MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$

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POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 - 3 + 3i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Koji su globalni ekstremi funkcije $g(x) = \sqrt{x^2 + 3}$ 10
3. Ispitati asimptote funkcije: $h(x) = \sqrt{x^2 + 2x}$. Zatim dovršiti ispitivanje toka i skicirati graf. 10(asimptote) 20(graf)
4. Odrediti i uvrštavanjem (kalkulator) provjeriti rezultat

   (a) $\lim_{x \to 0} \left(\frac{\sqrt{3+x} - \sqrt{3}}{x} \right) =$ 7+2

   (b) $\lim_{n \to \infty} \left(\frac{x^2 + 3}{x^2} \right) =$ 4+2
5. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 15+5

$$\begin{array}{rcrcrcrcr} 4x & - & y & + & z & + & 2u & = & -1 \\ 2x & + & y & & & - & 3u & = & 4 \\ x & - & y & + & 2z & + & u & = & 2 \\ 2x & + & y & + & z & - & 4u & = & 7 \end{array}$$

6. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\cos(2x^2 - 1))$. 10

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5.

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 0 & -3 & 4 \\ 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & -3 & 4 \\ 4 & -1 & 1 & 2 & -1 \\ 2 & 1 & 1 & -4 & 7 \end{array}\right]$$

$-2 \cdot \text{I} + \text{II}$

$$\begin{array}{ccccc} -2 & 2 & -4 & -2 & -4 \\ 2 & 1 & 0 & -3 & 4 \\ \hline 0 & 3 & -4 & -5 & 0 \end{array}$$

$-4 \cdot \text{I} + \text{III}$

$$\begin{array}{ccccc} -4 & 4 & -8 & -4 & -8 \\ 4 & -1 & 1 & 2 & -1 \\ \hline 0 & 3 & -7 & -2 & -9 \end{array}$$

$-2 \cdot \text{I} + \text{IV}$

$$\begin{array}{ccccc} -2 & 2 & -4 & -2 & -4 \\ 2 & 1 & 1 & -4 & 7 \\ \hline 0 & 3 & -3 & -6 & 3 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -3 & -6 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 3 & -4 & -5 & 0 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 1 & -1 & -2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ 1:3 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 3 & -7 & -2 & -9 \\ 0 & 3 & -4 & -5 & 0 \end{array}\right]$$

$-3 \cdot \text{II} + \text{III}$

$$\begin{array}{ccccc} 0 & -3 & 3 & 6 & -3 \\ 0 & 3 & -7 & -2 & -9 \\ \hline 0 & 0 & -4 & 4 & -12 \end{array}$$

$-3 \cdot \text{II} + \text{IV}$

$$\begin{array}{ccccc} 0 & -3 & 3 & 6 & -3 \\ 0 & 3 & -4 & -5 & 0 \\ \hline 0 & 0 & -1 & 1 & -3 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & -4 & 4 & -12 \\ 0 & 0 & -1 & 1 & -3 \end{array}\right] \begin{array}{l} \\ \\ \\ :(-4) \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \end{array}\right]$$

$\text{III} + \text{IV}$

$$\begin{array}{ccccc} 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 & -3 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$u = 0$

$1 \cdot z - 1 \cdot u = 3$

$1 \cdot z - 0 = 3$

$\boxed{z = 3}$

$1 \cdot y - 1 \cdot z - 2 \cdot u = 1$

$y - 3 - 0 = 1$

$y - 3 = 1$

$y = 1 + 3$

$\boxed{y = 4}$

$1 \cdot x - 1 \cdot y + 2 \cdot z + 1 \cdot u = 2$

$x - 4 + 6 + 0 = 2$

$x + 2 = 2$

$x = 2 - 2$

$\boxed{x = 0}$

BESKONAČNO RJEŠENJA NAŠLI STE, PROVJERILI SAMO JEDNO

PROVJERA:

$4x - y + z + 2u = -1$

$4 \cdot 0 - 4 + 3 + 2 \cdot 0 = -1$

$-4 + 3 = -1$

$\boxed{-1 = -1}$ //

$2x + y - 3u = 4$

$2 \cdot 0 + 4 - 3 \cdot 0 = 4$

$\boxed{4 = 4}$ //

$x - y + 2z + u = 2$

$0 - 4 + 2 \cdot 3 + 0 = 2$

$-4 + 6 = 2$

$\boxed{2 = 2}$ //

$2x + y + z - 4u = 7$

$2 \cdot 0 + 4 + 3 - 4 \cdot 0 = 7$

$4 + 3 = 7$

$\boxed{7 = 7}$ //

4.

b) $\lim_{n \to \infty} \left(\frac{x^2 + 3}{x^2} \right)^{1/x} \overset{1^\infty}{=} \lim_{n \to \infty} \frac{x + \frac{3}{x}}{x} = \frac{0}{0} = \infty$ (0 → 0)

L.H.A. $\lim_{x \to -\infty} f(-x) = \lim_{x \to -\infty} \sqrt{(-x)^2 + 2\cdot(-x)} = \infty$ NEMA L.H.A.

$\infty + \infty$

D.K.A

$k = \lim_{x \to +\infty} \left(\frac{f(x)}{x}\right) = \lim_{x \to +\infty} \frac{\sqrt{x^2+2x} /:x}{x /:x} = \lim_{x \to +\infty} \frac{\sqrt{\frac{x^2+2x}{x^2}}}{1} = \lim_{x \to +\infty} \frac{\sqrt{1+\frac{2}{x}}^{\,\to 0}}{1}$

$= \frac{1}{1} = 1$

$\boxed{k = 1}$

$l = \lim_{x \to +\infty} (f(x) - kx) = \lim_{x \to +\infty} (\sqrt{x^2+2x} - x) = \lim_{x \to +\infty} \sqrt{x^2+2x} - x \cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x}$

$\lim_{x \to +\infty} \frac{(\sqrt{x^2+2x})^2 - x^2}{\sqrt{x^2+2x}+x} = \lim_{x \to +\infty} \frac{2x /:x}{\sqrt{x^2+2x}+x /:x} = \lim_{x \to +\infty} \frac{2}{\sqrt{\frac{x^2+2x}{x^2}}+1}$

$\lim_{x \to +\infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1} = \lim_{x \to +\infty} \frac{2}{\sqrt{1}+1} = \frac{2}{2} = 1$ $\boxed{l = 1}$

($\frac{2}{x} \to 0$)

$y = kx + l$

$\boxed{y = x + 1}$ ✓

x	0	-1
y	1	0

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L.K.A.

$k = \lim_{x \to -\infty} \left(\frac{f(-x)}{x}\right) = \lim_{x \to -\infty} \frac{\sqrt{(-x)^2 + 2\cdot(-x)} /:x}{x /:x} = \lim_{x \to -\infty} \frac{\sqrt{\frac{x^2-2x}{x^2}}}{1}$

$\lim_{x \to -\infty} \frac{\sqrt{1-\frac{2}{x}}^{\,\to 0}}{1} = \frac{1}{1} = 1$ $\boxed{k = 1}$

$l = \lim_{x \to -\infty} (f(-x) - kx) = \lim_{x \to -\infty} \sqrt{(-x)^2 + 2\cdot(-x)} - 1x$

$\lim_{x \to -\infty} \sqrt{x^2-2x} - x \cdot \frac{\sqrt{x^2-2x}+x}{\sqrt{x^2-2x}+x} = \lim_{x \to -\infty} \frac{(\sqrt{x^2-2x})^2 - x^2}{\sqrt{x^2-2x}+x}$

$\lim_{x \to -\infty} \frac{-2x /:x}{\sqrt{x^2-2x}+x /:x} = \lim_{x \to -\infty} \frac{-2}{\sqrt{1-\frac{2}{x}}+1} = \frac{-2}{2} = -1$

$\boxed{l = -1}$

$y = kx + l$

$\boxed{y = x - 1}$ ✗

x	0	1
y	-1	0

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6. $f(x) = \ln(\cos(2x^2-1))$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (\cos(2x^2-1))'$$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (-\sin(2x^2-1)) \cdot (2x^2-1)'$$

$$f'(x) = \frac{1}{\cos(2x^2-1)} \cdot (-\sin(2x^2-1)) \cdot 4x$$

$$f'(x) = 4x \cdot \frac{-\sin(2x^2-1)}{\cos(2x^2-1)}$$ ✓

3. $h(x) = \sqrt{x^2+2x}$

$x^2 + 2x \geq 0$

$x(x+2) \geq 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4-0}}{2}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4}}{2}$$

$$x_1 = \frac{-2+2}{2} = 0$$

$$x_2 = \frac{-2-2}{2} = \frac{-4}{2} = -2$$

$D(f) = \langle -\infty, -2] \cup [0, +\infty \rangle$ ✓

N.T.

$f(x) = 0$

$\sqrt{x^2+2x} = 0 \ /^2$

$(\sqrt{x^2+2x})^2 = 0$

$x^2+2x = 0$

$x_1 = 0 \quad x_2 = -2$

$(0, 0)$

$(-2, 0)$

$f(0) = \sqrt{x^2+2x}$

$f(0) = \sqrt{0^2+2 \cdot 0}$

$f(0) = 0$

ASIMPTOTE:

V.A. NEMA

D.H.A. $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \sqrt{x^2+2x} = +\infty$ NEMA D.H.A.