MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: MARINO ŽUBČIĆ BROJ INDEKSA: 17-2-0216-2012	Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: 20

$$f'''(t) + 4f'(t) = t, \quad f(0) = 5,\ f'(0) = 2,\ f''(0) = 4.$$

2. Da li krivuljni integral u vektorskom polju $g = y\mathbf{i} - x\mathbf{j}$ ovisi o putu integracije? 20

3. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 5^2$, ravninom $z = -5$ i parabolom $z = x^2 + y^2$. Napomena: obzirom da je više takvih tijela traži se ono najmanje koje sadrži ishodište. 20

4. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 4z$, $z \le 5$. 20

5. Neka je C cilindar zadan sa $C = \{(x, y, z) : x^2 + z^2 \le 5,\ 1 \le y \le 4\}$. Izračunati plošni integral 20

$$\iint\limits_{\widehat{\partial C}} y^2 x \, dy dz$$

Ukupno: 60

④ $x^2 + y^2 = 4z,\ z \le 5$; $\iint ds = ?$

$$z = \frac{x^2}{4} + \frac{y^2}{4}$$

$$I = \sqrt{1 (\tfrac{2}{4}x)^2 + (\tfrac{2}{4}y)^2} = \int_0^{2\pi} d\varphi \int_0^{\sqrt{20}} \sqrt{1 + \frac{4}{16}\underbrace{(x^2+y^2)}_{r^2}} \cdot r\,dr$$

$$I = \int_0^{2\pi} d\varphi \int_0^{\sqrt{20}} \sqrt{1 + \frac{1}{4}r^2} \cdot r\,dr = \left| \begin{array}{l} 1 + \frac{1}{4}r^2 = t \\ \frac{1}{2} r\,dr = dt \\ r\,dr = 2dt \end{array} \right|$$

$$I = 2\pi \cdot \int_1^6 \cdot 2\sqrt{t}\,dt = 2\pi \cdot 2 \cdot \frac{r^{\frac{3}{2}}}{\frac{3}{2}} \Big|_1^6$$

$$I = 4\pi \cdot \frac{2}{3}\left(6^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{8\pi}{3} \cdot 13.69 \approx 114.747$$

③ $x^2+y^2=5^2$ $\quad x = r\cos\varphi \quad x^2+y^2=r^2$

$z=-5 \qquad y = r\sin\varphi$

$z = x^2+y^2 \qquad z=z$

$$V=\iiint dx\,dy\,dz \qquad dx\,dy\,dz = r\,dr\,d\varphi\,dz$$

$$V=\int_0^{2\pi}\int_0^{5}\int_{-5}^{r^2} r\,dz\,dr\,d\varphi \quad ✓$$

$\varphi \in [0, 2\pi]$

$r \in [0, 5]$

$z \in [-5, r^2]$

$$V=\int_0^{2\pi}\int_0^{\sqrt5}(r^2+5)\cdot r\,dr\,d\varphi$$

$$V=\int_0^{2\pi} \frac{r^4}{4}+5\frac{r^2}{2}\Big|_0^{\sqrt5} d\varphi$$

$$V=2\pi\cdot\left[\left(\frac{25}{4}+\frac{25}{2}\right)-(0+0)\right]$$

$$V=2\pi\cdot\frac{75}{4}$$

$$V=\frac{75}{2}\pi \quad ✓$$

⑤ $C=\{(x,y,z) : x^2+z^2\le 5,\ 1\le y\le 4$

$\operatorname{div}\vec{w} = y^2+0+0 = r^2$?

KAKO JE DOŠLO DO OVE GREŠKE ?

$$I=\int_0^{2\pi}d\varphi\int_0^{\sqrt5}dr\int_1^4 y^2\cdot r\,dy \quad ✓$$

$$I=2\pi\int_0^{\sqrt5}\cdot r\cdot\frac{y^3}{3}\Big|_1^4 dr$$

$$I=2\pi\cdot\frac{1}{3}\int_0^{\sqrt5} r\,(4^3-1^3)\,dr$$

$$I=\frac{2\pi}{3}\cdot 63\cdot\frac{r^2}{2}\Big|_0^{\sqrt5}$$

⑤ - nastavak

$$I = 21\pi(5-0)$$

$$I = 105\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: KANKO BRKIĆ   BROJ INDEKSA: 17-1-0091-2011

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: ~~20~~

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4. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 4z$, $z \le 5$. ~~20~~
5. Neka je C cilindar zadan sa $C = \{(x, y, z) : x^2 + z^2 \le 5,\ 1 \le y \le 4\}$. Izračunati plošni integral 20

$$\iint\limits_{\partial C} y^2 x \,\mathrm{d}y\mathrm{d}z$$

Ukupno: 35

① Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete.

$f'''(t)+4f'(t)=t, \quad f(0)=5, \quad f'(0)=2, \quad f''(0)=4.$

$s^3F(s)-s^2f(0)-sf'(0)-f''(0)+4fF(s)-4f = \frac{1}{s^2}$

$F(s)(s^3+4s)-5s^2-2s-4-20=\frac{1}{s^2}$

$F(s)(s^3+4s)=\frac{1}{s^2}+5s^2+2s+24$

$F(s)(s^3+4s)=\dfrac{5s^4+2s^3+24s^2+1}{s^2}$

$F(s)=\dfrac{5s^4+2s^3+24s^2+1}{s^3(s^2+4)}$

$\dfrac{A}{s^3}+\dfrac{B}{s^2}+\dfrac{C}{s}+\dfrac{D+E}{s^2+4} \quad / \cdot s^3(s^2+4)$

$A(s^2+4)+Bs(s^2+4)+Cs^2(s^2+4)+(Ds+E)\cdot s^3=$

$As^2+4A+Bs^3+4Bs+4Cs^4+4Cs^2+Ds^4+Es^3=$

$s^4(C+D)+s^3(B+E)+s^2(A+4C)+s(4B)+4A=$

$C + D = 5$

$D + E = 2$

$A + 4C = 24$

$4B = 0 \quad B = 0$

$4A = 1 \quad A = \frac{1}{4} \quad D = -\frac{15}{16}$

$E = 2 \quad C = \frac{95}{16}$

DAJE!

② $y = yi - xj$ $\quad g = \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix}$

RANKO BRKIĆ 16.09.2015.

Brodostrojarstvo

$\oint_c w\,dr = ?$ $\quad$ rot.w $= \begin{vmatrix} i & j & k \\ y & -x & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = k(1-(-1)) = 2k$ ✓

ovisi o kutu integracije

$\text{rot } g = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1-1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$

20

② Da li krivuljni integral u vektorskom polju $g = yi - xj$ ovisi o putu integracije?

$\cancel{g = yi - xj = \int}$

$\cancel{\int g = \int yi - \int xj}$

$\cancel{g \int 1 = yi \int 1 - xj \int 1}$

AR
5

RANKO BRKIĆ

Brodostrojarstvo

16.09.2015.

③

$x^2+y^2=5^2$

$z=-5$

$z=x^2+y^2$

$x=r\cos f$

$y=r\sin f$

$z=z \quad z\in[-5, r^2]$

$f\in[0, 2\pi]$

$r\in[0,5]$

$V=\int_0^{2\pi}\int_0^{5}\int_{-5}^{r^2} r\,dr\,df\,rz$ ✓ $r^2\cos^2 f + r^2\sin^2 f = r^2$

$z=r^2$

15

④ Izračunajte površinu oplošja paraboloida $x^2+y^2=4z$, $z\le 5$

$x^2+y^2=4z$

$z\le 5$

$$(x,y)=\begin{bmatrix} x \\ y \\ \frac{x^2}{4}+\frac{y^2}{4} \end{bmatrix}$$

PREVIŠE POGREŠAKA

$\varnothing$

$$\frac{\partial x}{x}=\begin{bmatrix}1\\0\\\frac{x}{2}\end{bmatrix} \quad \partial\left(\frac{y}{y}\right)=\begin{bmatrix}0\\1\\\frac{y}{2}\end{bmatrix} \quad \left|\partial\frac{x}{x}\cdot\partial\frac{y}{y}\right|=\begin{vmatrix} i & j & k \\ 1 & 0 & \frac{1}{2}x \\ 0 & 1 & \frac{1}{2}y\end{vmatrix}=i\begin{vmatrix}0 & \frac{1}{2}x \\ 1 & \frac{1}{2}y\end{vmatrix}-$$

$$j\begin{vmatrix}1 & \frac{1}{2}x \\ 0 & \frac{1}{2}y\end{vmatrix}+k\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}=-\frac{1}{2}xi-\frac{1}{2}yi+k$$

$$\bar{h}=\begin{vmatrix}-\frac{1}{2}x\\-\frac{1}{2}y\\1\end{vmatrix} \qquad \|\bar{h}\|=\sqrt{\left(-\frac{1}{2}x\right)^2+\left(-\frac{1}{2}y\right)^2+1}=\frac{\sqrt{x^2+y^2+4}}{2}$$

$x=r\cos\varphi$

$y=r\sin\varphi$

$$(r\cos\varphi)^2+(r\sin\varphi)^2=4\cdot 5$$

$r^2=20$

$r=\sqrt{20}$

$r\in[0,\sqrt{20}]$

$\varphi\in[0,2\pi]$

$$\iint dS=\int_0^{2\pi}\int_0^{\sqrt{20}}\frac{\sqrt{r^2+4}\; r\,dr\,d\varphi}{2}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: TOMISLAV GLAVAN **BROJ INDEKSA:** 17-0115-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: 20

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4. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 4z$, $z \leq 5$. ~~20~~ ~~5~~ ~~15~~ 20

5. Neka je C cilindar zadan sa $C = \{(x, y, z) : x^2 + z^2 \leq 5,\ 1 \leq y \leq 4\}$. Izračunati plošni integral 20

$$\iint\limits_{\widehat{\partial C}} y^2 x \, dy dz$$

Ukupno: 20

1) $f'''(t) + 4f'(t) = t$

$f(0) = 5$

$f'(0) = 2$

$f''(0) = 4$

$$s^3 f(s) - s^2 f(0) - s \cdot f'(0) - f''(0) + 4(s f(s) - f(0)) = \frac{1}{s^2}$$

$$s^3 f(s) - s^2 \cdot 5 - s \cdot 2 - 4 + 4(s \cdot f(s) - 5) = \frac{1}{s^2}$$

$$s^3 f(s) - 5s^2 - 2s - 4 + 4s \cdot f(s) - 20 = \frac{1}{s^2}$$

$$s^3 f(s) + 4s f(s) = \frac{1}{s^2} + 5s^2 + 2s + 4 + 20$$

$$f(s) \cdot (s^3 + 4s) = \frac{1}{s^2} + 5s^2 + 2s + 24$$

$$f(s)(s^3 + 4s) = \frac{1 + 5s^4 + 2s^3 + 24s^2}{s^2} \quad / (s^3 + 4s)$$

$$f(s) = \frac{\frac{1 + 5s^4 + 2s^3 + 24s^2}{s^2}}{\frac{(s^3 + 4s)}{1}} = \frac{1 + 5s^4 + 2s^3 + 24s^2}{s^5 + 4s^3}$$

$$\frac{1 + 5s^4 + 2s^3 + 24s^2}{s^3(s^2 + 4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2 + 4}$$

$$\frac{1+5s^4+2s^3+24s^2}{s^3(s^2+4)} = \frac{As^2(s^2+4)+Bs(s^2+4)+C(s^2+4)+(Ds+E)\cdot s^3}{s^3(s^2+4)}$$

$$\cancel{1}+\cancel{5s^4}+\cancel{2s^3}+\cancel{24}s^2 = \cancel{As^4}+\cancel{4As^2}+\cancel{Bs^3}+\cancel{4B}s+\cancel{Cs^2}+\cancel{4C}+\cancel{Ds^4}+\cancel{Es^3}$$

$A+D=5 \Rightarrow \frac{95}{16}+D=5 \Rightarrow D=5-\frac{95}{16} \Rightarrow \boxed{D=-\frac{15}{16}}$

$B+E=2 \Rightarrow 0+E=2 \Rightarrow \boxed{E=2}$

$4A+C=24 \Rightarrow 4A+\frac{1}{4}=24 \Rightarrow 4A=24-\frac{1}{4} \Rightarrow 4A=\frac{95}{4}/4$

$4B=0 \Rightarrow 4B=0/4$

$4C=1/4$ $\boxed{B=0}$

$\boxed{C=\frac{1}{4}}$ $\boxed{A=\frac{95}{16}}$

$$f(s)=\frac{\frac{95}{16}}{s}+\frac{0}{s^2}+\frac{\frac{1}{4}}{s^3}+\frac{-\frac{15}{16}s+2}{s^2+4} \Big/ \mathcal{L}^{-1}$$

$$f(t)=\frac{95}{16}+0+\frac{1}{4}\cdot\frac{t^{3-1}}{(3-1)!}-\frac{15}{16}\cos\sqrt{4}t+2\sin\sqrt{4}t$$

$$f(t)=\frac{95}{16}+\frac{1}{4}\cdot\frac{t^2}{2}-\frac{15}{16}\cos\sqrt{4}t+2\sin\sqrt{4}t$$

$$f(t)=\frac{95}{16}+\frac{t^2}{8}-\frac{15}{16}\cos\sqrt{4}t+2\sin\sqrt{4}t$$

PROVJERA:
$f'(t)=\frac{t}{4}+\frac{15}{8}\sin(2t)+4\cos(2t)$
$f'(0)=4$ X

Ø

ZAŠTO?

2) $g=y\vec{i}-x\vec{j}$

$g=(y,x)$

$\frac{y}{\partial x}=0;\ \frac{x}{\partial y}=0.$?

→ g je potencijalno polje pa NE ovisi o putu integracije

Ø

NISTE POKAZALI DA JE POTENCIJALNO POLJE!

TOMISLAV GLAVAN
17-0115-2011

4) $x^2+y^2=4z, \quad z\leq 5$

$$\iint \sqrt{1+\left(\frac{z}{\partial x}\right)^2+\left(\frac{z}{\partial y}\right)^2}\cdot dx\,dy$$

$$=\iint \sqrt{1+\left(\frac{1}{4}\cdot 2x\right)^2+\left(\frac{1}{4}\cdot 2y\right)^2}\,dx\,dy$$

$$=\iint \sqrt{1+\left(\frac{1}{16}\cdot 4x^2\right)+\left(\frac{1}{16}\cdot 4y^2\right)}\cdot dx\,dy$$

$$=\iint \sqrt{1+\left(\frac{4x^2}{16}\right)+\left(\frac{4y^2}{16}\right)}\quad dx\,dy$$

$$=\iint \sqrt{1+\frac{\cancel{4}(x^2+y^2)}{\cancel{16}\,4}}\cdot dx\,dy=\iint\sqrt{1+\frac{x^2+y^2 \Rightarrow 4z}{4}}\quad dx\,dy$$

$$=\int_0^{2\pi}\int_0^{\sqrt{20}} \sqrt{1+\frac{4z}{4}}\quad dx\,dy \qquad \times$$

$$=\int_0^{2\pi}\int_0^{\sqrt{20}} \sqrt{1+\frac{\frac{r^2}{4}}{4}}\cdot r\cdot dr\,d\varphi \qquad \checkmark$$

$$=\int_0^{2\pi}\int_0^{\sqrt{20}} \sqrt{1+\frac{r^2}{16}}\cdot r\cdot dr\cdot d\varphi$$

$4z=x^2+y^2 \,/\,4$

$z=\frac{x^2+y^2}{4}$

$z=\frac{x^2}{4}+\frac{y^2}{4}\,/'$

$$\boxed{\frac{z}{\partial x}=\frac{1}{4}\cdot 2x}$$

$$\boxed{\frac{z}{\partial y}=\frac{1}{4}\cdot 2y}$$

$x=r\cos\varphi$

$y=r\sin\varphi$

$z=\frac{r^2}{4}$

$\Downarrow$

$x^2+y^2=r^2$

$x^2+y^2=4z$

$4z=r^2\,/\,4$

$z=\frac{r^2}{4}$

GRANICA

$\frac{r^2}{4}=5\,/\,4$

$r^2=5\cdot 4 \Rightarrow r^2=20\,/\sqrt{}$

$r=\sqrt{20}$

$$= \int_0^{2\pi}\int_0^{\sqrt{20}} \sqrt{1+\frac{r^2}{16}} \cdot r \cdot dr\, d\varphi$$

$$= \int_0^{2\pi}\int_0^{\sqrt{20}} \sqrt{t} \cdot \cancel{r} \cdot \frac{16\,dt}{2\cdot\cancel{r}} \cdot d\varphi$$

$$= \int_0^{2\pi}\int_1^{\frac{9}{4}} \sqrt{t} \cdot \overset{8}{\frac{16}{2}}\, d\varphi$$

INTEGRIRATI $\left(= t^{\frac{1}{2}}\right) \int \Rightarrow \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \Rightarrow \frac{t^{\frac{3}{2}}}{\frac{3}{2}}$

$$= 8\int_0^{2\pi} \cdot \left.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right|_1^{\frac{9}{4}} \cdot d\varphi$$

$$= 8\int_0^{2\pi} \cdot \left(\frac{\left(\frac{9}{4}\right)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1^{\frac{3}{2}}}{\frac{3}{2}}\right) d\varphi$$

$$= 8\int_0^{2\pi} \cdot \frac{19}{12} \cdot d\varphi$$

$$= 8\cdot \frac{19}{12}\int_0^{2\pi} d\varphi$$

$$= \frac{38}{3} \cdot (2\pi - 0)$$

$$= \frac{38}{3} \cdot 2\pi$$

$$= \frac{76}{3}\pi$$

SUPSTITUCIJA

$$t = 1+\frac{r^2}{16} \;/\,'$$

$$= \frac{1}{16}\cdot 2r\,dr \;/\left(\frac{1}{16}\cdot 2r\right)$$

$$= \frac{dt}{\frac{1}{16}\cdot 2r}$$

$$= \frac{\frac{dt}{1}}{\frac{2r}{16}}$$

$$= \boxed{\frac{16dt}{2r}}$$

$$0 \Rightarrow 1+\frac{0^2}{16}$$

$$= 1+0 = 1$$

$$\boxed{0 \Rightarrow 1}$$

$$\sqrt{20} \Rightarrow 1+\frac{(\sqrt{20})^2}{16}$$

$$= 1+\frac{20}{16}$$

$$= 1+\frac{5}{4}$$

$$= \frac{9}{4}$$

$$\boxed{\sqrt{20} \Rightarrow \frac{9}{4}}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARIN MATEK BROJ INDEKSA: 17-1-0111-12

PROF. UGLEŠIĆ

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: ~~20~~

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$$\iint\limits_{\partial C} y^2 x \, dy dz$$

Ukupno: Ø

1. $f'''(t)+4f'(t)=t \quad f(0)=5,\ f'(0)=2,\ f''(0)=4$

$$s^3F(s)-s^2f(0)-sf'(0)-f''(0)+4\left(sF(s)-f(0)\right)=\frac{1}{s^2}$$

$$s^3F(s)-s^2\underbrace{f(0)}_{5}-s\underbrace{f'(0)}_{2}-\underbrace{f''(0)}_{4}+4sF(s)-4\underbrace{f(0)}_{5}=\frac{1}{s^2}$$

$$s^3F(s)-5s^2-2s-4+4sF(s)-20=\frac{1}{s^2}$$

$$s^3F(s)+4sF(s)=\frac{1}{s^2}+5s^2+2s+24$$

$$F(s)(s^3+4s)=\frac{1+5s^4+2s^3+24s^2}{s^2} \quad /:(s^3+4s)$$

$$F(s)=\frac{5s^4+2s^3+24s^2+1}{s^3(s^2+4)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{Ds+E}{s^2+4} \quad / s^3(s^2+4)$$

$$5s^4+2s^3+24s^2+1 = A\left(s^2(s^2+4)\right) + B\left(s(s^2+4)\right) + C\left(s^2+4\right) + (Ds+E)(s^3)$$

$$= A(s^4+4s^2) + B(s^3+4s) + C(s^2+4) + (Ds+E)s^3$$

$$= \underline{As^4} + \underline{4As^2} + \underline{Bs^3} + 4Bs + \underline{Cs^2} + 4C + \underline{Ds^4} + \underline{Es^3}$$

$$= s^4(A+D) + s^3(B+E) + s^2(4A+C) + s(4B) + 4C$$

$A+D=5$

$B+E=2$

$4A+C=24$

$4B=0 \Rightarrow B=0$ //

$4C=1 \Rightarrow C=\frac{1}{4}$ //

$4A+\frac{1}{4}=24$

$4A=\frac{95}{4}$

$A=\frac{95}{16}$ //

$\frac{95}{16}+D=5$

$D=-\frac{15}{16}$ //

$0+E=2$

$E=2$ //

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4} = \frac{95}{16}\cdot\frac{1}{s} + \cancel{0\cdot\frac{1}{s^2}} + \frac{1}{4}\cdot\frac{1}{s^3} + \frac{-\frac{15}{16}s+2}{s^2+4}$$

PROVJERA

$$= \frac{95}{16}\cdot\frac{1}{s} + \frac{1}{8}t^2 - \frac{15}{16}\cdot\frac{s}{s^2+2^2} + 2\cdot\frac{1}{s^2+2^2}$$

$$= \frac{95}{16} + \frac{1}{8}t^2 - \frac{15}{16}\cos(2t) + 2\sin(2t) \text{ //}$$ X

$f(0) = \frac{95}{16} + \cancel{\frac{1}{8}0^2} - \frac{15}{16}\cos 0 - 2\sin 0 = 5$ V

$f'(0) = \frac{1}{16}t + \frac{15}{16}\sin(2t) + 2\cos(2t) = 2$ V

$f'(0) = 4$ X

$f'(t) = \frac{1}{4}t + \frac{15}{8}\sin(2t) + 4\cos(2t)$

$f''(t) = \frac{1}{16}t + \frac{15}{16}\sin(2t)$

$f''(t) =$

~~$f''(0) = \frac{1}{16} + \frac{15}{16}\cos(2t) - 2\sin(2t)$~~

MARIN MATEK

3. $x^2+y^2=5^2$ $\qquad$ $z=x^2+y^2$

$z=r^2$ $\qquad$ $x=r\cos\varphi$

$y=r\sin\varphi$

$dx\,dy=r\,dr\,d\varphi$

$r^2=x^2+y^2$

$r^2=5$

$r=\sqrt{5}$

$\varphi\in[0,2\pi]$

$r\in[0,\sqrt{5}]$

$z\in[-5,r^2]$

$\times$

$$V=\int_0^{2\pi}\int_0^{\sqrt{5}}\int_{-5}^{r^2} r\,dz\,dr\,d\varphi=\int_0^{2\pi}\int_0^{\sqrt{5}} rz\Big|_{-5}^{r^2}dr\,d\varphi$$

$$=\int_0^{2\pi}\int_0^{\sqrt{5}}(r^3+5r)\,dr\,d\varphi$$

$$=\int_0^{2\pi}\int_0^{\sqrt{5}}(r^3+5r)\,dr\,d\varphi=\int_0^{2\pi}\left(\frac{r^4}{4}+5\frac{r^2}{2}\right)\Big|_0^{\sqrt{5}}d\varphi=\int_0^{2\pi}\left(\frac{25}{4}-\frac{0}{4}\right)+5\left(\frac{5}{2}-\frac{0}{2}\right)d\varphi$$

$$=\int_0^{2\pi}\left(\frac{25}{4}+\frac{25}{2}\right)d\varphi=\left(\frac{25}{4}\varphi+\frac{25}{2}\varphi\right)\Big|_0^{2\pi}=\left(\frac{25\cdot2\pi}{4}-\frac{25\cdot0}{4}\right)+\left(\frac{25\cdot2\pi}{2}-\frac{25\cdot0}{2}\right)$$

$$=\frac{25\pi}{2}+25\pi=\frac{75\pi}{2}\,//$$

5. $C = \{(x, y, z) : x^2 + z^2 \leq 5,\ 1 \leq y \leq 4\}$

$y \in [1, 4]$

$\int_1^4 \int \frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y} = \frac{y^2 x\, dy}{\partial x} - \frac{\partial dz}{\partial y}$

$P = y^2 x\, dy$

$Q = dz$

$= 1$

$x^2 \geq 5 - z^2$

$\int_1^4 \int_{\sqrt{5-z}}^{5} 1\, dx\, dy$

$\varnothing$

$x \geq 5 - z^2$

$x \geq \sqrt{5 - z}$

4. $x^2 + y^2 = 4z \qquad z \leq 5$

$\varnothing$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: JOSIP MARIĆ

BROJ INDEKSA: 17-2-0227-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: ~~20~~

$$f'''(t) + 4\,f'(t) = t, \quad f(0) = 5,\ f'(0) = 2,\ f''(0) = 4.$$

2. Da li krivuljni integral u vektorskom polju $g = y\mathbf{i} - x\mathbf{j}$ ovisi o putu integracije? 20

3. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 5^2$, ravninom $z = -5$ i parabolom $z = x^2 + y^2$. Napomena: obzirom da je više takvih tijela traži se ono najmanje koje sadrži ishodište. ~~20~~ ?

4. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 4z$, $z \le 5$. 20 ?

5. Neka je C cilindar zadan sa $C = \{(x, y, z) : x^2 + z^2 \le 5,\ 1 \le y \le 4\}$. Izračunati plošni integral 20

$$\iint\limits_{\widehat{\partial C}} y^2 x \, dy dz$$

Ukupno: $\cancel{0}$

1.

$$s^3 f(s) - s^2 f(0) - s f'(0) - f''(0) + 4 s f(s) - 4 f(0) = \frac{1}{s^2}$$

$$s^3 f(s) - 5s^2 - 2s - 4 + 4 s f(s) - 20 = \frac{1}{s^2}$$

$$s^3 f(s) + 4 s f(s) - 5 s^2 - 2s - 24 = \frac{1}{s^2}$$

$$s^3 f(s) + 4 s f(s) = \frac{1}{s^2} + 5s^2 + 2s + 24$$

$$f(s)\,[s^3 + 4s] = \frac{1}{s^2} + 5s^2 + 2s + 24 \quad /:(s^3+4s)$$

$$f(s) = \frac{\frac{1}{s^2} + 5s^2 + 2s + 24}{s^3 + 4s} = \frac{\frac{1 + s^2\cdot(5s^2) + s^2\cdot 2s + 24 s^2}{s^2}}{s^3 + 4s}$$

$$f(s) = \frac{\frac{1 + 5s^4 + 2s^3 + 24s^2}{s^2}}{\frac{s^3+4s}{1}} = \frac{1 + 5s^4 + 2s^3 + 24s^2}{s^2(s^3 + 4s)} = \frac{1 + 5s^4 + 2s^3 + 24s^2}{s^3(s^2+4)}$$

$$\cancel{f(s) = \frac{1 + 5s^4 + 2s^3 + 24s^2}{s^2(s^3+4s)}} \qquad \cancel{\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^3+4s}}$$

$$f(t) = \frac{1 + 5s^4 + 2s^3 + 24s^2}{s^3(s^2+4)}$$

$$f(s)=\frac{1+5s^4+2s^3+24s^2}{s^2(s^3+4s)}$$

$$\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3} \Big/ s^2(s^3+4s)$$

$$A\cdot[s(s^3+4s)]+B\cdot[s^3+4s]+C[s^2(4s)]=1+5s^4+2s^3+24s^2$$

$$A\cdot[s^4+4s^2]+B[s^3+4s]+C\cdot[4s^3]=1+5s^4+2s^3+24s^2$$

$$As^4+4As^2+Bs^3+4Bs+4Cs^3=5s^4+2s^3+24s^2+1$$

$$As^4+Bs^3+4Cs^3+4As^2+4Bs=5s^4+2s^3+24s^2+1$$

$$A=5$$

$$B+C=2 \Rightarrow C=1$$

$$B=1$$

$$f(t)=\frac{1+5s^4+2s^3+24s^2}{s^3(s^2+4)}$$

$$\frac{A}{s}+\frac{B}{s^2}+$$

$$\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{Ds+E}{(s^2+4)} \Big/ s^3(s^2+4)$$

$$A\cdot[s^2(s^2+4)]+B\cdot[s(s^2+4)]+C\cdot[(s^2+4)]+$$

$$+(Ds+E)\cdot s^3=5s^4+2s^3+24s^2+1$$

$$A\cdot(s^4+4s^2)+B(s^3+4s)+C\cdot(s^2+4)+(Ds+E)(\cdot s^3)=5s^4+2s^3+24s^2+1$$

$$As^4+4As^2+Bs^3+4Bs+Cs^2+4C+Ds^4+Es^3=5s^4+2s^3+24s^2+1$$

$$4A+D=5$$

$$B+E=2$$

$$A+C=24$$

$$4B+C=1 \Rightarrow 4B=1-C$$

~~2. LIST NIJE NAĐEN~~

~~STUDENTU PONUĐENO DA DOĐE~~

~~PISATI PONOVNO U PETAK U 14 SATI.~~

PRONAĐENO!

2. LIST

JOSIP MARIĆ

$$f(t)=\frac{1+5s^4+2s^3+24s^2}{s^3(s^2+4)}$$

$$\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{Ds+E}{(s+4)} \quad \Big/ : \left[s^3(s^2+4)\right]$$

$$A\left[s^2(s^2+4)\right]+B\left[s(s^2+4)\right]+C\left[s^2+4\right]+\left[Ds+E\right]\cdot\left[s^3(s+4)\right]=5s^4+2s^3+24s^2+1$$

$$A\left[s^4+4\right]+B\left[s^3+4s\right]+C\left[s^2+4\right]+\left[Ds+E\right]\cdot\left[s^4+4\right]=5s^4+2s^3+24s^2+1$$

$$As^4+4A+Bs^3+4Bs+Cs^2+4C+Ds^5.$$

Ø

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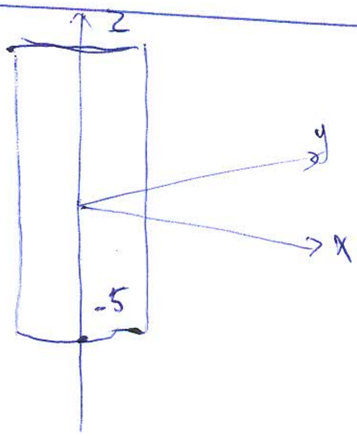
3. $x^2+y^2=5^2$

$z=-5$

$z=x^2+y^2$

$z=r^2\sin^2\varphi+r^2\cos^2\varphi$



Ø

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. 16.09.2015. UGMENI ISPIT: PROFESOR UGLEŠIĆ **IME I PREZIME:** TONI PERKOVIĆ **BROJ INDEKSA:** 172-0134-2011	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: ~~20~~

$$f'''(t) + 4f'(t) = t, \quad f(0) = 5,\ f'(0) = 2,\ f''(0) = 4.$$

2. Da li krivuljni integral u vektorskom polju $g = y\mathbf{i} - x\mathbf{j}$ ovisi o putu integracije? 20

3. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 5^2$, ravninom $z = -5$ i parabolom $z = x^2 + y^2$. Napomena: obzirom da je više takvih tijela traži se ono najmanje koje sadrži ishodište. ~~20~~

4. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 4z$, $z \leq 5$. 20

5. Neka je C cilindar zadan sa $C = \{(x, y, z) : x^2 + z^2 \leq 5,\ 1 \leq y \leq 4\}$. Izračunati plošni integral 20

$$\iint_{\partial C} y^2 x \,dy dz$$

Ukupno: ~~0~~

① $f'''(t) + 4f'(t) = t \qquad f(0)=5,\ f'(0)=2,\ f''(0)=4$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 4(sF(s) - f(0)) = \frac{1}{s^2}$$

$$s^3F(s) - s^2f(0) - \cancel{sf'(0)} - \cancel{f''(0)} + 4sF(s) - 4f(0) = \frac{1}{s^2}$$

$$s^3F(s) - 5s^2 - 2s - 4 + 4sF(s) - 20 = \frac{1}{s^2}$$

$$s^3F(s) + 4sF(s) = \frac{1}{s^2} + 5s^2 + 2s + 4 + 20$$

$$s^3F(s) + 4sF(s) = \frac{1}{s^2} + 5s^2 + 2s + 24 \rightarrow \frac{1 + 5s^4 + 2s^3 + 24s^2}{s^2}$$

$$F(s)(s^3 + 4s) = \frac{5s^4 + 2s^3 + 24s^2 + 1}{s^2}$$

$$F(s) = \frac{\frac{5s^4 + 2s^3 + 24s^2 + 1}{s^2}}{s^3 + 4s}$$

$$F(s) = \frac{5s^4 + 2s^3 + 24s^2 + 1}{s^2(s^3 + 4s)}$$

$$F(s) = \frac{5s^4 + 2s^3 + 24s^2 + 1}{s^3(s^2 + 4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2 + 4} \quad / \cdot s^3(s^2+4)$$

$$5s^4 + 2s^3 + 24s^2 + 1 = As^2(s^2+4) + Bs(s^2+4) + C(s^2+4) + Ds^4 + Es^3$$

$$5s^4 + 2s^3 + 24s^2 + 1 = As^4 + 4As^2 + Bs^3 + 4Bs + Cs^3 + 4C + Ds^4 + Es^3$$

$$5s^4 + 2s^3 + 24s^2 + 1 = (A+D)s^4 + (B+C+E)s^3 + (4A)s^2 + 4Bs + 4C \longrightarrow$$

DRUGA STRANA

1. ZADATAK - NASTAVAK

$A+D=5$
$B+C+E=2$
$4A=24$
$4B=0$
$4C=1$

$\boxed{B=0}$
$\boxed{C=\frac{1}{4}}$

$B+C+E=2$
$0+\frac{1}{4}+E=2$
$E=2-\frac{1}{4}$
$\boxed{E=\frac{7}{4}}$

$4A=24$
$\boxed{A=6}$

$A+D=5$
$6+D=5$
$D=5-6$
$\boxed{D=-1}$

$$F(s)=\frac{6}{s}+\cancel{\frac{0}{s^2}}+\frac{\frac{1}{4}}{s^3}+\frac{(-1)\cdot s+\frac{7}{4}}{s^2+4}$$

$$F(s)=\frac{6}{s}+\frac{\frac{1}{4}}{s^3}-\frac{s}{s^2+4}+\frac{\frac{7}{4}}{s^2+4}$$

$$F(s)=6\cdot\frac{1}{s}+\frac{1}{4}\cdot\frac{1}{s^3}-\frac{s}{s^2+4}+\frac{7}{4}\cdot\frac{1}{s^2+4}$$

$$F(t)=6\cdot 1+\frac{1}{4\cdot 2}\cdot\frac{2!}{s^3}-\cos(2t)+\frac{7}{4}\cdot\sin(2t)$$

$$F(t)=6+\frac{1}{8}\cdot t^2-\cos(2t)+\frac{7}{4}\sin(2t)$$

$\emptyset$

PROVJERA:

$f'(t)=\frac{1}{4}t+2\sin(2t)+\frac{7}{2}\cos(2t)$

$f''(t)=\frac{1}{4}+4\cos(2t)-7\sin(2t)$

$f'''(t)=-8\sin(2t)-14\cos(2t)$

$f'''+4f'=-8\cancel{\sin(2t)}-14\cancel{\cos(2t)}+t+8\cancel{\sin(2t)}+14\cancel{\cos(2t)}=t$ ✓

$f(0)=6-1=5$ ✓

$f'(0)=\frac{7}{2}\neq 2$ ✗

$f''(0)=\frac{1}{4}+4\neq 4$ ✗

3. $x^2+y^2=5^2$, $z=-5$, $z=x^2+y^2$

$x^2+y^2=5^2$
$x^2+y^2=r^2$
$\boxed{r=5}$

$r\in[0,5]$

$\emptyset$

$r\,dr\,d\varphi$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: VALENTINO ŠARE BROJ INDEKSA: 17-2-0149-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: ~~20~~

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2. Da li krivuljni integral u vektorskom polju $g = y\mathbf{i} - x\mathbf{j}$ ovisi o putu integracije? 20

3. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 5^2$, ravninom $z = -5$ i parabolom $z = x^2 + y^2$. Napomena: obzirom da je više takvih tijela traži se ono najmanje koje sadrži ishodište. 20

4. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 4z$, $z \leq 5$. ~~20~~

5. Neka je C cilindar zadan sa $C = \{(x, y, z) : x^2 + z^2 \leq 5,\ 1 \leq y \leq 4\}$. Izračunati plošni integral 20

$$\iint\limits_{\partial C} y^2 x \,\mathrm{d}y\mathrm{d}z$$

Ukupno: 0

1) $f'''(t) + 4f'(t) = t$

$f(0) = 5$

$f'(0) = 2$

$f''(0) = 4$

$s^3 f(s) - s^2 f(0) - s f'(0) - f''(0) + s f(s) - f(0) = \frac{1}{s^2}$

$s^3 f(s) - s^2 \cdot 5 - s \cdot 2 - 4 + s f(s) - 5 = \frac{1}{s^2}$

$s^3 f(s) - 5s^2 - 2s - 4 + s f(s) - 5 = \frac{1}{s^2}$

$f(s)(s^3 + s) = \frac{1}{s^2} + 5s^2 + 2s + 4 + 5$

$f(s)(s^3 + s) = \frac{1}{s^2} + 5s^2 + 2s + 9$

$f(s)(s^3 + s) = \frac{1 + 5s^2(s^2) + 2s(s^2) + 9(s^2)}{s^2}$

$f(s)(s^3 + s) = \frac{1 + 5s^4 + 2s^3 + 9s^2}{s^2} = \frac{5s^4 + 2s^3 + 9s^2 + 1}{s^2}$

$$f(s)(s^3+s) = \frac{5s^4+2s^3+9s^2+1}{s^2} \quad \Big/ (s^3+s)$$

$s^2(s^3+s)$

$s^5+s^3 = s^3(s^2+1)$

$$f(s) = \frac{5s^4+2s^3+9s^2+1}{s^3(s^2+1)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{Ds+E}{(s^2+1)} \quad \Big/ s^3(s^2+1)$$

$$= A(s^2+1) + Bs(s^2+1) + Cs^2(s^2+1) + (Ds+E)(s^3)$$

$$= As^2 + A + Bs^3 + Bs + Cs^4 + Cs^2 + Ds^4 + Es^3$$

$5s^4 + 2s^3 + 9s^2 + s \cdot 0 + 1 =$

$C+D=5$

$B+E=2$

$A+C=9$

$\boxed{B=0}$

$\boxed{A=1}$

$A+C=9$

$1+C=9$

$\boxed{C=8}$

$B+E=2$

$0+E=2$

$\boxed{E=2}$

$C+D=5$

$8+D=5$

$\boxed{D=-3}$

$$= \frac{1}{s^3} + \frac{0}{s^2} + \frac{8}{s} + \frac{-3+2}{(s^2+1)} = \frac{1}{s^3} + 0 + 8 \cdot \frac{1}{s} - \frac{1}{s^2+1}$$

$$f(t) = \frac{t^2}{2} + 8 \cdot 1 - \sin t$$

$$\boxed{f(t) = \frac{t^2}{2} + 8 - \sin t}$$

$$\frac{1}{s^3} = \frac{t^{3-1}}{(3-1)!} = \frac{t^2}{2}$$

$!\ 2 \cdot 1 = 2$

PROVJERA:

$f'(t) = \cancel{t} - \cos t$

$f''(t) = 1 + \sin t$

$f'''(t) = \cos t$

$f'''(t) + 4f'(t) = \cos t + 4t - \cancel{4\cos t} = 4t - 3\cos t \neq t$

VALENTINO ŠARE

4)

$x^2 + y^2 = 4z$ $\quad 4 \cdot 5 = 20z$

$z \leq 5$

$\varphi(0, 2\pi)$

$r\cos^2 + r\sin^2 = 4z$ $\quad r(-2, 2)$

$z($

$r^2 (\underbrace{\sin + \cos}_{1}) = 4z$

$r^2 = 4z \quad / \sqrt{}$

$r = \pm 2z$

$\emptyset$

VALENTINO ŠARE