MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. PIŠITE DVOSTRANO! Obavezno popuniti sva polja ispod⇊ $\mathcal{A}$

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: JURE PERIĆ VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0269083660

PROF. UGLEŠIĆ

1. Riješiti integrale:

   (a) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}\,\mathrm{d}x$,

   (b) $\displaystyle\int_0^{\pi} \left(x\cos x + \mathrm{e}^{1-3x}\right)\,\mathrm{d}x =$.

2. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin\left(x^2\right)\mathrm{d}x$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) 20

3. Riješiti $y'' + 2y' + 5y = x^2\mathrm{e}^{3x} + \sin(2x)$ i provjeriti rješenje.

4. Riješiti $x^2 + yy' = 1$, uz početni uvjet $y(0) = 1$.

5. U koordinatnoj ravnini skicirati domenu funkcije $f(x,y) = \arccos(x-y)$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije. 20

Ukupno: 40

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $\displaystyle\int_0^2 \sin(x^2)\,dx$

$\displaystyle\int_0^{0.5} \sin(x^2)\,dx$

0	0.25	0.5
0	1	2
0	0,062	0,247

$d = 0{,}5 - 0 = 0{,}5$

$S_1 = \frac{0{,}5}{6}\left(0 + 4\cdot 0{,}062 + 0{,}247\right) = 0{,}041$

$\displaystyle\int_{0{,}5}^{1} \sin(x^2)\,dx$

0,5	0,75	1
0	1	2
0,247	0,533	0,841

$d = 1 - 0{,}5 = 0{,}5$

$S_2 = \frac{0{,}5}{6}\left(0{,}247 + 4\cdot 0{,}533 + 0{,}841\right) = 0{,}268$

$\displaystyle\int_{1}^{1{,}5} \sin(x^2)\,dx$

1	1,25	1,5
0	1	2
0,841	0,999	0,778

$d = 1{,}5 - 1 = 0{,}5$

$S_3 = \frac{0{,}5}{6}\left(0{,}841 + 4\cdot 0{,}999 + 0{,}778\right) = 0{,}467$

$\int_{1,5}^{2} \sin(x^2)\,dx$

1,5	1,75	2
0	1	2
0,778	0,079	-0,756

$S = \frac{0,5}{6}\left(0,778 + 4\cdot 0,079 + (-0,756)\right)$

$S_4 = 0,028$

$d = 2 - 1,5 = 0,5$

$S_{uk} = S_1 + S_2 + S_3 + S_4 = 0,041 + 0,268 + 0,467 + 0,028 = \underline{0,804}$

---

5. $f(x,y) = \arccos(x-y)$ $\qquad -1 \leq x-y \leq 1$ $\qquad K: \underline{\mathbb{R}}$

$\arccos(x-y) = 0 \;/\cdot\cos$

$x - y = 1$

$-y = -x + 1 \quad /\cdot(-1)$

$y = x - 1$

x	0	1
y	-1	0

$c = 1$

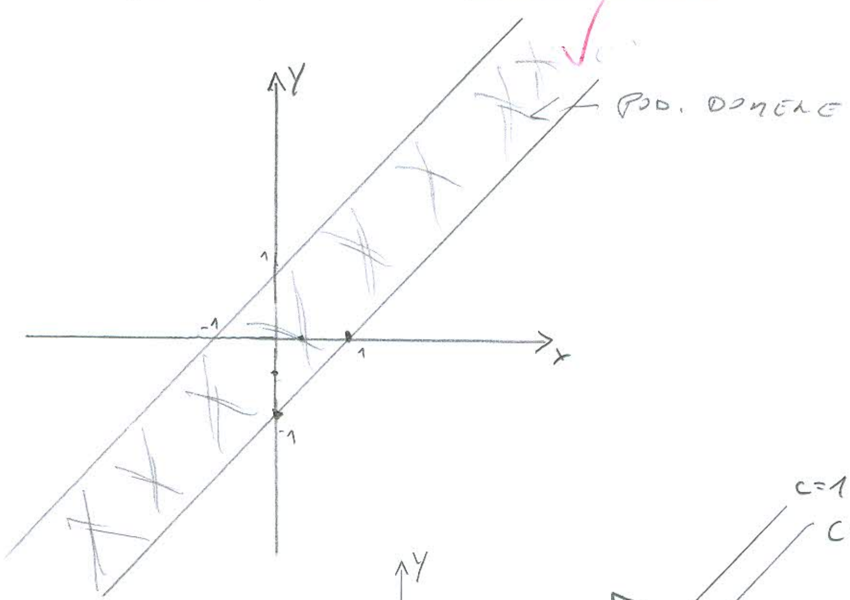
$\arccos(x-y) = 1 \;/\cdot\cos$

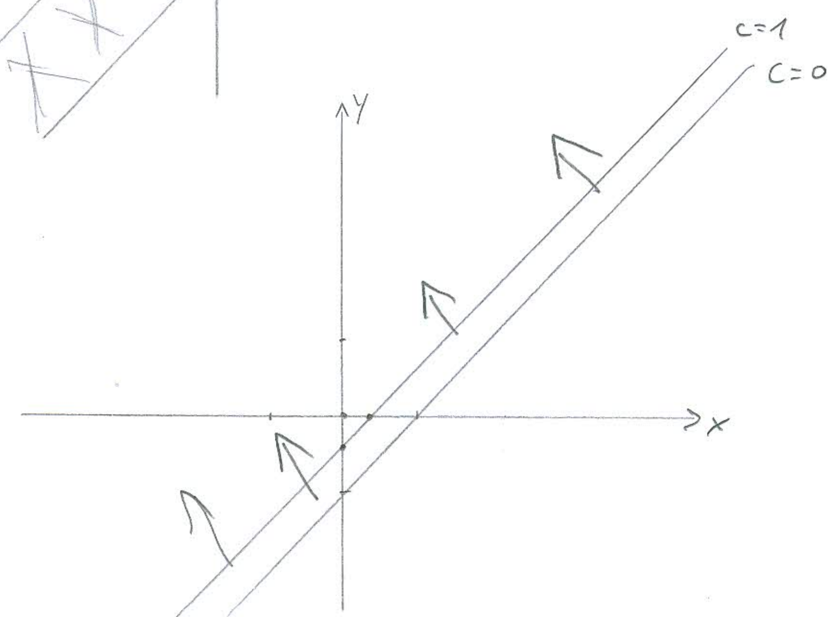
$x - y = 0,54$

$-y = -x + 0,54 \;/\cdot(-1)$

$y = x - 0,54$

x	0	1
y	-0,54	0,46





MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ŠIME ZELENČIĆ VRIJEME POČETKA:	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0370-2014 0269086384 PROF. UGLEŠIĆ	

1. Riješiti integrale:

   (a) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}\,\mathrm{d}x,$ 10

   (b) $\displaystyle\int_0^{\pi} \left(x\cos x + \mathrm{e}^{1-3x}\right)\,\mathrm{d}x =.$ 10

2. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin\left(x^2\right)\mathrm{d}x$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) Ø

3. Riješiti $y'' + 2y' + 5y = x^2\mathrm{e}^{3x} + \sin(2x)$ i provjeriti rješenje.

4. Riješiti $x^2 + yy' = 1$, uz početni uvjet $y(0) = 1$.

5. U koordinatnoj ravnini skicirati domenu funkcije $f(x,y) = \arccos(x-y)$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije.

Ukupno: 20

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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a) $$\int \frac{1+2x^2}{x^2(1+x^2)}dx = \int \frac{1+x^2+x^2}{x^2(1+x^2)}dx = \int \frac{\cancel{1+x^2}}{x^2\cancel{(1+x^2)}}dx + \int \frac{\cancel{x^2}}{\cancel{x^2}(1+x^2)}dx =$$

$$= \int \frac{1}{x^2}dx + \int \frac{1}{1+x^2}dx = \int x^{-2}dx + \frac{1}{1}\arctan\frac{x}{1} + C$$

$$= \frac{x^{-1}}{-1} + \operatorname{arctang} x + C = -\frac{1}{x} + \arctan x + C$$ ✓

$$\int_0^{\pi} (x\cos x + e^{1-3x})\,dx = I$$

1. $$\int_0^{\pi} x\cos x\,dx = \left| \begin{array}{ll} u = x & dv = \cos x\,dx \\ du = dx & v = \sin x \end{array} \right.$$

$$= \left[x\cdot\sin x - \int \sin x\,dx \right]_0^{\pi}$$

$$x\cdot\sin x + \cos x \Big|_0^{\pi}$$

$$(\pi\cdot\sin\pi + \overset{-1}{\cos\pi}) - (0\cdot\sin 0 + \cos 0) = \underline{\underline{-2}}$$ ✓

2. $$\int_0^{\pi} e^{1-3x}\,dx = \left| \begin{array}{l} 1-3x = t \\ -3dx = dt \end{array} \right|$$

$$= \int_0^{\pi} e^t\cdot\frac{dt}{-3} = -\frac{1}{3}\int_0^{\pi} e^t dt = \left[-\frac{1}{3}\cdot e^{1-3x} \right]_0^{\pi} =$$

$$= \left(-\frac{1}{3}\cdot e^{1-3\cdot\pi}\right) - \left(-\frac{1}{3}\cdot e^{1-3\cdot 0}\right) = 0{,}9060208215$$ ✓

$$I = I_1 + I_2 = -2 + 0{,}9060208215 = -1.0939791$$ ✓

② $\int_0^2 \sin(x^2)\,dx = \left| \cancel{\begin{matrix} x^2 = t \\ 2x\,dx = dt \\ dx = \frac{dt}{2x} \end{matrix}} \right| \int_0^2 \sin x \cdot x\,dx$

$u = \sin x \quad dv = x\,dx$

$du = \cos x \quad v = \frac{x^2}{2}$

$\sin x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \cos x\,dx$

$\sin x \cdot \frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^3}{3} \cdot (-\sin x)$

$\sin x \cdot \frac{x^2}{2} - \frac{x^3}{6} \cdot (-\sin x)$

$= \int_0^2 \sin t \cdot \frac{dt}{2x} = \frac{1}{2}\int_0^2 \sin t \cdot \frac{dt}{x} = \frac{1}{2}\int_0^2 \sin x^2 \cdot \frac{1}{x} \cdot dt$?

$= \left[\frac{1}{2} \cdot (-\cos x^2) \cdot \ln x\right]_0^2$

$3{,}031 - 0$

$= 3{,}031$

0,2265356164

$= \left(-\frac{1}{2} \cdot \cos x^2 \cdot \ln x\right)\Big|_0^2 = \left(-\frac{1}{2} \cdot \cos 2^2 \cdot \ln 2\right) - \cancel{\left(-\frac{1}{2} \cdot \cos 0^2 \cdot \ln 0\right)}$

$=$

ŠIME ZELENČIĆ

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ A

Ime i prezime: LARA ŠKEPICA **Vrijeme početka:**

Matični broj studenta (iznad slike u indeksu):

17-2-0382-2014 prof. Uglešić

Popunjava nastavnik Broj ↓ bodova

1. Riješiti integrale:

   (a) $\int \frac{1+2x^2}{x^2(1+x^2)} \mathrm{d}x,$ ∅

   (b) $\int_0^{\pi} \left(x\cos x + \mathrm{e}^{1-3x}\right) \mathrm{d}x =.$ ∅

2. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin\left(x^2\right) \mathrm{d}x$. (bodovanje: 20 za rel. grešku$\leq$ 1%, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) ∅

3. Riješiti $y'' + 2y' + 5y = x^2\mathrm{e}^{3x} + \sin(2x)$ i provjeriti rješenje.

4. Riješiti $x^2 + yy' = 1$, uz početni uvjet $y(0) = 1$. ∅

5. U koordinatnoj ravnini skicirati domenu funkcije $f(x, y) = \arccos(x - y)$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije. ∅

Ukupno: ∅

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$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1\. a) $\int \frac{1+2x^2}{x^2(1+x^2)} dx$ $\qquad x^2(1+x^2) = x^2 + x^4$

$= \left[\begin{matrix} x = t \\ 1dx = dt \end{matrix}\right] = \int \frac{1+2\cancel{t^2}}{\cancel{t^2}(1+t^2)} dt = \int \frac{3dt}{1+t^2} = 3\int \frac{dt}{1+t^2}$

$= 3\left(\frac{1}{1}\arctan\frac{t}{1}\right) = 3 \arctan t = 3 \arctan x$

5) $\int_0^{\pi} (x\cos x + e^{1-3x})\,dx = \int_0^{\pi} x\cos x\,dx + \int_0^{\pi} e^{1-3x}\,dx$

$= [x\sin x]_0^{\pi} + \left[-\frac{1}{3}e^{1-3x}\right]_0^{\pi} = (\cancel{0,1122} - 7,3121\times10^{-5}) - (0 - 0,9061)$

$\approx \boxed{0,906}$

$\emptyset$

$\int_0^{\pi} x\cos x\,dx = \begin{bmatrix} x = t \\ 1dx = dt \end{bmatrix} = \int_0^{\pi} t\cos t\,dt = t\int_0^{\pi} \cos t\,dt = [t\sin t]_0^{\pi}$

$= [x \sin x]_0^{\pi}$

$\int_0^{\pi} e^{1-3x}\,dx = \begin{bmatrix} 1-3x = t \\ -3dx = dt \\ dx = -\frac{1}{3}dt \end{bmatrix} = \int_0^{\pi} e^t \cdot -\frac{1}{3}dt = -\frac{1}{3}\int_0^{\pi} e^t\,dt$

$= \left[-\frac{1}{3}e^t\right]_0^{\pi} = \left[-\frac{1}{3}e^{1-3x}\right]_0^{\pi}$

2. $\int_0^2 \sin(x^2)\,dx$

k	0	1	2
x_k	0	0,5	1
f_k	0	0,2474	0,84147

k	0	1	2
x_k	1	1,5	2
f_k	0,84147	0,77807	−0,7568

$d = 2 - 0 = \boxed{2}$

$S_1 = \frac{2}{6}(f_0 + 4\cdot f_1 + f_2)$

$S_1 = \frac{1}{3}(0 + \underbrace{4\cdot 0,2474}_{0,9896} + 0,84147)$

$S_1 = 0,61036$

$S_2 = \frac{2}{6}(0,84147 + 4\cdot 0,77807 - 0,7568)$

$S_2 = 1,06565$

$S = S_1 + S_2 = 1,67601$

LARA [illegible]

$D(f)$:

$K(f) [-1, 1]$

5. $f(x,y) = \arccos(x-y)$

$c=-1 \to \arccos(x-y) = -1$ ✗

$x - y = \cos(-1)$

$-y = \cos(-1) - x$

$y = -\cos x + 1$

x	0	1	2	3	-1	-2
y	0	0,4597	1,416	1,92999	0,45969	1,416

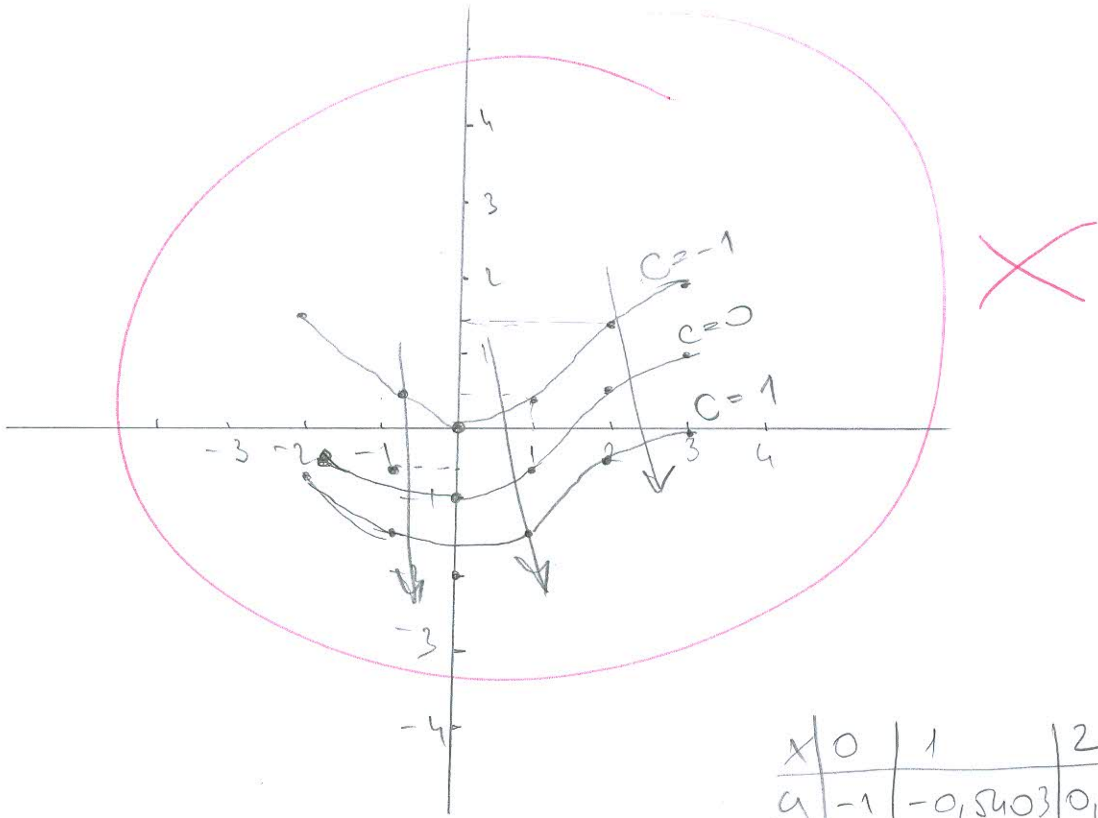
$c = 0 \to \arccos(x-y) = 0$

$y = -\cos x$ ✗

$c = 1 \to \arccos(x-y) = 1$

$-y = \cos(-x) + 1$

$y = -\cos x - 1$ ✗



x	0	1	2	3
y	-1	-0,5403	0,416	0,9895

-1	-2	-3
-0,5403	0,416	

x	0	1	2
y	-2	-1,540	-0,5238

3	-1	-2
-0,01	-1,540	-0,5839

LARA SLJEPICA

4. $x^2 + yy' = 1$ $\quad$ $y(0) = 1$

$yy' = 1 - x^2$

$ydy = 1 - x^2$

0