MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: DOMINIK MIŠEVIĆ **BROJ INDEKSA:** 17-2-0396-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

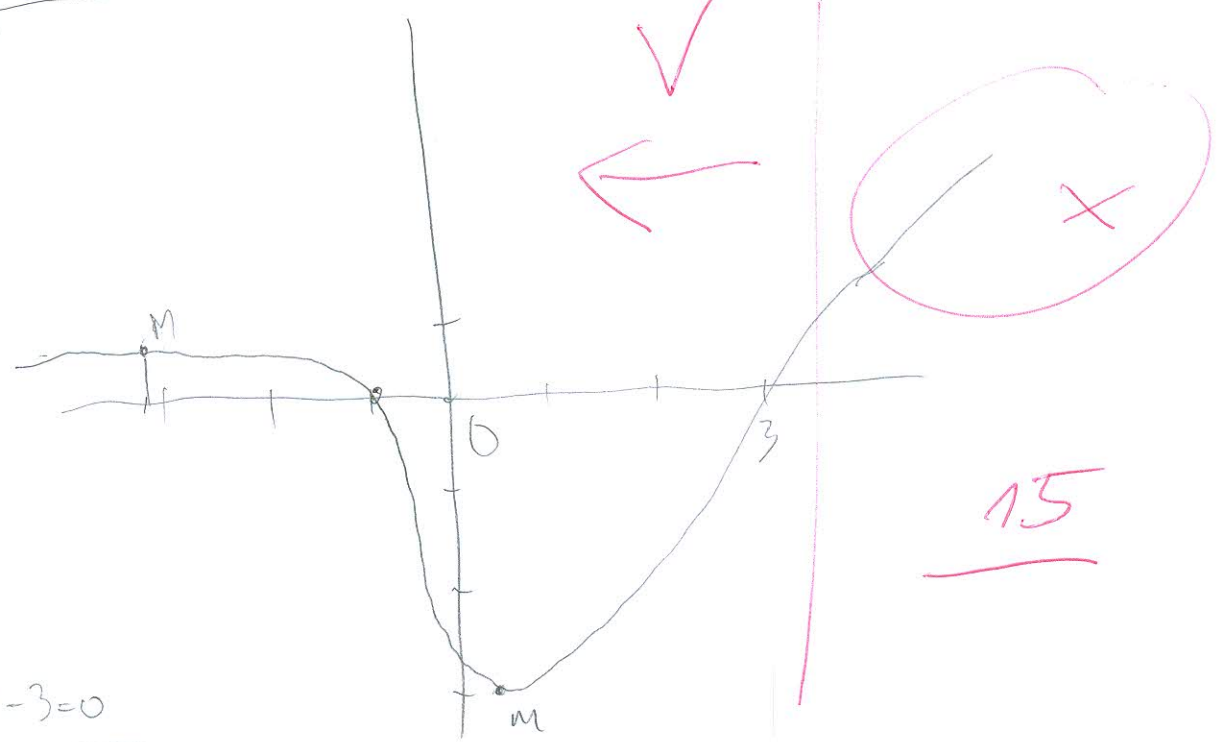
1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 3 \\ 2x + 5y - z + 2u &= 3 \\ 3x - y - 2z + u &= 2 \\ x - y + 3z - 5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 50

4. graf



15

$h(x)=0$

$x^2-2x-3=0$

$x_{1,2} = \dfrac{2 \pm \sqrt{4+12}}{2} = \dfrac{2 \pm 4}{2}$

$x_1 = -1$

$x_2 = 3$

6. $\lim_{x\to\infty} \frac{e^x}{x} = \lim_{x\to\infty} \frac{e^x}{1} = \frac{e^x}{1} = \frac{\infty}{1} = \infty$ ✓

∅

PROVJERA UVRŠTAVANJEM.

ZAKRIVLJENOST $g(x) = \ln(4-x^2)$

$$g'(x) = \frac{2x}{x^2-4}$$

$$g''(x) = \frac{2x'\cdot(x^2-4) - 2x(x^2-4)'}{(x^2-4)^2}$$

$$= \frac{2\cdot(x^2-4) - 2x(2x)}{(x^2-4)^2} = \frac{2x^2-8-4x^2}{(x^2-4)^2}$$

$$= \frac{-2x^2-8}{(x^2-4)^2}$$

$g''(x) = 0$

$$\frac{-2x^2-8}{(x^2-4)^2} = 0$$

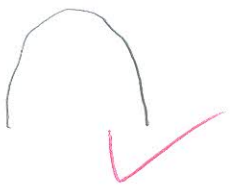
$$-2x^2-8=0$$

$$-2x^2 = 8 \mid \cdot(-2)$$

$$x^2 = -4$$

$$x = \sqrt{-4} \notin \mathbb{R}$$

funkcija je zakrivljena



	-2		0		2
$g''(x)$		$-$		$-$	
$g(x)$		$\cap$		$\cap$	

$$g''(-1) = \frac{-2(-1)^2-8}{(-1)^2-4)^2} = -\frac{10}{9} < 0$$

$$g''(1) = \frac{-2(1)^2-8}{(1^2-4)^2} = \frac{-10}{9} < 0$$

DOMINIK MIŠEVIĆ

3.) $g(x) = \ln(4-x^2)$

$4 - x^2 > 0$

$(2-x)(2+x) > 0$ N.T. $2, -2$

	$-\infty$		-2		2		$+\infty$
$2-x$		$+$		$+$	0	$-$	
$2+x$		$-$	0	$+$		$+$	
		$-$		$\oplus$		$-$	

$Df \langle -2, 2 \rangle$ ✓

PARNOST

$g(-x) = \ln(4-(-x)^2)$

$= \ln(4-x^2)$

$= g(x) \Rightarrow$ funkcija je parna ✓

$f'(x) = \frac{1}{4-x^2} \cdot (4-x^2)'$ ($-2x$)

$= \frac{-2x}{4-x^2} = \frac{2x}{x^2-4} = \frac{2x}{(x-2)(x+2)}$

$f'(x) = 0$

$\frac{2x}{x^2-4} = 0$

$2x = 0$

$x = 0 \rightarrow$

$M(0, \ln 4)$

$\langle -2, 0 \rangle$ funkcija raste

$\langle 0, 2 \rangle$ funkcija pada

	-2		-1		0		1		2
$f'(x)$		$+$		$+$	0	$-$		$-$	
$f(x)$		↗		↗		↘		↘	

$f'(-1) = \frac{2 \cdot (-1)}{(-1)^2-4} = \frac{-2}{-3} = \frac{2}{3} > 0$

$f'(1) = \frac{2 \cdot 1}{(1)^2-4} = \frac{2}{-3} < 0$

$f(0) = \ln(4-0) = \ln 4$

DOMINIK MISEVIC

5. $x+2y-z+v=3$
$2x+5y-z+2v=3$
$3x-y-2z+v=2$
$x-y+3z-5v=3$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \begin{array}{l} \\ R_2-2R_1 \\ R_3-3R_1 \\ R_4-R_1 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -7 & 1 & -2 & -4 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \begin{array}{l} R_1-2R_2 \\ \\ R_3+7R_2 \\ R_4+3R_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -25 \\ 0 & 0 & 1 & -6 & 9 \end{array}\right] \cdot \tfrac{1}{8} \quad \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & -\frac{2}{8} & -\frac{25}{8} \\ 0 & 0 & 1 & -6 & 9 \end{array}\right] \begin{array}{l} R_1+3R_3 \\ R_2-R_3 \\ \\ R_4-R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & \frac{47}{8} \\ 0 & 1 & 0 & \frac{2}{8} & \frac{1}{8} \\ 0 & 0 & 1 & -\frac{2}{8} & -\frac{25}{8} \\ 0 & 0 & 0 & -\frac{23}{4} & \frac{97}{8} \end{array}\right] \cdot \left(-\tfrac{4}{23}\right) \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & \frac{47}{8} \\ 0 & 1 & 0 & \frac{2}{8} & \frac{1}{8} \\ 0 & 0 & 1 & -\frac{2}{8} & -\frac{25}{8} \\ 0 & 0 & 0 & 1 & -\frac{97}{46} \end{array}\right] \begin{array}{l} R_1-\frac{1}{4}R_4 \\ R_2-\frac{2}{8}R_4 \\ R_2+\frac{2}{8}R_4 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{45}{8} \\ 0 & 1 & 0 & 0 & -\frac{1}{8} \\ 0 & 0 & 1 & 0 & -\frac{23}{8} \\ 0 & 0 & 0 & 1 & -\frac{97}{46} \end{array}\right]$$

PROVJERA ?

4. $h(x) = \frac{x^2-2x-(2+1)}{x^2+1}$

$= \frac{x^2-2x-3}{x^2+1}$

$x^2+1 \neq 0$

vrijedi za svaki

$x \in \mathbb{R}$

$D_f \{\mathbb{R}\}$

$h'(x) = \frac{(x^2-2x-3)' \cdot (x^2+1) - (x^2-2x-3)(x^2+1)}{(x^2+1)^2}$

$= \frac{(2x-2)(x^2+1) - (x^2-2x-3)(2x)}{(x^2+1)^2}$

$= \frac{2x^3+2x-2x^2-2-2x^3-4x^2+6x}{(x+1)^2}$

$= \frac{2x^2+8x-2}{(x+1)^2} = \frac{2(x^2+4x-1)}{(x+1)^2}$

$f'(x)=0$

$2(x^2-4x-1)=0 \;/:2$

$x^2+4x-1=0$

$x = \frac{-4 \pm \sqrt{16+4}}{2} = \frac{-4 \pm \sqrt{20}}{2} = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$

$x_1 = -2+\sqrt{5} \quad x_2 = -2-\sqrt{5}$

$h'(-5) = \frac{2}{1,03} > 0$

$h'(-2) = -\frac{2}{5} < 0$

$h'(2) = \frac{22}{25} > 0$

	∞		$-2-\sqrt{5}$		$-2+\sqrt{5}$		$+\infty$
$f'(x)$		+		−		+	
$f(x)$		↗	M	↘	m	↗	

$M(-4,2; 1,24)$

$m(0,23; -3,24)$

2 $f(x) = x - \sqrt{x^2-2}$

DOMENA RACUNA

$x^2 - 2 \geq 0$

$(x+\sqrt{2})(x-\sqrt{2}) \geq 0$

N.T: $\sqrt{2}, -\sqrt{2}$

$-\infty \quad -\sqrt{2} \quad \sqrt{2} \quad +\infty$

	$-\infty$	$-\sqrt{2}$		$\sqrt{2}$	$+\infty$
$x-\sqrt{2}$	$-$	$-$	$\|$	0	$+$
$x+\sqrt{2}$	$-$	0	$+$	$\|$	$+$
	$+$	0	$-$	0	$+$

$x \in \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$

$D_f \subseteq \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$ ✓

$f'(x) = x' - (\sqrt{x^2-2})'$

$= 1 - \frac{1}{2\sqrt{x^2-2}} (x^2-2)'$

$= 1 - \frac{2x}{2\sqrt{x^2-2}} = 1 - \frac{x}{\sqrt{x^2-2}}$

ASIMPTOTE X

$f''(x) = 1' - \left(\frac{x}{\sqrt{x^2-2}}\right)'$

$= 0 - \frac{x' \cdot \sqrt{x^2-2} - x \cdot (\sqrt{x^2-2})'}{(\sqrt{x^2-2})^2} = \frac{\sqrt{x^2-2} - x \cdot \frac{1}{2\sqrt{x^2-2}} \cdot (x^2-2)'}{x^2-2}$ ✓

$= \frac{\sqrt{x^2-2} - \frac{2x^2}{2\sqrt{x^2-2}}}{x^2-2}$ ✓

$= \frac{\frac{x^2-2-x^2}{\sqrt{x^2-2}}}{x^2-2} = \frac{-2}{(x^2-2)\sqrt{x^2-2}}$