

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!

POPUNJAVA
NASTAVNIK
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bodova

IME I PREZIME: **LOVRE BUBALO**

VRIJEME POČETKA: **15:40**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0389-2014

PROF. UGLEŠIĆ

1. Izračunati:

(a) određeni integral $\int_0^{\pi} \left(-\frac{1}{4} \sin^2 t + \frac{\sqrt{3}}{4} \cos t \right) dt =$ 10

(b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. 10

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^1 \cos(x^2) dx$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\leq 3\%$, 8 za rel. grešku $\leq 6\%$) 20

3. Riješiti diferencijalnu jednačinu $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$.

4. Izračunaj $\int_0^2 \frac{3x dx}{x^2 - 2x + 1}$ /

5. Odredi ekstreme funkcije $f(x, y) = x^2 + y - e^y$. 20

Ukupno:

60

f	$\frac{df}{dx}$
x^α ($\alpha \neq 0$)	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ ($\alpha > 0$)	$\frac{1}{x \ln \alpha}$
e^x	e^x
a^x ($a > 0$)	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

$$\textcircled{1} \text{ a) } \int_0^{\pi} \left(-\frac{1}{4} \sin^2 t + \frac{\sqrt{3}}{4} \cos t \right) dt = \int_0^{\pi} -\frac{1}{4} \sin^2 t dt + \int_0^{\pi} \frac{\sqrt{3}}{4} \cos t dt =$$

$$= -\frac{1}{4} \int_0^{\pi} \frac{1 - \cos 2t}{2} dt + \frac{\sqrt{3}}{4} \int_0^{\pi} \cos t dt = -\frac{1}{4} \int_0^{\pi} \frac{1}{2} dt + \frac{1}{4} \int_0^{\pi} \frac{\cos 2t}{2} dt + \frac{\sqrt{3}}{4} \int_0^{\pi} \cos t dt =$$

$$= -\frac{1}{8} \int_0^{\pi} dt + \frac{1}{8} \int_0^{\pi} \cos 2t dt + \frac{\sqrt{3}}{4} \int_0^{\pi} \cos t dt = -\frac{1}{8} \int_0^{\pi} dt + \left[\frac{2t-s}{2t=6s/2} \right] \frac{1}{8} \int_0^{\pi} \cos(s) \frac{ds}{2} + \frac{\sqrt{3}}{4} \int_0^{\pi} \cos t dt =$$

$$= -\frac{1}{8} [t]_0^{\pi} + \frac{1}{16} [\sin(s)]_0^{\pi} + \frac{\sqrt{3}}{4} [\sin t]_0^{\pi} = -\frac{\pi}{8} + \frac{1}{16} (\sin 2\pi - \sin 0) + \frac{\sqrt{3}}{4} (\sin \pi - \sin 0) =$$

$$= -\frac{\pi}{8} + 0 + 0 = -\frac{\pi}{8} \quad \checkmark$$

$$b) y = 2t - t^2$$

$$y = t$$

$$TJEME = -\frac{b}{2a} = -\frac{2}{-2} = 1$$

$$y = -t^2 + 2t$$

$$y = y$$

$$2t - t^2 = t$$

y		t
0		0
1		1
-1		-1

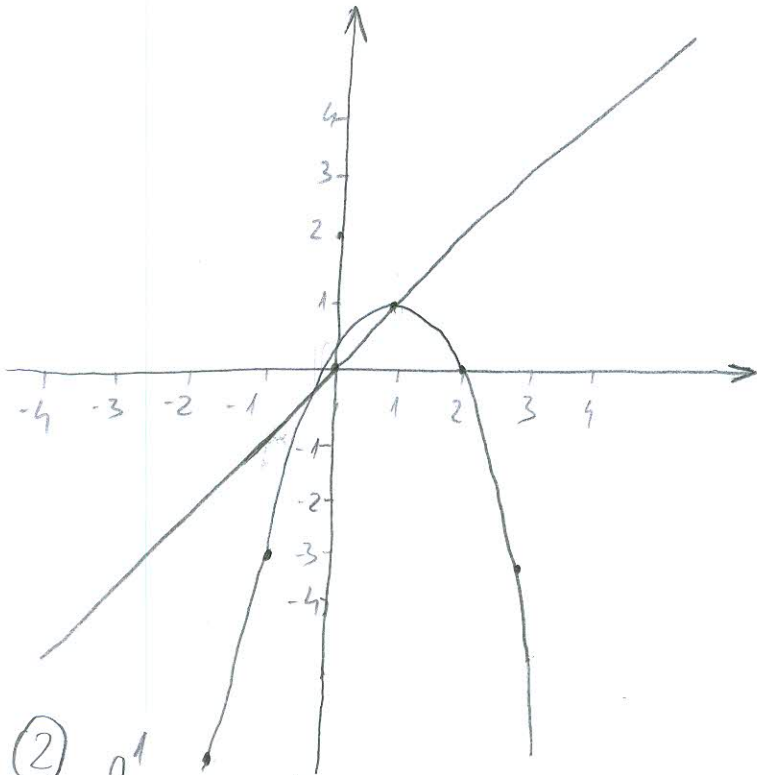
t		y
0		0
1		1
2		0
-2		-8
-1		-3

$$t - t^2 = 0$$

$$t(1-t) = 0$$

$$t_1 = 0 \quad 1-t = 0$$

$$t_2 = 1$$



gaji - gaji

$$\int_0^1 (2t - t^2 - t) dt = \int_0^1 (t - t^2) dt =$$

$$= \int_0^1 t dt - \int_0^1 t^2 dt = \left[\frac{t^{1+1}}{1+1} \right]_0^1 - \left[\frac{t^{2+1}}{2+1} \right]_0^1 =$$

$$= \left[\frac{t^2}{2} \right]_0^1 - \left[\frac{t^3}{3} \right]_0^1 = \left(\frac{1^2}{2} - \frac{0^2}{2} \right) - \left(\frac{1^3}{3} - \frac{0^3}{3} \right) =$$

$$= \frac{1}{2} - \frac{1}{3} = \boxed{0,16667} = \frac{1}{6} \quad \checkmark$$

$$\textcircled{2} \int_{-1}^1 \cos(x^2) dx =$$

$$d = 0 - (-1) = 1$$

k	0	1	2
x_k	-1	-0,5	0
f_k	0,5403	0,9689	1

$$d = 1 - 0 = 1$$

k	0	1	2
x_k	0	0,5	1
f_k	1	0,9689	0,5403

$$\cos(-1)^2 = 0,5403$$

$$\cos(-0,5)^2 = 0,9689$$

$$\cos(0)^2 = 1$$

$$S_{uk} = S_1 + S_2 = \boxed{1,8053} \quad \checkmark$$

20

$$S_1 = \frac{d}{6} (f_0 + 4 \cdot f_1 + f_2) =$$

$$= \frac{1}{6} (0,5403 + 4 \cdot 0,9689 + 1) = \underline{0,90265}$$

$$S_2 = \frac{1}{6} (1 + 4 \cdot 0,9689 + 0,5403) =$$

$$= 0,90265$$

$$(5) f(x, y) = x^2 + y - e^y$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial x} = 0$$

$$2x = 0$$

$$x = 0$$

$$\frac{\partial f}{\partial y} = 1 - e^y$$

$$\frac{\partial f}{\partial y} = 0$$

$$1 - e^y = 0$$

$$e^y = 1$$

$$y = 0$$

$T(0, 0)$

$$\frac{\partial f}{\partial x^2} = 2$$

$$\frac{\partial f}{\partial x \partial y} = 0 = \frac{\partial f}{\partial y \partial x}$$

$$\frac{\partial f}{\partial y^2} = -e^y$$

$$\Delta = \begin{vmatrix} \frac{\partial f}{\partial x^2} & \frac{\partial f}{\partial x \partial y} \\ \frac{\partial f}{\partial x \partial y} & \frac{\partial f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2 < 0$$

$$= -2 < 0$$

NEMA
EKSTREMA!!!

$$(4) \int_0^2 \frac{3x dx}{x^2 - 2x + 1} = \int_0^2 \frac{3x dx}{(x-1)^2}$$

$$\frac{1}{(x-1)^2} = \frac{3A}{x-1} + \frac{Bx}{x-1} \cdot \frac{1}{(x-1)(x-1)}$$

$$1 = 3A(x-1) + Bx(x-1)$$

$$1 = 3Ax - 3A + Bx^2 - Bx$$

$$-3A = 1$$

$$Bx^2 - Bx + 3Ax = 0$$

$$A = -\frac{1}{3}$$

$$Bx^2 - Bx - 1 = 0$$

$$B(x^2 - x) - 1 = 0$$

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