

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **FILIP GORŠEK**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **17 - 2 - 0369 - 2014**

PROF. NIKICA UGLEŠIĆ

- Izračunati površinu lika omeđenog parabolom $y = x^2 - 4x - 5$ i pravcem $y = x + 1$.
- Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = x^2 - y^2$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme?
- Pronaći:
 - koliko iznosi $\int_0^3 x^2 \ln x \, dx$,
 - partikularno rješenje diferencijalne jednačbe $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. Na kraju provjeri rješenje.
- Izračunati $\int_0^3 \frac{dx}{2-\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 8% ili manje.
- Nađi partikularno rješenje jednačbe $9y'' - 6y' + y = 0$ uz uvjet $x = 1, y = 0$ i $y' = 0$.

20

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0

Ukupno:

65

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

① $y_1 = x^2 - 4x - 5$

$y_2 = x + 1$

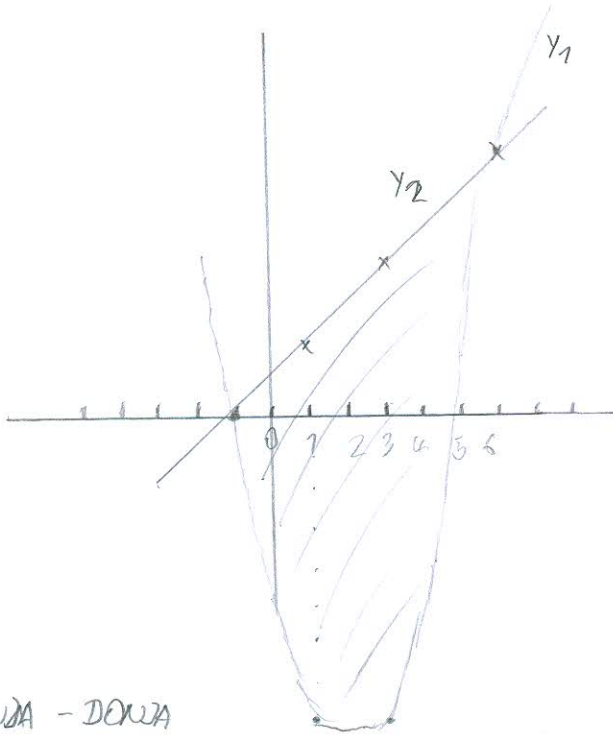
$x^2 - 4x - 5 = x + 1$

$x^2 - 5x - 6 = 0$

$x_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot (-6)}}{2}$

$x_1 = 6$

$x_2 = -1$



x	-1	1	3	6
y ₁	0	-8	-8	7

x	-1	1	3	6
y ₂	0	2	4	7

GORNJA - DOLNA

$$P = \int_{-1}^6 (x+1 - x^2 + 4x + 5) dx = \left. \frac{5x^2}{2} - \frac{x^3}{3} + 6x \right|_{-1}^6 = 54 + \frac{19}{6} = \frac{343}{6}$$



$$\int (5x - x^2 + 6) dx = \int 5x dx - \int x^2 dx + 6 \int dx = \frac{5x^2}{2} - \frac{x^3}{3} + 6x + C$$

$$\textcircled{3.} \text{ a) } \int_0^3 x^2 \ln x \, dx = \left[\begin{array}{l} u = \ln x \quad du = \frac{1}{x} dx \\ dv = x^2 dx \quad v = \frac{x^3}{3} \end{array} \right] = 6.88751 = 6.88751 \checkmark$$

$$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \frac{dx}{x}$$

$$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 \frac{dx}{x}$$

$$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 dx =$$

$$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \frac{x^3}{3} + C$$

$$\text{b) } x y y' = 1 - x^2 \quad / : x$$

$$y(1) = 1$$

$$2 \ln(1) - 1^2 + 2C = 1$$

$$y y' = \frac{1-x^2}{x}$$

$$-1 + 2C = 1$$

$$y \frac{dy}{dx} = \frac{1-x^2}{x} \cdot dx$$

$$2C = 2 \quad / : 2$$

$$\boxed{C = 1} \checkmark$$

$$\int y dy = \int \frac{1-x^2}{x} dx$$

$$\frac{y^2}{2} = \int \frac{1}{x} dx - \int x dx$$

$$\frac{y^2}{2} = \ln x - \frac{x^2}{2} + C \quad / \cdot 2$$

$$y^2 = 2 \ln x - x^2 + 2C \quad / \sqrt{\quad} \checkmark$$

$$y = \sqrt{2 \ln x - x^2 + 2C}$$

$$\boxed{y_p = \sqrt{2 \ln x - x^2 + 2}}$$

$$y' = \frac{1}{2} (2 \ln x - x^2 + 2) \cdot \left(\frac{2}{x} - 2x \right) \quad / \cdot 2$$

$$y' = (2 \ln x - x^2 + 2) \cdot \left(\frac{2}{x} - 2x \right)$$

4. $\int_0^3 \frac{dx}{2-\sqrt{x}}$

0 0.75 1.5 3

0	0.75	1.5
0.5	0.887854	1.289898

$$S_1 = \frac{1.5}{6} (0.5 + 4 \cdot 0.887854 + 1.289898) = 1.3293205$$

1.5	2.26	3
1.289898	2	3.73205

$$S_2 = \frac{1.5}{6} (1.289898 + 4 \cdot 2 + 3.73205) = 3.255487$$

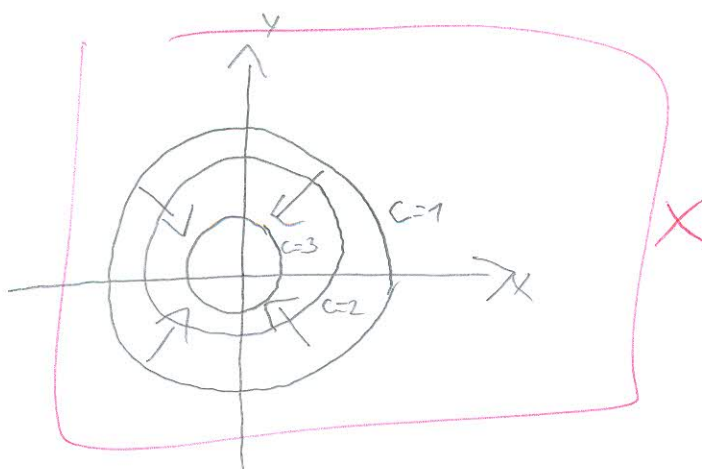
$$S_1 + S_2 = 4.5848065$$

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② $f(x,y) = x^2 - y^2$

$D(f) = \mathbb{R}^2$ ✓

$K(f) = \langle 0, +\infty \rangle$ ✗



$C=1, \quad x^2 - y^2 = 1$
 $-y^2 = -x^2 + 1 \quad | \cdot (-1)$
 $y^2 = x^2 - 1$

$C=2 \quad x^2 - y^2 = 2$
 $y^2 = x^2 - 2$

$C=3 \quad x^2 - y^2 = 3$
 $y^2 = x^2 - 3$

EKSTREMI $f(x,y) = x^2 - y^2$

$T(0,0)$

$\frac{\partial f}{\partial x} = 2x \rightarrow x=0$

$\frac{\partial f}{\partial y} = -2y \rightarrow y=0$

$A \cdot C - (B)^2 = 2 \cdot (-2) - 0 = -4 < 0$

A) $\frac{\partial^2 f}{\partial x^2} = 2$

B) $\frac{\partial^2 f}{\partial x \partial y} = 0$

$T(0,0)$ JE SEDLÁSTÁ TOČKA
 NEMÁ EKSTREMA ✓

C) $\frac{\partial^2 f}{\partial y^2} = -2$

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(5.) $9y'' - 6y' + y = 0$

$y(x=1) = 0$

$y'(x=1) = 0$

FILIP GORSEK

$9r^2 - 6r + 1 = 0$

$r_{1,2} = \frac{6 \pm \sqrt{36 - 4 \cdot 9 \cdot 1}}{18} =$

$r_1 = r_2 = \frac{1}{3}$

$y_H = e^{rx} (C_1 + C_2 x)$

$r = \frac{1}{3} \checkmark$

$y_H = e^{1/3 x} (C_1 + C_2 x)$

$A = 0$

$y_H = e^{1/3} (C_1 + C_2 x) = 0 \quad | : e^{1/3}$

$C_1 + C_2 x = 0$

$C_1 = -C_2 x$

$y'_H = \frac{1}{3} e^{1/3 x} \cdot (C_1 + C_2 x) + e^{1/3 x} \cdot 0$

$y'_H = \frac{1}{3} e^{1/3 x} \cdot (C_1 + C_2 x) \rightarrow \frac{1}{3} e^{1/3 \cdot 1} \cdot (C_1 + C_2 x)$

$\frac{1}{3} e^{1/3} \cdot (C_1 + C_2 x) = 0 \quad | \cdot 3$

$e^{1/3} \cdot (C_1 + C_2 x) = 0$

$e^{1/3} C_1 + e^{1/3} C_2 x$

$C_2 x = -C_1$

DA JE --

