

**MATEMATIKA 2:** Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: JURE PERIĆ

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0269083660

PROF. UGLEŠIĆ

A

1. Riješiti integrale:

(a)  $\int \frac{1+2x^2}{x^2(1+x^2)} dx,$

(b)  $\int_0^{\pi} (x \cos x + e^{1-3x}) dx =.$

2. Numeričkom integracijom odrediti vrijednost  $\int_0^2 \sin(x^2) dx$ . (bodovanje: 20 za rel. grešku  $\leq 1\%$ , 15 za rel. grešku  $\leq 3\%$ , 8 za rel. grešku  $\leq 6\%$ )

20

3. Riješiti  $y'' + 2y' + 5y = x^2 e^{3x} + \sin(2x)$  i provjeriti rješenje.

4. Riješiti  $x^2 + yy' = 1$ , uz početni uvjet  $y(0) = 1$ .

5. U koordinatnoj ravnini skicirati domenu funkcije  $f(x, y) = \arccos(x - y)$  i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije.

20

Ukupno:

40

$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x (\alpha > 0)$	$\frac{1}{x \ln a}$
$e^x$	$e^x$
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$	

2.  $\int_0^2 \sin(x^2) dx$

$\int_0^{0.5} \sin(x^2) dx$

0	0.25	0.5
0	1	2
0	0,062	0,247

$d = 0.5 - 0 = 0.5$

$S_1 = \frac{0.5}{6} (0 + 4 \cdot 0,062 + 0,247) = 0,1041$

$\int_{0.5}^1 \sin(x^2) dx$

$d = 1 - 0.5 = 0.5$

$S_2 = \frac{0.5}{6} (0,247 + 4 \cdot 0,533 + 0,841) = 0,268$

$\int_1^{1.5} \sin(x^2) dx$

$d = 1.5 - 1 = 0.5$

$S_3 = \frac{0.5}{6} (0,841 + 4 \cdot 0,999 + 0,778) = 0,467$

0,5	0,75	1
0	1	2
0,247	0,533	0,841

1	1,25	1,5
0	1	2
0,841	0,999	0,778

$$\int_{1,5}^2 \sin(x^2) dx$$

1,5	1,75	2
0	1	2
0,778	0,079	-0,756

$$S = \frac{0,5}{6} (0,778 + 4 \cdot 0,079 + (-0,756))$$

$$S_5 = 0,028$$

$$d = 2 - 1,5 = 0,5$$

$$S_{\text{ok}} = S_1 + S_2 + S_3 + S_4 = 0,041 + 0,268 + 0,467 + 0,028 = 0,804 \checkmark$$

5.  $f(x,y) = \arccos(x-y)$

$$-1 \leq x-y \leq 1$$

K: TR

$$\arccos(x-y) = 0 \quad / \cdot \cos$$

$$x-y = 1$$

$$-y = -x + 1 \quad / \cdot (-1)$$

$$y = x - 1$$

x	0	1
y	-1	0

$$c=1$$

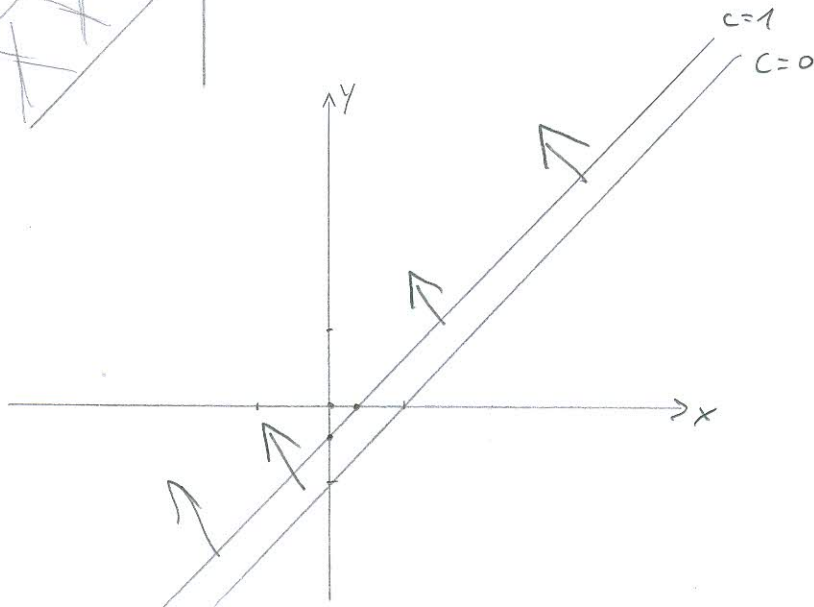
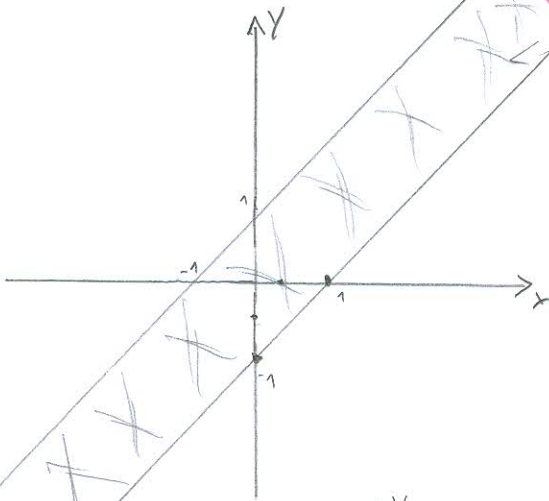
$$\arccos(x-y) = 1 \quad / \cdot \cos$$

$$x-y = 0,54$$

$$-y = -x + 0,54 \quad / \cdot (-1)$$

$$y = x - 0,54$$

x	0	1
y	-0,54	0,56



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IME I PREZIME: **ŠIME ZELENČIĆ**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0370-2014  
0269086384

PROF.  
UGLEŠIĆ

A

1. Riješiti integrale:

(a)  $\int \frac{1+2x^2}{x^2(1+x^2)} dx,$

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2. Numeričkom integracijom odrediti vrijednost  $\int_0^2 \sin(x^2) dx$ . (bodovanje: 20 za rel. grešku  $\leq 1\%$ , 15 za rel. grešku  $\leq 3\%$ , 8 za rel. grešku  $\leq 6\%$ )

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3. Riješiti  $y'' + 2y' + 5y = x^2 e^{3x} + \sin(2x)$  i provjeriti rješenje.

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$\ln x$	$\frac{1}{x}$	$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$	$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln  x + \sqrt{x^2 \pm a^2}  + C$
$e^x$	$e^x$	$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
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$\tan x$	$\frac{1}{\cos^2 x}$			
$\cot x$	$\frac{-1}{\sin^2 x}$			
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$			
$\arctan x$	$\frac{1}{1+x^2}$			

a)  $\int \frac{1+2x^2}{x^2(1+x^2)} dx = \int \frac{1+x^2+x^2}{x^2(1+x^2)} dx = \int \frac{1+x^2}{x^2(1+x^2)} dx + \int \frac{x^2}{x^2(1+x^2)} dx =$

$= \int \frac{1}{x^2} dx + \int \frac{1}{1+x^2} dx = \int x^{-2} dx + \frac{1}{1} \arctan \frac{x}{1} + C$

$= \frac{x^{-1}}{-1} + \arctan x + C = -\frac{1}{x} + \arctan x + C$

✓

$$\int_0^{\pi} (x \cos x + e^{1-3x}) dx = I$$

$$\int_0^{\pi} x \cos x dx = \begin{cases} u = x & dv = \cos x dx \\ du = dx & v = \sin x \end{cases}$$

$$\left[ x \cdot \sin x - \int \sin x dx \right]_0^{\pi}$$
$$x \cdot \sin x + \cos x \Big|_0^{\pi}$$

$$(\pi \cdot \sin^{\pi} \pi + \cos \pi) - (0 \cdot \sin 0 + \cos 0) = \underline{\underline{-2}} \quad \checkmark$$

$$\textcircled{2} \int_0^{\pi} e^{1-3x} dx = \begin{cases} 1-3x = t \\ -3dx = dt \end{cases}$$

$$= \int_0^{\pi} e^t \cdot \frac{dt}{-3} = -\frac{1}{3} \int_0^{\pi} e^t dt = \left[ -\frac{1}{3} \cdot e^{1-3x} \right]_0^{\pi} =$$

$$= \left( -\frac{1}{3} \cdot e^{1-3 \cdot \pi} \right) - \left( -\frac{1}{3} \cdot e^{1-3 \cdot 0} \right) = 0,9060208215 \quad \checkmark$$

$$I = I_1 + I_2 = -2 + 0,9060208215 = -1,0939791 \quad \checkmark$$

$$\textcircled{2} \int_0^2 \sin(x^2) dx = \left| \begin{array}{l} x^2 = t \\ 2x dx = dt \\ dx = \frac{dt}{2x} \end{array} \right| \int_0^2 \sin x \cdot x dx \quad \begin{array}{l} u = \sin x \quad du = x dx \\ du = \cos x \quad u = \frac{x^2}{2} \end{array}$$

$$\sin x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \cos x dx$$

$$= \int_0^2 \sin t \cdot \frac{dt}{2x} = \frac{1}{2} \int_0^2 \sin t \cdot \frac{dt}{x} = \frac{1}{2} \int_0^2 \sin x^2 \cdot \frac{1}{x} \cdot dt$$

$$\sin x \cdot \frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^3}{3} \cdot (-\sin x)$$

$$\sin x \cdot \frac{x^2}{2} - \frac{x^3}{6} \cdot (-\sin x)$$

$$= \left[ \frac{1}{2} \cdot (-\cos x^2) \cdot \ln x \right]_0^2$$

$$3,031 - 0$$

$$= 3,031$$

$$= \left( -\frac{1}{2} \cdot \cos x^2 \cdot \ln x \right) \Big|_0^2 = \left( -\frac{1}{2} \cdot \cos 2^2 \cdot \ln 2 \right) - \left( -\frac{1}{2} \cdot \cos 0^2 \cdot \ln 0 \right)$$

0,2265356164

=

ŠIME ŽELENIĆ

