MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 3,\ f'(0) = 3,\ f''(0) = 4.$$

2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{3}\mathbf{i} + (\cos(t) + 4)\,\mathbf{j} + \sin t\,\mathbf{k}$, $t \in [0, 3\pi]$. Zadano je skalarno polje $f(x, y, z) = 1 + z$. Izračunaj $\int_C f\,ds$. 20

3. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y\,dx dy$. 20

4. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 4 + x^2$, $z = -y^2$. 20

5. Izračunati $\iint_S (x^2 + y^2) dS$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \le z \le 4$. 20

Ukupno: 40

1. $f'''(t) + 2f''(t) + f'(t) + 2f(t) = t$

$f(0) = 3$, $f'(0) = 3$, $f''(0) = 4$

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 2s^2F(s) - 2sf(0) - 2f'(0) + sF(s) - f(0) + 2F(s) = \frac{1}{s^2}$

$s^3F(s) - 3s^2 - 3s - 4 + 2s^2F(s) - 6s - 6 + sF(s) - 3 + 2F(s) = \frac{1}{s^2}$

$F(s)(s^3 + 2s^2 + s + 2) = \frac{1}{s^2} + 3s^2 + 3s + 4 + 6 + 6s + 3$

$F(s) = \frac{3s^4 + 9s^3 + 13s^2 + 1}{s^2(s^3 + 2s^2 + s + 2)}$

$F(s) = \frac{3s^4 + 9s^3 + 13s^2 + 1}{s^2(s^2 + 1)(s + 2)}$

$s = 0$ n.t. $\quad s = -2$ n.t. $\quad s^2 = -1$ imagin

$(s^3 + 2s^2 + s + 2)$ faktorizacija

$(s^3 + s) + (2s^2 + 2) = s(s^2 + 1) + 2(s^2 + 1)$

$(s^2 + 1)(s + 2) = s^2 + 2s^2 + s + 2$

$$\frac{3s^4+9s^3+13s^2+1}{s^2(s^2+1)(s+2)} = \frac{A}{s}+\frac{B}{s^2}+\frac{Cx+D}{s^2+1}+\frac{E}{s+2}$$

$s=0$, $s=-2$

$$A(s^3+s)(s+2)+B(s^2+1)(s+2)+(Cs+D)(s^3+2s^2)+E(s^4+s^2)$$
$$= 3s^4+9s^3+13s^2+1$$

$s=0$ .... $2B=1$ $\boxed{B=\frac{1}{2}}$

$s=-2$ .... $20E = 3\cdot 16 - 9\cdot 8 + 13\cdot 4 + 1$

$18 - 81 + 52 + 1$

$20E = 20$ $\boxed{E=1}$

s^4 $3 = A + C + E$ $\quad 2 = A + C$ $\quad -A = C-2$ $\quad -C = A-2$

$\boxed{A = 2-C}$ $\quad \boxed{C = 2-A}$

$\boxed{C=0}$

s^3 $9 = 2A + B + 2C + D$

$9 = 2A + \frac{1}{2} + 2C + D$

$\frac{17}{2} = 2A + 2C + D \longrightarrow \frac{17}{2} = 2A + 4 - 2A - \frac{A}{2} + \frac{11}{2}$

$13 = A + 2B + 2D + E$

$13 = A + 1 + 2D + 1$

$11 = A + 2D$

$\frac{17}{2} - \frac{8}{2} - \frac{11}{2} = -\frac{A}{2} \;/\cdot 2$

$-2 = -A$ $\quad \boxed{A=2}$

$-2D = A - 11 \;/:-2$

$\boxed{D = -\frac{A}{2} + \frac{11}{2}}$

$D = -1 + \frac{11}{2} \rightarrow \boxed{\frac{9}{2}}$

$$F(s) = \frac{2}{s} + \frac{1}{2}\cdot\frac{1}{s^2} + \frac{9}{2}\cdot\frac{1}{s^2+1} + \frac{1}{s+2}$$

$$\boxed{f(t) = 2 + \frac{1}{2}t + \frac{9}{2}\sin t + e^{-2t}}$$

$$s^3F(s) - s^2(f_0) - sf'(0) - f''(0) + 2s^2F(s) - 2sf(0) - 2f'(0) + sF(s) - f(0) + 2F(s) = \frac{1}{s^2}$$
(3) (3) (4) (3) (3)

$$F(s)(s^3+2s^2+s+2) - 3s^2 - 3s - 4 - 6s - 6 - 3 = \frac{1}{s^2}$$

$$F(s)(s^3+2s^2+s+2) - 3s^2 - 9s - 13 = \frac{1}{s^2}$$

$$F(s)(s^3+2s^2+s+2) = \frac{1+3s^4+9s^3+13s^2}{s^2(s^2+1)(s+2)} \quad / \cdot N$$

$s=0$ $\quad s=-2$

$s^3+2s^2+s+2=($

s^3+s+2s^2+2

$s(s^2+1)+2(s^2+1)$

$(s^2+1)(s+2)$

$$F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+1}$$

$$A(s^2+2s)(s^2+1) + B(s+2)(s^2+1) + C(s^4+s^2) + (Ds^3+Es^2)(s+2)$$
$$= 3s^4 + 9s^3 + 13s^2 + 1$$

$s=0 \dots 2B = 1 \quad \boxed{B = \frac{1}{2}}$

$s=-2 \dots 20C = 3\cdot 16 - 9\cdot 8 + 13\cdot 4 + 1$

$20C = 48 - 72 + 52 + 1$

$C = \frac{101-72}{20} = \boxed{\frac{29}{20}}$

$3 = A + C + D$

$3 - \frac{29}{20} = A + D \quad 4 - \frac{29}{20} = D = \boxed{\frac{51}{20}}$

$9 = 2A + B + 2D + E \quad 9 + 2 - \frac{10}{20} - \frac{102}{20} = E$

$13 = A + 2B + C + 2E$

$0 = 2A + B$

$-B = 2A$

$-\frac{1}{2} = 2A$

$\boxed{A = -1}$

$\frac{220-10-102}{20} = \boxed{\frac{108}{20} = E}$

$\frac{54}{10} = \frac{27}{5} = E$

$$F(s) = -\frac{1}{s} + \frac{\frac{1}{2}}{s^2} + \frac{\frac{29}{20}}{s+2} + \frac{\frac{51}{20}s}{s^2+1} + \frac{\frac{27}{5}}{s^2+1}$$

provjera

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$f(t) = 2 + \frac{1}{2}t + \frac{9}{2}\sin t + e^{-2t}$ X

$f'(t) = \frac{1}{2} + \frac{9}{2}\cos t - 2e^{-2t}$

$f''(t) = -\frac{9}{2}\sin t + 4e^{-2t}$

$f'''(t) = -\frac{9}{2}\cos t - 8e^{-2t}$

$f(0) = 3$

$f'(0) = 3$

$f''(0) = 4$

ODJ

$2(2 + \frac{1}{2}t + \frac{9}{2}\sin t + e^{-2t}) +$

$\frac{1}{2} + \frac{9}{2}\cos t - 2e^{-2t} +$

$2(-\frac{9}{2}\sin t + 4e^{-2t}) - \frac{9}{2}\cos t - 8e^{-2t}$

$= 4 + \frac{1}{2} + \ldots \neq t$!!!

$-\frac{9}{2}\cos t - 8e^{-2t} + -9\sin t + 8e^{-2t} + \frac{1}{2} + \frac{9}{2}\cos t - 2e^{-2t}$

$+4 + t + 9\sin t + 2e^{-2t} = t$

$4 + \frac{1}{2} + t \neq t$

$\frac{9}{2} + t \neq t$

PROVJERA:

X

$f(t) = -1 + \frac{1}{2} + \frac{29}{20}e^{-2t} + \frac{51}{20}\cos t + \frac{27}{5}\sin t$?

$f'(t) = \frac{1}{2} - \frac{58}{20}e^{-2t} - \frac{51}{20}\sin t + \frac{27}{5}\cos t$

$f''(t) = \frac{116}{20}e^{-2t} - \frac{51}{20}\cos t - \frac{27}{5}\sin t$

$f'''(t) = -\frac{232}{20}e^{-2t} + \frac{51}{20}\sin t - \frac{27}{5}\cos t$

$+\frac{232}{20}e^{-2t} - \frac{102}{20}\cos t - \frac{54}{5}\sin t + \frac{1}{2} - \frac{58}{20}e^{-2t}$

$-\frac{51}{20}\sin t + \frac{27}{5}\cos t + 2 + 1 + \frac{58}{20}e^{-2t} + \frac{102}{20}\cos t +$

$\frac{54}{5}\sin t = t$

$3 + \frac{1}{2}$

$t = \frac{7}{2}$

X

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MATEMATIKA III

2. $r(t) = \frac{t}{3}i + (\cos t + 4)j + \sin t\, k \quad t \in [0, 3\pi]$

$f(x, y, z) = 1 + z$

$\|r'(t)\| = \sqrt{\left(\frac{1}{3}\right)^2 + (-\sin t)^2 + (\cos t)^2} = \sqrt{\frac{1}{4} + 1} = \sqrt{5}$

$$\int_0^{3\pi} (1 + \sin t)\sqrt{5}\, dt = \left(\sqrt{5}t - \sqrt{5}\cos t\right)\Big|_0^{3\pi}$$

$\sqrt{5} \cdot 3\pi - \sqrt{5}\cos 3\pi - (0 - \sqrt{5} \cdot \cos 0) = 21{,}105$

$= \sqrt{5} \cdot 3\pi - \sqrt{5} \cdot 0{,}98 + \sqrt{5} \cdot 1 =$

$= \sqrt{5}(3\pi - 0{,}98 + 1) \approx \sqrt{5} \cdot 3\pi$

$= 21{,}1192$ ✓