MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: FILIP MEŠTROVIĆ **BROJ INDEKSA:** 0171256000

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 3,\; f'(0) = 3,\; f''(0) = 4\,.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{3}\mathbf{i} + (\cos(t) + 4)\,\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 3\pi]$. Zadano je skalarno polje $f\,(x, y, z) = 1 + z$. Izračunaj $\int_C f\mathrm{d}s$. 20

3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y\,\mathrm{d}x\mathrm{d}y$. 20

4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 4 + x^2$, $z = -y^2$. 20

5. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \le z \le 4$. 20

Ukupno: 40

1. $f'''(t) + 2f''(t) + f'(t) + 2f(t) = t$

$f(0) = 3$

$f'(0) = 3$

$f''(0) = 4$

$s^3F(s) - s^2\underset{=3}{f(0)} - s\underset{=3}{f'(0)} - \underset{=4}{f''(0)} + 2s^2F(s) - 2s\underset{=3}{f(0)} - 2f'(0) + sF(s) - \underset{=3}{f(0)}$

$\cancel{s^3F(s)} - 3s^2 - 3s - 4 + 2s^2F(s) - 6s - 6 + \cancel{sF(s)} - 3 + \cancel{2F(s)} = \frac{1}{s^2}$ $+2F(s) = \frac{1}{s^2}$

$F(s)(s^3 + 2s^2 + s + 2) = \frac{1}{s^2} + 3s^2 + 3s + 4 + 6 + 6s + 3$

$F(s) = \dfrac{3s^4 + 9s^3 + 13s^2 + 1}{s^2(s^3 + 2s^2 + s + 2)}$

$-8 + 8 \quad -2 + 2$

$\boxed{s = 0}$ n.t. $\quad \boxed{s = -2}$ n.t

$F(s) = \dfrac{3s^4 + 9s^3 + 13s^2 + 1}{s^2(s^2 + 1)(s + 2)}$

$s = 0 \quad s^2 = -1$ imagin $\quad s = -2$

$(s^3 + 2s^2 + s + 2)$ faktorizacija

$(s^3 + s) + (2s^2 + 2) = s(s^2 + 1) + 2(s^2 + 1)$

$(s^2 + 1)(s + 2) = \boxed{s^2 + 2s^2 + s + 2}$

$$\frac{3s^4+9s^3+13s^2+1}{s^2(s^2+1)(s+2)} = \frac{A}{s}+\frac{B}{s^2}+\frac{Cx+D}{s^2+1}+\frac{E}{s+2}$$

$0 \quad\quad -2$

$$A(\overset{-8}{s^3}+\overset{-2}{s})(\overset{-2}{s}+2)+B(s^2+1)(s+2)+(C\overset{-2}{s}+D)(\overset{-8+8}{s^3+2s^2})+E(\overset{2016+4}{s^4+s^2})$$

$$=3s^4+9s^3+13s^2+1$$

$s=0 \ldots\ldots \quad 2B=1 \quad \boxed{B=\frac{1}{2}}$

$s=-2 \ldots\ldots \quad 20E = 3\cdot 16 - 9\cdot 8 + 13\cdot 4 + 1$

$48 - 81 + 52 + 1$

$20E = 20 \quad \boxed{E=1}$

$s^4 \quad 3 = A + C + E \qquad 2 = A + C \qquad -A = C - 2 \qquad -C = A - 2$

$\boxed{A = 2 - C} \quad \boxed{C = 2 - A} \quad \boxed{C = 0}$

$s^3 \quad 9 = 2A + B + 2C + D$

$9 = 2A + \frac{1}{2} + 2C + D$

$\frac{17}{2} = 2A + 2C + D \longrightarrow \frac{17}{2} = \cancel{2A} + 4 - \cancel{2A} - \frac{A}{2} + \frac{11}{2}$

$13 = A + 2B + 2D + E$

$13 = A + 1 + 2D + 1$

$11 = A + 2D$

$-2D = A - 11 \ / :-2$

$\boxed{D = -\frac{A}{2} + \frac{11}{2}}$

$\frac{17}{2} - \frac{8}{2} - \frac{11}{2} = -\frac{A}{2} \ / \cdot 2$

$-2 = -A \quad \boxed{A = 2}$

$D = -1 + \frac{11}{2} = \boxed{\frac{9}{2}}$

$$F(s) = \frac{2}{s} + \frac{1}{2}\cdot\frac{1}{s^2} + \frac{9}{2}\cdot\frac{1}{s^2+1} + \frac{1}{s+2}$$

$$\boxed{f(t) = 2 + \frac{1}{2}t + \frac{9}{2}\sin t + e^{-2t}}$$

$$s^3F(s) - s^2\underset{3}{f(0)} - s\underset{3}{f'(0)} - \underset{4}{f''(0)} + 2s^2F(s) - 2s\underset{3}{f(0)} - 2\underset{3}{f'(0)} + sF(s) - f(0) + 2F(s) = \frac{1}{s^2}$$

$$F(s)(s^3+2s^2+s+2) - 3s^2 - 3s - 4 - 6s - 6 - 3 = \frac{1}{s^2}$$

$$F(s)(s^3+2s^2+s+2) - 3s^2 - 9s - 13 = \frac{1}{s^2}$$

$$\cancel{F(s)(s^3+2s^2+s+2)} = \frac{1+3s^4+9s^3+13s^2}{s^2(s^2+1)(s+2)} \Big/ \cdot N$$

$s=0$ $\quad$ $s=-2$

$s^3+2s^2+s+2=($

s^3+s+2s^2+2 $\qquad F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+1}$

$s(s^2+1)+2(s^2+1)$

$(s^2+1)(s+2)$

$16+4$ $\quad$ $-8+4$

$$A(s^2+2s)(s^2+1) + B(s+2)(s^2+1) + C(s^4+s^2) + (Ds^3+Es^2)(s+2)$$
$$= 3s^4+9s^3+13s^2+1$$

$s=0 \dots\; 2B=1 \quad \boxed{B=\frac{1}{2}}$ $\qquad s=-2 \dots\; 20C = 3\cdot16 - 9\cdot8 + 13\cdot4 + 1$

$20C = 48 - 72 + 52 + 1$

$C = \frac{101-72}{20} = \boxed{\frac{29}{20}}$

$3 = A + C + D$ $\qquad$ $\frac{80}{29}$ $\frac{}{51}$

$3 - \frac{29}{20} = A + D \qquad 4 - \frac{29}{20} = D = \boxed{\frac{51}{20}}$

$9 = 2A + B + 2D + E \qquad 9+2-\frac{10}{20}-\frac{102}{20} = E$ $\qquad$ $\frac{101}{72}$ $\frac{}{29}$

$13 = A + 2B + C + 2E$

$0 = 2A + B$ $\qquad$ $\frac{220-10-102}{20} = \boxed{\frac{108}{20} = E}$

$-B = 2A$ $\qquad$ $20\cdot11$ $\frac{20}{220}$ $\qquad$ $\frac{220}{112}$ $\frac{}{108}$

$-\frac{1}{2} = 2A$ $\qquad$ $\frac{54}{10} = \frac{27}{5} = E$

$\boxed{A=-1}$

$$F(s) = -\frac{1}{s} + \frac{\frac{1}{2}}{s^2} + \frac{\frac{29}{20}}{s+2} + \frac{\frac{51}{20}s}{s^2+1} + \frac{\frac{27}{5}}{s^2+1}$$

provyera

FILIP MEŠTROVIĆ 0171256000

$$f(t) = 2 + \frac{1}{2}t + \frac{9}{2}\sin t + e^{-2t}$$

X

$f(0) = 3$

$f'(0) = 3$

$f''(0) = 4$

ODJ

$2(2+\frac{1}{2}t+\frac{9}{2}\sin t+e^{-2t})+$

$\frac{1}{2}+\frac{9}{2}\cos t-2e^{-2t}+$

$2(-\frac{9}{2}\sin t+4e^{-2t})-\frac{9}{2}\cos t-8e^{-2t}$

$= 4+\frac{1}{2}+ \ldots \neq t$!!!

0

$$f'(t) = \frac{1}{2} + \frac{9}{2}\cos t - 2e^{-2t}$$

$$f''(t) = -\frac{9}{2}\sin t + 4e^{-2t}$$

$$f'''(t) = -\frac{9}{2}\cos t - 8e^{-2t}$$

$$\cancel{-\frac{9}{2}\cos t} \cancel{-8e^{-2t}} + \cancel{-9\sin t} + \cancel{8e^{-2t}} + \frac{1}{2} + \cancel{\frac{9}{2}\cos t} \cancel{-2e^{-2t}}$$

$$+4 + t + \cancel{9\sin t} + \cancel{2e^{-2t}} = t$$

$$4 + \frac{1}{2} + t \neq t$$

$$\frac{9}{2} + t \neq t$$

PROVJERA:

X

$$f(t) = -1 + \frac{1}{2}t + \frac{29}{20}e^{-2t} + \frac{51}{20}\cos t + \frac{27}{5}\sin t$$

?

$$f'(t) = \frac{1}{2} - \frac{58}{20}e^{-2t} - \frac{51}{20}\sin t + \frac{27}{5}\cos t$$

$$f''(t) = \frac{116}{20}e^{-2t} - \frac{51}{20}\cos t - \frac{27}{5}\sin t$$

$$f'''(t) = -\cancel{\frac{232}{20}e^{-2t}} + \cancel{\frac{51}{20}\sin t} - \cancel{\frac{27}{5}\cos t}$$

$$+\cancel{\frac{232}{20}e^{-2t}} - \cancel{\frac{102}{20}\cos t} - \cancel{\frac{54}{5}\sin t} + \frac{1}{2} - \cancel{\frac{58}{20}e^{-2t}}$$

$$-\cancel{\frac{51}{20}\sin t} + \cancel{\frac{27}{5}\cos t} + 2 + 1 + \cancel{\frac{58}{20}e^{-2t}} + \cancel{\frac{102}{20}\cos t} +$$

$$\cancel{\frac{54}{5}\sin t} = t \qquad 3 + \frac{1}{2}$$

$$\boxed{t = \frac{7}{2}}$$

0

X

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MATEMATIKA III

(2.)

$$r(t) = \underset{x}{\frac{t}{3}}\, i + \underset{y}{(\cos t + 4)}\, j + \underset{z}{\sin t}\, k \quad t \in [0, 3\pi]$$

$$f(x,y,z) = 1 + z$$

$$\|r'(t)\| = \sqrt{\left(\frac{1}{3}\right)^2 + (-\sin t)^2 + (\cos t)^2} = \sqrt{\frac{1}{4} + 1} = \sqrt{5}$$

$$\int_0^{3\pi} (1 + \sin t)\sqrt{5}\, dt = \left(\sqrt{5}\, t - \sqrt{5} \cos t\right)\Big|_0^{3\pi}$$

$$\sqrt{5} \cdot 3\pi - \sqrt{5} \cos 3\pi - (0 - \sqrt{5} \cdot \cos 0) = \underline{21{,}105}$$

$$= \sqrt{5} \cdot 3\pi - \sqrt{5} \cdot 0{,}98 + \sqrt{5} \cdot 1 =$$

$$= \sqrt{5}\,(3\pi - 0{,}98 + 1) \approx \underline{\sqrt{5} \cdot 3\pi}$$

$$= 21{,}1192$$ ✓

(4) $x=1$, $x=-1$, $y=1$, $y=-1$ $z=4+x^2$

$z=-y^2$

$$\int_{-1}^{1} dx \int_{-1}^{1} dy \int_{-y^2}^{4+x^2} dz =$$

$$= \int_{-1}^{1} dx \int_{-1}^{1} z \Big|_{-y^2}^{4+x^2} dy = \int_{-1}^{1} dx \int_{-1}^{1} 4+x^2+y^2 \, dy =$$

$$= \int_{-1}^{1} \left(4y + x^2y + \frac{y^3}{3}\right)\Big|_{-1}^{1} = \int_{-1}^{1} \left[\left(4+x^2+\frac{1}{3}\right) - \left(-4-x^2-\frac{1}{3}\right)\right] dx =$$

$$= \int_{-1}^{1} \left(8+2x^2+\frac{2}{3}\right) dx = \left(\frac{26}{3}x + \frac{2x^3}{3}\right)\Big|_{-1}^{1} =$$

$$= \frac{26}{3} + \frac{2}{3} - \left(-\frac{26}{3} - \frac{2}{3}\right) =$$

$$= \frac{52}{3} + \frac{4}{3} = \boxed{\frac{56}{3}}$$

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: MAURO MIŠLOV   BROJ INDEKSA: 17-2-0170-2012.

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Ukupno: ∅

1. $f'''(t) + 2f''(t) + f'(t) + 2f(t) = t \qquad f(0)=3,\ f'(0)=3\ \ f''(0)=4$

$$s^3F(s) - s^2\underbrace{f(0)}_{=3} - s\underbrace{f'(0)}_{=3} - \underbrace{f''(0)}_{=4} + 2s^2F(s) - s\underbrace{f(0)}_{=3} - \underbrace{f'(0)}_{=3} + sF(s) - \underbrace{f(0)}_{=3} + 2F(s) = \frac{1}{s^2}$$

$$s^3F(s) - 3s^2 - 3s - 4 + 2s^2F(s) - 6s - 6 + s\cdot F(s) - 3 + 2F(s) = \frac{1}{s^2}$$

$$s^3F_s + 2s^2F(s) + sF(s) + 2F(s) = \frac{1}{s^2} + 3s^2 + 3s + 4 + 6s + 6 + 3$$

$$F_s(s^3 + 2s^2 + s + 2) = \frac{1}{s^2} + 3s^2 + 9s + 13$$

$$F_s(s^3 + 2s^2 + s + 2) = \frac{1 + 3s^4 + 9s^3 + 13s^2}{s^2}$$

$$F_s\ s^2(s+2) = \frac{\dfrac{1 + 3s^4 + 9s^3 + 13s^2}{s^2}}{s^2(s+2)} \Big/ s^2(s+2)$$

$$F_s = \frac{1 + 3s^4 + 9s^3 + 13s^2}{s^4(s+2}$$

$$\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{D}{s^4}+\frac{E}{s+2}$$

$$1+3s^4+9s^3+13s^2=As^3(s+2)+Bs^2(s+2)+Cs(s+2)+D(s+2)+Es^4$$

$$=As^4+2As^3+Bs^3+2Bs^2+Cs^2+2Cs+Ds+2D+Es^4$$

$$=s^4(A+E)+s^3(2A+B)+s^2(2B+C)+s(2C+D)+2D$$

$$1+3s^4+9s^3+13s^2= \quad A+E=3$$

$$2A+B=9$$

$$2B+C=13$$

$$2C+D=0$$

$$2D=1$$

$$D=\frac{1}{2}$$

$$2C+\frac{1}{2}=0$$

$$2C=-\frac{1}{2}\ /:2$$

$$C=-\frac{1}{4}$$

$$2B+-\frac{1}{4}=13$$

$$2B=13+\frac{1}{4}$$

$$2B=\frac{53}{4}$$

$$B=\frac{53}{8}$$

$$E=\frac{41}{8}$$

$$-\frac{17}{8}+E=3$$

$$E=3+\frac{17}{8}$$

$$2A+\frac{53}{4}=9-\frac{53}{4}$$

$$2A=-\frac{17}{4}$$

$$A=-\frac{17}{8}$$

$$F_s=\frac{-\frac{17}{8}}{s}+\frac{\frac{53}{8}}{s^2}+\frac{-\frac{1}{4}}{s^3}+\frac{\frac{1}{2}}{s^4}+\frac{\frac{41}{8}}{s+2}$$

$$F_s=-\frac{17}{8}\frac{1}{s}+\frac{53}{8}\frac{1}{s^2}+-\frac{1}{4}\frac{1}{s^3}+\frac{1}{2}\cdot\frac{1}{s^4}+\frac{41}{8}\cdot\frac{1}{s+2}$$

$$F_s=-\frac{17}{8}+\frac{53}{8}t+\left(-\frac{1}{4}t\right)+\left(\frac{1}{2}\cdot t\right)+\frac{41}{8}\cdot e^{-2t}$$

PROVJERA:

$f'(t)=$?

MAURO MISLOV

4. $x=1,\ x=-1,\ y=1,\ y=-1,\ z=4+x^2,\ z=-y^2$

$$\int_{-1}^{1}\int_{-1}^{1}\int_{4+x^2}^{-y^2} dz\, dy\, dx$$

5.

$x = r\cos\varphi$

$y = r\sin\varphi$

$z \leq 4$

$$z = \sqrt{r^2\sin\varphi^2 + r^2\cos\varphi^2}$$

$$z = \sqrt{r^2(\underbrace{-\sin^2\varphi + \cos^2\varphi}_{=1})}$$

$$z = \sqrt{r^2}$$

$$z = r$$

$$\int_0^{2\pi}\int_0^{4}(r\cos\varphi^2 + r\sin\varphi^2)\cdot r \cdot dr \cdot d\varphi$$

$(r^2\cos^2\varphi + r^2\sin^2\varphi)\cdot r$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. IME I PREZIME: PETAR JELAVIĆ MITROVIĆ UGLEŠIĆ BROJ INDEKSA: 17-2-0245-2012	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno: 20

5. $\iint_S (x^2+y^2)\,ds$ $\quad z \le 4$

$S \ldots \quad z = \sqrt{x^2+y^2} \quad \left(\cancel{z'_y = \frac{y}{\sqrt{x^2+y^2}}}\right)$

$\left(\cancel{z'_x = \frac{x}{\sqrt{x^2+y^2}}} \quad \cancel{z'_y}\right)$

$z'_x = \frac{x}{\sqrt{x^2+y^2}} \qquad z'_y = \frac{y}{\sqrt{x^2+y^2}}$

$ds = \sqrt{1 + z'^2_x + z'^2_y}\,dx\,dy = \sqrt{1 + \frac{x^2+y^2}{x^2+y^2}}\,dx\,dy = \sqrt{2}\,dx\,dy$

$ds = \sqrt{2}\,dx\,dy$

$S_{xy} = \qquad 4 = \sqrt{x^2+y^2} \iff x^2+y^2 = 16$

ŠTO JE OVO?

$I = \iint_{S_{xy}} (x^2+y^2)\sqrt{2}\,dx\,dy = \left\{ \begin{array}{l} x = \rho\cos\varphi \\ y = \rho\sin\varphi \\ J = \rho \end{array} \right. \quad \varphi \in [0, 2\pi] \quad \rho \in [0, 4]$

$= \sqrt{2}\int_0^{2\pi} d\varphi \int_0^4 \rho^2\,\rho\,d\rho = \sqrt{2}\,\varphi\Big|_0^{2\pi} \cdot \frac{\rho^4}{4}\Big|_0^4 =$

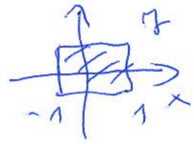
$= \sqrt{2}\cdot 2\pi \cdot \frac{16\cdot 16}{4}$

$= \sqrt{2}\pi \cdot 128$

0

④ = $x = 1$ $\quad x = -1 \quad y = -1 \quad y = 1$

$z_2 = 4 + x^2 \qquad z_1 = -y^2$ $\qquad V_{xy}$:



$V = ?$ $\qquad V = \iint\limits_{V_{xy}} (z_2 - z_1)\,dx\,dy$

$= \iint\limits_{V_{xy}} (4 + x^2 - (-y^2))\,dx\,dy = \int_{-1}^{1} dx \int_{-1}^{1} (4 + x^2 + y^2)\,dy$

$= \int_{-1}^{1} dx \left(4y + x^2 y + \frac{y^3}{3}\right)\Big|_{-1}^{1} =$

$= \left(\cancel{\int_{-1}^{1}} dx\,\cancel{4}y\,\cancel{+xx^2}\right)$

$= \int_{-1}^{1} dx \left(4 + x^2 + \frac{1}{3} + 4 + x^2 + \frac{1}{3}\right)$

$= \int_{-1}^{1} dx \left(8 + \frac{3}{3} + 2x^2\right) =$

$\left(\frac{26}{3}x + 2\frac{x^3}{3}\right)\Big|_{-1}^{1} = \left(\frac{26}{3} + \frac{2}{3}\right) - \left(-\frac{26}{3} - \frac{3}{3}\right)$

$= 2\left(\frac{28}{3}\right) = \frac{56}{3}$

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IME I PREZIME: VESNA ŠARIĆ

BROJ INDEKSA:

prof. UGLEŠIĆ

POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno:

Ø

VESNA ŠARIĆ MATEMATIKA (3) 16. 09. 2015

(1)

$$f'''(t)+2f''(t)+f'(t)+2f(t)=t \qquad f(0)=3,\ f'(0)=3,\ f''(0)=4$$

$$(s^3F(s)-s^2\underbrace{f(0)}_{3}-s\underbrace{f'(0)}_{3}-\underbrace{f''(0)}_{4}+2\left(s^2F(s)-s\underbrace{f(0)}_{3}-\underbrace{f'(0)}_{3}\right)+\left(sF(s)-\underbrace{f(0)}_{3}\right)+2F(s)=\frac{1}{s^2}$$

$$(s^3F(s)-3s^2-3s-4)+2(s^2F(s)-3s-3)+(sF(s)-3+2F(s))=\frac{1}{s^2}$$

$$(\cancel{s^3F(s)}-3s^2-3s-4+\cancel{2s^2F(s)}-6s-6+\cancel{sF(s)}-3+2F(s))=\frac{1}{s^2}$$

$$F(s)(s^3+2s^2+s+2)=\frac{1}{s^2}+3s^2+3s+4+6s+6+3$$

$$F(s)(s^3+2s^2+s+2)=\frac{1}{s^2}+3s^2+9s+13$$

$$F(s)(s^3+2s^2+s+2)=\frac{1+3s^4+9s^3+13s^2}{s^2}$$

$$F(s)=\frac{\frac{1+3s^4+9s^3+13s^2}{s^2}}{\frac{(s^3+2s^2+s+2)}{1}}$$

$$F(s)=\frac{1+3s^4+9s^3+13s^2}{s^2(s^3+2s^2+s+2)}$$

$$F(s)=\frac{1+3s^4+9s^3+13s^2}{s^2(s+2)(s^2+1)}$$

$$=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+2}+\frac{Ds+E}{s^2+1}$$

$$=A(s(s+2)(s^2+1)+B((s+2)(s^2+1))+C(s^2(s^2+1))+(Ds+E)(s^2(s+2))$$

$$= A(s(s^3+2s^2+s+2)+B(s^3+2s^2+s+2)+C(s^4+s^2)+(Ds+E)(s^3+2s^2)$$

$$= A(s^4+2s^3+s^2+2s)+B(s^3+2s^2+s+2)+C(s^4+s^2)+(Ds+E)(s^3+2s^2)$$

$$= As^4+2As^3+As^2+2As+Bs^3+2Bs^2+Bs+2B+Cs^4+Cs^2+Ds^4+Es^3+2Ds^3+2Es^2$$

uz s^4 $\quad 3 = A+C+D$

uz s^3 $\quad 9 = 2A+B+E+2D$

uz s^2 $\quad 13 = A+2B+C+2E$

uz s $\quad 0 = 2A+B$

uz 1 $\quad 1 = 2B \Rightarrow \boxed{B=\frac{1}{2}}$

$$0 = 2A+\frac{1}{2}$$

$$2A+\frac{1}{2}=0$$

$$2A=-\frac{1}{2}$$

$$\boxed{A=-\frac{1}{4}}$$

$$A+C+D=3$$

$$-\frac{1}{4}+C+D=3$$

$$C=3+\frac{1}{4}-D$$

$$\boxed{C=\frac{13}{4}-D}$$

$$2A+B+E+2D=9$$
$$A+2B+C+2E=13$$

$$\cancel{2}\cdot\left(-\frac{1}{\cancel{4}_2}\right)+\frac{1}{2}+E+2D=9$$
$$-\frac{1}{4}+2\cdot\frac{1}{2}+\left(\frac{13}{4}-D\right)+2E=13$$

$$-\frac{1}{2}+\frac{1}{2}+E+2D=9$$
$$-\frac{1}{4}+1+\frac{13}{4}-D+2E=13$$

$$E+2D=9$$
$$-\frac{1}{4}+\frac{4}{4}+\frac{13}{4}-D+2E=13$$

$$E+2D=9$$
$$4-D+2E=13$$

$$E+2D=9$$
$$2E-D=9 \quad /\cdot 2$$

$$E+2D=9$$
$$4E-2D=18$$

$$5E=27$$

$$\boxed{E=\frac{27}{5}}$$

$$= \frac{-\frac{1}{4}}{s}+\frac{\frac{1}{2}}{s^2}+\frac{\frac{11}{20}}{s+1}+\frac{\frac{9}{5}s+\frac{27}{5}}{s^2+1} \Big/ \mathcal{L}^{-1}$$

$$\boxed{= -\frac{1}{4}+\frac{1}{2}t+\frac{11}{20}e^{-t}+\frac{9}{5}\cdot\cos t+\frac{27}{5}\sin t}$$

PROVJERA:

$$f(0) = -\frac{1}{4}+\frac{11}{20}+\frac{9}{5} = \frac{-5+20+36}{20} = \frac{51}{20}$$

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$A+C+D=3$

$-\frac{1}{4}+C+D=3$

$C=3+\frac{1}{4}-D$

$C=\frac{13}{4}-D$

$2A+B+E+2D=9$

$A+2B+\left(\frac{13}{4}-D\right)+2E=13$

$2\cdot\left(-\frac{1}{4}\right)+\frac{1}{2}+E+2D=9$

$-\frac{1}{4}+2\cdot\frac{1}{2}+\frac{13}{4}-D+2E=13$

$-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}+E+2D=9$

$-\frac{1}{4}+1+\frac{13}{4}-D+2E=13$

$E+2D=9$

$\frac{-1+4+13}{4}-D+2E=13$

$E+2D=9$

$4-D+2E=13$

$E+2D=9$

$-D+2E=13-4$

$E+2D=9 \quad |\cdot(-2)$

$2E-D=9 \quad |\cdot 2$

$E+2D=9$

$4E-2D=18$

$5E=27$

$\boxed{E=\frac{27}{5}}$

$-\cancel{2E}-4D=-18$

$\cancel{2E}-D=9$

$-5D=-9$

$D=\frac{9}{5}$

$2A+B+E+2D=9$

$2\cdot\left(-\frac{1}{4}\right)+\frac{1}{2}+\frac{27}{5}+2D=9$

$-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}+\frac{27}{5}+2D=9$

$2D=9-\frac{27}{5}$

$2D=\frac{45}{5}-\frac{27}{5}$

$2D=\frac{18}{5}$

$D=\frac{9}{5}$

$\frac{18}{5}/2=\frac{18}{10}$

$C=\frac{13}{4}-\frac{9}{5}$

$C=\frac{45-36}{20}$

$C=\frac{11}{20}$

$A+C+D=3$

$-\frac{1}{4}+\frac{11}{20}+\frac{9}{5}=3$

$\frac{-80+11+36}{20}=3$

$-69+36=3$

$\frac{33}{20}=2$