$

FILIP GORŠEK

4. $\int_0^3 \frac{dx}{2-\sqrt{x}}$

0 0.75 1.5 3

0	0.75	1.5
0.5	0.881854	1.289898

$S_1 = \frac{1.5}{6}\left(0.5 + 4 \cdot 0.881854 + 1.289898\right) = 1.3293285$

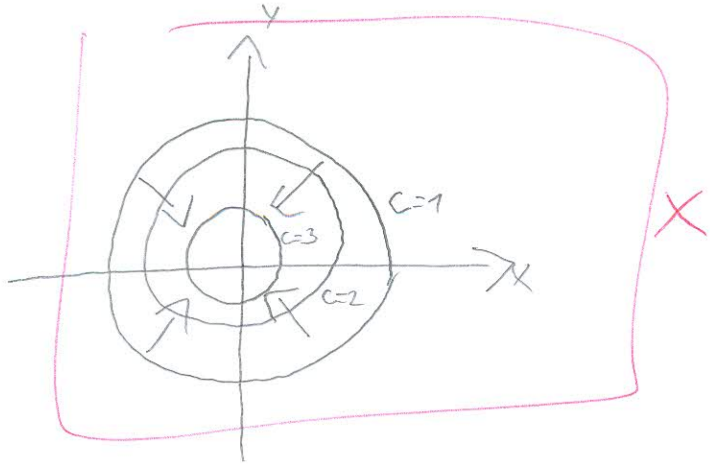
1.5	2.25	3
1.289898	2	3.73205

$S_2 = \frac{1.5}{6}\left(1.289898 + 4 \cdot 2 + 3.73205\right) = 3.255487$

$S_1 + S_2 = 4.5848155$ //

15

2. $f(x,y) = x^2 - y^2$ $\qquad D(f) = \mathbb{R}^2$ $\qquad K(f) = \langle 0, +\infty \rangle$



$C=1, \quad x^2 - y^2 = 1$

$-y^2 = -x^2 + 1 \;/\cdot(-1)$

$y^2 = x^2 - 1$

$C=2 \quad x^2 - y^2 = 2$

$y^2 = x^2 - 2$

$C=3 \quad x^2 - y^2 = 3$

$y^2 = x^2 - 3$

EKSTREMI $\quad f(x,y) = x^2 - y^2$

$\frac{\partial f}{\partial x} = 2x \rightarrow x = 0$ $\qquad \boxed{T(0,0)}$

$\frac{\partial f}{\partial y} = -2y \rightarrow y = 0$

A) $\frac{\partial^2 f}{\partial x^2} = 2$ $\qquad$ B) $\frac{\partial f}{\partial x \partial y} = 0$

C) $\frac{\partial^2 f}{\partial y^2} = -2$

$A \cdot C - (B)^2 = 2 \cdot (-2) - 0 = -4 < 0$

T(0,0) JE SEDLASTA TOČKA

NEMA EKSTREMA

10

FILIP GORSEK

(5.) $9y''-6y'+y=0$ $\quad y(1)=0 \quad y'(1)=0$

$9r^2-6r+1=0$ $\quad r_{1,2}=\frac{6\pm\sqrt{36-4\cdot 9\cdot 1}}{18}=$ $\quad r_1=r_2$ ✓

$r=\frac{1}{3}$ ✓

$Y_H=e^{r_1x}(C_1+C_2x)$

$Y_H=e^{1/3x}(C_1+C_2x)$

$A=0$

$Y_H=e^{1/3}(C_1+C_2x)=0 \;/:e^{1/3}$

$C_1+C_2x=0$

$C_1=-C_2x$

$Y'_H=\frac{1}{3}e^{1/3x}\cdot(C_1+C_2x)+e^{1/3x}\cdot 0$

$Y'_H=\frac{1}{3}e^{1/3x}\cdot(C_1+C_2x)\longrightarrow \frac{1}{3}e^{1/3\cdot 1}\cdot(C_1+C_2x)$

$\frac{1}{3}e^{1/3}\cdot(C_1+C_2x)=0 \;/\cdot 3$

$e^{1/3}\cdot(C_1+C_2x)=0$

$e^{1/3}C_1+e^{1/3}C_2x$

$C_2x=-C_1$ $\quad$ DALJE ... ?

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ D

IME I PREZIME: JAKOV ZUBČIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0273-2014 USTMENI: prof. Uglešić

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati površinu lika omeđenog parabolom $y = x^2 - 4x - 5$ i pravcem $y = x + 1$. 20
2. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = x^2 - y^2$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme? 10
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   (a) koliko iznosi $\int_0^3 x^2 \ln x \, dx$, ∅
   (b) partikularno rješenje diferencijalne jednadžbe $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. Na kraju provjeri rješenje. 10
4. Izračunati $\int\limits_0^3 \frac{dx}{2-\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 8% ili manje.
5. Nađi partikularno rješenje jednadžbe $9y'' - 6y' + y = 0$ uz uvjet $x = 1$, $y = 0$ i $y' = 0$. 20

Ukupno: 60

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
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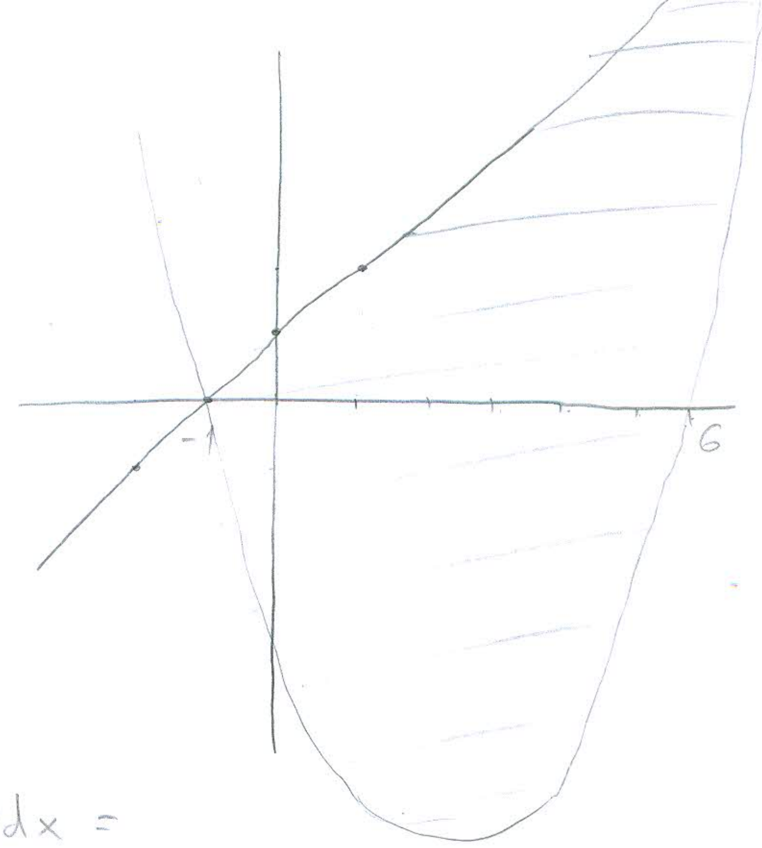
JAKOV ŽUBČIĆ

1. $y = x^2 - 4x - 5$

$y = x + 1$

x	0	1	-1	-2
y	1	2	0	1



$$x^2 - 4x - 5 = x + 1$$
$$x^2 - 4x - 5 - x - 1 = 0$$
$$x^2 - 5x - 6 = 0$$
$$x = \frac{5 \pm \sqrt{25+24}}{2}$$
$$x = \frac{5 \pm 7}{2}$$
$$x_1 = -1$$
$$x_2 = 6$$

$$\int (x^2 - 4x - 5)\,dx =$$
$$\frac{x^3}{3} - \frac{4x^2}{2} - 5x = \frac{x^3}{3} - 2x^2 - 5x$$

$$\int (x+1)\,dx = \frac{x^2}{2} + x$$

$$\left[\frac{x^2}{2} + x - \frac{x^3}{3} + 2x^2 + 5x \right]_{-1}^{6} =$$

$$\frac{36}{2} + 6 \cancel{-72} \cancel{+72} + 30 - \left(\frac{1}{2} - 1 + \frac{1}{3} + 2 - 5 \right) =$$
$$54 - (-3,16) = 57,16$$

5. $9y'' - 6y' + y = 0$ $\quad x = 1$

$y = 0$

$y' = 0$

$x' = 0$

$9r^2 - 6r + 1 = 0$

$r = \frac{6 \pm \sqrt{36-36}}{18}$

$r_{1,2} = \frac{1}{3}$

$y = C_1 \cdot e^{\frac{1}{3}x} + C_2 \cdot x \cdot e^{\frac{1}{3}x}$

$y' = \frac{1}{3} C_1 \cdot e^{\frac{1}{3}x} + \frac{1}{3} C_2 x e^{\frac{1}{3}x} + C_2 e^{\frac{1}{3}x}$

$0 = C_1 \cdot e^{\frac{1}{3}} + C_2 \cdot e^{\frac{1}{3}}$

$-C_1 \cdot \cancel{e^{\frac{1}{3}}} = C_2 \cdot \cancel{e^{\frac{1}{3}}}$

$\boxed{-C_1 = C_2}$

$y' = \frac{1}{3} C_1 \cdot e^{\frac{1}{3}x} + \frac{1}{3} C_2 x e^{\frac{1}{3}x} + C_2 \cdot e^{\frac{1}{3}x}$

$0 = \frac{1}{3} C_1 + C_2$

$0 = \frac{1}{3} C_1 - C_1$

$0 = -\frac{2}{3} C$

PARTIKULARNO RJ : $\boxed{y = 0}$ ✓

JAKOV ZUBČIĆ

4. $\int_0^3 \frac{dx}{2-\sqrt{x}} = \begin{bmatrix} x = t^2 \\ 2t\,dt = dx \end{bmatrix} = \int \frac{2t\,dt}{2-t} = 2\int \frac{t}{2-t}\,dt$

$\varnothing$

4. $\int_0^3 \frac{dx}{2-\sqrt{x}}$

x	0	1	2
	0	1,5	3
	$\frac{1}{2}$	0,76	0,268

$S = \frac{3}{6}\left(\frac{1}{2} + 4 \cdot 0,76 + 0,268\right) = 1,904$

$\varnothing$

JAKOV ZUBČIĆ

3 a) $xyy' = 1-x^2 \quad , \quad y(1) = 1$

$$yy' = \frac{1-x^2}{x}$$

$$y\frac{dy}{dx} = \frac{1-x^2}{x} \quad / \cdot dx$$

$$y\,dy = \left(\frac{1-x^2}{x}\right)dx$$

$$\frac{y^2}{2} = \ln x - \frac{x^2}{2} + C$$

$$\frac{1}{2} = \ln 1 - \frac{1}{2} + C$$

$$\frac{1}{2} + \frac{1}{2} = C$$

$$\boxed{C = 1}$$

$$\int \frac{1-x^2}{x}\,dx =$$

$$\int \frac{1}{x}\,dx - \int x\,dx =$$

$$= \ln|x| - \frac{x^2}{2}$$

$$\frac{y^2}{2} = \ln x - \frac{x^2}{2} + 1$$

PROVJERA:

$$\frac{1}{2} = \underbrace{\ln 1}_{0} - \frac{1}{2} + 1$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{2y}{2} = \frac{1}{x} - x$$

$$yy' = \frac{1}{x} - x \quad / \cdot x$$

$$xyy' = 1 - x^2$$

2. $f(x,y) = x^2 - y^2$

$\boxed{D : R^2}$

$\boxed{K : R^2}$

$\frac{\partial f}{\partial x} = 2x$

$\frac{\partial f}{\partial y} = -2y$

$T_0(0,0)$

$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = -4 < 0$

$\frac{\partial^2 f}{\partial x^2} = 2$

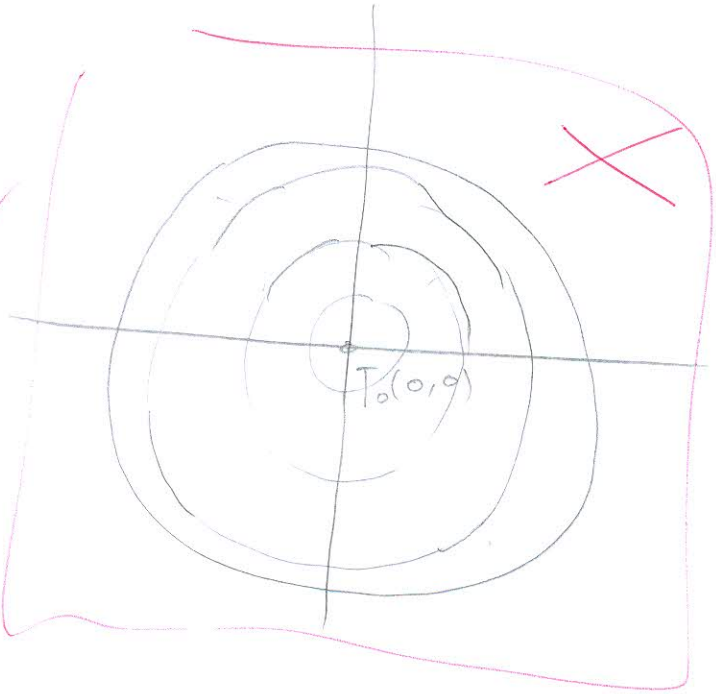
$T_0(0,0)$ - stacionarna točka

$\frac{\partial^2 f}{\partial y^2} = -2$

10

$\frac{\partial f}{\partial xy} = 0$

f nema lok. eks.



JAKOV ZUBČIĆ

3. a) $\int_0^3 x^2 \cdot \ln x \, dx = \begin{bmatrix} \ln x = u \,/\,' & dv = x^2 dx \,/\int \\ du = \frac{1}{x} dx & v = 2x \end{bmatrix}$

$= 2x \cdot \ln x - \int 2\cancel{x} \cdot \frac{1}{\cancel{x}} \, dx$

$= 2x \cdot \ln x - \int 2 \, dx = \left[2x \cdot \ln x - 2x\right]_0^3$

$\left[6 \cdot \ln 3 - 6 - \cancel{(2 \cdot 0 \cdot \ln 0 - 0)}\right]$

$= 0{,}59$ NEPR: INT.

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ D	POPUNJAVA NASTAVNIK
IME I PREZIME: TOMISLAV BULIĆ **VRIJEME POČETKA:**	Broj ↓ bodova
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0271-2013 PROFESOR UGLEŠIĆ	

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5. Nađi partikularno rješenje jednadžbe $9y'' - 6y' + y = 0$ uz uvjet $x = 1$, $y = 0$ i $y' = 0$. ∅

Ukupno: 55

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
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$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

TOMISLAV BULIĆ

① $y=x^2-4x-5 \qquad y=x+1$

$x^2-4x-5=x+1$

$x^2-4x-5-x-1=0$

$x^2-5x-6=0$

$x_{1,2}=\dfrac{5\pm\sqrt{25+24}}{2}$

$x_{1,2}=\dfrac{5\pm7}{2}$

$x_1=-1 \qquad x_2=6$

$\int_{-1}^{6}(x+1)-(x^2-4x-5)\,dx$

$\int_{-1}^{6}x+1-x^2+4x+5\,dx$ ✓

$\int_{-1}^{6}-x^2+5x+6\,dx$ ✓

$-\int_{-1}^{6}x^2dx+5\int_{-1}^{6}xdx+6\int_{-1}^{6}dx$

$-\left.\dfrac{x^3}{3}\right|_{-1}^{6}+5\left.\dfrac{x^2}{2}\right|_{-1}^{6}+6\left.x\right|_{-1}^{6}+C$

$-\left(\dfrac{6^3}{3}-\dfrac{-1^3}{3}\right)+5\left(\dfrac{6^2}{2}-\dfrac{-1^2}{2}\right)+6\left(6-(-1)\right)$

$-72{,}333+92{,}5+42$

$=\underline{62{,}16667}$ ✓

③ a) $\int_{0}^{3}x^2\ln x\,dx$

$u=\ln|x| \qquad dv=x^2$

$du=\dfrac{1}{x} \qquad v=\dfrac{x^3}{3}$

$\left.\ln|x|\dfrac{x^3}{3}\right|_0^3-\int_0^3\dfrac{x^3}{3}\cdot\dfrac{1}{x}dx$

$\left.\ln|x|\dfrac{x^3}{3}\right|_0^3-\int_0^3\dfrac{x^3}{3x}dx$

→

$$\Big|_0^3 \ln|x| \frac{x^3}{3} - \frac{1}{3}\int_0^3 x^2\,dx$$

$$\Big|_0^3 \ln|x| \frac{x^3}{3} - \frac{1}{3}\Big|_0^3 \frac{x^3}{3}$$

$$\left(\ln|3| \frac{3^3}{3} - \ln|0| \frac{0^3}{3}\right) - \frac{1}{3}\left(\left(\frac{3^3}{3}\right) - \left(\frac{0^3}{3}\right)\right)$$

$$9{,}88751 \quad - \frac{1}{3}(9)$$

$$9{,}88751 - 3 = \underline{\underline{6{,}88751}}$$

b) $xyy' = 1 - x^2 \qquad y(1) = 1$

$$xy\frac{dy}{dx} = 1 - x^2 \Big/ \frac{dx}{x}$$

$$y\,dy = \frac{1-x^2}{x}\,dx$$

$$\int y\,dy = \int \frac{1}{x}\,dx - \int x\,dx$$

$$\frac{y^2}{2} = \int x^{-1}\,dx - \int x\,dx$$

$$\frac{y^2}{2} = \int \frac{dx}{x} - \frac{x^2}{2}$$

$$\frac{y^2}{2} = \ln|x| + c - \frac{x^2}{2} \Big/ \cdot 2$$

$$\underline{y^2 = 2\ln|x| + 2c - x^2}$$

$$1^2 = 2\ln|1| + 2c - 1^2$$

$$1 = 2c - 1$$

$$-2c = -2$$

$$\boxed{c = 1}$$

$$y^2 = 2\ln|x| + 2 - x^2 \Big/ \sqrt{\ }$$

$$y = \sqrt{2\ln|x| + 2 - x^2}$$

④ TOMISLAV BULIĆ

$\int_0^3 \frac{dx}{2-\sqrt{x}}$ $f_0(0)$ $f_1(\frac{3}{2})$ $f_2(3)$

$d=3$

$S \frac{3}{6}\left(\left(\frac{1}{2-\sqrt{0}}\right)+4\left(\frac{1}{2-\sqrt{\frac{3}{2}}}\right)+\left(\frac{1}{2-\sqrt{3}}\right)\right)$

$\frac{3}{6}\left(\frac{1}{2}+4\cdot(1{,}289897949)+3{,}732050808\right.$

$\frac{3}{6}\left(\frac{1}{2}+5{,}159591796+3{,}732050808\right)$

$\frac{3}{6}\cdot 9{,}391642604 = \underline{4{,}695821302}$ ✓ 15

⑤ $9y''-6y'+y=0$ $x=1$ $y=0$ $y'=0$

$r^2=ar+b=0$

$r^2=-6r+1=0$

$-6r=-1$

$r^2=$ $r=\frac{1}{6}$ ~~$r^2 \frac{1}{6}\sqrt{\ } \quad r=\frac{\sqrt{1}}{6}$~~

$r=$ $\boxed{r=\frac{\pm 1}{6}}$

~~$9y''-6y'+y=0 \,/:9$~~

~~$y''-\frac{2}{3}y'+\frac{y}{9}=0$~~

~~$r^2=ar+b=0$~~

~~$r^2=-\frac{2}{3}r+\frac{1}{9}=0$~~

~~$-\frac{2}{3}r=-\frac{1}{9}$~~

$y(x)=C_1e^{r_1x}+C_2e^{r_2x}$

$y(x)=C_1e^{\frac{1}{6}x}+C_2e^{-\frac{1}{6}x}$

$y(0)=C_1e^{0}+C_2e^{0}$

$y(0)=C_1+C_2=1$

×

$y'(x)=C_1\frac{1}{6}e^{\frac{1}{6}x}+C_2\frac{1}{6}e^{-\frac{1}{6}x}$

0

$y'(0)=\frac{1}{6}C_1 + -\frac{1}{6}C_2$

$y'=0$

~~$y(0)=C_1\,1{,}181+C_2$~~

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ D	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Josip Gaula **VRIJEME POČETKA:** 17.35	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0385-2014	
PROF. UGLEŠIĆ	

1. Izračunati površinu lika omeđenog parabolom $y = x^2 - 4x - 5$ i pravcem $y = x + 1$. 20
2. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = x^2 - y^2$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme? 8
3. Pronaći:
   (a) koliko iznosi $\int_0^3 x^2 \ln x \, dx$, ∅
   (b) partikularno rješenje diferencijalne jednadžbe $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. Na kraju provjeri rješenje. ∅
4. Izračunati $\int\limits_0^3 \frac{dx}{2-\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 8% ili manje. ~~∅~~ 15
5. Nađi partikularno rješenje jednadžbe $9y'' - 6y' + y = 0$ uz uvjet $x = 1$, $y = 0$ i $y' = 0$. ∅

Ukupno: 43

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

Josip Gaula

(1) $y = x^2 - 4x - 5$

$y = x + 1$

x	0	1	2
y	1	2	3

$x_0 = \frac{-b}{2a}$

$x_0 = \frac{4}{2 \cdot 1}$ $x_0 = 2$

$x^2 - 4x - 5 = x + 1$

$x^2 - 4x - x - 5 - 1 = 0$

$x^2 - 5x - 6 = 0$

x	-2	0	2	4	6
y	7	-5	-9	-5	7

$x_{1,2} = \frac{+5 \pm \sqrt{(-5)^2 + 4 \cdot 6 \cdot 1}}{2 \cdot 1}$

$x_{1,2} = \frac{5 \pm \sqrt{25 + 24}}{2}$

$x_{1,2} = \frac{5 \pm \sqrt{49}}{2}$

$x_{1,2} = \frac{5 \pm 7}{2}$

$x_1 = \frac{5+7}{2} \Rightarrow x_1 = \frac{12}{2} \Rightarrow x_1 = 6$

$x_2 = \frac{5-7}{2} \Rightarrow x_2 = \frac{-2}{2} \Rightarrow x_2 = -1$

$y = -2^2 - 4(-2) - 5$

$y = 4 + 8 - 5 \Rightarrow y = 7$

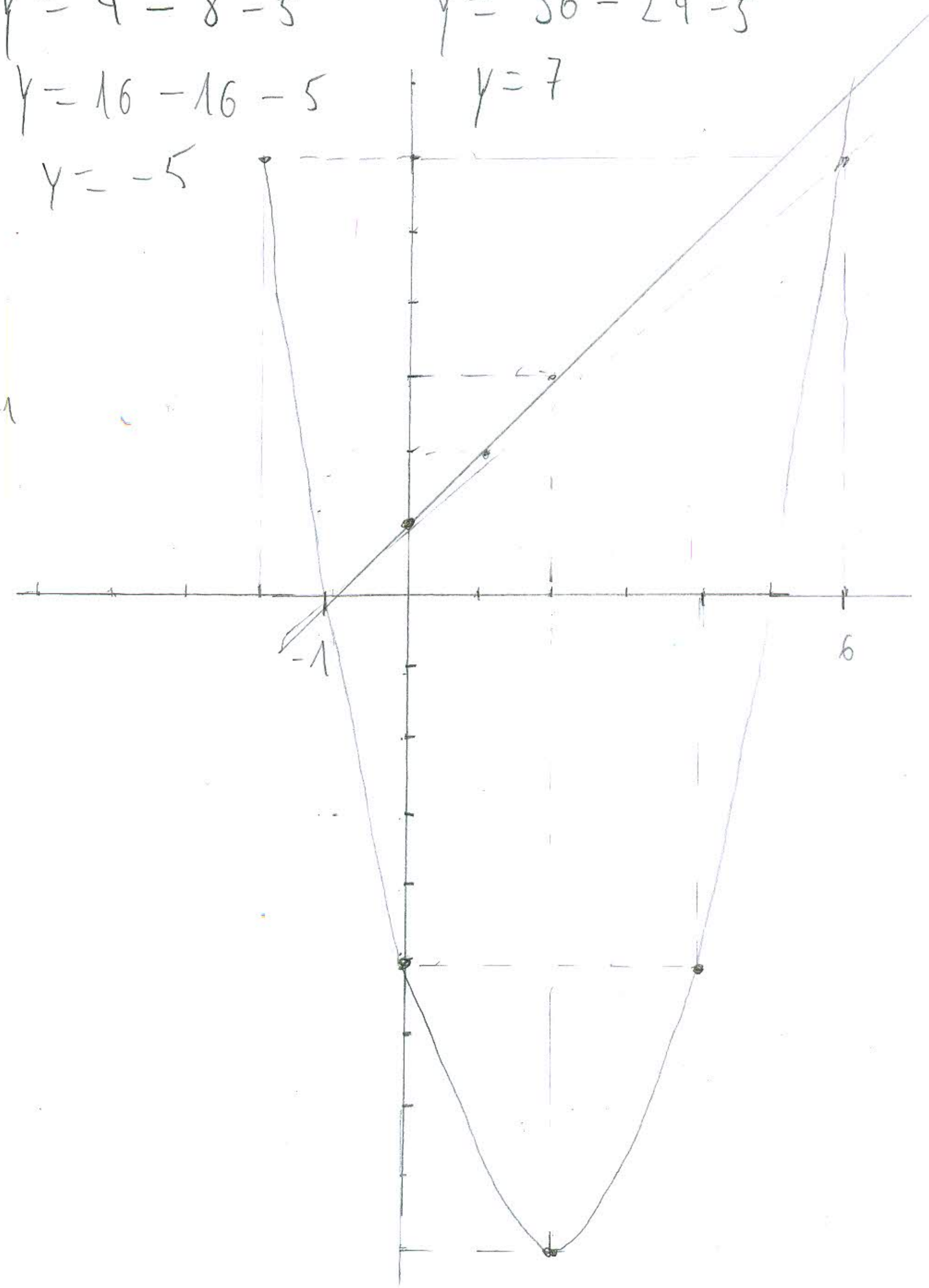
$y = 4 - 8 - 5$

$y = 16 - 16 - 5$

$y = -5$

$y = 36 - 24 - 5$

$y = 7$



$$P = \int_{-1}^{6} (x + 1 - x^2 + 4x + 5)\,dx \Rightarrow \int_{-1}^{6} (-x^2 + 5x + 6)\,dx \Rightarrow$$

$$\left[-\left[\frac{x^3}{3} \right]_{-1}^{6} + 5\left[\frac{x^2}{2} \right]_{-1}^{6} + 6\left[x \right]_{-1}^{6} \right] \Rightarrow$$

$$\Rightarrow \left[-\left[\frac{6^3}{3} - \frac{-1^3}{3} \right] + 5\left[\frac{6^2}{2} - \frac{-1^2}{2} \right] + 6\left[6+1 \right] \right] \Rightarrow$$

$$\Rightarrow -\left[\frac{216}{3} + \frac{1}{3} \right] + 5\left[\frac{36}{2} - \frac{1}{2} \right] + 6\left[7 \right] \Rightarrow$$

$$\Rightarrow -\left[\frac{217}{3} \right] + 5\left[\frac{35}{2} \right] + 42 \Rightarrow -\frac{217}{3} + \frac{175}{2} + 42 \Rightarrow$$

$$\Rightarrow \frac{-434 + 525 + 252}{6} \Rightarrow \frac{343}{6} \Rightarrow 57{,}16666667$$

$$4)\ \int_0^3 \frac{dx}{2-\sqrt{x}} = \left| \begin{array}{l} \sqrt{x} = t /^2 \\ x = t^2 dt \\ dx = 2t\,dt \end{array} \right| \Rightarrow \int_0^3 \frac{2t\,dt}{2-t^2} = -\int_0^3 \frac{2t\,dt}{2-t^2} = \left| \begin{array}{l} 2-t^2 = e \\ -2t\,dt = de /\cdot(-1) \\ 2t\,dt = -de \end{array} \right|$$

$$\Rightarrow -\int_0^3 \frac{de}{e} \Rightarrow -\left[\ln|e| + C\right]_0^3 \Rightarrow -\left[\ln|2-t^2| + C\right]_0^3 \Rightarrow$$

$$\Rightarrow -\left[\ln|2-\sqrt{x^2}| + C\right]_0^3 \Rightarrow -\left[\ln|2-\sqrt{3^2}| - \ln|2-\sqrt{0^2}|\right] \Rightarrow$$

$$\Rightarrow -\left[\ln|1| - \ln|2|\right] \Rightarrow 0,69314718 \quad \times \quad \emptyset$$

PROVJERA: NUMERIČKOM INTEGRACIJOM ~?

Josip Gavula

(5) $9y'' - 6y' + y = 0$

$9r^2 - 6r + 1 = 0$

$r_{1,2} = \frac{6 \pm \sqrt{36 - 36}}{18} \Rightarrow r_{1,2} = \frac{6 \pm \sqrt{0}}{18}$

$r = \frac{1}{3}$

$C_1^{x^2} - C_2^{x} = 0$

$x = 1 \quad y = 0 \quad y' = 0$

$2C_1 x - C_2^x = 0$

$9y'' - 6 \cdot 0 + 0 = 0$

$2C_1^x = C_2^x$

$9y'' = 0 \,/ : 9$

$2C_1^x = 0$

$y'' = 0$

$C_2^x = 0$

RJEŠENJE = ?

Ø

Josip Gavula

(3) (a) $\int_0^3 x^2 \ln x = \left| \begin{matrix} u = \ln x & dv = x^2 \\ du = \frac{1}{x} dx & v = \frac{x^3}{3} \end{matrix} \right|$

$\Rightarrow \left[\ln x \frac{x^3}{3} - \int \frac{x^{\cancel{3}2}}{3} \frac{1}{\cancel{x}} dx \right]_0^3 \Rightarrow \ln x \frac{x^3}{3} - \frac{1}{3} \int x^2 dx \Rightarrow$

$\Rightarrow \left[\ln x \frac{x^3}{3} - \frac{1}{3} \left[\frac{x^3}{3} \right] \right]_0^3 \Rightarrow \left[\ln 3 \frac{3^3}{3} - \underbrace{\ln 0 \frac{0^3}{3}}_{N/P} - \frac{1}{3} \left[\frac{3^3}{3} - \frac{0^3}{3} \right] \right] \Rightarrow$ ×

$\Rightarrow \ln 3 \cdot 9 + \infty - \frac{1}{3} [9] \Rightarrow \ln 27 + \infty - 3 \Rightarrow 3{,}29583 + \infty - 3 \Rightarrow +\infty$

N/P ×

k	0	1	2
x_k	0	1,5	3
f_k	$+\infty$		

$\Rightarrow \lim_{b \to 0} \left[\ln x \frac{x^3}{3} - \frac{1}{3} \left[\frac{x^3}{3} \right] \right]_b^3 - \lim_{a \to 0} \left[\ln x \frac{x^3}{3} - \frac{1}{3} \left[\frac{x^3}{3} \right] \right]_a^3 \Rightarrow$

$\lim_{b \to 0} \left[\ln 3 \frac{3^3}{3} - \ln b \frac{b^3}{3} - \frac{1}{3} \left[\frac{3^3}{3} - \frac{b^3}{3} \right] \right] - \lim_{a \to 0} \left[\ln 3 \frac{3^3}{3} - \ln a \frac{a^3}{3} - \frac{1}{3} \left[\frac{3^3}{3} - \frac{a^3}{3} \right] \right]$

$\lim_{b \to 0} \ln 27 + \infty - 3 = - \lim_{a \to 0} \ln 27 + \infty - 3 \Rightarrow +\infty - \infty \Rightarrow$ N/P $\emptyset$

$\lim_{x \to 0} \ln x \cdot x^3 = \lim_{x \to 0} \frac{\ln x}{\frac{1}{x^3}}$

$\overset{L'H}{=} \lim_{x \to 0} \frac{\frac{1}{x}}{\frac{-3}{x^4}}$

$= \lim_{x \to 0} -\frac{1}{3} \frac{x^4}{x}$

$= \lim_{x \to 0} -\frac{1}{3} x^3 = 0$

(b) $x y y' = 1 - x^2 \quad y(1) = 1$

$x r = 1 - x^2$

$\emptyset$

(2.) $f(x,y) = x^2 - y^2$ $\qquad x^2 - y^2 = \quad y^2 = x^2 \Rightarrow 0$ $\qquad Df: \mathbb{R}^2 \{0\}$ ×

$Kf \langle 0, +\infty \rangle$ ×

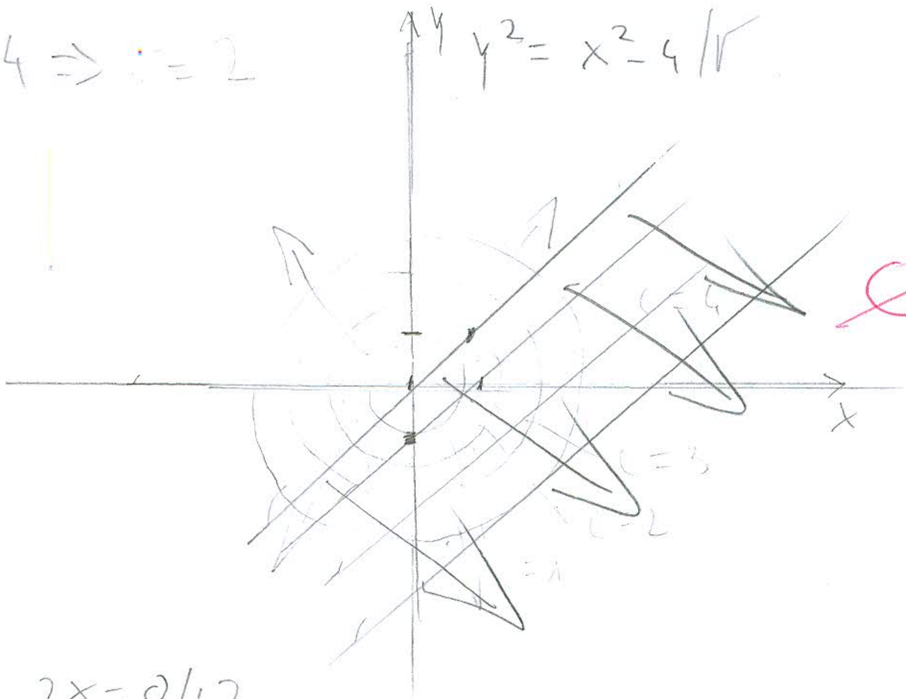
$c = 0 = x^2 - y^2 = 0 \Rightarrow r = 0 \Rightarrow y^2 = x^2 / \sqrt{\ } \Rightarrow y = x$ $\quad \begin{array}{cc} 0 & 1 \\ \hline 1 & 1 \end{array}$

$c = 1 \quad x^2 - y^2 = 1 \Rightarrow r = \quad y^2 = x^2 - 1 / \sqrt{\ } \quad y = x - 1$ $\quad \begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}$

$c = 2 \quad x^2 - y^2 = 2 \Rightarrow \quad 1{,}142 \quad y^2 = x^2 - 2 / \sqrt{\ }$

$c = 3 \quad x^2 - y^2 = 3 \Rightarrow r = 1{,}7321 \quad y^2 = x^2 - 3 / \sqrt{\ }$

$c = 4 \quad x^2 - y^2 = 4 \Rightarrow r = 2 \quad y^2 = x^2 - 4 / \sqrt{\ }$



∅

$\frac{\partial f}{\partial x} = 2x \Rightarrow 0 \qquad 2x = 0 / : 2$

$x = 0$

$T(0,0)$

$\frac{\partial f}{\partial y} = -2y \Rightarrow 0 \qquad -2y = 0 / : (-2)$

$y = 0$

$\frac{\partial^2 f}{\partial x^2} = 2 \qquad \frac{\partial^2 f}{\partial y^2} = -2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 0 \qquad \frac{\partial^2 f}{\partial y \partial x} = 0$

$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = -4 < 0$ $\quad$ NEMA EKSTREMA ✓

8

(4) $\int_0^3 \frac{dx}{2-\sqrt{x}}$

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k	0	1	2	3	4
x_k	0	0,75	1,5	2,25	3
f_k	$\frac{1}{2}$	0,8818	1,2898	2	3,7321

$S_2 = \frac{1,5}{6}(1,2898 + 4 \cdot 2 + 3,7321) \Rightarrow S_2 = 0,25(13,02194879) \Rightarrow$
$S_2 = 3,255487199$

$\Delta S = S_1 + S_2 \Rightarrow \Delta S = 1,3293 + 3,2554 \Rightarrow \Delta S = 4,584815657$

15

$\frac{dx}{2-\sqrt{0,75}} \Rightarrow \frac{1}{1,133974596} \Rightarrow 0,88185397$

$d = 1,5 - 0$
$d = 1,5$

$\frac{dx}{2-\sqrt{1,5}} \Rightarrow \frac{1}{0,775255128} \Rightarrow 1,28989795$

$d = 3 - 1,5$
$d = 1,5$

$\frac{dx}{2-\sqrt{2,25}} \Rightarrow \frac{1}{\frac{1}{2}} \Rightarrow 2$

$\frac{dx}{2-\sqrt{3}} = \frac{1}{0,267949192} \Rightarrow 3,732050814$

$S_1 = \frac{1,5}{6}\left(\frac{1}{2} + 4 \cdot 0,8818 + 1,2898\right) \Rightarrow S_1 = \frac{1,5}{6}\left(\frac{1}{2} + 3,52741388 + 1,2898\right)$

$S_1 = \frac{1,5}{6}(5,31731383) \Rightarrow S_1 = 0,25(5,31731383) \Rightarrow S_1 = 1,329328438$

4. $\int_0^3 \frac{dx}{2-\sqrt{x}} = \left| \begin{matrix} \cancel{2-}\sqrt{x} = t/^2 \\ x = t^2 \\ \cancel{x}dx = 2t\,dt \end{matrix} \right| \Rightarrow 2\int_0^3 \frac{t\,dt}{2-t^2} \Rightarrow 2\int_0^3 \frac{t-1+1}{2-t^2} =$

$\Rightarrow 2\int_0^3 \frac{t-1}{t+1} - \int \frac{1}{1+t} \Rightarrow 2\int_0^3 t - |\ln|t||\Big|_0^3 \Rightarrow$ ✗

$2\left[\frac{t^2}{2} - |\ln|t||\right]_0^3 \Rightarrow 2\left[\frac{\sqrt{3}^2}{2} - \frac{\sqrt{0}^2}{2} - |\ln|\sqrt{3}-0||\right] \Rightarrow$

$\Rightarrow 2\left[\frac{3}{2} - 0{,}5494\right] \Rightarrow 2\left[0{,}9506\right] \Rightarrow 1{,}9012$ ✗

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ D	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: TONI GRBIĆ VRIJEME POČETKA:	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0288-2014 prof. Uglešić	

1. Izračunati površinu lika omeđenog parabolom $y = x^2 - 4x - 5$ i pravcem $y = x + 1$. — $\emptyset$
2. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = x^2 - y^2$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme? $\emptyset$
3. Pronaći:
   (a) koliko iznosi $\int_0^3 x^2 \ln x \, dx$, — $\emptyset$
   (b) partikularno rješenje diferencijalne jednadžbe $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. Na kraju provjeri rješenje. — $\emptyset$
4. Izračunati $\int\limits_0^3 \frac{dx}{2-\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 8% ili manje. $\emptyset$
5. Nađi partikularno rješenje jednadžbe $9y'' - 6y' + y = 0$ uz uvjet $x = 1$, $y = 0$ i $y' = 0$.

Ukupno: $\emptyset$

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

TONI GRBIĆ

(4) $\int_0^3 \frac{dx}{2-\sqrt{x}} =$

n	0	1	2	3	4	5	6
x_n	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
f_n	$\frac{1}{2}$	0,773	1	1,289	1,707	2,387	3,732

$S_1 = \frac{3}{6}\left(\frac{1}{2} + 4(0,773) + 1\right) = 2,296$

VIDI GAUTA

$S_2 = \frac{3}{6}\left(1 + 4(1,289) + 1,707\right) = 3,9315$

$S_3 = \frac{3}{6}\left(1,707 + 4(2,387) + 3,732\right) = 7,4935$

$S_{uk} = 13,721$

0

(5) $9y'' - 6y' + y = 0$ $\qquad x = 1$; $y' = 0$

$y = 0$

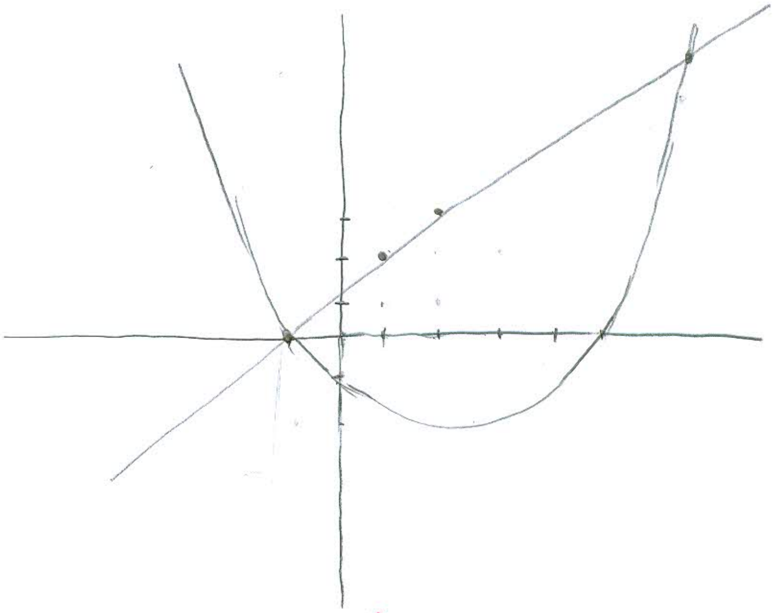
$y = C_1 \cdot e^{rx}$

$\emptyset$

TOMI GRBIC

1) $y = x^2 - 4x - 5$

$y = x + 1$



$x^2 - 4x - 5 = 0$

$x_{1,2} = \frac{4 \pm \sqrt{16 - 4 \cdot (-5)}}{2} = \begin{matrix} x_1 = 5 \\ x_2 = -1 \end{matrix}$

$x^2 - 4x - 5 = x + 1$

$x^2 - 4x - 5 - x - 1 = 0$

$x^2 - 5x - 6 = 0$

$x_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot (-6)}}{2} = \begin{matrix} x_1 = 6 \\ x_2 = -1 \end{matrix}$

$P = \int_{-1}^{6} (x+1) - (x^2 - 4x - 5)\, dx$

$$\int_{-1}^{6} x + 1 - x^2 - 4x - 5 \, dx = \frac{x^2}{2} + x - \frac{x^3}{3} - 4\frac{x^2}{2} - 5x \Big|_{-1}^{6} =$$

×

$= 154{,}16$ ×

0

② $f(x,y)=x^2-y^2$

$Df(x): \mathbb{R}^2$

$Df(y): \mathbb{R}^2$

?

③a) $\int_0^3 x^2 \ln x\,dx = \begin{bmatrix} u=\ln x & dv=x^2 \\ du=\frac{1}{x}dx & v=\frac{x^3}{3} \end{bmatrix}$

$u\cdot v-\int v\cdot du$

$\ln x\cdot\frac{x^3}{3}-\int\frac{x^3}{3}\cdot\frac{1}{x}dx$

$\ln x\cdot\frac{x^3}{3}-\int\frac{x^3}{3x}dx \begin{cases} x^3=t \\ dx=3x^2dt \Rightarrow dt= \end{cases}$

$\emptyset$

b) $xyy'=1-x^2 /:x \quad , y(1)=1$

$yy'=\frac{1}{x}-\frac{x^2}{x}$

$y\frac{dy}{dx}=\frac{1}{x}-\frac{x^2}{x} /\cdot dx$

$\int y\,dy=\int\frac{1}{x}dx-\int x\,dx$ ✓

$\frac{y^2}{2}=\ln|x|-\frac{x^2}{2}+C /\cdot 2$

$y^2=2\ln x-x^2+2C /\sqrt{\ }$

$y=\sqrt{2\ln|x|-x^2}+2C$ ×

$1=\sqrt{2\ln|1|-1^2}+2C$

$2C=\sqrt{2\ln|1|-1^2}-1$

$\boxed{C=0}$ ×

$\emptyset$