MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$

IME I PREZIME: Luka Žilić **BROJ INDEKSA:** 12-2-0208-2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x \to 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$ 7+2

   (b) $\lim_{n \to \infty} \left(\frac{x^2 + 3}{x^2} \right) =$ 4+2
5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno: 45

LUKA ŽILIĆ

5.

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right]$$

$-2 \cdot \text{I} + \text{II}$

$$\begin{array}{ccccc} -2 & 2 & -4 & -2 & -4 \\ 2 & 1 & 0 & -3 & 4 \\ \hline 0 & 3 & -4 & -5 & 0 \end{array}$$

$-4 \cdot \text{I} + \text{III}$

$$\begin{array}{ccccc} -4 & 4 & -8 & -4 & -8 \\ 4 & -1 & 1 & 2 & -1 \\ \hline 0 & 3 & -7 & -2 & -9 \end{array}$$

$-2 \cdot \text{I} + \text{IV}$

$$\begin{array}{ccccc} -2 & 2 & -4 & -2 & -4 \\ 2 & 1 & 1 & -4 & 7 \\ \hline 0 & 3 & -3 & -6 & 3 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 1 & -1 & -2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ 1:3 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \end{array}\right]$$

$-3 \cdot \text{II} + \text{III}$

$$\begin{array}{ccccc} 0 & -3 & 3 & 6 & -3 \\ 0 & 3 & -7 & -2 & -9 \\ \hline 0 & 0 & -4 & 4 & -12 \end{array}$$

$-3 \cdot \text{II} + \text{IV}$

$$\begin{array}{ccccc} 0 & -3 & 3 & 6 & -3 \\ 0 & 3 & -4 & -5 & 0 \\ \hline 0 & 0 & -1 & 1 & -3 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & -4 & 4 & -12 \\ 0 & 0 & -1 & 1 & -3 \end{array}\right] \begin{array}{l} \\ \\ \\ :(-4) \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array}\right]$$

$\text{III} + \text{IV}$

$$\begin{array}{ccccc} 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$u = 0$

$1 \cdot z - 1 \cdot u = 3$

$1 \cdot z - 0 = 3$

$\boxed{z = 3}$

$1 \cdot y - 1 \cdot z - 2 \cdot u = 1$

$y - 3 - 0 = 1$

$y - 3 = 1$

$y = 1 + 3$

$\boxed{y = 4}$

$1 \cdot x - 1 \cdot y + 2 \cdot z + 1 \cdot u = 2$

$x - 4 + 6 + 0 = 2$

$x + 2 = 2$

$x = 2 - 2$

$\boxed{x = 0}$

BESKONAČNO RJEŠENJA NAŠLI STE, PROVJERILI SAMO JEDNO

PROVJERA:

$4x - y + z + 2u = -1$

$4 \cdot 0 - 4 + 3 + 2 \cdot 0 = -1$

$-4 + 3 = -1$

$\boxed{-1 = -1}$ //

$2x + y - 3u = 4$

$2 \cdot 0 + 4 - 3 \cdot 0 = 4$

$\boxed{4 = 4}$ //

$x - y + 2z + u = 2$

$0 - 4 + 2 \cdot 3 + 0 = 2$

$-4 + 6 = 2$

$\boxed{2 = 2}$ //

$2x + y + z - 4u = 7$

$2 \cdot 0 + 4 + 3 - 4 \cdot 0 = 7$

$4 + 3 = 7$

$\boxed{7 = 7}$ //

4.

b) $\lim_{n \to \infty} \left(\frac{x^2 + 3}{x^2} \right)^{1/x} \overset{1^\infty}{=} \lim_{n \to \infty} \frac{x + \frac{3}{x}}{x} = \frac{0}{0} = \infty$ (0 → ∞)

L.H.A. $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \sqrt{(-x)^2+2\cdot(-x)} = \infty$ (NEMA L.H.A.) $\quad \infty + \infty$

D.K.A. $k = \lim\limits_{x\to+\infty}\left(\frac{f(x)}{x}\right) = \lim\limits_{x\to+\infty} \frac{\sqrt{x^2+2x}\,/:x}{x\,/:x} = \lim\limits_{x\to+\infty} \frac{\sqrt{\frac{x^2+2x}{x^2}}}{1} = \lim\limits_{x\to+\infty} \frac{\sqrt{1+\frac{2}{x}}}{1}$ ($\frac{2}{x}\to 0$)

$= \frac{1}{1} = 1$

$\boxed{k=1}$

$l = \lim\limits_{x\to+\infty}(f(x)-kx) = \lim\limits_{x\to+\infty}\left(\sqrt{x^2+2x}-x\right) = \lim\limits_{x\to+\infty} \sqrt{x^2+2x}-x \cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x}$

$\lim\limits_{x\to+\infty} \frac{\left(\sqrt{x^2+2x}\right)^2 - x^2}{\sqrt{x^2+2x}+x} = \lim\limits_{x\to+\infty} \frac{2x\,/:x}{\sqrt{x^2+2x}+x\,/:x} = \lim\limits_{x\to+\infty} \frac{2}{\sqrt{\frac{x^2+2x}{x^2}}+1}$

$\lim\limits_{x\to+\infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1} = \lim\limits_{x\to+\infty} \frac{2}{\sqrt{1}+1} = \frac{2}{2} = 1$ ($\frac{2}{x}\to 0$) $\quad \boxed{l=1}$

$y = kx + l$

$\boxed{y = x+1}$ ✓

x	0	-1
y	1	0

5

L.K.A. $k = \lim\limits_{x\to-\infty}\left(\frac{f(-x)}{x}\right) = \lim\limits_{x\to-\infty} \frac{\sqrt{(-x)^2+2\cdot(-x)}\,/:x}{x\,/:x} = \lim\limits_{x\to-\infty} \frac{\sqrt{\frac{x^2-2x}{x^2}}}{1}$

$\lim\limits_{x\to-\infty} \frac{\sqrt{1-\frac{2}{x}}}{1} = \frac{1}{1} = 1$ ($\frac{2}{x}\to 0$) $\quad \boxed{k=1}$

$l = \lim\limits_{x\to-\infty}(f(-x)-kx) = \lim\limits_{x\to-\infty} \sqrt{(-x)^2+2\cdot(-x)} - 1x$

$\lim\limits_{x\to-\infty} \sqrt{x^2-2x} - x \cdot \frac{\sqrt{x^2-2x}+x}{\sqrt{x^2-2x}+x} = \lim\limits_{x\to-\infty} \frac{\left(\sqrt{x^2-2x}\right)^2 - x^2}{\sqrt{x^2-2x}+x}$

$\lim\limits_{x\to-\infty} \frac{-2x\,/:x}{\sqrt{x^2-2x}+x\,/:x} = \lim\limits_{x\to-\infty} \frac{-2}{\sqrt{1-\frac{2}{x}}+1} = \frac{-2}{2} = -1$

$\boxed{l=-1}$

$y = kx + l$

$\boxed{y = x-1}$ ✗

x	0	1
y	-1	0

LUKA ĐILIĆ

6. $f(x) = \ln(\cos(2x^2-1))$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (\cos(2x^2-1))'$$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (-\sin(2x^2-1)) \cdot (2x^2-1)'$$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (-\sin(2x^2-1)) \cdot 4x$$

$$f'(x) = 4x \cdot \frac{-\sin(2x^2-1)}{\cos(2x^2-1)}$$ ✓

3. $h(x) = \sqrt{x^2+2x}$

$x^2 + 2x \geq 0$

$x(x+2) \geq 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4-0}}{2}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4}}{2}$$

$$x_1 = \frac{-2+2}{2} = 0$$

$$x_2 = \frac{-2-2}{2} = \frac{-4}{2} = -2$$

$D(f) = \langle -\infty, -2] \cup [0, +\infty \rangle$ ✓

N.T.

$f(x) = 0$

$\sqrt{x^2+2x} = 0 \ /^2$

$(\sqrt{x^2+2x})^2 = 0$

$x^2+2x = 0$

$x_1 = 0 \quad x_2 = -2$

$(0, 0)$

$(-2, 0)$

$f(0) = \sqrt{x^2+2x}$

$f(0) = \sqrt{0^2+2 \cdot 0}$

$f(0) = 0$

ASIMPTOTE:

V.A. NEMA

D.H.A. $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \sqrt{x^2+2x} = +\infty$ NEMA D.H.A.

1. $z^3 - 3 + 3i = 0$

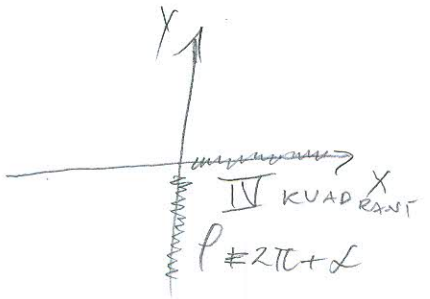
$z^3 = 3 - 3i$

$w = x + yi$

$w = 3 - 3i$

$|w| = \sqrt{3^2 + (-3)^2} = \sqrt{18}$

$tg\,\varphi = \frac{y}{x} = \frac{-3}{3} = -1$



$\varphi = 2\pi + \left(-\frac{\pi}{4}\right) = \frac{7\pi}{4}$

$k = 0, 1, 2$

$\pi = 180°$

$z_0 = \sqrt[3]{\sqrt{18}} \cdot \left(\cos\frac{\varphi + 2k\pi}{3} + i\sin\frac{\varphi + 2k\pi}{3}\right)$

$= \sqrt[6]{18} \cdot \left(\cos\frac{\frac{7\pi}{4}}{3} + i\sin\frac{\frac{7\pi}{4}}{3}\right) = \sqrt[6]{18}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

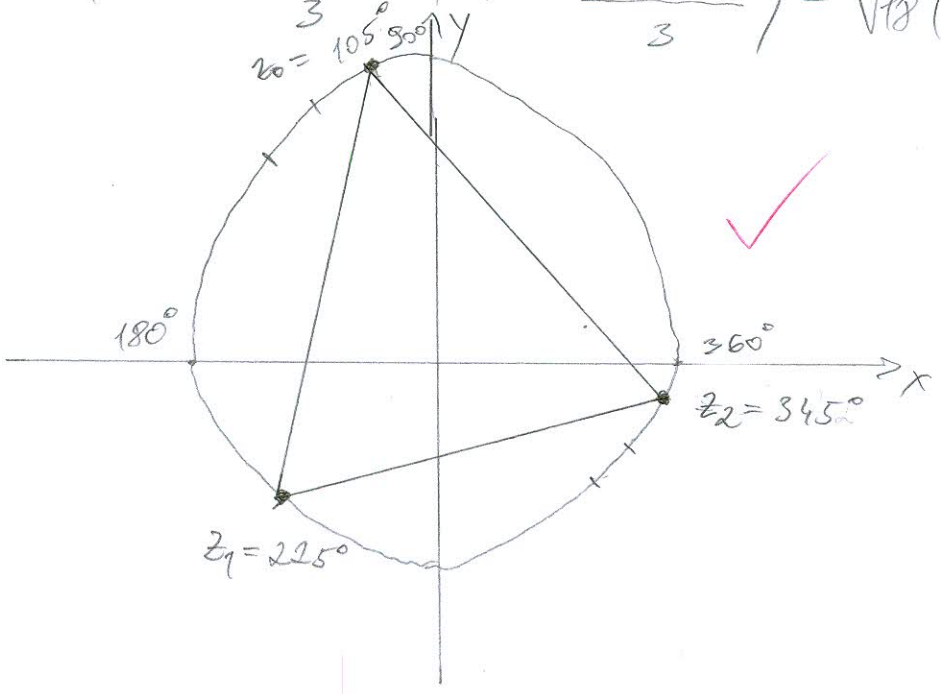
105°

$z_1 = \sqrt[3]{\sqrt{18}}\left(\cos\frac{\frac{7\pi}{4} + 2\pi}{3} + i\sin\frac{\frac{7\pi}{4} + 2\pi}{3}\right) = \sqrt[6]{18}\left(\cos\frac{15\pi}{12} + i\sin\frac{15\pi}{12}\right)$

225°

$z_2 = \sqrt[3]{\sqrt{18}}\left(\cos\frac{\frac{7\pi}{4} + 4\pi}{3} + i\sin\frac{\frac{7\pi}{4} + 4\pi}{3}\right) = \sqrt[6]{18}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$

345°



(3.) $f(x) = \sqrt{x^2+2x}$ LUKA ŽIČIĆ

$$f'(x) = \frac{1}{2\sqrt{x^2+2x}} \cdot (x^2+2x)'$$

$$f'(x) = \frac{1}{2\sqrt{x^2+2x}} \cdot (2x+2)$$

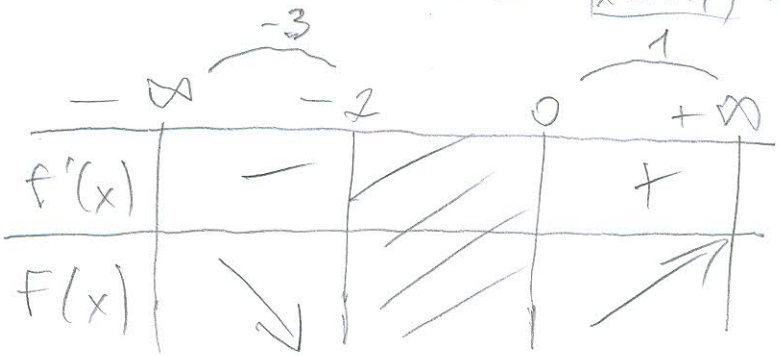
$$F'(x) = \frac{2x+2}{2\sqrt{x^2+2x}}$$

$f'(x) = 0$

$2x+2=0$

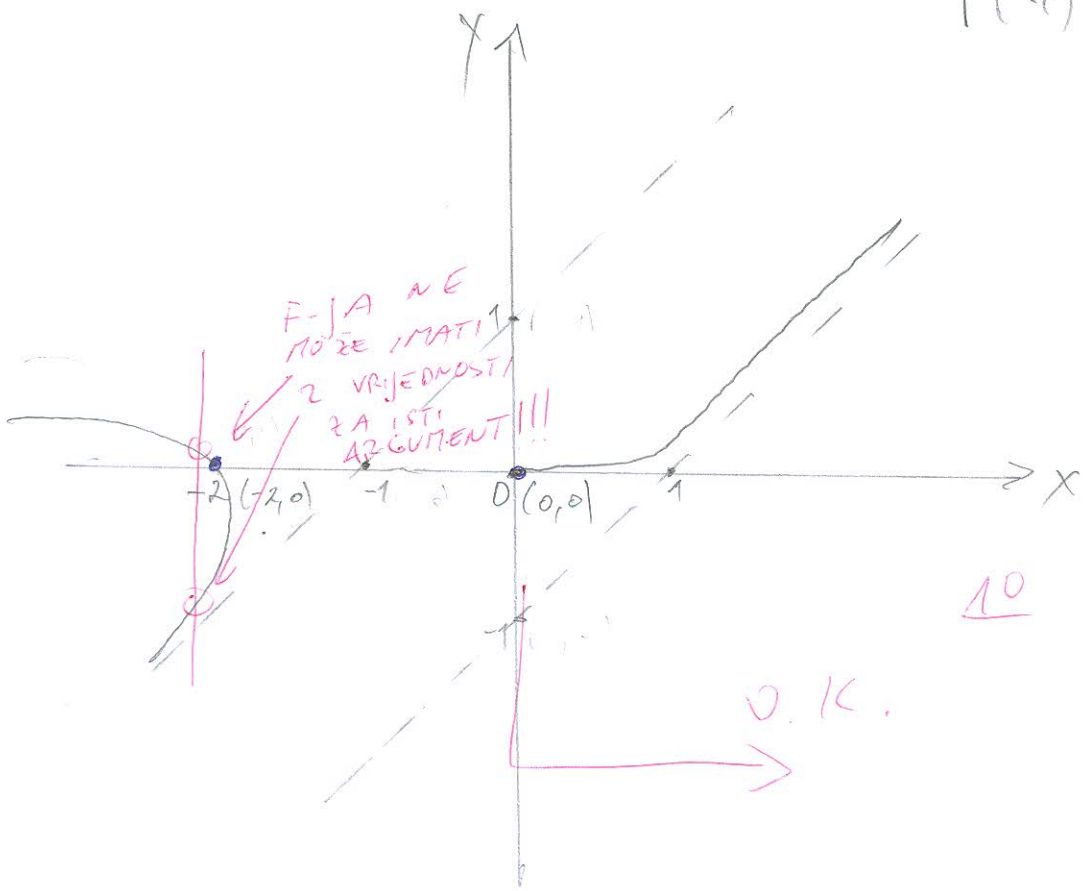
$2x = -2 \mid \cdot \frac{1}{2}$

$x = -1$ NEMA STACIONARNIH TOČKI JER $\boxed{x=-1}$ NIJE U DOMENI FUNKCIJE



$$f'(-3) = \frac{2\cdot(-3)+2}{+} = \frac{-}{+} = -$$

$$f(1) = \frac{2\cdot 1+2}{+} = \frac{+}{+} = +$$



1°

D.K.

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$

IME I PREZIME: DRAGAN ASIĆ **BROJ INDEKSA:** 17-2-0286-2013

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3

2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2+3}$ 10

3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2+2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) =$ 7+2

   (b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) =$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2-1))$. 10

Ukupno: 29

$f(x) = \sqrt{x^2+2x}$

$x^2+2x \geq 0$

$= \mathbb{R}$ ✗

. Nema

L.A. $k(x) = \sqrt{x^2+2x}$

$$\lim_{x\to+\infty}\left(\frac{\sqrt{x^2+2x}}{1}\cdot\frac{\sqrt{x^2+2x}}{\sqrt{x^2+2x}}\right)$$

$$\lim_{x\to+\infty}\left(\frac{\sqrt{x^2+2x}^2}{\sqrt{x^2+2x}}\right) = \lim_{x\to\infty}\frac{x^2+2x\ /:x^2}{\sqrt{x^2+2x}\ /:x^2} = \frac{\frac{x^2}{x^2}+\frac{2x}{x^2}}{\sqrt{\frac{x^2}{x^4}+\frac{2x}{x^4}}} = \frac{1}{0} = +\infty$$ Nema D.H.A

D.H.A. $\lim_{x\to-\infty}\left(\frac{\sqrt{x^2+2x}}{1}\cdot\frac{\sqrt{x^2+2x}}{\sqrt{x^2+2x}}\right) = \lim_{x\to-\infty}\frac{x^2+2x\ /:x^2}{\sqrt{x^2+2x}\ /:x^2} = -\frac{1}{0} = -\infty$ Nema L.H.A.

D.K.A. $k = \lim_{x\to\infty}\frac{\sqrt{x^2+2x}\ /:x}{x\ /:x} = \frac{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}}{\frac{x}{x}} = \frac{1}{1}$ $k=1$ ✓

$(a-b)\cdot(a+b) = a^2-b^2$

$n = \lim_{x\to\infty} f(x) - k\cdot x$

$n = \lim_{x\to\infty}\left(\sqrt{x^2+2x}-1\cdot x\right)\cdot\frac{\sqrt{x^2+2x}+1x}{\sqrt{x^2+2x}+1x} = \frac{x^2+2x-x^2}{\sqrt{x^2+2x}+1x} = \frac{2x\ /:x}{\sqrt{x^2+2x}+1x\ /:x} = \frac{2}{2} = 1$ ✓

$n=1$

$y = 1-1x$ D.K.A.

x	0	2
y	1	-1

L.K.A. $k = \lim_{x\to-\infty}\frac{\sqrt{x^2+2x}\ /:x}{x\ /:x} = \frac{-1}{1} = -1$ $k = \#1$ ✗

$n = \lim_{x\to-\infty}\left(\sqrt{x^2+2x}+1x\right)\cdot\frac{\sqrt{x^2+2x}-1x}{\sqrt{x^2+2x}-1x} = \frac{\sqrt{x^2+2x}-x^2}{-\sqrt{x^2+2x}-x} = \frac{2x\ /:x}{\sqrt{x^2+2x}-x\ /:x} = \frac{\frac{2x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}-\frac{x}{x}} = \frac{2}{2}$

$n=1$

$y = -1+1x$ L.K.A.

x	0	2
y	-1	1

$h(x) = \sqrt{x^2+2x}$

$$h'(x) = \frac{1}{2\sqrt{x^2+2x}} \cdot (x^2+2x)'$$

$$h'(x) = \frac{2x+2}{2\sqrt{x^2+2x}}$$

$h'(x) = 0$

$$\frac{2x+2}{2\sqrt{x^2+2x}} = 0 \quad / \cdot 2\sqrt{x^2+2x}$$

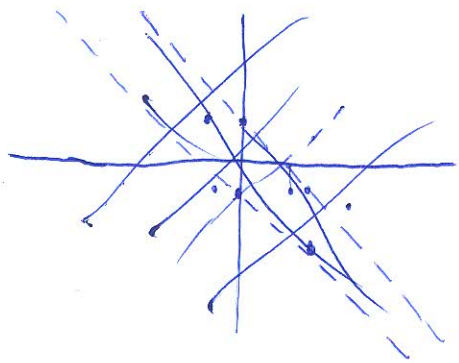
$2x+2 = 0$

$2x = -2 \quad / :2$

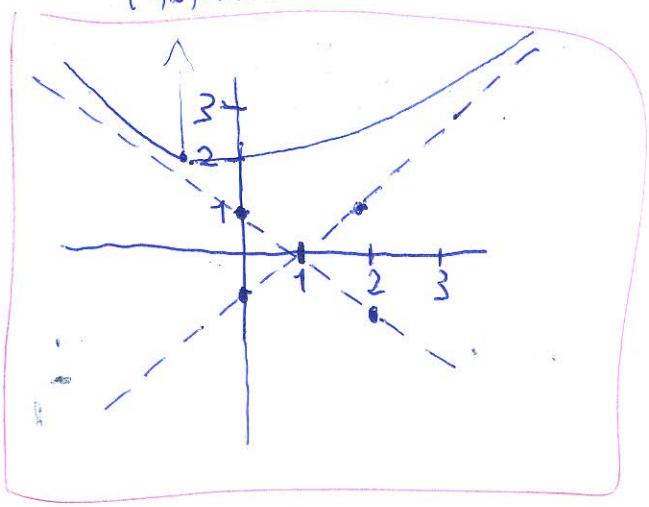
$x = -1$

	$-\infty$		-1		∞
$h'(x)$		$-$	0	$+$	
$h(x)$		$\searrow$	min	$\nearrow$	

$(-1,2)$ EKSTREM MINIMUM



$(-1,2)$ MINIMUM



5

DRAGAN ASIĆ

② $g(x) = \sqrt{x^2+3}$

$D(g): x^2+3 \geq 0$

$x^2 \geq -3 \mid \sqrt{\ }$

$x \geq \sqrt{-3}$ $\sqrt{-3}$ - Nije realan broj

$D(g) = \mathbb{R}$

V.A. Nema, jer nema prekida domene

$g(x) = \sqrt{x^2+3}$

$g'(x) = \frac{1}{2\sqrt{x^2+3}} \cdot (x^2+3)'$

$g'(x) = \frac{1}{2\sqrt{x^2+3}} \cdot 2x = \frac{2x}{2\sqrt{x^2+3}}$

$g'(x) = \frac{x}{\sqrt{x^2+3}}$

$g'(x) = 0$

$\frac{x}{\sqrt{x^2+3}} = 0 \mid \cdot \sqrt{x^2+3}$

$x = 0$ $(0, \sqrt{3}) \approx (0, 1.732)$ $(0, 1.73)$ = Globalni minimum ✓

x	$-\infty$		0		∞
$g'(x)$		$-$	0	$+$	
$g(x)$	$\searrow$		$\sqrt{3}$		$\nearrow$

4. a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right)=\lim_{x\to 0}\frac{\sqrt{3+0}-\sqrt{3}}{0}=\frac{0}{0}$ Neodredeni slučaj

$\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}\ /:x}{x\ /:x}\right)=\frac{0}{1}$ ✗

$\lim_{x\to 0^-}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right)=0.29$

$\lim_{x\to 0^+}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right)=0.28$

b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right)$

$\lim_{n\to\infty}\left(\frac{\infty^2+3}{\infty^2}\right)=\frac{\infty}{\infty}$

$\lim_{n\to\infty}\left(\frac{x^2+3\ /:x^2}{x^2\ /:x^2}\right)=\frac{1}{1}=1$

$\lim_{n\to 1}\frac{x^2+3}{x^2}=\frac{1^2+3}{1^2}=1$ ✓

6. $f(x)=\ln|\cos(2x^2-1)|$

$f'(x)=\frac{1}{\cos(2x^2-1)}\cdot(\cos(2x^2-1))'$

$f'(x)=\frac{1}{\cos(2x^2-1)}\cdot(-\sin(2x^2-1))\cdot(2x^2-1)'$

$f'(x)=\frac{1}{\cos(2x^2-1)}\cdot(-\sin 2x^2-1)\cdot 4x$

$f'(x)=\frac{(-\sin 2x^2-1)\cdot 4x}{\cos(2x^2-1)}=\frac{-4x\cdot\sin(2x^2+1)}{\cos(2x^2-1)}$ ✓

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IME I PREZIME: Dino Renko ustmeni : asistent Kosor

BROJ INDEKSA: [illegible] 0269091526

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x \to 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$ 7+2

   (b) $\lim_{n \to \infty} \left(\frac{x^2 + 3}{x^2} \right) =$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno: 29

$\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) = \frac{\sqrt{3+0}-\sqrt{3}}{0}$ Dino Ranto

$= \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{2\sqrt{3+x}}+\frac{1}{2\sqrt{3}}}{1} = \frac{\frac{2}{2\sqrt{3}}}{\frac{1}{1}} = \frac{2}{2\sqrt{3}}$

$(\sqrt{3+x})' = \frac{1}{2\sqrt{3+x}} \cdot 1$

$(\sqrt{3})' = \frac{1}{2\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$ ✓

[ ]

(b) $\lim_{x\to\infty}\left(\frac{x^2+3}{x^2}\right) \overset{L'H}{=} \lim_{x\to\infty}\left(\frac{2x}{2x}\right) = 1$ ✓

(5)

$4x - y + z + 2u = -1$
$2x + y + 0 - 3u = 4$
$x - y + 2z + u = 2$
$2x + y + z + 4u = 7$

$$\begin{bmatrix} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{bmatrix} \begin{matrix} \\ +1R\cdot(-4) \\ +1R\cdot(-2) \\ \\ +1R\cdot(-2) \end{matrix} \approx$$

$$\approx \begin{bmatrix} 0 & 3 & 7 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \\ 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -3 & -6 & 3 \end{bmatrix} :3 \approx \begin{bmatrix} 0 & 3 & 4 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \\ 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \end{bmatrix} \begin{matrix} +4R\cdot(-3) \\ +4R\cdot(-3) \\ +4R\approx \end{matrix}$$

$$\approx \begin{bmatrix} 0 & 0 & 10 & -8 & -12 \\ 0 & 0 & -1 & 1 & -3 \\ 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \end{bmatrix} :2 + 2R(\cdot 4) \quad +2R \quad +2R(\cdot 2)$$

$$\approx \begin{bmatrix} 0 & 0 & 1 & 0 & -18 \\ 0 & 0 & -1 & 1 & -3 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 & -5 \end{bmatrix} \begin{matrix} \\ +1R \\ \approx \\ +1R(\cdot 3) \end{matrix}$$

$$\approx \begin{bmatrix} 0 & 0 & 1 & 0 & -24 \\ 0 & 0 & 0 & 1 & -21 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 74 \end{bmatrix} \begin{matrix} z = -24 \\ u = -21 \\ x = 0 \\ y = 74 \end{matrix}$$

$$\approx \begin{bmatrix} 0 & 0 & 1 & 0 & -18 \\ 0 & 0 & 0 & 1 & -21 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -59 \end{bmatrix} z = -18$$

provjera:

$$\begin{cases} 4x - y + z + 2u = -1 \\ 4 \cdot 0 - 17 + (-24) + 2 \cdot (-27) = 1 \end{cases}$$

$$4 \cdot 0 - (-59) + (-18) + 2 \cdot (-21) = -1$$

$$-1 = -1 \quad \text{W}$$

$$2x + y + 0 - 3u = 4$$

$$2 \cdot 0 + (-59) + 0 - 3 \cdot (-21) = 4$$

NISTE NAPRAVILI POTPUNU PROVJERU!

$$4 = 4 \quad \text{W}$$

$x = 0$
$y = -59$
$u = -21$
$z = -18$

$$\begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ -59 \\ -18 \\ -21 \end{bmatrix}$$

POGREŠNO ✗

(6.) $f(x) = \ln(\cos(2x^2 - 1))$

$$f'(x) = \left[\ln(\cos(2x^2-1))\right]' = \frac{1}{\cos(2x^2-1)} \cdot \left[-\sin(2x^2-1)\right] \cdot 4x$$

$(\ln x)' = \frac{1}{x}$

$$= -\frac{\sin(2x^2-1)}{\cos(2x^2-1)} \cdot 4x = -\tan(2x^2-1) \cdot 4x$$

$\cos x' = -\sin x$

$\ln(\cos(x))$

$\frac{1}{\cos x} \cdot (-\sin x) \cdot 1$

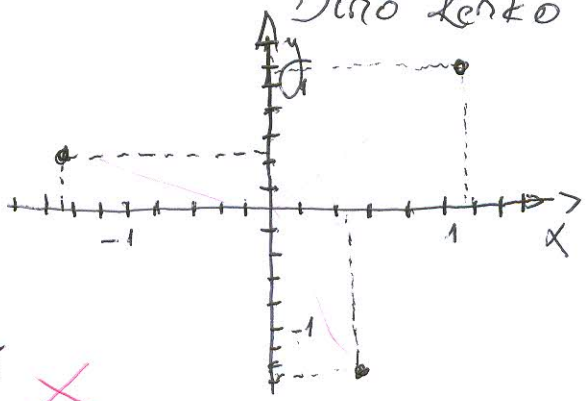
Dino Lenko

(1.) $z^3 - 3 + 3i = 0$

$z^3 = 3 - 3i$

$r = |z| = \sqrt{x^2+y^2} = 4.24$

$(x>0)\ \varphi = \pi + \arctan(y/x) = 2.35$ ×



$w_1 = \sqrt[3]{4.24}\left(\cos\frac{2.35}{3} + i\sin\frac{2.35}{3}\right) = 1.14 + 1.14i$

$w_2 = 1.62\left(\cos\frac{2.35+2\pi}{3} + i\sin\frac{2.35+2\pi}{3}\right) = -1.56 + 0.42i$

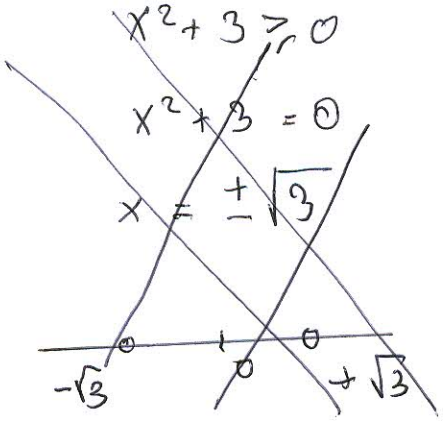
$w_3 = 1.62\left(\cos\frac{2.35+4\pi}{3} + i\sin\frac{2.35+4\pi}{3}\right) = 0.41 - 1.56i$

Dino Rento

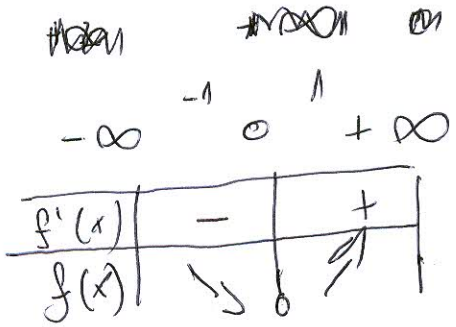
(2.) $g(x) = \sqrt{x^2+3}$

$D_g = \mathbb{R}$

$g'(x) = \frac{1}{2\sqrt{x^2+3}} \cdot 2x = \frac{x}{\sqrt{x^2+3}}$



$\frac{x}{\sqrt{x^2+3}} = 0 \rightarrow x = 0$



nema globalnih ekstrema

GLOBALNI MINIMUM

(3.) $k = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2+2x}}{x} = \left\{\begin{matrix} x \mapsto -x \\ -\infty \mapsto +\infty \end{matrix}\right\} =$

$= \lim_{x\to\infty} \frac{\sqrt{(-x)^2-2x}}{-x} = \lim_{x\to\infty} \frac{\sqrt{x^2-2x}}{-x} = \lim$?

SJECIŠTA

$y = 0, \quad \sqrt{x^2+2x} = 0$

$x^2+2x = 0$

$x_1 = 0$

$x_2 = -2$

$x = 0, \quad \sqrt{0} = 0$

~~$x = 0$~~

DERIVACIJA

$h'(x) = \frac{x}{\sqrt{x^2+2x}}$

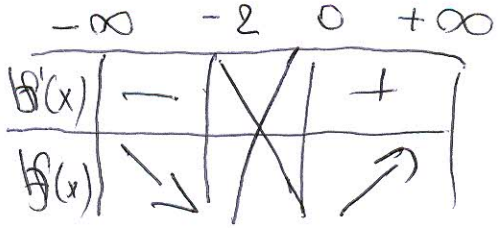
$\frac{x}{\sqrt{x^2+2x}} = 0$

$x = 0$

DRUGA DERIVACIJA

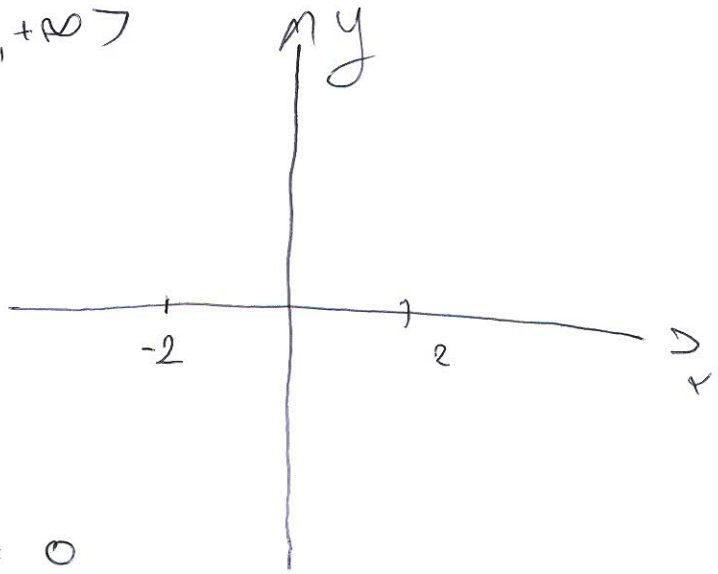
$h''(x) =$

GRAF ?



③ $h(x) = \sqrt{x^2+2x}$

$D_f \;]-\infty, -2] \cup [0, +\infty[$



$x^2+2x \geq 0$

$x^2+2x = 0$

$x_1 = 0 \;,\; x_2 = -2$

V.A.

$\lim_{x \to -2_-} (f(x) = \sqrt{4-4} = 0$

$\lim_{x \to 0_+} f(x) = \sqrt{0_+} = 0$

K. A.

$k = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\sqrt{x^2+2x}}{x} \overset{L'H}{=} \lim_{x \to \infty} \frac{\frac{1}{2\sqrt{x^2+2x}} \cdot 2x}{1}$

$= \lim_{x \to \infty} \frac{x}{\sqrt{x^2+2x}} \cdot \frac{\sqrt{x^2-2x}}{\sqrt{x^2-2x}} = \lim_{x \to \infty} \frac{x\sqrt{x^2-2x}}{\sqrt{x^2+2x}\cdot\sqrt{x^2-2x}}$

$[\infty + \infty]$

$\lim_{x \to \infty} = \frac{x\sqrt{x^2-2x}}{x^2-2x} = \lim_{x \to \infty} \frac{\sqrt{x^2-2x}}{x-2}$

$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{\sqrt{x^2+2x}\,/:x}{x\,/:x} = \lim_{x \to \infty} \frac{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}}{\frac{x}{x}} =$

$= \lim_{x \to \infty} \frac{\sqrt{1+\frac{2}{x}}}{1} = \frac{\sqrt{1}}{1} = 1 \qquad k = 1$

5

~~$\lim_{x \to \infty} [f(x) - kx] = l$~~ $l = \lim_{x \to \infty} [f(x) - kx] = \lim_{x \to \infty} \sqrt{x^2+2x} - x$

$l = 1 \qquad y = x + 1 \;(u + \infty) \rightarrow$

Dino Lanko

$$\lim_{x\to\infty} \frac{\sqrt{x^2-2x}}{x-2} =$$

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+2x}} \overset{L'H}{=} \frac{1}{\frac{2x}{2\sqrt{x^2+2x}}} = \frac{\sqrt{x^2+2x}}{x} =$$

$$\frac{\sqrt{x^2+2x}\ /:x}{x\ /:x} = \frac{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}}{\frac{x}{x}} = \frac{\sqrt{1+\frac{2}{x}}}{1} = \frac{1}{1} = 1$$

$$\lim_{x\to\infty} \sqrt{x^2+2x} - x \qquad (\infty - \infty)$$

$$\lim_{x\to\infty} \sqrt{x^2+2x} - x \cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x} \qquad \frac{1}{2\sqrt{x^2+2x}} - 1$$

$$\frac{(\sqrt{x^2+2x}-x)(\sqrt{x^2+2x}+x)}{\sqrt{x^2+2x}+x} \qquad \frac{x}{\sqrt{x^2+2x}} - 1$$

$$x^2+2x+x\sqrt{x^2+2x}-x\sqrt{x^2+2x}-x^2 \qquad \frac{1}{\frac{1x}{2\sqrt{x^2+2x}}} = \frac{\sqrt{x^2+2x}\ /:x}{x\ /:x}$$

$$\frac{2x}{\sqrt{x^2+2x}+x} \overset{L'H}{=} \frac{2}{\frac{x}{\sqrt{x^2+2x}}+\frac{1}{1}} = \frac{2}{\frac{x+\sqrt{x^2+2x}}{\sqrt{x^2+2x}}}$$

$$\frac{2\sqrt{x^2+2x}}{x+\sqrt{x^2+2x}} \qquad \frac{\sqrt{x^2-2x}}{-x} \cdot \frac{\sqrt{x^2+2x}}{\sqrt{x^2+2x}} = \frac{x^2-4x^2}{-x\sqrt{x^2+2x}}$$

$z = -24$

$u = -24$

$x = 0$

$y = 44$

$4x - y + z + 2u = -1$

$2x + y + 0 - 3u = 4$

$z = -18$

$u = -21$

$x = 0$

$y = -59$

$x^2 + 2x = 0$

$x_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 0}}{2}$

$-\infty \quad -\sqrt{3} \quad \sqrt{3} \quad +\infty$

x	-2			
$f(x)$	$+$	$+$	$+$	$+$

$x^2 + 3 = 0$

$x_{1,2} = \frac{\pm \sqrt{-4 \cdot 1 \cdot 3}}{2}$

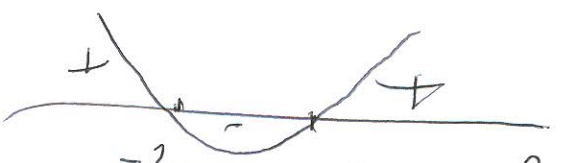
$x_{1,2} = \frac{-2 \pm 2}{2}$

$-4/2$

$x_1 = 0 \quad x_2 = -2$

$\frac{x}{\sqrt{x^2+3}} \cdot \frac{\sqrt{x^2-3}}{\sqrt{x^2-3}} = \frac{x\sqrt{x^2-3}}{}$

0.47085

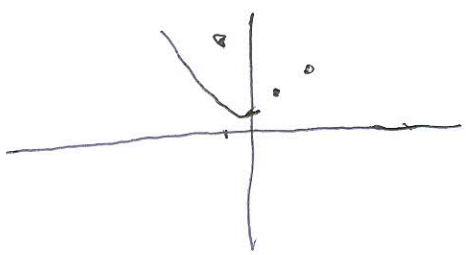


$\left((x^2)^{\frac{1}{2}}\right)^2 - \left((2x)^{\frac{1}{2}}\right)^2$

$x^2 - 2x$

$x = 3$

$y = -3$



$\sqrt{9+9} = 4.24$

$\sqrt[3]{3-3i}$

$r = |R|$

$\varphi = \pi/2 + \arctan y/x$

$\varphi = \pi + \arctan y/x$

$\pi/2 + \arctan \frac{y}{x}$

$W_{k,1} = \sqrt[n]{r}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right)$

2 cos $= -0.96538$

3 cos =

$-\infty \quad -1 \quad 1 \quad +\infty$

$f'(x)$	$-$	$+$
$f(x)$	$\searrow$	$\nearrow$

$$\frac{*}{\sqrt{x^2+2x}-1} = \left(\frac{x}{\sqrt{x^2+2x}}\right)' = \frac{\sqrt{x^2+2x} - \frac{x^2}{\sqrt{x^2+2x}}}{x^2+2x}$$

Dividendo

$$f = x \quad f' = 1$$

$$g = \sqrt{x^2-2x} \quad g' = \frac{x}{\sqrt{x^2-2x}}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\sqrt{x^2+2x}$$

$$\frac{x}{\sqrt{x^2+2x}}$$

$$\frac{\left(\sqrt{x^2+2x}\right)^2 - x^2}{\sqrt{x^2+2x}^{-1/2}} = \frac{\left(\sqrt{x^2+2x}\right)^2 - x^2}{x^2+2x^1}$$

$$x^{-1/2}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ ℰ5

IME I PREZIME: ANTE NEGRO **BROJ INDEKSA:** 17-2-0399-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2+3}$ 10 4
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2+2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf) 12
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) =$ 7+2

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5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2-1))$. 10

Ukupno: 32

5.

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \begin{matrix} \\ \\ (-2) \\ (-2) \\ (-4) \end{matrix} \sim \left[\begin{array}{cccc|c} 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \\ 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] \begin{matrix} \\ \\ \\ (-1) \\ (-1) \end{matrix} \sim$$

$$\left[\begin{array}{cccc|c} 0 & 0 & -4 & 4 & -12 \\ 0 & 0 & -1 & 1 & -3 \\ 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] \begin{matrix} (:4) \\ \\ \\ (:3) \end{matrix} \sim \left[\begin{array}{cccc|c} 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & -1 & 1 & -3 \\ 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \end{array}\right] \begin{matrix} \\ \\ \\ (1) \end{matrix}$$

$$\left[\begin{array}{cccc|c} 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & -1 & 1 & -3 \\ 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \end{array}\right] \begin{matrix} (1) \\ (-1) \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 \end{array}\right] (1)$$

$$\left[\begin{array}{cccc|c} 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 1 \end{array}\right]$$

$4x - y + z + 2u = -1$

$0 - (1+3u) + 3 - u + 2u = -1$

$0 - 1 - 3u + 3 - u + 2u = -1$ ← POKAZUJE DA RJEŠENJE NIJE DOBRO!

$-2u = -1 - 3 + 1$

$-2u = -3$

$u = \frac{3}{2}$

$z = 3 - \frac{3}{2} = \frac{3}{2} \to z = \frac{3}{2}$

$y = 1 + 3\cdot\frac{3}{2} = \frac{11}{2} \to y = \frac{11}{2}$

$x = 0$

$4x - y + z + 2u = -1$

$0 - \frac{11}{2} + \frac{3}{2} + 3 = -1$

$-1 = -1$ ✓

$2x + y - 3u = 4$

$0 + \frac{11}{2} - 3\cdot\frac{3}{2} =$

$$\begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{11}{2} \\ \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}$$

$-z + u = -3$

$x = 0$

$y - 3u = 1$

$-z = -3 + u$

$z = 3 - u$

$y = 1 + 3u$

6/4

6. $f(x) = \ln(\cos(2x^2 - 1))$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (-\sin(2x^2-1)) \cdot 4x$$

$$f'(x) = -\frac{4x(\sin(2x^2-1))}{(\cos(2x^2-1))}$$ ✓

4.

a) $\lim_{x \to 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = \frac{\sqrt{3-0} - \sqrt{3}}{0} = \frac{0}{0} = 0$ ✗

x	$\frac{\sqrt{3+x} - \sqrt{3}}{x}$
1	0.26
0.9	0.269
0.5	0.27
0.1	0.28
↓	↓
0	0

x	$\frac{\sqrt{3+x} - \sqrt{3}}{x}$
0.001	0.288
0.00001	0.2886
0.1×10^{-8}	0.28868
0.1×10^{-20}	0

RASTE DO 0.28, NA KONCU TEŽI K 0

b) $\lim_{x \to \infty} \left(\frac{x^2 + 3}{x^2} \right) = \frac{\frac{x^2}{x^2} + \frac{3}{x^2}}{\frac{x^2}{x^2}} = 1$ ✓

x	$\left(\frac{x^2+3}{x^2}\right)$
1	4
10	1.03
100	1.0003
1000	1.000003
↓	↓
∞	1

✓

ANTE NEGRO

17-2-0399-2014

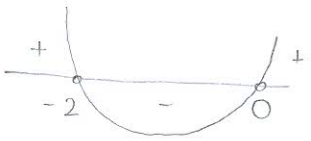
3. $h(x) = \sqrt{x^2+2x}$

$D_f: \; x^2+2x \geq 0$

$x(x+2) \geq 0$

$x_1 = 0 \qquad x+2=0$

$x_2 = -2$



$D_f = \langle -\infty, -2] \cup [0, +\infty \rangle$

4

D.K.A ✓ L.K.A. ?

V.A.

$\lim\limits_{x \to -2-} \sqrt{x^2+2x} = 0$

$\lim\limits_{x \to 0+} \sqrt{x^2+2x} = 0$

K.A.

$k = \lim\limits_{x \to +\infty} \sqrt{x^2+2x} = 1$

$l = \lim\limits_{x \to -\infty} \sqrt{x^2+2x} = 1$

$Y = X + 1$

X	Y
0	1
1	2
2	3

$c = f_x - k_x = \lim\limits_{x \to +\infty} \sqrt{x^2+2x} - x \cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x} = \lim\limits_{x \to +\infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x} = \lim\limits_{x \to +\infty} \frac{2x}{\sqrt{x^2+2x}+x} = \frac{2}{2} = 1$

$c = f(x) - l_x = \lim\limits_{x \to -\infty} \sqrt{x^2+2x} - x \cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x} = \lim\limits_{x \to +\infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x} = \lim\limits_{x \to -\infty} \frac{2x}{\sqrt{x^2+2x}+x} = \frac{2}{2} = 1$

$f(x) = 0$

$\sqrt{x^2+2x} = 0 \; / ^2$

$x^2+2x = 0$

$x(x+2) = 0$

$x_1 = 0 \qquad x_2 = -2$

$f'(x) = \frac{1}{2\sqrt{x^2+2x}} \cdot (2x+2)$

$f'(x) = \frac{2(x+1)}{2\sqrt{x^2+2x}} = \frac{x+1}{\sqrt{x^2+2x}}$

$f'(x) = 0$

$x + 1 = 0$

$x = -1$

	$-\infty$		-2		-1		0	
$f'(x)$		−		−	0	+		+
$f(x)$		↘		↘	min	↗		↗

NEMA STACIONARNIH TOČAKA

-1 NEPARNOG DOMENA

$f''(x) = \frac{1\left(\sqrt{x^2+2x}\right) - (x+1)\left[\frac{1}{2\sqrt{x^2+2x}} \cdot (2x+2)\right]}{\left(\sqrt{x^2+2x}\right)^2}$

$f''(x) = \frac{\sqrt{x^2+2x} - \frac{(x+1)(2x+2)}{2\sqrt{x^2+2x}}}{\left(\sqrt{x^2+2x}\right)^2}$

$f''(x) = \frac{\sqrt{x^2+2x} - \frac{2x^2+2x+2x+2}{2\sqrt{x^2+2x}}}{\left(\sqrt{x^2+2x}\right)^2}$

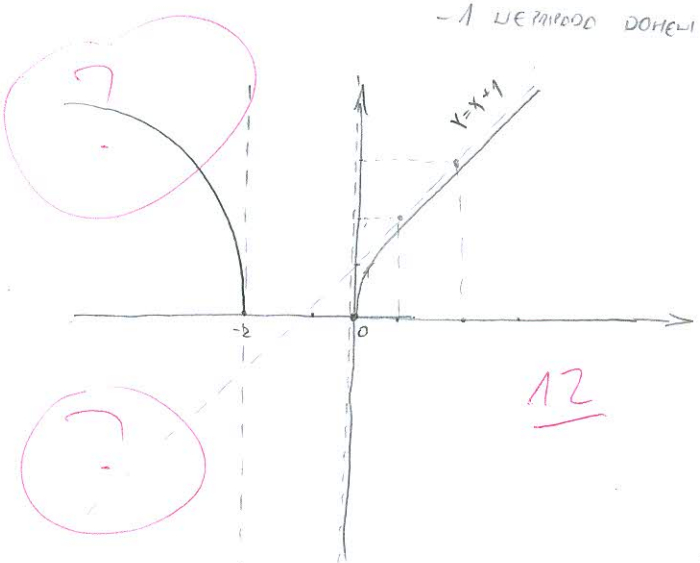
$f''(x) = \frac{\sqrt{x^2+2x} - \frac{2(x^2+2x+1)}{2\sqrt{x^2+2x}}}{\left(\sqrt{x^2+2x}\right)^2}$

$f''(x) = \frac{\frac{x^2+2x-x^2-2x-1}{\sqrt{x^2+2x}}}{\left(\sqrt{x^2+2x}\right)^2}$

$f''(x) = \frac{-\frac{1}{\sqrt{x^2+2x}}}{\left(\sqrt{x^2+2x}\right)^2} = -\frac{1}{\left(\sqrt{x^2+2x}\right)^3} = -\frac{1}{(x^2+2x)\sqrt{x^2+2x}}$

$f''(x) = 0 \qquad -\frac{1}{(x^2+2x)\sqrt{x^2+2x}} = 0$

$-1 \neq 0$ NEMA TOČAKA INFLEKSIJE



12

2.) $g(x) = \sqrt{x^2+3}$

D.

$x^2+3 \geq 0$

$x^2+3=0$

$x^2=-3$

$D_f = \mathbb{R}$

$f'(x) = \frac{1}{2\sqrt{x^2+3}} \cdot 2x = \frac{2x}{2\sqrt{x^2+3}} = \frac{x}{\sqrt{x^2+3}}$

$f'(x) = x = 0$

---

3. ? $x=0$

$-2+u=3$

$x-3u=1$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$

IME I PREZIME: LUCIAN SORIĆ **BROJ INDEKSA:** 17-2-0404-2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat
   (a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) =$ 7+2
   (b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) =$ 4+2
5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno: 14

4. b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) = \lim_{n\to\infty}\frac{x^2+3}{x^2}\ /:x^2 = \frac{1\left(\frac{3}{x^2}\right)}{1} = 1$ ✓

a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) = \lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3}}{x} = \frac{\sqrt{3+0}-\sqrt{3}}{0} = 0$ ✗

2. $g(x) = \sqrt{x^2+3} = (x^2+3)^{\frac{1}{2}}$

$g'(x) = \frac{1}{2}(2x)^{-\frac{1}{2}}$

$g'(x) = x^{-\frac{1}{2}}$

$g(x) = \sqrt{x}$

(6) $f(x) = \ln(\cos(2x^2-1))$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \left(-\sin(2x^2-1)\right) \cdot 4x$$

$$f'(x) = \frac{-\sin(2x^2-1)}{\cos(2x^2-1)} \cdot 4x \quad \checkmark$$

$$f'(x) = -\tan \cdot 4x \quad \times$$

$$f'(x) = -4x \tan$$

LUCIAN SORIĆ

5. $4x - y + z + 2u = -1$

$2x + y - 3u = 4$

$x - y + 2z + u = 2$

$2x + y + z - 4u = 7$

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \text{ ZAMJENA R3 i R1} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \begin{array}{l} /\cdot(-2) \\ /\cdot(-4) \\ + \\ + \\ + \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] \begin{array}{l} \text{ZAMJENA R4} \\ \\ \\ \text{ZAMJENA R2} \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -3 & -6 & 3 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \end{array}\right] /:3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \end{array}\right] \begin{array}{l} + \\ /\cdot 1 \\ /\cdot(-3) \\ /\cdot(-3) \\ + \\ + \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & -4 & 4 & -12 \\ 0 & 0 & -1 & 1 & -3 \end{array}\right] /:(-4)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array}\right] \begin{array}{l} + \\ + \\ /\cdot 1 \\ /\cdot 1 \\ /\cdot(-1) \\ + \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

NEMA RJEŠENJA ✗

$4 \cdot 0 - 4 + 3 + 2 \cdot 0 = -1$

$-1 = -1$ //

$2 \cdot 0 + 4 - 3 \cdot 0 = 4$

$4 = 4$ //

$0 - 4 + 2 \cdot 3 + 0 = 2$

$2 = 2$ //

$2 \cdot 0 + 4 + 3 - 4 \cdot 0 = 7$

$7 = 7$ //

(3.) $h(x) = \sqrt{x^2 + 2x}$

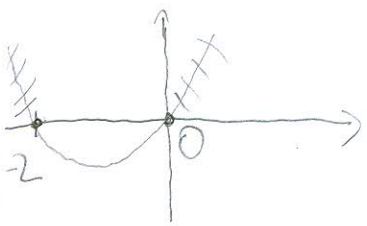
1) DOMENA

$x^2 + 2x \geq 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x_1 = 0 \quad x_2 = -2$



$D(h) = \langle -\infty, -2] \cup [0, +\infty \rangle$

2) H.A

$\lim_{x \to -2} h(x) = \lim_{x \to -2} \sqrt{x^2+2x} = \sqrt{(-2)^2 + 2(-2)} = \sqrt{8}$

$\lim_{x \to 0} h(x) = \lim_{x \to 0} \sqrt{x^2+2x} = \sqrt{0^2 + 2 \cdot 0} = 0$ ✗

V.A

$\lim_{x \to +\infty} h(x) = \lim_{x \to +\infty} \sqrt{x^2+2x} = 0 \rightarrow$ V.A. ✗

(3) $h(x) = \sqrt{x^2+2x} = (x^2+2x)^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}(2x+2)^{-\frac{1}{2}}$

$h'(x) = (x+1)^{-\frac{1}{2}}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{E}5$

IME I PREZIME: IVAN PAŽANIN **BROJ INDEKSA:** 0112054304

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2+3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2+2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right)^{\frac{1}{x}}$ 7+2

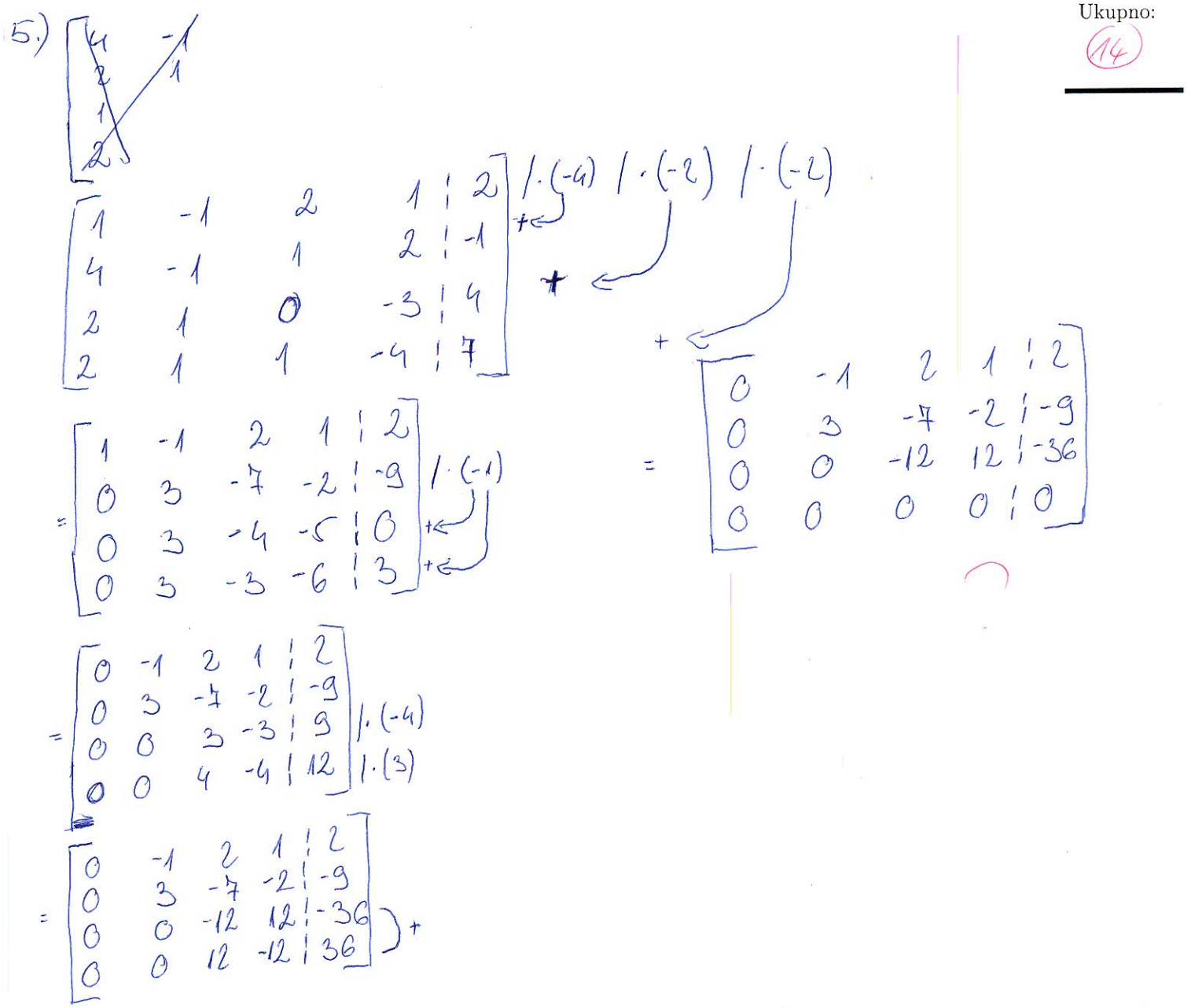
   (b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right)^{x^2}$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcr} 4x & - & y & + & z & + & 2u = & -1 \\ 2x & + & y & & & - & 3u = & 4 \\ x & - & y & + & 2z & + & u = & 2 \\ 2x & + & y & + & z & - & 4u = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2-1))$. 10

Ukupno: 14



IVAN PAZAVIN

3) $h(x) = \sqrt{x^2 + 2x}$

VERT: $\lim \frac{f(x)}{x}$

KOSE:

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{E}5$

IME I PREZIME: MATE ĆARAC **BROJ INDEKSA:** 0269075987

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2+3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2+2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) =$ 7+2

   (b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) =$ 4+2
5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2-1))$. 10

Ukupno: 14

6. $f(x) = \ln(\cos(2x^2-1))$

$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (-\sin(2x^2-1)) \cdot 4x$ ✓

3. $h(x) = \sqrt{x^2+2x}$

$\lim_{x\to-\infty} \frac{\sqrt{x^2+2x}}{1} \cdot \frac{1/x}{1/x} = \frac{1+0}{0} = \frac{1}{0} = +\infty$

NEMA H.A.

$k = \lim_{x\to\infty} \frac{\sqrt{x^2+2x}}{x} \cdot \frac{1/x}{1/x} = \frac{1+0}{1} = \frac{1}{1} = 1$

$l = \lim_{x\to-\infty} \sqrt{x^2+2x} - x = \frac{\sqrt{x^2+2x} - x}{1} \cdot \frac{-1/x}{-1/x} = \frac{1+0-1}{0} = 0$

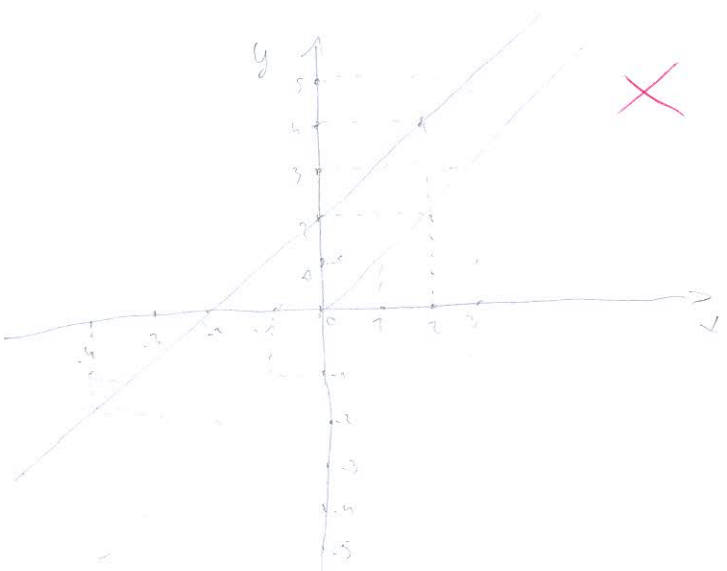
$y = x + 0$ ✗

$$k = \lim_{x \to -\infty} \frac{\sqrt{x^2+2x}\,/x}{1\,/x} = -1$$

$$l = \lim_{x \to -\infty} \frac{\sqrt{x^2+2x} + x\,/x}{1\,/x} = \frac{-2}{0} = -2$$

$y = 1x + 2$ ×

x	1	2	3
$y = 1x + 0$	1	2	3



×

x	1	2	3
$y = 1x + 2$	1	4	5

$$f\,x = \sqrt{x^2+2x}$$

$$f'_x = \frac{1}{2 \cdot \sqrt{x^2+2x}}$$

$$f'_x = 0$$

$$\sqrt{x^2+2x} = 0$$

$$x^2 + 2x = 0$$

$$x^2 = 0 \,/\sqrt{\ }$$

$$x = 0$$

$$2x = 0$$

$$x = -2$$

MATE PARAC

(4.) b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) = \frac{x^2+3 \mid :x^2}{x^2 \mid :x^2} = \frac{1+0}{1} = \frac{1}{1} = 1$ ✓

a) $\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3}}{x} = \left(\frac{0}{0}\right) \overset{L'H}{=} \frac{\frac{1}{3+1}-\frac{1}{3}}{x} = \frac{0}{0} = 0$ ✗ ✗

$\lim_{x\to 0,1}\frac{\sqrt{3+x}-\sqrt{3}}{x} = \frac{\sqrt{3+0,1}-\sqrt{3}}{0,1} = 0,28$

$\lim_{x\to 0,01}\frac{\sqrt{3+x}-\sqrt{3}}{x} = 0,28$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{E}5$

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: STIPE NINČEVIĆ

BROJ INDEKSA: 17-2-0454-2014 0018116795

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2+3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2+2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
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   (a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) =$ 7+2

   (b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) =$ 4+2
5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcr} 4x & - & y & + & z & + & 2u = & -1 \\ 2x & + & y & & & - & 3u = & 4 \\ x & - & y & + & 2z & + & u = & 2 \\ 2x & + & y & + & z & - & 4u = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2-1))$. 10

Ukupno: 15

5.

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \end{array}\right] \begin{array}{l} \\ -2 \\ -2 \\ -4 \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -3 & -6 & 3 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \end{array}\right] /:3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & 9 \end{array}\right] \begin{array}{l} \\ 1 \\ -3 \\ -3 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & -4 & 4 & 6 \end{array}\right] /:(-1) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 18 \end{array}\right]$$

$x_4 = 0$

$y - 3u = 4$

$z - u = 3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -4 & 4 & 6 \end{array}\right] \begin{array}{l} \\ -1 \\ 1 \\ 4 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 18 \end{array}\right]$$

$x = 0$

(3.) $h(x) = \sqrt{x^2+2x}$

$x^2+2x \geq 0$

$x(x+2) \geq 0$

$x \geq 0 \quad x \geq -2$

DOMENA

$x \in [0, +\infty\rangle$

$\sqrt{x^2+2x} = 0 \,/^2$

$x^2+2x = 0$

$x(x+2) = 0$

$x = 0 \quad x = -2$

NULTOČKE

$\emptyset$

ASIMPTOTE ?

SIME NINČEVIĆ
MATEMATIKA 1

5.

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] /:4 \quad \left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{2}{4} & -\frac{1}{4} \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \begin{array}{l} \\ -2 \\ -1 \\ -2 \end{array} \quad \left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{2}{4} & -\frac{1}{4} \\ 0 & \frac{9}{2} & -\frac{1}{2} & -4 & \frac{9}{2} \\ 0 & -\frac{9}{4} & \frac{7}{4} & \frac{1}{2} & \frac{9}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & -5 & \frac{15}{2} \end{array}\right] /:\frac{3}{2}$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 3 \\ 0 & -\frac{9}{4} & \frac{7}{4} & \frac{1}{2} & \frac{9}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & -5 & \frac{15}{2} \end{array}\right] \begin{array}{l} \\ \frac{1}{4} \\ \frac{9}{4} \\ -\frac{3}{2} \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{6} & -\frac{1}{6} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 3 \\ 0 & 0 & \frac{3}{2} & -\frac{3}{2} & \frac{18}{4} \\ 0 & 0 & 1 & -1 & 3 \end{array}\right] /:\frac{3}{2} \quad \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{6} & -\frac{1}{6} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 3 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -1 & 3 \end{array}\right] \begin{array}{l} -\frac{1}{6} \\ \frac{1}{3} \\ -1 \end{array}$$

$$\left[\begin{array}{cccc|c} x & y & z & u & \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x = 0$

$y - 3u = 4 \quad (y = 3u + 4)$

$z - u = 3 \quad (z = 3 + u)$

$2x + y - 3u = 4$

$2x + 3u - 3u = 4$

$2x = 0$

$-\frac{\frac{1}{2}}{\frac{3}{2}} = -\frac{2}{6} = -\frac{1}{3}$ $\quad -\frac{1}{12} + \frac{1}{4} = \frac{-1+3}{12} = \frac{2}{12} = \frac{1}{6}$

$-\frac{\frac{1}{4}}{\frac{3}{2}} = -\frac{8}{3}$ $\quad \frac{\frac{9}{4}}{\frac{3}{2}} = \frac{9}{3} = 3$ $\quad -\frac{8}{12} + \frac{1}{2} = \frac{-8+6}{12} = -\frac{2}{12} = -\frac{1}{6}$

$-\frac{1}{3} \cdot \left(-\frac{3}{2}\right) = \frac{3}{6} = \frac{1}{2}$ $\quad -\frac{1}{4} + \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

$-\frac{8}{3} \cdot \left(-\frac{3}{2}\right) = \frac{24}{6} = 4$ $\quad -\frac{8}{12} - \frac{1}{4} + \frac{7}{4} = \frac{6}{4} = \frac{3}{2}$

$-\frac{9}{2} + \frac{15}{2} = \frac{6}{2} = 3$ $\quad -\frac{24}{12} = -2 + \frac{1}{2} = -\frac{3}{2}$

$\frac{\frac{18}{4}}{\frac{3}{2}} = \frac{6}{2} = 3$

$2x + y + z - 4u = 7$

$0 + 3u + 4 + 3 + u - 4u = 7$

$4u + 7 - 4u = 7$

PROVJERA:

$4 \cdot 0 - (3u + 4) + (3 + u) + 2u = -1$ ✓

$2 \cdot 0 + (3u + 4) + (3 + u) - 4u = 7$ ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ ε5

IME I PREZIME: TONI BABIĆ **BROJ INDEKSA:** 17-1-0272-2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3

2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10

3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x \to 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$ 7+2

   (b) $\lim_{n \to \infty} \left(\frac{x^2 + 3}{x^2} \right) =$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno: 14

TONI BABIĆ

(2) $g(x) = \sqrt{x^2+3}$

$x^2+3 \geq 0$

$x^2 \geq -3$

$x = \pm\sqrt{-3}$

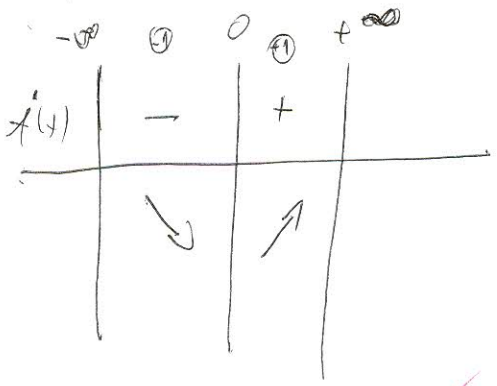
NEMA RJEŠENJA

$D(f) = x \in \mathbb{R}$

$g'(x) = \frac{1}{2\sqrt{x^2+3}} * 2x = \frac{2x}{2\sqrt{x^2+3}}$

$2x = 0 \quad / : 2$

$x = 0$



T(0, 1.732) ✓

③ $h(x) = \sqrt{x^2+2x}$

① $x^2+2x \geq 0$

$x^2+2x = 0$

$x_{1/2} = \frac{-2 \pm \sqrt{4}}{2}$

$x_1 = \frac{-2+2}{2} = \frac{0}{2} = 0$

$x_2 = \frac{-2-2}{2} = -2$

$D(h) = \langle -\infty, -2 \rangle \cup \langle 0, +\infty \rangle$

④ $h'(x) = \frac{1}{2\sqrt{x^2+2x}} \cdot (2x+2)$

$h'(x) = \frac{2x+2}{2\sqrt{x^2+2x}}$

$2x+2 = 0$

$2x = -2$

$x = -1$

② NULTOČKE

$\sqrt{x^2+2x} = 0 \quad /^2$

$x^2+2x = 0$

$x_1 = 0$

$x_2 = -2$

$N_1(0,0)$

$N_2(-2,0)$

$x = 0$

$y = \sqrt{0^2+2\cdot 0}$

$y = 0$

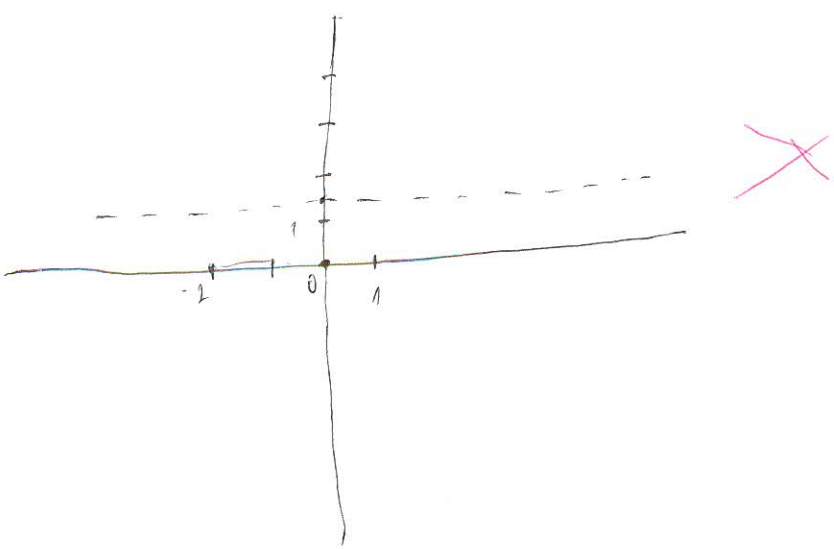
③ ASIMPTOTE

H. A ...

$\lim_{x \to \infty} \sqrt{x^2+2x} = \lim_{x \to \infty} \frac{\sqrt{x^2+2x}}{x} = \frac{\infty}{\infty}$

$\overset{L.H.}{\Rightarrow} \lim_{x \to \infty} \frac{2x+2}{2\sqrt{x^2+2x}} \overset{L.H.}{\Rightarrow} \lim_{x \to \infty} \frac{2}{\sqrt{2}} =$

$= 1.414$ //



TONI BABIĆ

(4.) a) $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$

b) $\lim_{n \to \infty} \frac{x^2+3}{x^2}$

a) $\lim_{x \to 0} \frac{\sqrt{3+0} - \sqrt{3}}{0} \Rightarrow \lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} \cdot \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}} = \lim_{x \to 0} \frac{3+x-3}{x \cdot (\sqrt{3+x} + \sqrt{3})}$

$\lim_{x \to 0} \frac{3+x \to}{x \cdot (\sqrt{3+x} + \sqrt{3})} \overset{L.H.}{\Rightarrow} \frac{1}{1 \cdot \frac{1}{2}}$ ✗

b) $\lim_{n \to \infty} \frac{x^2+3}{x^2} \overset{LH}{\Rightarrow} \lim_{n \to \infty} \frac{2x}{2x} \overset{L.H}{\Rightarrow} \lim_{n \to \infty} \frac{2}{2} = 1$ ✓

(5.)

$$x - y + 2z + v = 1$$
$$2x + y - 3v = 4$$
$$4x - y + z + 2v = -1$$
$$2x + y + z - 4v = 7$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \begin{array}{l} / \cdot (-2) \\ / \cdot (-4) \\ / \cdot (-2) \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -5 \\ 0 & 3 & 3 & -6 & 3 \end{array}\right] \begin{array}{l} \\ \\ / \cdot (-1) \\ \end{array} =$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 0 & -3 & 3 & -5 \\ 0 & 0 & 7 & -1 & 3 \end{array}\right] /:3 = \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 0 & -1 & -1 & \frac{5}{3} \\ 0 & 0 & 7 & -1 & 3 \end{array}\right] \cdot (-7) =$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 0 & -1 & -1 & \frac{5}{3} \\ 0 & 0 & 0 & 6 & -\frac{26}{3} \end{array}\right] /:3 = \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 0 & -1 & -1 & \frac{5}{3} \\ 0 & 0 & 0 & 2 & -26 \end{array}\right]$$

$2v = -26$

$v = -13$

$-1z - 1v = \frac{5}{3}$

$-z = \frac{5}{3} + (-13)$

$z = \frac{34}{3}$

→

$3y - 4z - 5v = 0$

$3y = 4 \cdot \frac{34}{3} - 5 \cdot (-13)$

$3y = \frac{331}{3} \quad / : 3$

$y = \frac{331}{9}$

$x - \frac{331}{9} + 2 \cdot \frac{34}{3} + -13 = 2$

$x = \frac{331}{9} - 2 \cdot \frac{34}{3} + 13 + 2$

$x = \frac{262}{9}$

PROVJERA

$\frac{262}{9} - \frac{331}{9} + 2 \cdot \frac{34}{3} - 13 = 2$

$2 = 2$ //

MIJE POTPUNA PROVJERA !

(6.) $f(x) = \ln(\cos(2x^2 - 1))$

$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot -\sin 4x = \frac{-\sin 4x}{\cos(2x^2-1)}$ //

×

① $z^3 - 3 + 3i = 0$

$z^3 = 3 - 3i$

$z^3 = r([redacted] \cos \frac{\pi}{2} \cdot$ ϕ

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$

IME I PREZIME: PINO KOTLAR **BROJ INDEKSA:** 17-1-0261-2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2+3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2+2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
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   (b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) =$ 4+2
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$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2-1))$. 10

Ukupno: 14

5.

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \begin{array}{l} \cdot(-2)\cdot(-4)\cdot \\ + \\ + \\ + \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] (-1)$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] \begin{array}{l} \\ \\ \\ +4 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 5 & -2 & 11 \\ 0 & 1 & 3 & -3 & 3 \\ 0 & 0 & -16 & -7 & -36 \\ 0 & 0 & -12 & -15 & -24 \end{array}\right] \cdot(-\frac{1}{16}) \sim \left[\begin{array}{cccc|c} 1 & 0 & 5 & -2 & 11 \\ 0 & 1 & 3 & -3 & 9 \\ 0 & 0 & 1 & \frac{7}{16} & \frac{9}{4} \\ 0 & 0 & -12 & -15 & -24 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & & \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & & \\ 0 & 0 & 0 & & \end{array}\right]$$

4.

a) $\lim_{x \to 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = \left[\frac{0}{0} \right] = 0$ $\left(\frac{\sqrt{3+0} - \sqrt{3}}{0} \right) = \frac{\sqrt{3} - \sqrt{3}}{0} = \frac{0}{0} = 0$ ✗

b) $\lim_{x \to \infty} \left(\frac{x^2 + 3}{x^2} \; /:x^2 \; /:x^2 \right) = 1$ ✓

$$\frac{1 + \frac{1}{x^2}}{1}$$

PINO KOTLAR 17-1-0261-2014

$\frac{2}{4} + \frac{16}{4} = \frac{18}{4} = \frac{9}{2}$

5

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] /:4 \sim \left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ 0 & \frac{3}{2} & -\frac{2}{4} & -4 & \frac{9}{2} \\ 0 & & & & \end{array}\right]$$

1.

$z^3 - 3 + 3i = 0$

$z^3 = 3 - 3i$

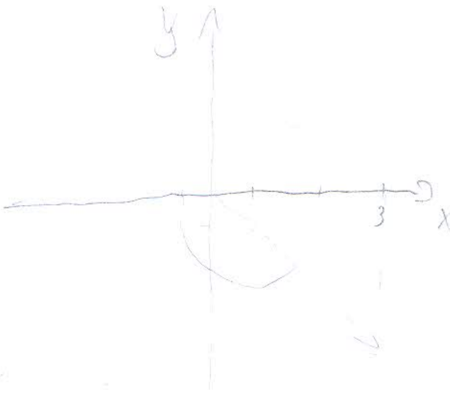
$z = \sqrt[3]{3 - 3i}$

$w = 3 - 3i$

$|w| = \sqrt{3^2 - 3i^2}$

$|w| = \sqrt{9 + 9}$

$|w| = \sqrt{18}$



$k = 0, 1, 2, 3, \dots, n-1$

$k = 0$

$z_1 = \sqrt[n]{|z_1|}\left(\cos\frac{\varphi \cdot 2k\pi}{n} + i\sin\frac{\varphi \cdot 2k\pi}{n}\right)$

$z_1 = 1.62\,(\cos 1.83 + i\sin 1.83)$

$z_1 = 1.62\,(-0.26 + 0.97i)$

$z_1 = -0.42 + 1.57i$

$k = 1$

$z_2 = 1.62\,(\cos 11.51 + i\sin 11.51)$

$z_2 = 1.62\,(0.49 - 0.87i)$

$z_2 = 0.79 - 1.41i$

$\varphi = 2\pi - \arctan\frac{y}{x}$

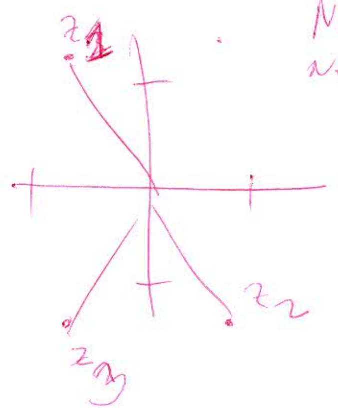
$\varphi = 6.283 - 0.785 =$

$= 5.498$

$k = 2$

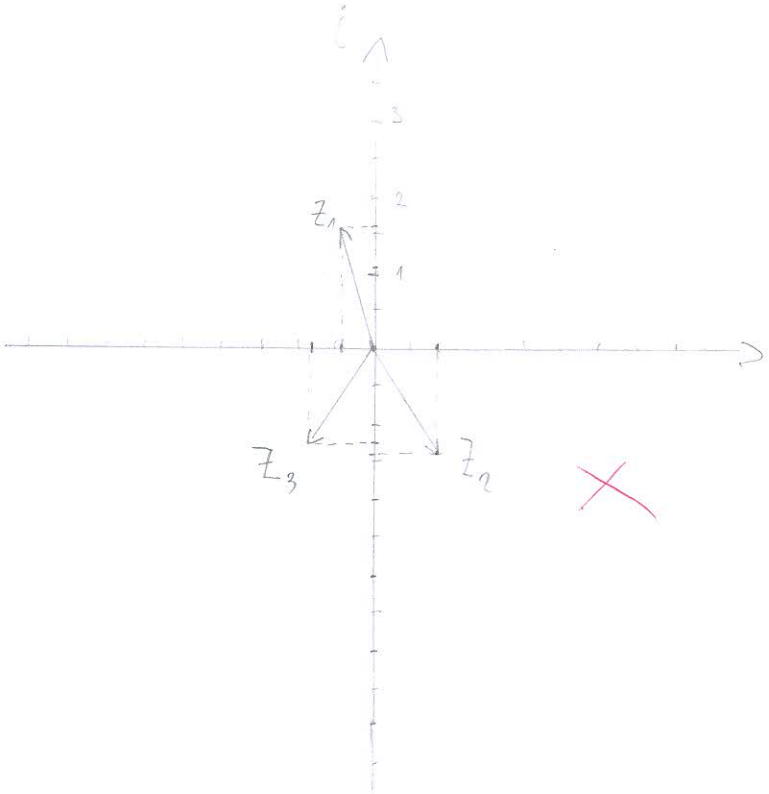
$z_3 = 1.62\,(-0.54 - 0.84)$

$z_3 = -0.87 - 1.36$



NEŠTO NE ŠTIMA!

4.



6.

$$f(x) = \ln(\cos(2x^2 - 1))$$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot -\sin(2x^2-1) \cdot 4x$$

$$f'(x) = \frac{4x}{\cos(2x^2-1)} \cdot -\sin(2x^2-1)$$

$$f'(x) = \frac{-4x\sin(2x^2-1)}{\cos(2x^2-1)}$$

$$f'(x) = -4x\tan(2x^2-1)$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$

IME I PREZIME: KLAUDIO KOTLAR

BROJ INDEKSA: 17-2-0236-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x \to 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$ 7+2

   (b) $\lim_{n \to \infty} \left(\frac{x^2 + 3}{x^2} \right) =$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno: 10

KLASIČNO REŠEVAR

⑥ $f'(x) = \left[\ln(\cos(2x^2-1))\right]'$

$= \frac{1}{\cos(2x^2-1)} \cdot (-\sin(2x^2-1)) \cdot 4x - 0$ ✓

$= \frac{-\sin(2x^2-1)}{-\cos(2x^2-1)} \cdot 4x$ ✗ ???

④ a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) = \frac{0}{0}$ NEODREDEN OBLIK

$\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) \overset{L-H}{=} \lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right)' = \lim_{x\to 0}\left(\frac{(3+x)^{\frac{1}{2}} - 3^{\frac{1}{2}}}{x}\right)'$ ✗ ✗

$\lim_{x\to 0}\frac{\frac{1}{2}(3+x)^{-\frac{1}{2}} \cdot 0 \cdot 1}{1} = \frac{0}{1} = 0$

② $y(x) = \sqrt{x^2+3}$

$y'(x) = \left[(x^2+3)^{\frac{1}{2}}\right]'$

$= \frac{1}{2}(x^2+3)^{-\frac{1}{2}} \cdot 2x$

$\frac{1}{2} \cdot (x^2+3)^{-\frac{1}{2}} \cdot 2x = 0$

$\sqrt{x^2+3} \geq 0$

$\sqrt{x^2+3} = 0 \;/^2$

$x^2+3 = 0$

$x^2 = 3 \;/\sqrt{\ }$

$x_1 = 1.73$

$x_2 = -1.73$

$x \in [-1.73, 1.73]$

x	$-\infty$		-1.73		1.73		$+\infty$
$f(x)$		+	N/D	+			
$f'(x)$		+	N/D	+			
		↘		↗			

$D_f: \langle -\infty, -1.73] \cup [1.73, +\infty\rangle$ ✗

GLOB. MIN. FUNK. JE V TOČKI $(-1.73, 0)$

GLOB. MAX. NEMA JER FUNKCIJA TEŽI V BESKONAČNO. ✗

$x_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 0}}{2 \cdot 1}$

$x_{1,2} = \frac{-3 \pm \sqrt{9}}{2} = \frac{-3 \pm 3}{2}$

$x_1 = 0$

$x_2 = -3$

$h(x) = \sqrt{x^2+2x}$

① DOMENA

$\sqrt{x^2+2x} \geq 0$

$D_f = \langle -\infty, -2] \cup [0, +\infty \rangle$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{-2 \pm \sqrt{2^2-4\cdot 1\cdot 0}}{2\cdot 1}$

$x_{1,2} = \frac{-2 \pm \sqrt{4}}{2}$

$x_1 = 0$

$x_2 = -2$

② ASIMPTOTE

V. A.

$\lim_{x \to 0^+} \sqrt{x^2+2x} = \sqrt{1^2+2\cdot 1} = +\infty$

$\lim_{x \to 0^-} \sqrt{x^2+2x} = \sqrt{(-1)^2+2\cdot(-1)} = -\infty$

NEMA V. A.

H. A.

$\lim_{x \to +\infty} \frac{\sqrt{x^2+2x}}{1} \cdot \frac{\sqrt{x^2+2x}}{\sqrt{x^2+2x}} = \lim_{x\to+\infty} \frac{\left(\sqrt{x^2+2x}\right)^2}{\sqrt{x^2+2x}} = \lim_{x\to+\infty} \frac{x^2+2x \;/:x^2}{\sqrt{x^2+2x}\;/:x^2} = \lim_{x\to+\infty} \frac{\frac{x^2}{x^2}+\frac{2x}{x^2}}{\sqrt{\frac{x^2}{x^3}+\frac{2x}{x^3}}} = \lim_{x\to+\infty} \frac{1+\frac{2}{x}}{\sqrt{\frac{1}{x}+\frac{2}{x^2}}} = \frac{1}{0}$

NEMA H. A.

K. A.

$k = \lim_{x\to\infty} \frac{f_x}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2+2x}}{x}\;/:x = \lim_{x\to\infty} \frac{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}}{\frac{x}{x}} = \frac{\sqrt{1}}{1} = 1$

$\lim_{x\to\infty} [f_x - k\cdot x] = \lim_{x\to\infty} \sqrt{x^2+2x} - 1\cdot x$

$= \lim_{x\to\infty} \sqrt{x^2+2x} - x \cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x}$

$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+2x}\right)^2 - (x)^2}{\sqrt{x^2+2x}+x} = \lim_{x\to\infty} \frac{x^2+2x-x^2\;/:x}{\sqrt{x^2+2x}+x\;/:x} = \lim_{x\to\infty} \frac{\frac{2x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}+\frac{x}{x}}$

$= \frac{0}{\sqrt{1}+1} = 0$

③ SVOJSTVA

$f(x) = f(-x)$

$f(-x) = -f(x)$

$\sqrt{x^2+2x} = \sqrt{(-x)^2-2x}$

$= \sqrt{x^2-2x} = \sqrt{-(x^2-2x)}$

$= \sqrt{(-x)^2+2x}$

$= \sqrt{x^2+2x}$

FUNK. NIJE NITI PARNA NITI NEPARNA

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$

IME I PREZIME: KREŠIMIR PILIĆA

BROJ INDEKSA: 12-2-0285-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3

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   (b) $\lim_{n \to \infty} \left(\frac{x^2 + 3}{x^2} \right) =$ 4+2 4

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno: ~~0~~ (4)

Kraj

(4) a) $\lim_{x \to 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = \frac{\frac{\sqrt{3+x}}{x} - \frac{\sqrt{3}}{x}}{x} = 0$ ✗

b) $\lim_{n \to \infty} \left(\frac{x^2+3}{x^2} \right) = \frac{\frac{x^2}{x^2} + \frac{3}{x^2}}{\frac{x^2}{x^2}} = 1$ ✗ ✓

KREŠIMIR PIPLICA

② $g(x) = \sqrt{x^2+13}$

Domena

$x^2+13 \geq 0$

$x^2 \geq -13$

$D_f = \mathbb{R}$

$f'(x) = \sqrt{x^2+13}$

$= \frac{1}{2\sqrt{x^2+13}} \cdot 2x$

$= \frac{2x}{2\sqrt{x^2+13}}$

$f'(x) = 0$

$\frac{2x}{2\sqrt{x^2+13}} = 0 \quad / \cdot 2\sqrt{x^2+13}$

$2x = 0$

$x = 0$

③ $h(x) = \sqrt{x^2+2x} =$

① Domena

$x^2+2x \geq 0 \quad / \cdot x$

$3x \geq 0$

$D_f = \mathbb{R}$

② nultočke

⑥ $f(x) = \ln(\cos(2x^2-1)) = \frac{(\cos(2x^2-1))' \cdot 4x^2+4}{x}$

⑤

$$\begin{aligned} 4x - y + z + 2v &= -1 \\ 2x + y \quad - 3v &= 4 \\ x - y + 2z + v &= 2 \\ 2x + y + z - 4v &= 7 \end{aligned}$$

$$\sim \left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right]$$

$\cdot(-2) + \text{II}$

$\cdot(2) + \text{III}$, $\cdot(-1) + \text{IV}$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -3 & -5 \\ 0 & 2 & -1 & -5 & 5 \end{array}\right]$$

② $g(x) = \sqrt{x^2+3}$

① Porque

$Dg = \mathbb{R}$

②

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ ε5

IME I PREZIME: ANTE IVKOVIĆ **BROJ INDEKSA:** 17-1-0264-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2+3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2+2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) =$ 7+2

   (b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) =$ 4+2
5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcr} 4x & - & y & + & z & + & 2u = & -1 \\ 2x & + & y & & & - & 3u = & 4 \\ x & - & y & + & 2z & + & u = & 2 \\ 2x & + & y & + & z & - & 4u = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2-1))$. 10

Ukupno: (4)

5.

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \cdot\frac{1}{4} \left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \cdot(-2)\ \cdot(-1)\ \cdot(-2)$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ 0 & \frac{3}{2} & -\frac{1}{2} & -4 & \frac{9}{2} \\ 0 & -\frac{3}{4} & \frac{7}{4} & \frac{1}{2} & \frac{5}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & -5 & \frac{15}{2} \end{array}\right] \cdot\frac{2}{3} \qquad \left[\begin{array}{cccc|c} 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 3 \\ 0 & -\frac{3}{4} & \frac{7}{4} & \frac{1}{2} & \frac{9}{4} \\ 0 & \frac{3}{2} & \frac{1}{2} & -5 & \frac{15}{2} \end{array}\right] \cdot\left(\frac{1}{4}\right),\left(\frac{3}{4}\right),\left(-\frac{3}{2}\right)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{6} & -\frac{1}{6} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 3 \\ 0 & 0 & \frac{3}{2} & -\frac{3}{2} & \frac{9}{2} \\ 0 & 0 & 1 & -1 & 3 \end{array}\right] \qquad \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{6} & -\frac{1}{6} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} & 3 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & \frac{3}{2} & -\frac{3}{2} & \frac{9}{2} \end{array}\right] \cdot\left(-\frac{1}{6}\right),\left(\frac{1}{3}\right),\left(-\frac{3}{2}\right)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

MATRICA IMA PARAMETARSKO RJEŠENJE

KOJE?

0

③ $h(x) = \sqrt{x^2+2x}$

$x^2+2x \geq 0$

$x(x+2) \geq 0$

$D = [-2, +\infty)$

$x \geq 0$

$x+2 \geq 0$

$x \geq -2$

V. A. nema

H. A.

$\lim_{x \to \pm\infty} \sqrt{x^2+2x} =$

$h'(x) = \frac{1}{2\sqrt{x^2+2x}} \cdot (2x+2) = \frac{2(x+1)}{2\sqrt{x^2+2x}} = \frac{x+1}{\sqrt{x^2+2x}}$

ANA NEONIC

4.

d) $\lim_{n\to\infty} \left(\frac{x^2+3}{x^2}\right)^{:x^2} :x^2 = \lim_{x\to\infty} \frac{1+\left(\frac{3}{x^2}\right)^{\to 0}}{1} = \frac{1}{1} = 1$ ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{E}5$

IME I PREZIME: MATEO PEDIŠIĆ

BROJ INDEKSA: 17-1-0306-2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) =$ 7+2

   (b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) =$ 4+2
5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcr} 4x & - & y & + & z & + & 2u = & -1 \\ 2x & + & y & & & - & 3u = & 4 \\ x & - & y & + & 2z & + & u = & 2 \\ 2x & + & y & + & z & - & 4u = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno: 4

1) $z^3 - 3 + 3i = 0$

$z^3 = 3 - 3i$

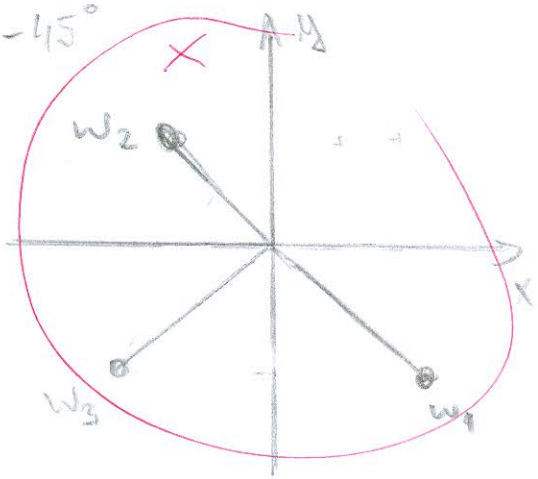
$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-3)^2}$

$r = \sqrt{9+9} = \sqrt{18}$

$r = 4,24$

$\varphi = \operatorname{arctg}\frac{y}{x} = \frac{-3}{3} = \operatorname{arctg} -1$

$\varphi = -45°$



$k=0$

$w_1 = \sqrt[3]{4,24}\left(\cos\frac{-45}{3} + i\sin\frac{-45}{3}\right) =$

$= 1,56 - 0,42i$ NEGDJE GREŠKA

$k=1$

$w_2 = \sqrt[3]{4,24}\left(\cos\frac{-45+2\pi}{3} + i\sin\frac{-45+2\pi}{3}\right)$

$= -0,42 + 1,56i$

$k=2$

$w_3 = \sqrt[3]{4,24}\left(\cos\frac{-45+4\pi}{3} + i\sin\frac{-45+4\pi}{3}\right)$

$= -1,15 - 1,15i$

MATEO PEDIŠIĆ

6) $f(x) = \ln(\cos(2x^2-1))$

$f(x)' = \frac{1}{\cos(2x^2-1)} \cdot \sin(2x^2-1) \cdot 4x - 1$ ✗

$f(x)' = \frac{4x-1 \cdot \sin(2x^2-1)}{\cos(2x^2-1)}$

$f(x)' = \frac{4x-1 \cdot \sin}{\cos}$

4) a) $\lim_{x \to 0} \left(\frac{\sqrt{3}+x-\sqrt{3}}{x}\right) = \frac{\sqrt{3}+0-\sqrt{3}}{0} = \frac{0}{0}$ NEODREĐENI OBLIK

DALJE...?

$\lim_{x \to 0^+} \left(\frac{\sqrt{3}+0^+-\sqrt{3}}{0^+}\right) = 1$ ✗

$\lim_{x \to 0^-} \left(\frac{\sqrt{3}+0^--\sqrt{3}}{0^-}\right) = -1$ ✗

b) $\lim_{x \to \infty} \left(\frac{x^2+3}{x^2}\right)^{1/x^2} = \lim_{x \to \infty} \left(\frac{\frac{x^2}{x^2}+\frac{3}{x^2}}{\frac{x^2}{x^2}}\right) = \lim_{x \to \infty} \frac{1+0}{1} = 1$ ✓

3.) $h(x) = \sqrt{x^2+2x}$ MATEO PEDIŠIĆ

$x^2 + 2x \geq 0$

$x(x+2) \geq 0$

$x \geq 0 \quad x+2 \geq 0$

$x \geq -2$

$D(f) = [-2, +\infty\rangle$ ~~X~~

V.A.

$\lim_{x \to -2} \sqrt{x^2+2x} = \sqrt{(-2)^2+2\cdot(-2)} = 0$ NEMA V.A

H.A

$\lim_{x \to \infty} \sqrt{x^2+2x} = \lim_{x \to \infty} \frac{\sqrt{x^2+2x}}{1} \cdot \frac{\sqrt{x^2-2x}}{\sqrt{x^2-2x}} : \lim_{x \to \infty} \frac{x^2+2x \,/:x^2}{\sqrt{x^2-2x}\,/:x^2}$

$\lim_{x \to \infty} \frac{1+0}{0} = \infty$ NEMA H.A

$\lim_{x \to \infty} \frac{\sqrt{x^2+2x}\,/:x}{x\,/:x} = \lim_{x \to \infty} \frac{\sqrt{1}}{1} = 1 \quad k = 1$ KOSA ASIMPTOTA JE $y = x$ ~~X~~

$\lim_{x \to \infty} \sqrt{x^2+2x} - k \cdot x = \lim_{x \to \infty} \frac{\sqrt{x^2+2x} - x}{1} \cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x} = \frac{x^2+2x-x^2\,/:x^2}{\sqrt{x^2+2x}+x\,/:x^2}$

$\lim_{x \to \infty} \frac{0}{0} = 0$

5.)

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \begin{matrix} \\ (-2) \\ (-4) \\ (-2) \end{matrix} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] /:3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1,33 & -1,66 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] \begin{matrix} \\ (1) \\ (-3) \\ (-3) \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & 0,67 & -0,66 & 2 \\ 0 & 1 & -1,33 & -1,66 & 0 \\ 0 & 0 & -3 & 3 & -9 \\ 0 & 0 & 1 & -1 & 3 \end{array}\right] /:(-3)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0,67 & -0,66 & 2 \\ 0 & 1 & -1,33 & -1,66 & 0 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -3 & 3 & -9 \end{array}\right] \begin{matrix} \\ (3) \\ (1,33) \\ (-0,67) \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & 0 & & \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \end{array}\right] \begin{matrix} \\ (1) \\ (-3) \\ (-3) \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & -4 & 4 & -12 \\ 0 & 0 & -1 & 1 & -3 \end{array}\right] /:(-4)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & +1 & 3 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array}\right] \begin{matrix} \\ \\ (-1) \\ (1) \end{matrix} \begin{matrix} \\ \\ (1) \\ (1) \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

OVA MATRICA NEMA RJEŠENJA

X

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{E}5$

IME I PREZIME: ANTUN Sinić **BROJ INDEKSA:** 00 1595

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3

2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10

3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)

4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x\to 0}\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right) =$ $\frac{\sqrt{\frac{3}{x}+1}-\frac{\sqrt{3}}{x}}{1} = 1$ 7+2

   (b) $\lim_{n\to\infty}\left(\frac{x^2+3}{x^2}\right) =$ $\frac{1+\frac{3}{x^2}}{1} = 1$ 4+2

5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno: 0

5.

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 0 & 1 & -3 & 0 & -5 \\ 0 & 3 & -4 & -5 & 0 \\ 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 0 & 1 & -3 & 0 & -5 \\ 0 & 0 & 5 & -5 & 15 \\ 1 & 0 & 11 & 1 & 17 \\ 0 & 0 & 6 & -6 & 18 \end{array}\right] : 5 \sim$$

$$\left[\begin{array}{cccc|c} 0 & 1 & -3 & 0 & -5 \\ 0 & 0 & 1 & -1 & 3 \\ 1 & 0 & 11 & 1 & 17 \\ 0 & 0 & 6 & -6 & 18 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & -1 & 3 \\ 1 & 0 & 0 & 2 & 14 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$y - 3u = 4$

$z - u = 3$

$x + 2u = 14$

$y = 3u + 4$

$z = u + 3$

$x = -2u + 14$

PROVJERA?

$2 \cdot (-2u+14) + (3u+4) + (u+3) + 2u = 32 \neq X$

1. $z^3 - 3 + 3i = 0$

$z^3 = 3 - 3i$

2. $g(x) = \sqrt{x^2+3}$

$\sqrt{x^2+3} = 0 \quad /^2$

$x^2 + 3 = 0$

$x^2 = -3 \quad /\sqrt{\ }$

$x_1 = \sqrt{-3} = 3i$

$x_2 = -\sqrt{-3} = -3i$

3. $h(x) = \sqrt{x^2 + 2x}$

$h(x) > 0$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$

IME I PREZIME: MATEO MATKOVIĆ **BROJ INDEKSA:** 0269092453

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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$$\begin{array}{rcrcrcrr} 4x & - & y & + & z & + & 2u = & -1 \\ 2x & + & y & & & - & 3u = & 4 \\ x & - & y & + & 2z & + & u = & 2 \\ 2x & + & y & + & z & - & 4u = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

Ukupno:

4.a) $\lim_{x \to \infty} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) = 0$

$\lim_{x \to \infty} \frac{\sqrt{x+3} - \sqrt{3}}{x}$

$\lim_{x \to \infty} \frac{\sqrt{x+3}}{x}$

$\lim_{x \to \infty} \frac{\sqrt{x}}{x}$