

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: ANTONIO BEGIĆ

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

PROFESOR UGLJEŠIĆ

1. Grafički prikazati funkciju $f(x, y) = \ln(y - x)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije.

16
φ

2. Odrediti $\int_0^{\pi} \left(-\frac{1}{4} \sin^2 t + \frac{\sqrt{3}}{4} \cos t \right) dt$.

3. Pronaći koliko iznosi:

(a) površina između krivulja $y^2 = 2x + 5$ i $py = -\sqrt{3}x$,

(b) integral $\int_{-1}^{+\infty} \frac{dx}{1+x^2}$?

10
φ

4. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = e^x$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

5. Izračunati $\int_0^4 \frac{dx}{\sqrt{x^2 + 2x + 2}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje.

15

Ukupno:

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f	$\frac{df}{dx}$
x^α ($\alpha \neq 0$)	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ ($\alpha > 0$)	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x ($\alpha > 0$)	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

$\lim_{x \rightarrow +\infty} \arctan x = \frac{\pi}{2}$

① $f(x,y) = \ln|y-x|$

$D = \mathbb{R}^2 \setminus \{y-x > 0\}$
 GDJE JE OVO RAUMINI???

Kodomena $\mathbb{R} \checkmark$ (BEGIĆ ANTONIO)

$C=1 \quad \ln|y-x|=1 \quad / \exp$

$y-x = e^1$
 $\rightarrow y = e^1 + x$

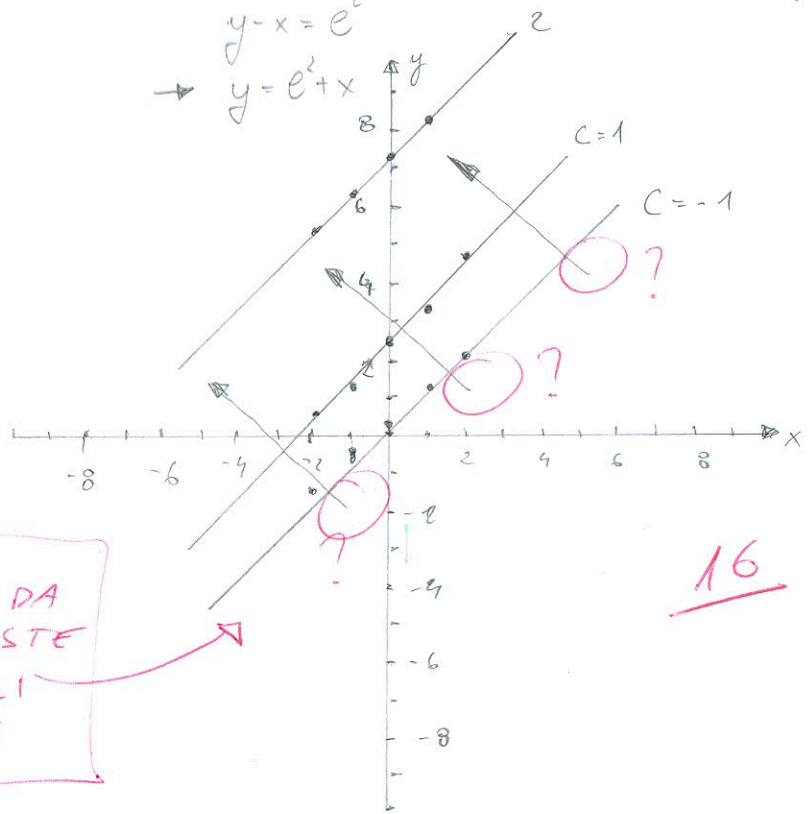
$C=2 \quad \ln|y-x|=2 \quad / \exp$

$y-x = e^2$
 $\rightarrow y = e^2 + x$

\rightarrow od $C(-1)$ to $C(2)$

$C=-1 \quad \ln|y-x|=-1 \quad / \exp$

$y-x = e^{-1}$
 $y = e^{-1} + x$



ČINI SE DA DOMENU MISTE PREPOZNALI U RAVNINI

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③ a)

$y^2 = 2x + 5$ zamjena varijabli

$y = -\sqrt{3}x$

$y^2 - 2x = 0$

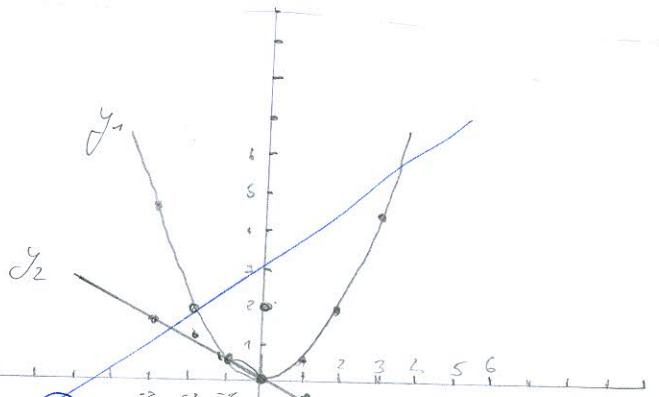
$x = -\sqrt{3}y$

$x^2 - 2y = 0$

$y = \frac{x}{-\sqrt{3}}$

$-2y = x^2 \quad / : -2$

$y_1 = \frac{x^2}{2}$



~~KRIVO PREPISAO~~

~~ZADATAK~~

$\frac{x^2}{2} + \frac{x}{\sqrt{3}} = 0 \quad / \cdot 2$

$x^2 + \frac{2x}{\sqrt{3}} = 0 \quad / \cdot \sqrt{3}$

$\sqrt{3}x^2 + 2x = 0$

$a = \sqrt{3} \quad b = 2 \quad c = 0$

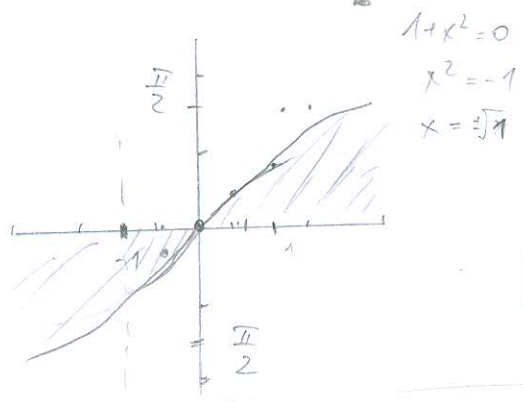
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 0}}{2\sqrt{3}} = -1,1547$

$-\frac{x^2}{2\sqrt{3}} - \frac{x^3}{6} = 0 + 0,1283 \rightarrow 0,1283$

$x_1 = -1,1547$

$x_2 = 0$

(6) $\int_{-1}^{+\infty} \frac{dx}{1+x^2} = \arctg x \Big|_{-1}^{+\infty} \rightarrow \lim_{b \rightarrow +\infty} (\arctg b - \arctg -1) = +\infty - \frac{1}{4}\pi \neq X$



$1+x^2=0$
 $x^2=-1$
 $x=\pm i$

$\int_{-1}^0 \frac{dx}{1+x^2} = \frac{\pi}{4} \Rightarrow \int_0^{+\infty} \frac{dx}{1+x^2} = +\infty$

Integral se može djelomično riješiti
 jedan dio je $\frac{\pi}{4}$ a drugi konvergira u $+\infty$

(5) $\int_0^4 \frac{dx}{\sqrt{x^2+2x+2}} = \int_0^4 \frac{dx}{\sqrt{x^2+2x+1+1}} = \int_0^4 \frac{dx}{\sqrt{(x+1)^2+1^2}} = \int_0^4 \frac{dx}{x+1+1} = \int_0^4 \frac{dx}{x+2} = \left| \begin{matrix} x+2=t \\ d+ = dt \end{matrix} \right|$

$\int_2^6 \frac{dt}{t} = \ln(t) \Big|_2^6 \rightarrow \ln|6-2| \rightarrow \boxed{1,386}$

NUMERICKI
 $d = b - a = 4 - 0 = 4$

0	2	4
1	1	1
0,707	0,316	0,196

$S \rightarrow \frac{4}{6} (f_0 + 4f_1 + f_2) \rightarrow \frac{4}{6} (0,707 + 4(0,316) + 0,196) \rightarrow \boxed{1,45}$ ✓

(3) $\int_0^{\pi} (-\frac{1}{4} \sin^2 t + \frac{\sqrt{3}}{4} \cos t) dt = \int_0^{\pi} (-\frac{1}{4} \sin^2 x + \frac{\sqrt{3}}{4} \cos x) dx = \left| \begin{matrix} \sin x = t \\ \cos dx = dt \\ \pi \rightarrow 0 \\ 0 \rightarrow \pi \end{matrix} \right| = 0$ nema rezultata

NE MOGU SE SKONCENTRIRATI S SLOVOM t!

$\int_0^{\pi} -\frac{1}{4} t^2 + \frac{\sqrt{3}}{4} dt = -\frac{1}{4} \cdot \frac{t^3}{3} + \frac{\sqrt{3}}{4} x \Big|_0^{\pi} = -\frac{t^3}{12} + \frac{\sqrt{3}}{4} x = -\frac{\sin^3 x}{12} + \frac{\sqrt{3}}{4} x$

KADA BI BIO NEODREĐEN ALI MIJE PA MU JE REZULTAT VEĆ NA SUPSTITUCIJI JEDNAK 0

BEARÉ ANTONIO

a) $y^2 = 2x + 5$ $y = -\sqrt{3}x$
 $y \leftrightarrow x$

$$x^2 = 2y + 5$$

$$x = -\sqrt{3}y$$

$$2y + 5 - x^2 = 0$$

$$2y = x^2 - 5 \quad | :2$$

$$y_1 = \frac{x^2 - 5}{2}$$

$$\frac{x^2 - 5}{2} = -\frac{x}{\sqrt{3}}$$

$$\frac{x^2 - 5}{2} + \frac{x}{\sqrt{3}} = 0 \quad | \cdot 2 \cdot \sqrt{3}$$

$$\sqrt{3}(x^2 - 5) + 2x = 0$$

$$\sqrt{3}x^2 - 5\sqrt{3} + 2x = 0$$

$$a = \sqrt{3} \quad b = 2 \quad c = -5\sqrt{3}$$

$$x_{1,2} = -2,887 \quad \checkmark$$

$$= \sqrt{3} \quad \checkmark$$

$y_2 - y_1$

$$\int_{-2,887}^{\sqrt{3}} -\frac{x}{\sqrt{3}} - \left(\frac{x^2 - 5}{2} \right) = \int_{-2,887}^{\sqrt{3}} -\frac{x}{\sqrt{3}} - \frac{x^2}{2} + \frac{5}{2} = \int_{-2,887}^{\sqrt{3}} -x \cdot \frac{1}{\sqrt{3}} - x^2 \cdot \frac{1}{2} + \frac{5}{2}$$

$$\left. -\frac{x^2}{2} \cdot \frac{1}{\sqrt{3}} - \frac{x^3}{3} \cdot \frac{1}{2} + \frac{5}{2} x \right|_{-2,887}^{\sqrt{3}} \rightarrow -\frac{x^2}{2\sqrt{3}} - \frac{x^3}{6} + \frac{5}{2}x \Big|_{-2,887}^{\sqrt{3}} \rightarrow 2,598 + 5,613 = \boxed{8,211} \quad \checkmark$$

$$x_{1,2} = \frac{-2 \pm \sqrt{64}}{2\sqrt{3}}$$

$$x_1 = \frac{-10}{2\sqrt{3}} = -\frac{5}{\sqrt{3}}$$

$$x_2 = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

