MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C

IME I PREZIME: ANTONIO BEGIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

PROFESOR UGLJEŠIĆ

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Grafički prikazati funkciju $f(x,y) = \ln(y-x)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. 16

2. Odrediti $\int\limits_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) dt$. Ø

3. Pronaći koliko iznosi:

   (a) površina između krivulja $y^2 = 2x + 5$ i $py = -\sqrt{3}x$, 10

   (b) integral $\int\limits_{-1}^{+\infty} \frac{dx}{1+x^2}$? Ø

4. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = e^x$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

5. Izračunati $\int\limits_0^4 \frac{dx}{\sqrt{x^2+2x+2}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. ~~X~~ 15

Ukupno: 41

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

$$\boxed{\lim_{x\to+\infty} \arctan x = \frac{\pi}{2}}$$

(1.) $f(x,y)=\ln|y-x|$ $\quad D=\mathbb{R}^2 \setminus \{y-x>0\}$ $\quad$ Kodomena $\mathbb{R}$ ✓ $\quad$ BEGIĆ ANTONIO

GDJE JE OVO U RAVNINI ???

$C=1 \qquad \ln|y-x|=1 \ /\exp$

$y-x=e^1$

$\rightarrow y=e^1+x$

$C=2 \qquad \ln|y-x|=2 \ /\exp$

$y-x=e^2$

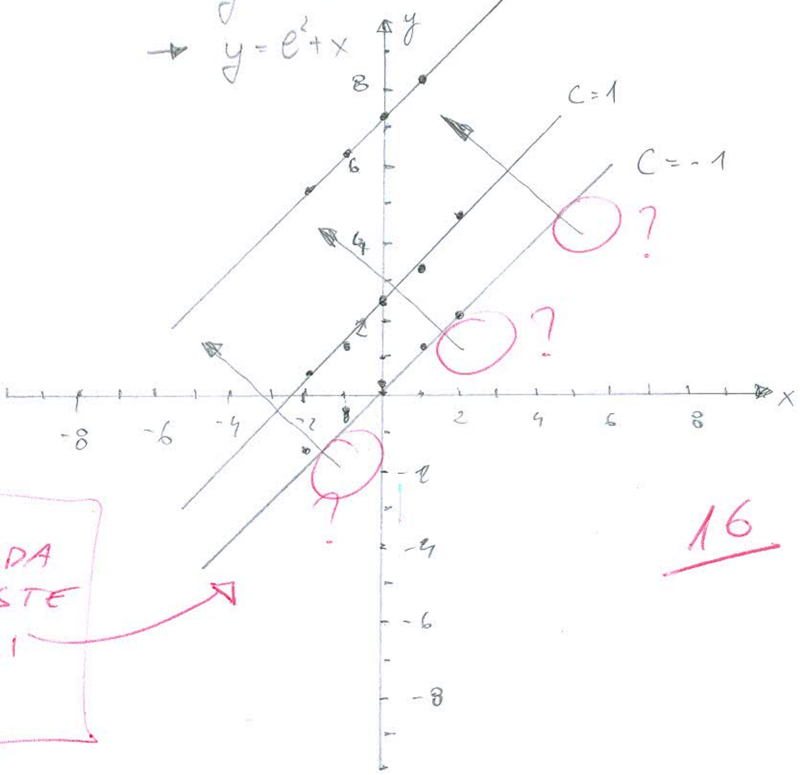
$\rightarrow y=e^2+x$

$\rightarrow$ Od $C(-1)$ do $C(2)$

$C=-1 \qquad \ln|y-x|=-1 \ /\exp$

$y-x=e^{-1}$

$y=e^{-1}+x$



ČINI SE DA DOMENU NISTE PREPOZNALI U RAVNINI

16

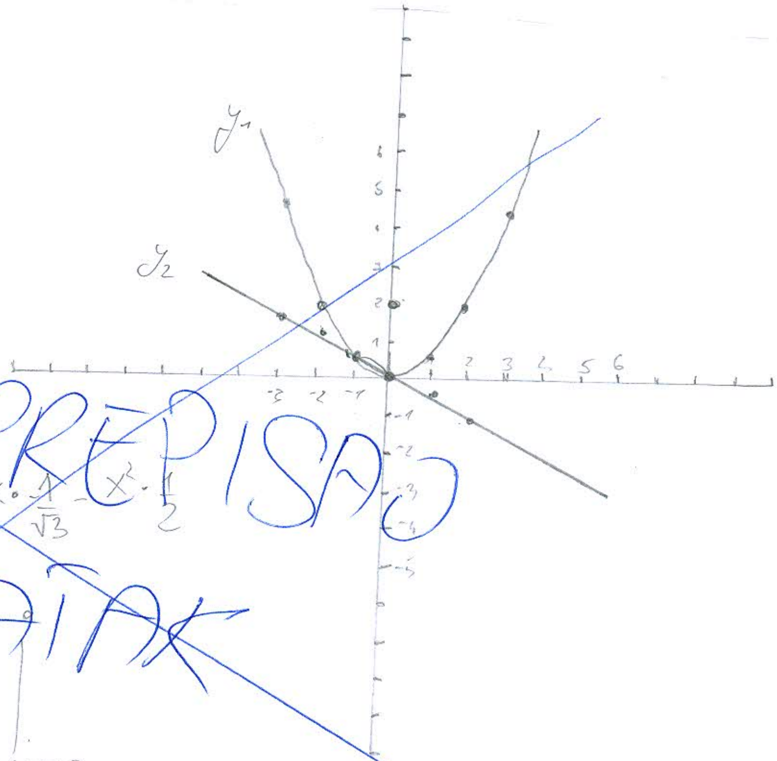
(3.) (a.) $y^2=2x+5$ zamjena varijabli $\qquad y=-\sqrt{3}x$

$y^2-2x=0 \qquad x=-\sqrt{3}\cdot y$

$x^2-2y=0 \qquad y_2=\frac{x}{-\sqrt{3}}$

$-2y=-x^2 \ /:-2$

$y_1=\frac{x^2}{2}$



KRIVO PREPISAO ZADATAK

$\frac{x^2}{2}=-\frac{x}{\sqrt{3}} \qquad \int_{-1,1547}^{0}\left(\frac{x}{-\sqrt{3}}-\frac{x^2}{2}\right)=\int_{-1,154}\left(x\cdot\frac{1}{\sqrt{3}}-x^2\cdot\frac{1}{2}\right)$

$\frac{x^2}{2}+\frac{x}{\sqrt{3}}=0 \ /\cdot 2$

$\frac{x^2}{2}\cdot\frac{1}{\sqrt{3}}-\frac{x^3}{3}\cdot\frac{1}{2}\Big|_{-1,1547}^{0}$

$x^2+2\frac{x}{\sqrt{3}}=0 \ /\cdot\sqrt{3}$

$x^2\sqrt{3}+2x=0$

$a=\sqrt{3} \quad b=2 \quad c=0 \qquad -\frac{x^2}{2\sqrt{3}}-\frac{x^3}{6}\Big|_{-1,1547}^{0}=0+0,1283 \rightarrow 0,1283$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}=-1,1547$

0

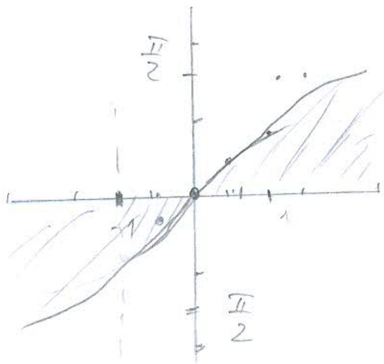
$x_1=-1,1547$

$x_2=0$

(5.) $\int_{-1}^{+\infty} \frac{dx}{1+x^2} = \operatorname{arctg} x \Big|_{-1}^{+\infty} \rightarrow \lim_{b \to +\infty} \left(\operatorname{arctg} x - \operatorname{arctg} -1 \right) = +\infty - \frac{1}{4}\pi$ X

X

$1+x^2=0$
$x^2=-1$
$x=\sqrt{-1}$



$\int_{-1}^{0} \frac{dx}{1+x^2} = \frac{\pi}{4}$ a $\int_{0}^{+\infty} \frac{dx}{1+x^2} = +\infty$

$\varnothing$

Integral se može djelomično rješiti jedan dio je $\frac{\pi}{4}$ a drugi konvergira u $+\infty$

$\varnothing$

(5) $\int_{0}^{4} \frac{dx}{\sqrt{x^2+2x+2}} = \int_{0}^{4} \frac{dx}{\sqrt{x^2+2x+1+1}} = \int_{0}^{4} \frac{dx}{\sqrt{(x+1)^2+1^2}} = \int_{0}^{4} \frac{dx}{x+1+1} = \int_{0}^{4} \frac{dx}{x+2} =$ $\left| \begin{array}{l} x+2=t \\ dx=dt \end{array} \right|$

$1 \quad 2 \quad 2$

$\cancel{x+2}$

$\int_{2}^{6} \frac{dt}{t} = \ln(t) \Big|_{2}^{6} \rightarrow \ln|6-2| \rightarrow \boxed{1{,}386}$

$\varnothing$

NUMERIČKI

$\begin{array}{ccc} 0 & 2 & 4 \\ | & | & | \\ 0{,}707 & 0{,}316 & 0{,}196 \end{array}$

$\rightarrow \quad d = b-a = 4-0 = 4$

$S \rightarrow \frac{4}{6}\left(f_0 + 4(f_1) + f_2\right) \rightarrow \frac{4}{6}\left(0{,}707 + 4(0{,}316) + 0{,}196\right) \rightarrow \boxed{1{,}45}$ ✓

1,5

(3.) $\int_{0}^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) dt = \int_{0}^{\pi} \left(-\frac{1}{4}\sin^2 x + \frac{\sqrt{3}}{4}\cos x\right) dx =$ $\left| \begin{array}{l} \sin x = t \\ \cos dx = dt \\ b = \pi \rightarrow 0 \\ a = 0 \rightarrow 0 \end{array} \right|$ $= 0$ nema Rezultata

$\varnothing$

$t \leftrightarrow x$

NE MOGU SE SKONCENTRIRAT S SLOVOM t!

$\int_{0}^{0} -\frac{1}{4}t^2 + \frac{\sqrt{3}}{4} dt = -\frac{1}{4} \cdot \frac{t^3}{3} + \frac{\sqrt{3}}{4}x \Big|_{0}^{0} = -\frac{t^3}{12} + \frac{\sqrt{3}}{4}x = -\frac{\sin^3 x}{12} + \frac{\sqrt{3}}{4}x$

X

KADA BI BIO NEODREĐEN ALI NIJE PA MU JE REZULTAT VEĆ NA SUPSTITUCIJI JEDNAK 0!

X

BEGIĆ ANTONIO

ⓐ $y^2 = 2x + 5$ $\quad y = -\sqrt{3}\,x$

$y \leftrightarrow x$

$x^2 = 2y + 5$ $\quad x = -\sqrt{3} \cdot y$

$2y + 5 - x^2 = 0$ $\quad y_2 = -\frac{x}{\sqrt{3}}$

$2y = x^2 - 5 \quad /:2$

$y_1 = \frac{x^2-5}{2}$

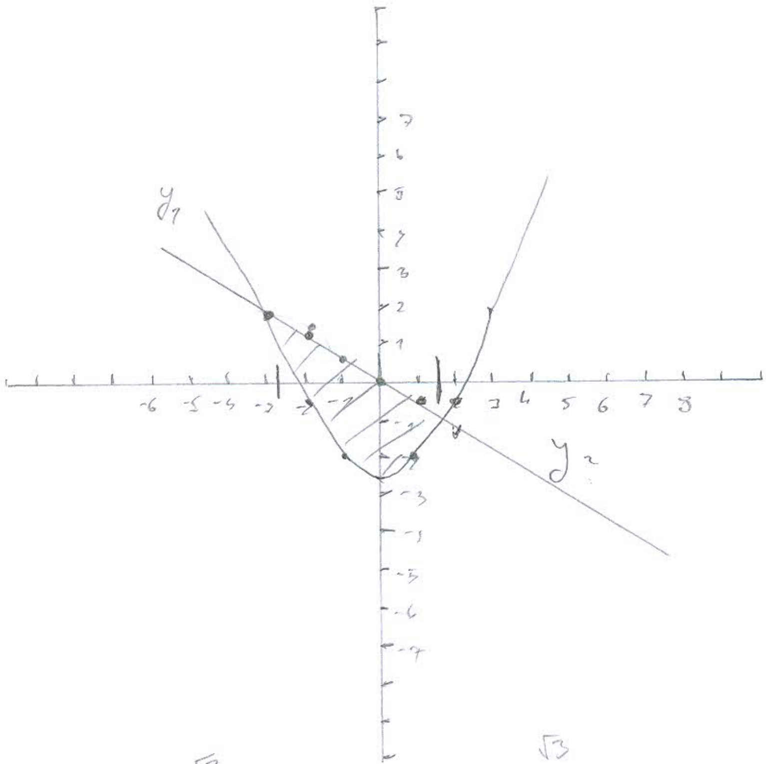
$x_{1,2} = \frac{-2 \pm \sqrt{64}}{2\sqrt{3}}$

$x_1 = \frac{-10}{2\sqrt{3}} = \frac{-5}{\sqrt{3}}$

$x_2 = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\frac{x^2-5}{2} = -\frac{x}{\sqrt{3}}$

$\frac{x^2-5}{2} + \frac{x}{\sqrt{3}} = 0 \quad / \cdot 2 \cdot \sqrt{3}$



$\sqrt{3}(x^2-5) + 2x = 0$

$\sqrt{3}x^2 - 5\sqrt{3} + 2x = 0$

$a = \sqrt{3} \quad b = 2 \quad c = -5\sqrt{3}$

$x_{1,2} = -2{,}887$ ✓

$= \sqrt{3}$ ✓

$y_2 - y_1$

$\int_{-2{,}887}^{\sqrt{3}} -\frac{x}{\sqrt{3}} - \left(\frac{x^2-5}{2}\right) = \int_{-2{,}887}^{\sqrt{3}} -\frac{x}{\sqrt{3}} - \frac{x^2}{2} + \frac{5}{2} = \int_{-2{,}887}^{\sqrt{3}} -x \cdot \frac{1}{\sqrt{3}} - x^2 \cdot \frac{1}{2} + \frac{5}{2}$

$-\frac{x^2}{2} \cdot \frac{1}{\sqrt{3}} - \frac{x^3}{3} \cdot \frac{1}{2} + \frac{5}{2}x \Big|_{2{,}887}^{\sqrt{3}} \rightarrow -\frac{x^2}{2\sqrt{3}} - \frac{x^3}{6} + \frac{5}{2}x \Big|_{-2{,}887}^{\sqrt{3}} \rightarrow 2{,}598 + 5{,}613 = \boxed{8{,}211}$ ✓

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MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C	POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: LUKA LUKAČIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0365-2014

prof. Uglešić

1. Grafički prikazati funkciju $f(x,y) = \ln(y-x)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. ~~10~~ Ø

2. Odrediti $\int\limits_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) \mathrm{d}t.$ 20

3. Pronaći koliko iznosi:

   (a) površina između krivulja $y^2 = 2x+5$ i $y = -\sqrt{3}x$, Ø

   (b) integral $\int\limits_{-1}^{+\infty} \frac{\mathrm{d}x}{1+x^2}$? Ø

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5. Izračunati $\int\limits_0^4 \frac{\mathrm{d}x}{\sqrt{x^2+2x+2}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. 15

Ukupno: 35

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

LUKA LUKAČIĆ

1.

$f(x,y) = \ln(y-x)$

kodomena: $\langle 0, +\infty \rangle$

domena: $\mathbb{R}^2$

LUKA LUKAČIĆ

②
$$\underbrace{-\frac{1}{4}\int_0^{\pi}\sin^2 t\,dt}_{\underline{1.}} + \underbrace{\frac{\sqrt{3}}{4}\int_0^{\pi}\cos t\,dt}_{\underline{2.}}$$

20

$\underline{1.}$
$$-\frac{1}{4}\int_0^{\pi}\frac{1-\cos(2t)}{2}\,dt = -\frac{1}{4}\int_0^{\pi}\left(\frac{1}{2} - \frac{\cos(2t)}{2}\right)dt$$

$$= -\frac{1}{4}\left(\frac{1}{2}\int_0^{\pi}dt - \frac{1}{2}\int_0^{\pi}\cos(2t)\,dt\right)$$

$$= -\frac{1}{4}\left(\frac{1}{2}x\Big|_0^{\pi} - \frac{1}{4}\sin(2t)\Big|_0^{\pi}\right)$$

$$= -\frac{1}{4}\left(\frac{\pi}{2} - 0\right) = \cancel{\ldots} -\frac{\pi}{8}$$

$\underline{2.}$
$$\frac{\sqrt{3}}{4}\sin t\Big|_0^{\pi} = 0$$

$$\int_0^{\pi}\left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right)dt = -\frac{\pi}{8}$$

ŽUKA ŽUKAČIĆ

3. a) $y^2 = 2x + 5 \qquad y = -\sqrt{3}x$

$$\sqrt{2x+5} = -\sqrt{3}x \quad /^2$$

$$2x + 5 = 3x^2$$

$$3x^2 - 2x - 5 = 0$$

$$x_{1,2} = \frac{+2 \pm \sqrt{4 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3}$$

$$= \frac{2 \pm \sqrt{64}}{6}$$

$$= \frac{2 \pm 8}{6}$$

$$\boxed{x_1 = \frac{5}{3}} \qquad \boxed{x_2 = -1}$$

$$2\left(\left. -\sqrt{3}x \right|_{-1}^{5/3} + \left. \sqrt{2x+5} \right|^{5/3} \right)$$

$$= 2\left[\left(-\frac{5\sqrt{3}}{3} - \sqrt{3} \right) + \left(\frac{5\sqrt{3}}{3} - \sqrt{3} \right) \right]^{-1}$$

$$= 6{,}928$$

LUKA LUKAČIĆ

3. b)

$$\int_{-1}^{+\infty} \frac{dx}{1+x^2} = \left| \operatorname{arctg} x \right|_{-1}^{+\infty} = ?$$

nepravi integral

$\varnothing$

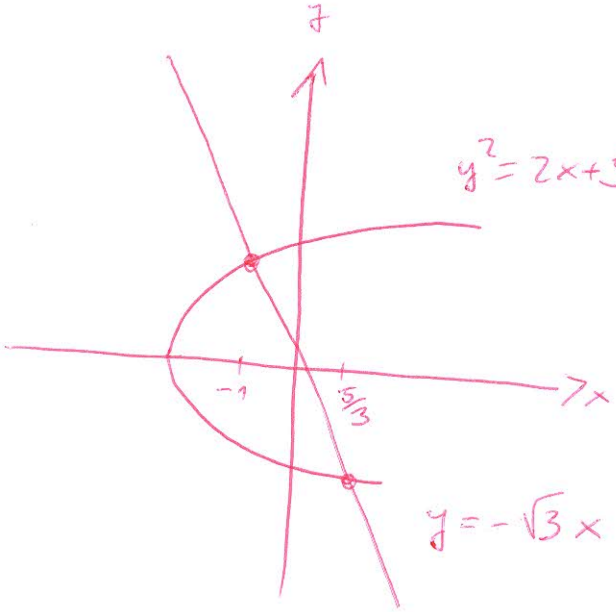
3.a)

$$2\left(\int_{-1}^{5/3} -\sqrt{3}\,x\,dx + \int_{-1}^{5/3} \sqrt{2x+5}\,dx\right)$$ ×

$$= 2\left(\frac{-\sqrt{3}}{2}\left|\cancel{x^2}\right|_{-1}^{5/3} + \frac{1}{2}\left|\sqrt{2x+5}\right|_{-1}^{5/3}\right)$$

$$= 2\left(-\frac{17\sqrt{3}}{9} + \frac{\sqrt{3}}{3}\right) = 5{,}388$$ ∅

VIDI BEGIĆ



ZORA ZORAČIĆ

(4)

$y'' - 5y' + 4y = e^x$ $\quad y(0) = 5$

$\quad y'(0) = 8$

$= A \cdot e^x$

$y_p = e^x$

$y'_p = e^x$

$y''_p = e^x$

$1e^x - 5 \cdot e^x + 4 \cdot e^x = Ae^x$

$0 \cdot e^x = A e^x$

$\boxed{A = 0}$

$\boxed{y_p = 0 \cdot e^x = 0}$ ⟸ partikularno rješenje

$\emptyset$

~~$y'' - 5y' +$~~

~~$A \cdot e^x = 5$~~

LUKA LUKAČIĆ

(5.) $$\int_0^4 \frac{dx}{\sqrt{x^2+2x+2}}$$

$$y = \frac{1}{\sqrt{x^2+2x+2}}$$

$d = 4-0 = 4$

	f_0	f_1	f_2
	0	2	4
	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{10}}{10}$	$\frac{\sqrt{26}}{26}$

$$S = \frac{4}{6}\left(\frac{\sqrt{2}}{2} + 4 \cdot \frac{\sqrt{10}}{10} + \frac{\sqrt{26}}{26}\right)$$

$$\boxed{S = 1{,}4454}$$

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MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ C	Popunjava nastavnik Broj ↓ bodova

Ime i prezime: SEBASTIJAN KOŠTA **Vrijeme početka:**

Matični broj studenta (iznad slike u indeksu): 17-2-0094-2011

PROF. UGLEŠIĆ

1. Grafički prikazati funkciju $f(x,y) = \ln(y-x)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije.
2. Odrediti $\int\limits_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) \mathrm{d}t$. $\varnothing$
3. Pronaći koliko iznosi:
   (a) površina između krivulja $y^2 = 2x+5$ i p$y = -\sqrt{3}x$,
   (b) integral $\int\limits_{-1}^{+\infty} \frac{\mathrm{d}x}{1+x^2}$? $\varnothing$
4. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = \mathrm{e}^x$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.
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Ukupno: $\varnothing$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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Tablica nekih integrala

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$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3.) b) $\int\limits_{-1}^{+\infty} \frac{dx}{1+x^2} = \begin{bmatrix} t = 1+x^2 \\ dt = 2x\,dx \end{bmatrix} = \cancel{\int \frac{2x\,dx}{t}} = \ln|1+x^2| \Big|_{-1}^{+\infty}$ ×

$1+x^2 \neq 0$

$x^2 \neq -1$

$= \ln|1+1^2| - \ln|1+(-1)|$

$= \ln 2 - \ln 0$ $\varnothing$

$\lim\limits_{x\to+\infty} \frac{dx}{1+x^2} =$

3) a) $y^2 = 2x+5$

$y = -\sqrt{3x}$

2) $\int_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) dt = -\frac{1}{4} + \frac{\sqrt{3}}{4}\int_0^{\pi} \sin^2 t + \cos t\, dt \; \left[\begin{matrix} \cancel{x = \cos t} \\ \cancel{dx = \sin t\, dt} \end{matrix}\right]$

$= \cancel{-\frac{1}{4} + \frac{\sqrt{3}}{4}\int \sin^2 t +} \quad = -\frac{1}{4} + \frac{\sqrt{3}}{4}\int_0^{\pi} 1 - \cos^2 t + \cos t\, dt$

$= -\frac{1}{4} + \frac{\sqrt{3}}{4}\int_0^{\pi} 1 - \cos t\, dt = -\frac{1}{4} + \frac{\sqrt{3}}{4} + 1 - \sin t \Big|_0^{\pi} = -\frac{1}{4} + \frac{\sqrt{3}}{4} + 1 - \sin\pi - \left(-\frac{1}{4} + \frac{\sqrt{3}}{4} + 1 - \sin 0\right)$

$= -\frac{1}{4} + 0{,}43 + 1 - 0{,}1 + \frac{1}{4} - \frac{\sqrt{3}}{4} - 1 = -0{,}25 + 0{,}43 + 1 - 0{,}1 + 0{,}25 - 0{,}43 - 1 = -0{,}1 = 0$

5) $\int_0^4 \frac{dx}{\sqrt{x^2+2x+2}} = \left[\begin{matrix} t = x^2+2x+2 \\ dt = 2x+2\, dx \end{matrix}\right]$

$x = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot 2}}{2\cdot 1}$

$x = \frac{-2 \pm \sqrt{4-8}}{2}$

$x = \frac{-2 \pm \sqrt{-4}}{2}$

USMENI ISPIT KOD PROF. UGLEŠIĆA

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: MATKO DONADIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0247-2014

1. Grafički prikazati funkciju $f(x, y) = \ln(y - x)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije.

2. Odrediti $\int\limits_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t \right) \mathrm{d}t.$

3. Pronaći koliko iznosi:

   (a) površina između krivulja $y^2 = 2x + 5$ i $y = -\sqrt{3}x$,

   (b) integral $\int\limits_{-1}^{+\infty} \frac{\mathrm{d}x}{1 + x^2}$?

4. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = \mathrm{e}^x$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

5. Izračunati $\int\limits_0^4 \frac{\mathrm{d}x}{\sqrt{x^2 + 2x + 2}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje.

Ukupno:

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
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$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

3 a) $y^2 = 2x + 5 \quad / \sqrt{\ }$

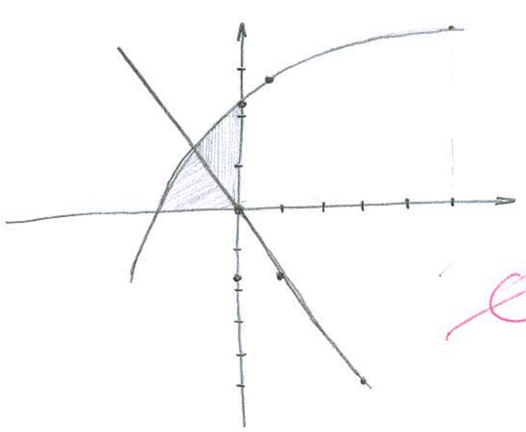
$y = \sqrt{2x+5}$

x	y
0	2.23
1	2.65
5	3.87

$y = -\sqrt{3}\,x$

x	y
0	0
1	-1.73
3	-5.1

SKICA



(2) $\int_0^{\pi}\left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right)dt = \int_0^{\pi} -\frac{1}{4}\sin^2 t\,dt + \int_0^{\pi}\frac{\sqrt{3}}{4}\cos t\,dt$

$-\frac{1}{4}\int_0^{\pi}\sin^2 t\,dt + \frac{\sqrt{3}}{4}\int_0^{\pi}\cos t\,dt = -\frac{1}{4}\left(-\cos^2 t\right)\Big|_0^{\pi} + \frac{\sqrt{3}}{4}$

$\emptyset$