

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

F4

IME I PREZIME:

MATKO DONADIĆ

BROJ INDEKSA:

17-1-0247-2014

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 3 = 0$. Prikaži ih u kompleksnoj ravnini i provjeri uvrštavanjem! Dalje izračunaj: $\left(\frac{z_1 - z_2}{z_2 + 3}\right)$.

2
4+3+8 15

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5 15

$$\begin{aligned} x_1 - 2x_2 + 3x_3 - 4x_4 &= 8 \\ x_2 - x_3 + x_4 &= -2 \\ x_1 + 3x_2 - 3x_4 &= 6 \\ -7x_2 + 3x_3 + x_4 &= -2 \end{aligned}$$

3. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2 + 4) + \sin(2x - 3)$.

5+15 20

4. Odrediti tok funkcije $f(x) = x - \frac{1}{x}$.

15(graf) 15

5. Odrediti i provjeriti uvrštavanjem: $\lim_{x \rightarrow -4} \frac{x^2 - 3}{x^2 + 8x + 16} =$

4+1 4

6. Odredi derivaciju funkcije $f(x) = \frac{4}{\sin(5x)}$

10 10

7. Odrediti tangentu na funkciju $f(x) = \cos x$ tamo gdje je $x = \frac{\pi}{4}$. Nacrtati graf funkcije i nacrtati izračunatu tangentu.

15+3+2 20

1) $z^2 - z + 3 = 0$

$$z_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot 3}}{2} = \frac{1 \pm \sqrt{-11}}{2}$$

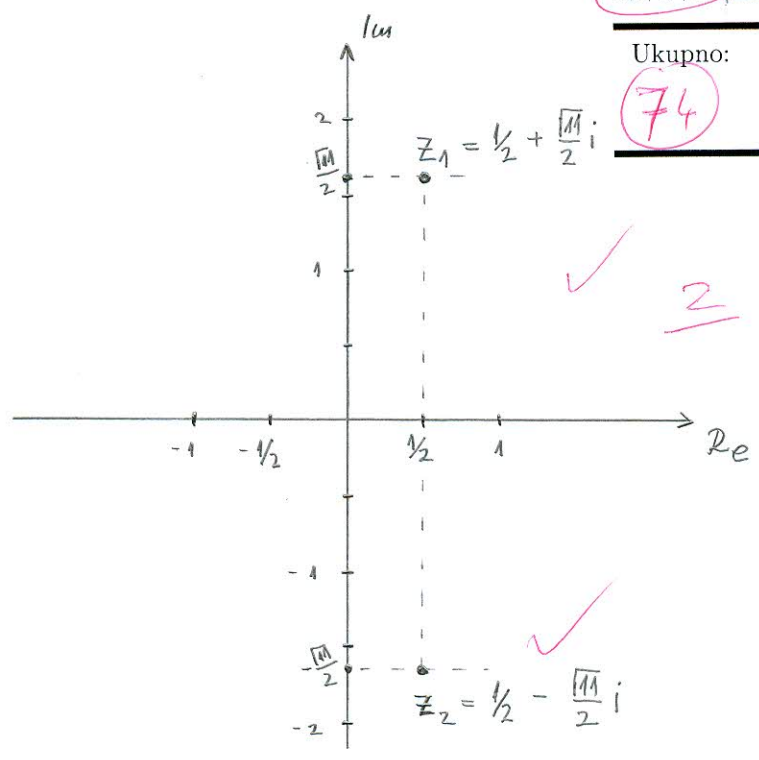
$$z_1 = \frac{1 + \sqrt{11}i}{2} \quad z_2 = \frac{1 - \sqrt{11}i}{2}$$

$$z_1 = \frac{1}{2} + \frac{\sqrt{11}}{2}i \quad z_2 = \frac{1}{2} - \frac{\sqrt{11}}{2}i$$

$$\left(\frac{z_1 - z_2}{z_2 + 3}\right) = \left(\frac{\frac{1}{2} + \frac{\sqrt{11}}{2}i - \frac{1}{2} + \frac{\sqrt{11}}{2}i}{\frac{1}{2} - \frac{\sqrt{11}}{2}i + 3}\right)$$

$$= \left(\frac{\sqrt{11}i}{\frac{7}{2} - \frac{\sqrt{11}}{2}i}\right) = \frac{-\sqrt{11}i}{\frac{7}{2} + \frac{\sqrt{11}}{2}i}$$

$$= -\frac{11}{30} - \frac{7\sqrt{11}}{30}i$$



Ukupno:
74

2

$$\textcircled{5} \lim_{x \rightarrow -4} \frac{x^2 - 3}{x^2 + 8x + 16} = \left[\frac{13}{0} \right] \stackrel{\text{L.H.}}{=} \frac{2x}{2x + 8} = \left[\frac{8}{0} \right] \stackrel{\text{L.H.}}{=} \frac{2}{2} = 1 \quad \times$$

$$\lim_{x \rightarrow -4} \frac{x^2 - 3}{x^2 + 8x + 16} = \frac{(-3.99)^2 - 3}{(-3.99)^2 + (8 \cdot -3.99) + 16} = \frac{12.92}{1 \cdot 10^{-4}} \quad \checkmark \quad \text{Približava se } 1 \quad //$$

$$\textcircled{6} \text{ Derivacija } f(x) = \frac{4}{\sin(5x)}$$

$$f'(x) = \frac{4' \cdot \sin(5x) - 4 \cdot (\sin(5x))'}{\sin^2(5x)} = \frac{0 - 4 \cdot \cos(5x) \cdot 5}{\sin^2(5x)}$$

$$f'(x) = \frac{-20 \cos(5x)}{\sin^2(5x)} \quad \checkmark$$

$$\textcircled{7} f(x) = \cos x \quad x_0 = \frac{\pi}{4}$$

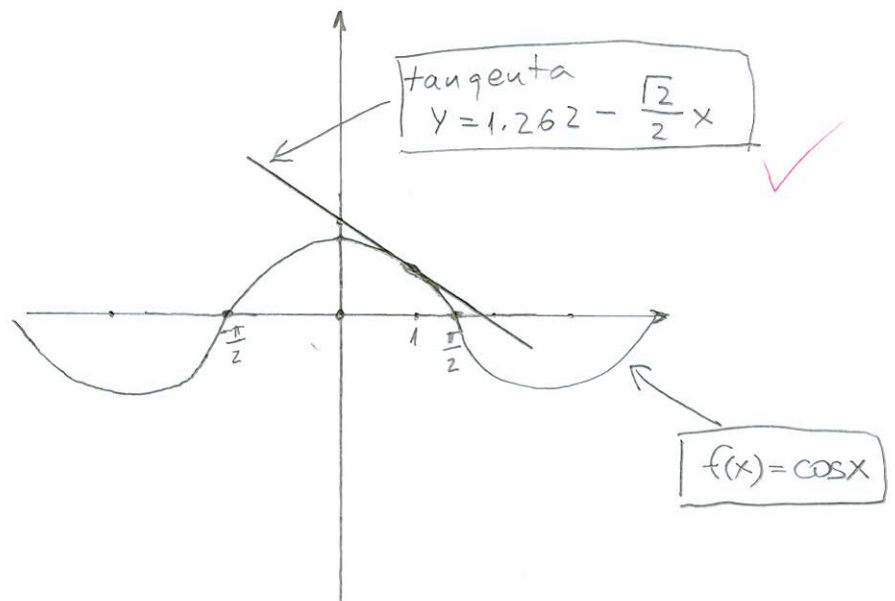
$$f'(x) = -\sin x$$

$$y(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$y(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \left(x - \frac{\pi}{4} \right)$$

$$y = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} x + 0.555$$

$$y = 1.262 - \frac{\sqrt{2}}{2} x$$



x	0	1	2
y	1.262	0.555	-0.15

$$\textcircled{2} \begin{bmatrix} 1 & -2 & 3 & -4 & | & 8 \\ 0 & 1 & -1 & 1 & | & -2 \\ 1 & 3 & 0 & -3 & | & 6 \\ 0 & -7 & 3 & 1 & | & -2 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 1 & -2 & 3 & -4 & | & 8 \\ 0 & 1 & -1 & 1 & | & -2 \\ 0 & 5 & -3 & 1 & | & -2 \\ 0 & -7 & 3 & 1 & | & -2 \end{bmatrix} \begin{array}{l} +2R_2 \\ -5R_2 \\ +7R_2 \end{array}$$

$$= \begin{bmatrix} 1 & 0 & 1 & -2 & | & 4 \\ 0 & 1 & -1 & 1 & | & -2 \\ 0 & 0 & 2 & -4 & | & 8 \\ 0 & 0 & -4 & 8 & | & -16 \end{bmatrix} \cdot \frac{1}{2} = \begin{bmatrix} 1 & 0 & 1 & -2 & | & 4 \\ 0 & 1 & -1 & 1 & | & -2 \\ 0 & 0 & 1 & -2 & | & 4 \\ 0 & 0 & -4 & 8 & | & -16 \end{bmatrix} \begin{array}{l} -R_3 \\ +R_3 \\ +4R_3 \end{array}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & -1 & | & 2 \\ 0 & 0 & 1 & -2 & | & 4 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

JEDNAŽBA IMA BESKONAČNO
MNOGO RJEŠENJA

KOJA? ~~∅~~

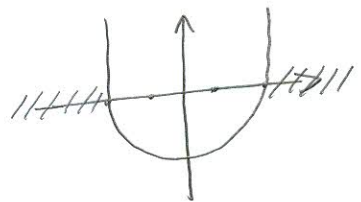
$$\textcircled{3} f(x) = \ln(x^2 + 4) + \sin(2x - 3)$$

Domena: $\ln(x^2 + 4)$

$$x^2 + 4 > 0$$

$$x^2 = 4$$

$$x = \pm 2$$



$$D: x \in \langle -\infty, -2 \rangle \cup \langle 2, +\infty \rangle$$

$$f'(x) = \frac{1}{(x^2+4)} \cdot (x^2+4)' + \cos(2x-3) (2x-3)'$$

$$f'(x) = \frac{1}{x^2+4} \cdot 2x + \cos(2x-3) \cdot 2 \quad \checkmark$$

$$f'(x) = \frac{2x}{x^2+4} + 2\cos(2x-3)$$

④ $f(x) = x - \frac{1}{x}$

Domena: $x \neq 0$

$\langle -\infty, 0 \rangle \cup \langle 0, +\infty \rangle$

Asimptote:

$\lim_{x \rightarrow \infty} x - \frac{1}{x} = \infty - \frac{1}{\infty} = \infty$ Nova DHA

$\lim_{x \rightarrow \infty} \frac{x - \frac{1}{x}}{x} = \frac{\frac{x^2 - 1}{x}}{\frac{x}{1}} = \frac{x^2 - 1}{x^2} = \left[\frac{\infty}{\infty} \right] \stackrel{L.H.}{=} \frac{2x}{2x} \stackrel{L.H.}{=} \frac{2}{2} = 1$ $a=1$

$\lim_{x \rightarrow \infty} x - \frac{1}{x} - x = \lim_{x \rightarrow \infty} -\frac{1}{x} = 0$ $y=x$ DKA

$\lim_{x \rightarrow -\infty} x - \frac{1}{x} = \lim_{x \rightarrow -\infty} -x + \frac{1}{x} = -\infty + \frac{1}{\infty} = -\infty$ Nova LHA

$\lim_{x \rightarrow -\infty} \frac{x - \frac{1}{x}}{x} = \lim_{x \rightarrow -\infty} \frac{\frac{x^2 - 1}{x}}{\frac{x}{1}} = \lim_{x \rightarrow -\infty} \frac{x^2 - 1}{x^2} = \left[\frac{\infty}{\infty} \right] \stackrel{L.H.}{=} \frac{2x}{2x} = \frac{2}{2} = 1$ $a=1$

$\lim_{x \rightarrow -\infty} x - \frac{1}{x} - x = \lim_{x \rightarrow -\infty} -x + \frac{1}{x} + x = \lim_{x \rightarrow -\infty} \frac{1}{x} = 0$ $y=x$ LKA

$\lim_{x \rightarrow 0_+} 0_+ - \frac{1}{0_+} = 0_+ - \infty = -\infty$
 $\lim_{x \rightarrow 0_-} 0_- - \frac{1}{0_-} = 0_- - (-\infty) = \infty$ } V.A. $\rightarrow 0$

SJECIŠTA S OSIMA:

$f(0) = 0 - \frac{1}{0} \Rightarrow$ Nikad

$f(x) = 0$
 $\frac{x^2 - 1}{x} = 0$
 $x^2 - 1 = 0$
 $x^2 = 1$
 $x = \pm 1$

1^o DERIVACIJA

$$f(x) = x - \frac{1}{x} = x - x^{-1}$$

$$f'(x) = 1 - (-1) \cdot x^{-2} = \frac{1}{1} + \frac{1}{x^2} = \frac{x^2 + 1}{x^2}$$

Domena derivacije

$$x^2 \neq 0$$

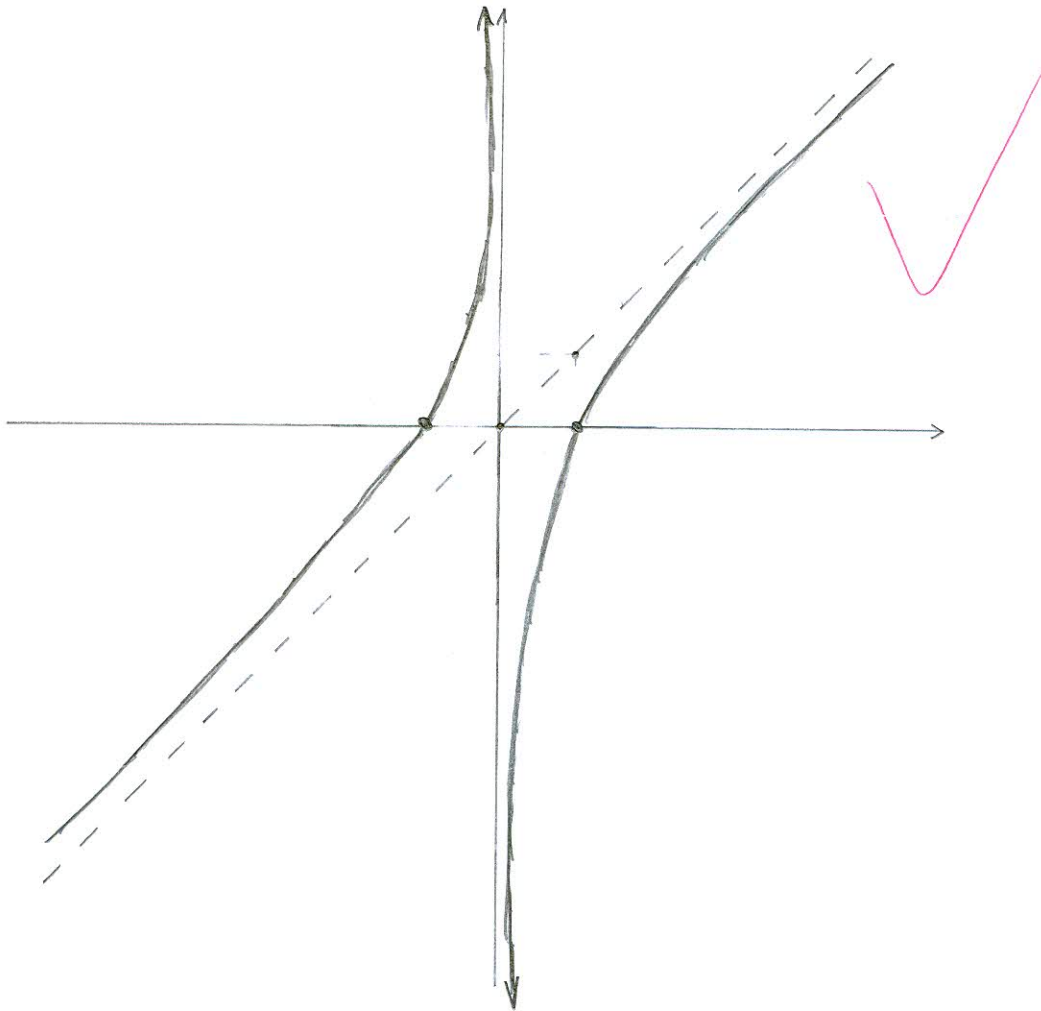
$$x \neq 0$$

$$x^2 + 1 = 0$$

$$x^2 \neq -1$$

NIKAD

	$-\infty$	0	∞
$f'(x)$	+		+
$f(x)$	\nearrow		\nearrow



2^o DERIVACIJA

$$f(x) = \frac{x^2 + 1}{x^2}$$

$$f'(x) = \frac{(x^2 + 1)' \cdot x^2 - (x^2 + 1) \cdot (x^2)'}{x^4}$$

$$f'(x) = \frac{2x \cdot x^2 - (x^2 + 1) \cdot 2x}{x^4}$$

$$f'(x) = \frac{2x^3 - 2x^3 - 2x}{x^4} = \frac{-2x}{x^4}$$

Domena: $x^4 \neq 0$

$$x \neq 0$$

$$-2x = 0$$

$$x = 0$$

	$-\infty$	0	∞
$f''(x)$	+		-
$f(x)$	\cup		\cap

