MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B

Popunjava nastavnik
Broj ↓ bodova

IME I PREZIME: LOVRE BUBALO **VRIJEME POČETKA:** 15:40

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0389-2014 PROF. UGLEŠIĆ

1. Izračunati:

   (a) određeni integral $\int_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) \mathrm{d}t =$. 10

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. 10

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos\left(x^2\right) \mathrm{d}x$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) 20

3. Riješiti diferencijalnu jednadžbu $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$.

4. Izračunaj $\int_0^2 \frac{3x\,\mathrm{d}x}{x^2 - 2x + 1}$ ∅

5. Odredi ekstreme funkcije $f(x, y) = x^2 + y - e^y$. 20

Ukupno: 60

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1 a) $\int_0^\pi \left(-\frac14 \sin^2 t + \frac{\sqrt3}{4}\cos t\right)dt = \int_0^\pi -\frac14 \sin^2 t\,dt + \int_0^\pi \frac{\sqrt3}{4}\cos t\,dt =$

$= -\frac14\int_0^\pi \frac{1-\cos 2t}{2}dt + \frac{\sqrt3}{4}\int_0^\pi \cos t\,dt = -\frac14\int_0^\pi \frac12 dt + \frac14\int_0^\pi \frac{\cos 2t}{2}dt + \frac{\sqrt3}{4}\int_0^\pi\cos t\,dt =$

$= -\frac18\int_0^\pi dt + \frac18\int_0^\pi \cos(2t)\,dt + \frac{\sqrt3}{4}\int_0^\pi \cos t\,dt = -\frac18\int_0^\pi dt + \begin{bmatrix}2t=s\\ 2dt=ds\ /:2\\ dt=\frac{ds}{2}\end{bmatrix} \frac18\int_0^\pi \cos(s)\frac{ds}{2} + \frac{\sqrt3}{4}\int_0^\pi\cos t\,dt =$

$= -\frac18[t]_0^\pi + \frac1{16}[\sin(s)]_0^\pi + \frac{\sqrt3}{4}[\sin t]_0^\pi = -\frac\pi8 + \frac1{16}(\sin 2\pi - \sin 0) + \frac{\sqrt3}{4}(\sin\pi - \sin 0) =$

$= -\frac\pi8 + 0 + 0 = -\frac\pi8$ ✓

b) $y = 2t - t^2$

$y = t$

TJEME $= -\frac{b}{2a} = -\frac{2}{-2} = 1$

$y = y$

$y = -t^2 + 2t$

$2t - t^2 = t$

y	t
0	0
1	1
-1	-1

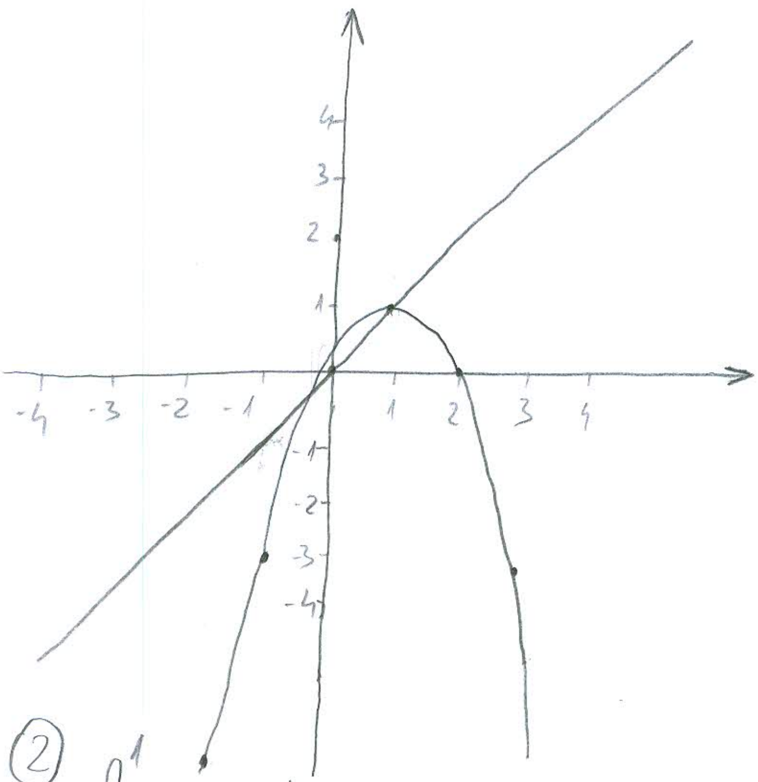
t	y
0	0
1	1
2	0
-2	-8
-1	-3

$t - t^2 = 0$

$t(1-t) = 0$

$t_1 = 0 \qquad 1 - t = 0$

$t_2 = 1$



gornja - donja

$$\int_0^1 (2t - t^2 - t)\,dt = \int_0^1 (t - t^2)\,dt =$$

$$= \int_0^1 t\,dt - \int_0^1 t^2\,dt = \left[\frac{t^{1+1}}{1+1}\right]_0^1 - \left[\frac{t^{2+1}}{2+1}\right]_0^1 =$$

$$= \left[\frac{t^2}{2}\right]_0^1 - \left[\frac{t^3}{3}\right]_0^1 = \left(\frac{1^2}{2} - \frac{0^2}{2}\right) - \left(\frac{1^3}{3} - \frac{0^3}{3}\right) =$$

$$= \frac{1}{2} - \frac{1}{3} = \boxed{0,16667 = \frac{1}{6}}$$

(2) $\int_{-1}^{1} \cos(x^2)\,dx =$

$d = 0 - (-1) = 1$

k	0	1	2
x_k	-1	-0,5	0
f_k	0,5403	0,9689	1

$d = 1 - 0 = 1$

k	0	1	2
x_k	0	0,5	1
f_k	1	0,9689	0,5403

$\cos(-1)^2 = 0,5403$

$\cos(-0,5)^2 = 0,9689$

$\cos(0)^2 = 1$

$S_{uk} = S_1 + S_2 = \boxed{1,8055}$

20

$$S_1 = \frac{d}{6}\left(f_0 + 4 \cdot f_1 + f_2\right) =$$

$$= \frac{1}{6}\left(0,5403 + 4 \cdot 0,9689 + 1\right) = \underline{0,90265}$$

$$S_2 = \frac{1}{6}\left(1 + 4 \cdot 0,9689 + 0,5403\right) =$$

$$= 0,90265$$

LOVRE BUBALO

(5)

$$f(x,y) = x^2 + y - e^y$$

$\frac{\partial f}{\partial x} = 2x$ $\qquad \frac{\partial f}{\partial x} = 0 \qquad 2x = 0$

$x = 0$

$\frac{\partial f}{\partial y} = 1 - e^y$ $\qquad \frac{\partial f}{\partial y} = 0 \qquad 1 - e^y = 0$

$e^y = 1$

$y = 0$

$\Rightarrow T(0,0)$

$\frac{\partial f}{\partial x^2} = 2$ $\qquad \frac{\partial f}{\partial x \partial y} = 0 = \frac{\partial f}{\partial y \partial x}$

$\frac{\partial f}{\partial y^2} = -e^y$ $\qquad \boxed{\frac{\partial f}{\partial y^2} T_0 = -1}$

$$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2 < 0$$

NEMA EKSTREMA !!!

(4)

$$\int_0^2 \frac{3x\,dx}{x^2-2x+1} = \int_0^2 \frac{3x\,dx}{(x-1)^2} =$$

$$\frac{1}{(x-1)^2} = \frac{3A}{x-1} + \frac{Bx}{x-1} \Big/ \cdot (x-1)(x-1)$$

$1 = 3A(x-1) + Bx(x-1)$

$1 = 3Ax - 3A + Bx^2 - Bx$

$-3A = 1 \qquad Bx^2 - Bx + 3Ax = 0$

$A = -\frac{1}{3} \qquad Bx^2 - Bx - 1 = 0$

$B(x^2 - x) - 1 = 0$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ B

Ime i prezime: LEINA ADUM **Vrijeme početka:**

Matični broj studenta (iznad slike u indeksu):

PROF. UGLEŠIĆ

Popunjava nastavnik
Broj ↓
bodova

1. Izračunati:

   (a) određeni integral $\int_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) \mathrm{d}t =.$

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. 10

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos\left(x^2\right) \mathrm{d}x$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) 8

3. Riješiti diferencijalnu jednadžbu $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$.

4. Izračunaj $\int_0^2 \frac{3x\,\mathrm{d}x}{x^2 - 2x + 1}$ Ø

5. Odredi ekstreme funkcije $f(x, y) = x^2 + y - e^y$. Ø

Ukupno: 18

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

LEJNA ADUM

(4) $\int_0^2 \frac{3x\,dx}{x^2-2x+1}$ $\quad x^2-2x+1=0$

$x_{1,2}=\frac{-2\pm\sqrt{4-4}}{2}=\frac{-2}{2}=-1$ ✗ $\emptyset$

NEPRAVI INTEGRAL!!!

$\int_0^2 \frac{3x\,dx}{x^2-2x+1} = \left\{ \begin{matrix} x^2-2x+1=t \\ \dots dx = dt \end{matrix} \right\}$ ✗

$3\int_0^2 \frac{x\,dx}{x^2-2x+1} = 3\cdot\frac{1}{2}\int_0^2 \frac{2x-2+2}{x^2-2x+1}dx = \frac{3}{2}\int_0^2 \overset{(1)}{\frac{2x-2}{x^2-2x+1}}dx + \frac{3}{2}\int_0^2 \overset{(2)}{\frac{2}{x^2-2x+1}}dx$

$\Rightarrow 1^\circ \; \frac{3}{2}\int_0^2 \frac{2x-2\,dx}{x^2-2x+1} = \left\{ \begin{matrix} x^2-2x+1=t \\ 2x-2\,dx=dt \end{matrix} \right\} = \frac{3}{2}\int_0^2 \frac{dt}{t} = \frac{3}{2}\ln|t|\Big|_0^2 =$

$= \frac{3}{2}\ln|x^2-2x+1|\Big|_0^2 = \frac{3}{2}\ln|4-4+1| - \frac{3}{2}\ln|1| = 0$

$2^\circ \; \frac{3}{2}\int_0^2 \frac{2}{x^2-2x+1}dx = \frac{3}{2}\cdot 2\int \frac{dx}{x^2-2x+1} = 3\int_0^2 \frac{dx}{x^2-2x+1} =$

$= 3\int_0^2 \frac{dx}{x^2-2x+2-1} = 3\int_0^2 \frac{dx}{x^2-1-2(x-1)} = 3\int_0^2 \frac{dx}{(x-1)(x+1)-2(x-1)} =$

$= 3\int_0^2 \frac{dx}{(x-1)(x+1-2)} = 3\int_0^2 \frac{dx}{(x-1)(x-1)} = 3\int_0^2 \frac{dx}{(x-1)^2} = \left\{ \begin{matrix} x-1=t \\ dx=dt \end{matrix} \right\}$

$= 3\int_0^2 \frac{dt}{t^2} = 3\int_0^2 t^{-2}dt = 3\cdot\frac{t^{-2+1}}{-2+1}\Big|_0^2 = 3\cdot\frac{t^{-1}}{-1}\Big|_0^2 = 3\cdot -\frac{1}{t}\Big|_0^2 =$

$= 3\cdot -\frac{1}{x-1}\Big|_0^2 = -3\cdot\frac{-1}{2-1} - 0 = 3$ ✓

3. NASTAVAK

$$= e^x \left[x^2(3A+B) + Ax^3 + x(2B+C) + C \right]$$

$$y_p' = (e^x)' \cdot \left[x^2(3A+B) + Ax^3 + x(2B+C) + C \right] + e^x \left[2x(3A+B) + 3Ax^2 + 2B + C \right]$$

$$(x^2)'(3A+B) + x^2(3A+B)' = 2x(3A+B)$$

$$3Ax^2$$

$$x'(2B+C) + x \cdot 0 = 2B+C$$

$$y_p' = e^x \left[x^2(3A+B) + Ax^3 + x(2B+C) + C + 2x(3A+B) + 3Ax^2 + 2B + C) \right]$$

$$y_p'' = e^x \left[x^2(3A+B) + Ax^3 + x(2B+C) + C + x(6A+2B) + 3Ax^2 + 2B + C) \right]$$

$$y_p'' = e^x \underbrace{\left[x^2(6A+B) + x^3 A + x(6A+4B+C) + 2B + 2C \right]}_{\heartsuit}$$

$$e^x \cdot \heartsuit - xe^x(Ax^2 + Bx + C) =$$

$$e^x(\heartsuit - (Ax^3 + Bx^2 + Cx)) =$$

$$e^x \left[x^2(6A) + x(6A+4B) + 2B + 2C \right] = x^2 - xe^x$$

...

$$2B + 2C = 0$$

$$6A + 4B = e^x$$

DALJE...

LEINA ADUM

(5) $f(x,y) = x^2 + y - e^y$

$x^2 + y = e^y \; / \ln$

$\ln x^2 + \ln y = y$

$\ln x^2 + \ln y - y = 0$

$\frac{\partial f}{\partial x} = \frac{1}{x} \cdot x^2 = \frac{1}{x} \cdot 2x = 2$ ✗

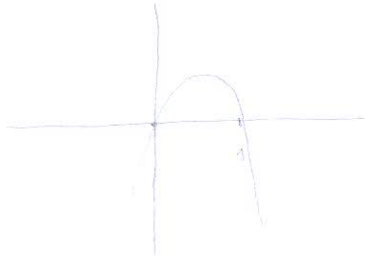
$\frac{\partial f}{\partial x \partial x} = 0 \qquad \frac{\partial f}{\partial x \partial y} =$

NEMA EKSTREMA

$\frac{\partial f}{\partial y} = \frac{1}{y} - 1 \qquad D = \begin{vmatrix} 0 & 0 \\ 0 & \end{vmatrix} = -\frac{1}{y^2}$

$\frac{\partial f}{\partial y \partial y} = y^{-2} = \frac{1}{y^2}$

1, b) $y = 2t - t^2 \qquad y = t$



$2t - t^2 = t$

$2t - t - t^2 = 0$

$t - t^2 = 0$

$t(t-1) = 0$

$t_1 = 0 \quad t_2 = 1$

$\int_0^1 2t - t^2 - t\,dt = \int_0^1 t - t^2\,dt = \frac{t^2}{2} - \frac{t^3}{3} \Big|_0^1 =$

$= \frac{1^2}{2} - \frac{1^3}{3} - 0 = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$ ✓

(3) $y'' - y = x^2 - xe^x \quad ; \quad y(0) = 0 \quad y'(0) = 1$

$r^2 - 1 = 0$

$r^2 = 1$

$r = \pm 1$

$y_H = C_1 e^{-x} + C_2 e^x$

$f(x) = x^2 e^{\alpha x} \quad P_N \cos \beta x + Q_M \sin \beta x) = x^2 - xe^x$

$P_N = Ax^2 + Bx + C$

$y_p = xe^x (Ax^2 + Bx + C)$

$y_p' = (xe^x)'(Ax^2 + Bx + C) + (xe^x)(Ax^2 + Bx + C)' =$

$= e^x \cdot xe^x (Ax^2 + Bx + C) + (xe^x)(2Ax + B) =$

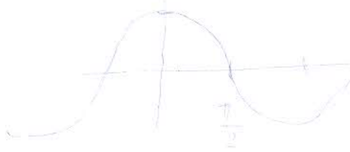
$= e^x(1+x)(Ax^2 + Bx + C) + (xe^x)(2Ax + B)$

$= e^x \left[(1+x)(Ax^2 + Bx + C) + x(2Ax + B) \right] =$

$= e^x \left[Ax^2 + Bx + C + Ax^3 + Bx^2 + Cx + 2Ax^2 + Bx \right] =$

$= e^x \left[x^2(A + 2A + B) + Ax^3 + x(2B + C) + C \right]$

(2) $\int_{-1}^{1} \cos(x^2)\,dx =$



k	0	1	2
x_k	-1	0	1
f_k	0,999	1	0,999

$$S = \frac{d}{6}\left(f_0 + 4f_1 + f_2\right) = \frac{2}{6}\left(0{,}999 + 4 + 0{,}999\right) = \frac{2}{6}\cdot 5{,}998 = 1{,}9$$

TOČNO RJEŠENJE = 1.809

APS. GREŠKA = 0.09

REL. GREŠKA = $\frac{0.09}{1.809}$

= 4.975 %

8

(1) a) $\int_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) dt =$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: DOMAGOJ GROZAJ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0056-2011	
PROF. UGLEŠIĆ	

1. Izračunati:

   (a) određeni integral $\int\limits_0^\pi \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) \mathrm{d}t =$. — $\emptyset$

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. — 10

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos\left(x^2\right) \mathrm{d}x$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$)

3. Riješiti diferencijalnu jednadžbu $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$.

4. Izračunaj $\int\limits_0^2 \frac{3x\,\mathrm{d}x}{x^2 - 2x + 1}$ — $\emptyset$

5. Odredi ekstreme funkcije $f(x, y) = x^2 + y - e^y$. — $\emptyset$

Ukupno: $\emptyset$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

4ROZAD

$$f(x,y) = x^2 + y - e^y$$

$$\frac{\delta f}{\delta x} = 2x$$

$2x = 0$
$x = 0$

$S = (0,0)$

$$\frac{\delta f}{\delta y} = -e^y + 1$$

$-e^y = -1$
$e^y = 1 \quad / \ln$
$y = \ln|1|$
$y = 0$

$$\frac{\delta f}{\delta x^2} = 2 \qquad \frac{\delta f}{\delta x \delta y} = 0$$

$$\frac{\delta f}{\delta y \delta x} = 0 \qquad \frac{\delta f}{\delta y^2} = 0$$

$S = (0,0)$
NISU EKSTREMI

$\emptyset$

$(-e^y - 1)'$

$-e^y = 0 \quad / \ln$
$-y = \ln|0|$
$-y = 0$

$$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} < 0$$

$$\int_0^2 \frac{3x\,dx}{x^2-2x+1}$$

$$=\int_0^2 \frac{3x}{(x-1)^2}$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x^2-2x+1=0$$

$$x_{12}=\frac{2\pm\sqrt{4-4\cdot 1\cdot 1}}{2a}$$

$$=\frac{2\pm\sqrt{0}}{2}$$

$$(x-1)^2=x^2-2x+1$$

4RO2AD

$y = 2x - x^2$ $\quad$ $y = x$

$-x^2 + 2x = 0 \quad / \cdot (-1)$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x_1 = 0 \quad x_2 = 2$

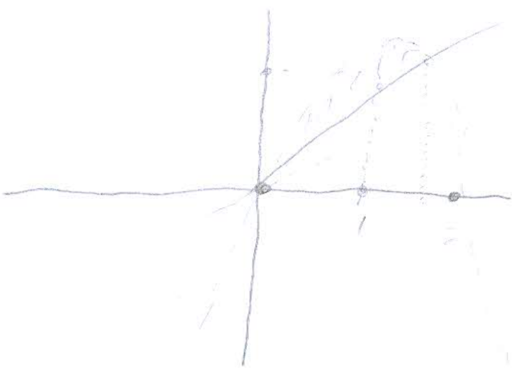
$x = 2x - x^2$

$x^2 + x - 2x = 0$

$x^2 - x = 0$

$x(x-1)$

$x = 0 \quad x = 1$



$P = \int_0^1 2x - x^2 - x \, dx$

$= \int_0^1 -x^2 + x \, dx$

$= -\frac{x^3}{3} + \frac{x^2}{2}$

$= \frac{1}{2} - \frac{1}{3}$

$\boxed{= \frac{1}{6}}$

GROZAN

$$\int_0^{\pi}\left(-\frac{1}{4}\sin^2 x + \frac{\sqrt{3}}{4}\cos x\right)dx$$

$$= -\frac{1}{4}\int_0^{\pi}\sin^2 x + \frac{\sqrt{3}}{4}\cos x \, dx$$

$$= -\frac{1}{4}\cdot\frac{\sqrt{3}}{4}\int_0^{\pi}\sin^2 x + \cos x \, dx$$

$$= -\frac{1}{4}\cdot\frac{\sqrt{3}}{4}\int_0^{\pi}\frac{1-\cos 2x}{2} + \cos x \, dx$$

$$= -\frac{\sqrt{3}}{16}\left[\int_0^{\pi}\frac{1-\cos 2x}{2} + \int_0^{\pi}\cos x \, dx\right]$$

$$= \frac{-\sqrt{3}}{16}\left[\frac{1}{2}\int_0^{\pi}dx - \frac{1}{2}\int_0^{\pi}\cos 2x + \int_0^{\pi}\cos x \, dx\right]$$

$$= \frac{-\sqrt{3}}{16}\left[\frac{1}{2}\cdot\frac{x^2}{2} + \frac{1}{2}\sin 2x - \sin x\right]_0^{\pi}$$

$$= \frac{-\sqrt{3}}{16}\left[\frac{1}{4}x^2 + \frac{1}{2}\sin 2x - \sin x\right]$$

$$= \frac{-\sqrt{3}}{16}\left[\frac{1}{4}\pi^2 + \frac{1}{2}\sin 2\pi - \sin\pi\right] + \frac{-\sqrt{3}}{16}\left[\frac{1}{4}0^2 + \frac{1}{2}\sin 20 - \sin 0\right]$$

2,4673

$= -0,2670$ $= -0,2670$

$t^2 = \sin x$

$2t\,dt = \cos x \, dx$

$dx = \frac{2t\,dt}{\cos x}$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\int\frac{1-\cos 2x}{2}$

$\frac{1}{2}\int dx - \frac{\cos 2x}{2}$

$= \frac{1}{2}\int dx - \frac{1}{2}\int\cos 2x$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ B	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: JURE ŠUŠIĆ **Vrijeme početka:**	
Matični broj studenta (iznad slike u indeksu): 0269087878	

prof. Uglešić

1. Izračunati:

   (a) određeni integral $\int_0^{\pi}\left(-\frac{1}{4}\sin^2 t+\frac{\sqrt{3}}{4}\cos t\right)\mathrm{d}t =.$ Ø

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. Ø

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1}\cos\left(x^2\right)\mathrm{d}x$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$)

3. Riješiti diferencijalnu jednadžbu $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. Ø

4. Izračunaj $\int_0^2 \frac{3x\,\mathrm{d}x}{x^2-2x+1}$ Ø

5. Odredi ekstreme funkcije $f(x, y) = x^2 + y - e^y$.

Ukupno: Ø

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

JURE SUSIC

NEPRAVI INTEGRAL!!!

4) $\int_0^2 \frac{3x\,dx}{x^2-2x+1} = \left[\begin{array}{l} t = x^2-2x+1 \\ dt = 2x-2\,dx \\ dx = \frac{dt}{2x-2} \end{array}\right]$

$= \int_0^2 \frac{3\cancel{x}\cdot\frac{dt}{2\cancel{x}-2}}{t} = \frac{3}{2}\int_0^2 \frac{\frac{3dt}{-2}}{\underset{1}{t}} = \frac{3}{2}\int_0^2 \frac{\frac{dt}{-2}}{t} = \frac{3}{2}\cdot\frac{\frac{2x-2dx}{2x-2}}{\underset{1}{x^2-2x+1}}\Big|_0^2$

$= \frac{3}{2}\cdot\frac{2x-2dx}{2x^3-4x^2+2x-2x^2+4x-2}\Big|_0^2 = \frac{3}{2}\cdot\frac{2x-2dx}{2x^3-6x^2+6x-2}\Big|_0^2 = \emptyset$

1. a) $\int_0^{\pi}\left(-\frac{1}{4}\sin^2 t+\frac{\sqrt{3}}{4}\cos t\right)dt = \left[\begin{array}{l} x = -\frac{1}{4}\sin^2 t+\frac{\sqrt{3}}{4}\cos t \\ dx = +\frac{1}{4}\,2\cos t+\frac{\sqrt{3}}{4}\sin t \\ dt = \frac{dx}{\frac{2}{4}\cos t+\frac{\sqrt{3}}{4}\sin t} \end{array}\right]$

$= \int_0^{\pi} x\cdot\frac{2}{4}\cos t+\frac{\sqrt{3}}{4}\sin t = \frac{x^2}{2}.$

1.) b) $y = 2t - t^2$ $\quad y = t$ $\quad 2t - t^2 = 0$

$t(2-t) = 0$

$$P = \int_0^2 2t - t^2 \, dt = \int_0^2 2\frac{t^2}{2} - \frac{t^3}{3} \, dt = \quad t_1 = 0 \quad 2 - t = 0$$

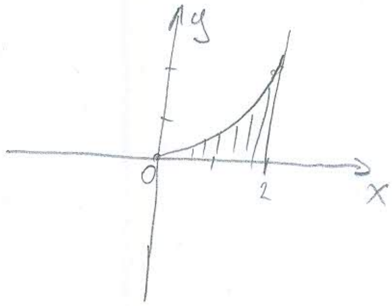
$-t = -2$

$t_2 = 2$

$$= \int_0^2 t^2 - \frac{t^3}{3} \, dt$$

$$= t^2 - \frac{t^3}{3} \Big|_0^2 = \left(2^2 - \frac{2^3}{3}\right) - \left(0^2 - \frac{0^3}{3}\right) = \frac{4}{3}$$

$\emptyset$



3.) $y'' - y = x^2 - xe^x$ $\quad y(0) = 0 \quad y'(0) = 1$

$$\int y'' - y = x^2 - xe^x$$

$$= \int y' - \frac{y^2}{2} = \frac{x^3}{3} - xe^x$$

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇓ B	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: STIPE KATALINIĆ **Vrijeme početka:**	
Matični broj studenta (iznad slike u indeksu): 17-2-0320-2013 UGLEŠIĆ	

1. Izračunati:

   (a) određeni integral $\int\limits_0^\pi \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) dt =.$

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. Ø

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos\left(x^2\right) dx$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$)

3. Riješiti diferencijalnu jednadžbu $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$.

4. Izračunaj $\int\limits_0^2 \frac{3x\,dx}{x^2 - 2x + 1}$ Ø

5. Odredi ekstreme funkcije $f(x, y) = x^2 + y - e^y$.

Ukupno: Ø

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x\, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

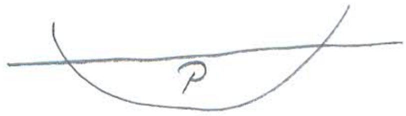
STIPE CATALI

b)

1) $y = 2t - t^2$ i $y = t$ $\qquad y - y_1 = \frac{y}{\quad}$

$t \to x$

$$y = 2x - x^2 = y = x$$



$$x - 2x + x^2 = 0$$
$$x^2 - 2x + x = 0$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$0 \qquad x = 1$$

$$\int_0^1 \left[(2x - x^2) - x\right] dx$$

$$\int_0^1 \left[2x - x^2 - x\right] dx$$

$$\int_0^1 \left[-x^2 - x\right] dx$$

$$= \int_0^1 -x^2 dx + \int_0^1 -x^1 dx$$

$$= \underbrace{\left[-\frac{x^3}{3} - \frac{-x^2}{2}\right]}_{F}\Bigg|_0^1$$

$$= F(b) - F(a)$$

$$= \left[\frac{-(1)^3}{3} - \frac{(1)^2}{2}\right] - \left[\frac{-(0)^3}{3} - \frac{0^2}{2}\right]$$

$$= \frac{5}{6}$$

$$= 0,83$$

Površina lika omeđenog krivulja

$$y = 2t - t^2 \text{ i } y = t$$

iznosi $= 0,83$

0

4. kvocunoj

$$\int_0^2 \frac{3x\,dx}{x^2-2x+1}$$

$$\int_0^2 \frac{3x\,dx}{(x-1)^2} = 3\int_0^2 \frac{x\,dx}{(x-1)^2} \quad \Bigg| \begin{array}{l} t = \frac{x}{x-1} \\ dt = \frac{x\cdot(x-1) - \cancel{x}}{(x-1)^2}dx \\ dt = \frac{x}{x-1}dx \end{array}$$

$t=2 \quad t = \frac{x}{x-1} = \frac{2}{2-1} = 2$

$t=0 \quad t = \frac{x}{x-1} = \frac{0}{x-0} = 0$

$a = x$
$b = 1$

$$\boxed{\begin{array}{c} a^2 - 2ab + b^2 \\ x^2 - 2\cdot x + 1 \end{array}}$$

$$= 3\int_0^2 dt\,dx$$

0

1. $\int_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right)dt$

∅