MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: DOMINIK MIŠEVIĆ **BROJ INDEKSA:** 17-2-0396-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

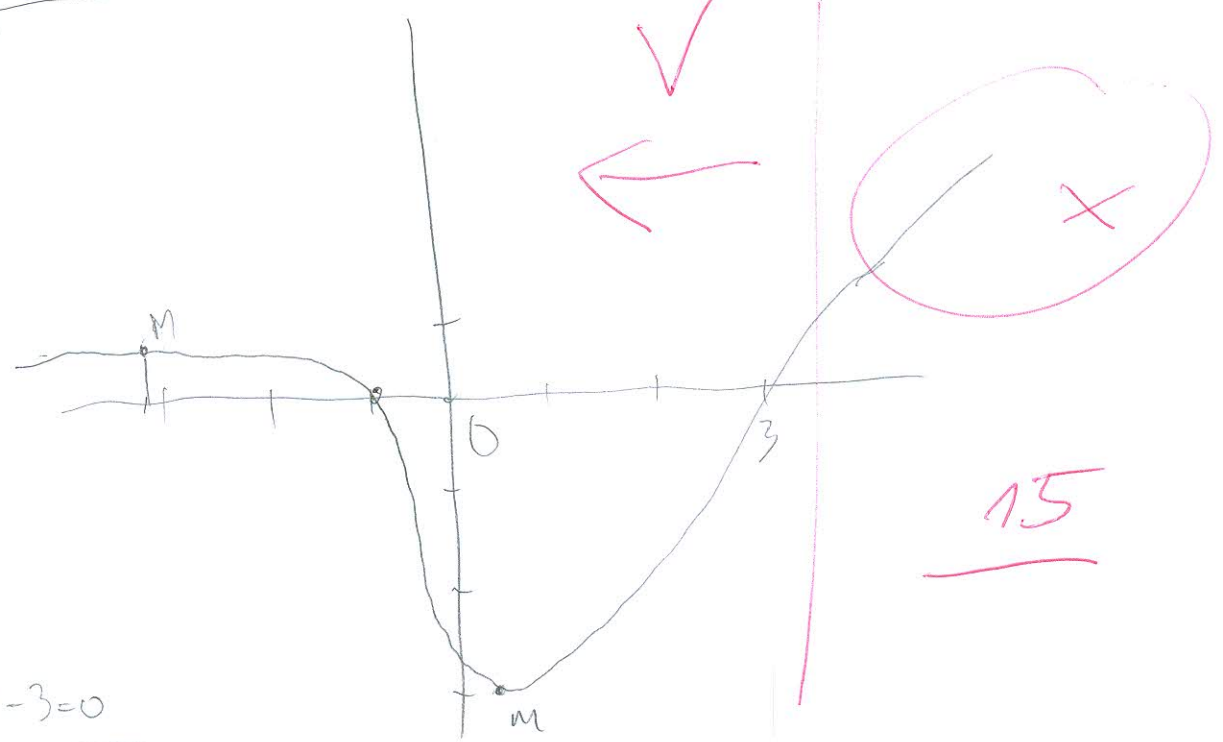
1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 3 \\ 2x + 5y - z + 2u &= 3 \\ 3x - y - 2z + u &= 2 \\ x - y + 3z - 5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 50

4. graf



15

$h(x)=0$

$x^2-2x-3=0$

$x_{1,2} = \dfrac{2 \pm \sqrt{4+12}}{2} = \dfrac{2 \pm 4}{2}$

$x_1 = -1$

$x_2 = 3$

6. $\lim_{x\to\infty} \frac{e^x}{x} = \lim_{x\to\infty} \frac{e^x}{1} = \frac{e^x}{1} = \frac{\infty}{1} = \infty$ ✓

∅

PROVJERA UVRŠTAVANJEM.

ZAKRIVLJENOST $g(x) = \ln(4-x^2)$

$$g'(x) = \frac{2x}{x^2-4}$$

$$g''(x) = \frac{2x'\cdot(x^2-4) - 2x(x^2-4)'}{(x^2-4)^2}$$

$$= \frac{2\cdot(x^2-4) - 2x(2x)}{(x^2-4)^2} = \frac{2x^2-8-4x^2}{(x^2-4)^2}$$

$$= \frac{-2x^2-8}{(x^2-4)^2}$$

$g''(x) = 0$

$$\frac{-2x^2-8}{(x^2-4)^2} = 0$$

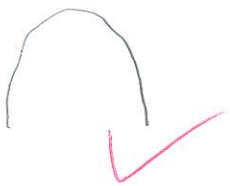
$$-2x^2-8=0$$

$$-2x^2 = 8 \mid \cdot(-2)$$

$$x^2 = -4$$

$$x = \sqrt{-4} \notin \mathbb{R}$$

funkcija je zakrivljena



	-2		0		2
$g''(x)$		$-$		$-$	
$g(x)$		$\cap$		$\cap$	

$$g''(-1) = \frac{-2(-1)^2-8}{(-1)^2-4)^2} = -\frac{10}{9} < 0$$

$$g''(1) = \frac{-2(1)^2-8}{(1^2-4)^2} = \frac{-10}{9} < 0$$

DOMINIK MIŠEVIĆ

3.) $g(x) = \ln(4-x^2)$

$4 - x^2 > 0$

$(2-x)(2+x) > 0$ N.T. $2, -2$

	$-\infty$		-2		2		$+\infty$
$2-x$		$+$		$+$	0	$-$	
$2+x$		$-$	0	$+$		$+$	
		$-$		$\oplus$		$-$	

$Df \langle -2, 2 \rangle$ ✓

PARNOST

$g(-x) = \ln(4-(-x)^2)$

$= \ln(4-x^2)$

$= g(x) \Rightarrow$ funkcija je parna ✓

$f'(x) = \frac{1}{4-x^2} \cdot (4-x^2)'$ ($-2x$)

$= \frac{-2x}{4-x^2} = \frac{2x}{x^2-4} = \frac{2x}{(x-2)(x+2)}$

$f'(x) = 0$

$\frac{2x}{x^2-4} = 0$

$2x = 0$

$x = 0 \rightarrow$

$M(0, \ln 4)$

$\langle -2, 0 \rangle$ funkcija raste

$\langle 0, 2 \rangle$ funkcija pada

	-2		-1		0		1		2
$f'(x)$		$+$		$+$	0	$-$		$-$	
$f(x)$		↗		↗		↘		↘	

$f'(-1) = \frac{2 \cdot (-1)}{(-1)^2-4} = \frac{-2}{-3} = \frac{2}{3} > 0$

$f'(1) = \frac{2 \cdot 1}{(1)^2-4} = \frac{2}{-3} < 0$

$f(0) = \ln(4-0) = \ln 4$

DOMINIK MISEVIC

5. $x+2y-z+v=3$
$2x+5y-z+2v=3$
$3x-y-2z+v=2$
$x-y+3z-5v=3$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \begin{array}{l} \\ R_2-2R_1 \\ R_3-3R_1 \\ R_4-R_1 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -7 & 1 & -2 & -4 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \begin{array}{l} R_1-2R_2 \\ \\ R_3+7R_2 \\ R_4+3R_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -25 \\ 0 & 0 & 1 & -6 & 9 \end{array}\right] \cdot \tfrac{1}{8} \quad \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & -\frac{2}{8} & -\frac{25}{8} \\ 0 & 0 & 1 & -6 & 9 \end{array}\right] \begin{array}{l} R_1+3R_3 \\ R_2-R_3 \\ \\ R_4-R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & \frac{47}{8} \\ 0 & 1 & 0 & \frac{2}{8} & \frac{1}{8} \\ 0 & 0 & 1 & -\frac{2}{8} & -\frac{25}{8} \\ 0 & 0 & 0 & -\frac{23}{4} & \frac{97}{8} \end{array}\right] \cdot \left(-\tfrac{4}{23}\right) \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & \frac{47}{8} \\ 0 & 1 & 0 & \frac{2}{8} & \frac{1}{8} \\ 0 & 0 & 1 & -\frac{2}{8} & -\frac{25}{8} \\ 0 & 0 & 0 & 1 & -\frac{97}{46} \end{array}\right] \begin{array}{l} R_1-\frac{1}{4}R_4 \\ R_2-\frac{2}{8}R_4 \\ R_2+\frac{2}{8}R_4 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{45}{8} \\ 0 & 1 & 0 & 0 & -\frac{1}{8} \\ 0 & 0 & 1 & 0 & -\frac{23}{8} \\ 0 & 0 & 0 & 1 & -\frac{97}{46} \end{array}\right]$$

PROVJERA ?

4. $h(x) = \frac{x^2-2x-(2+1)}{x^2+1}$

$= \frac{x^2-2x-3}{x^2+1}$

$x^2+1 \neq 0$

vrijedi za svaki

$x \in \mathbb{R}$

$D_f \{\mathbb{R}\}$

$h'(x) = \frac{(x^2-2x-3)' \cdot (x^2+1) - (x^2-2x-3)(x^2+1)}{(x^2+1)^2}$

$= \frac{(2x-2)(x^2+1) - (x^2-2x-3)(2x)}{(x^2+1)^2}$

$= \frac{2x^3+2x-2x^2-2-2x^3-4x^2+6x}{(x+1)^2}$

$= \frac{2x^2+8x-2}{(x+1)^2} = \frac{2(x^2+4x-1)}{(x+1)^2}$

$f'(x)=0$

$2(x^2-4x-1)=0 \;/:2$

$x^2+4x-1=0$

$x = \frac{-4 \pm \sqrt{16+4}}{2} = \frac{-4 \pm \sqrt{20}}{2} = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$

$x_1 = -2+\sqrt{5} \quad x_2 = -2-\sqrt{5}$

$h'(-5) = \frac{2}{1,03} > 0$

$h'(-2) = -\frac{2}{5} < 0$

$h'(2) = \frac{22}{25} > 0$

	∞		$-2-\sqrt{5}$		$-2+\sqrt{5}$		$+\infty$
$f'(x)$		+		−		+	
$f(x)$		↗	M	↘	m	↗	

$M(-4,2; 1,24)$

$m(0,23; -3,24)$

2 $f(x) = x - \sqrt{x^2-2}$

DOMENA RACUNA

$x^2 - 2 \geq 0$

$(x+\sqrt{2})(x-\sqrt{2}) \geq 0$

N.T: $\sqrt{2}, -\sqrt{2}$

$-\infty \quad -\sqrt{2} \quad \sqrt{2} \quad +\infty$

	$-\infty$	$-\sqrt{2}$		$\sqrt{2}$	$+\infty$
$x-\sqrt{2}$	$-$	$-$	$\|$	0	$+$
$x+\sqrt{2}$	$-$	0	$+$	$\|$	$+$
	$+$	0	$-$	0	$+$

$x \in \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$

$D_f \subseteq \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$ ✓

$f'(x) = x' - (\sqrt{x^2-2})'$

$= 1 - \frac{1}{2\sqrt{x^2-2}} (x^2-2)'$

$= 1 - \frac{2x}{2\sqrt{x^2-2}} = 1 - \frac{x}{\sqrt{x^2-2}}$

ASIMPTOTE X

$f''(x) = 1' - \left(\frac{x}{\sqrt{x^2-2}}\right)'$

$= 0 - \frac{x' \cdot \sqrt{x^2-2} - x \cdot (\sqrt{x^2-2})'}{(\sqrt{x^2-2})^2} = \frac{\sqrt{x^2-2} - x \cdot \frac{1}{2\sqrt{x^2-2}} \cdot (x^2-2)'}{x^2-2}$ ✓

$= \frac{\sqrt{x^2-2} - \frac{2x^2}{2\sqrt{x^2-2}}}{x^2-2}$ ✓

$= \frac{\frac{x^2-2-x^2}{\sqrt{x^2-2}}}{x^2-2} = \frac{-2}{(x^2-2)\sqrt{x^2-2}}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓ UGLEŠIĆ H2

IME I PREZIME: STIPE KATALINIĆ

BROJ INDEKSA: 17-2-0320-2013

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6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 25

2. $f(x) = x - \sqrt{x^2-2}$

$x^2 - 2 \geq 0$ $\quad D(f) = \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$ ✓

$x^2 \geq 2$

$x_1 = \sqrt{2}$

$x_2 = -\sqrt{2}$

$f(x) = x - \sqrt{x^2-2}$

$f'(x) = 1 - \dfrac{1}{2\sqrt{x^2-2}} \cdot 2x$

$f'(x) = 1 - \dfrac{2x}{2\sqrt{x^2-2}}$

$f''(x) = 0 - \dfrac{2\cdot(2\sqrt{x^2-2}) - 2x\cdot 2\sqrt{x^2-2}}{(2\sqrt{x^2-2})^2}$

ASIMPT.

Vertikalnih asimp nemo

H. ASIMP.

$\lim\limits_{x\to+\infty} x - \sqrt{x^2-2} \;/:x$

$\lim\limits_{x\to+\infty} \dfrac{x}{x} - \sqrt{\dfrac{x^2}{x^2} \cdot \dfrac{2}{x^2}}$

$\lim\limits_{x\to+\infty} = y = 1$ DESNA HOR. ASIMP. ✗

$f''(x) = \dfrac{-4\sqrt{x^2-2} - 4x\cdot\sqrt{x^2-2}}{(2\sqrt{x^2-2})^2}$ ✗

$f''(x) = \dfrac{-4x\sqrt{x^2-2} + \sqrt{x^2-2}}{\sqrt{x^2-2}}$ ✗

$\lim\limits_{x\to-\infty} x - \sqrt{x^2-2} \quad (x\to(-x),\ -x\to+\infty) \quad (-x) - \sqrt{x^2-2} \;/:x$

$y = -1$

LIJEVA HOR. ASIMP ✗

KOSE ASIM. NEMA

6. $\lim_{x \to \infty} \frac{e^x}{x}$

$\lim_{x \to 3} \frac{e^3}{3} = 6,67$

$4 = 13,64$

-1

Rješenje $\langle -\infty, +\infty \rangle$ X

STIPE KATALINIĆ

① $z^4 - (4-i)^2 = 0$

$z^4 = \sqrt{-4-i}$

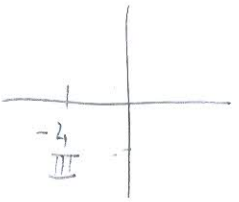
$z = \sqrt[4]{15}$ ×

$-(4-i)^2 = 0 \,/^2$

$-4-i = 0$

$w = \sqrt{(-4)^2 + (-1)^2}$

$w = \sqrt{15}$



$k = 0$

$z_1 = \sqrt[4]{15} \cdot \left(\cos\frac{\varphi + 2k\pi}{4} + i\sin\frac{\varphi + 2k\pi}{4}\right)$

$\varphi = \pi + \arctan\left|\frac{y}{x}\right|$

$\varphi = \pi + \arctan\left|\frac{1}{4}\right|$

$\varphi = 3{,}39$

$z_1 = 1{,}3 + 1{,}48i$

$k = 1$

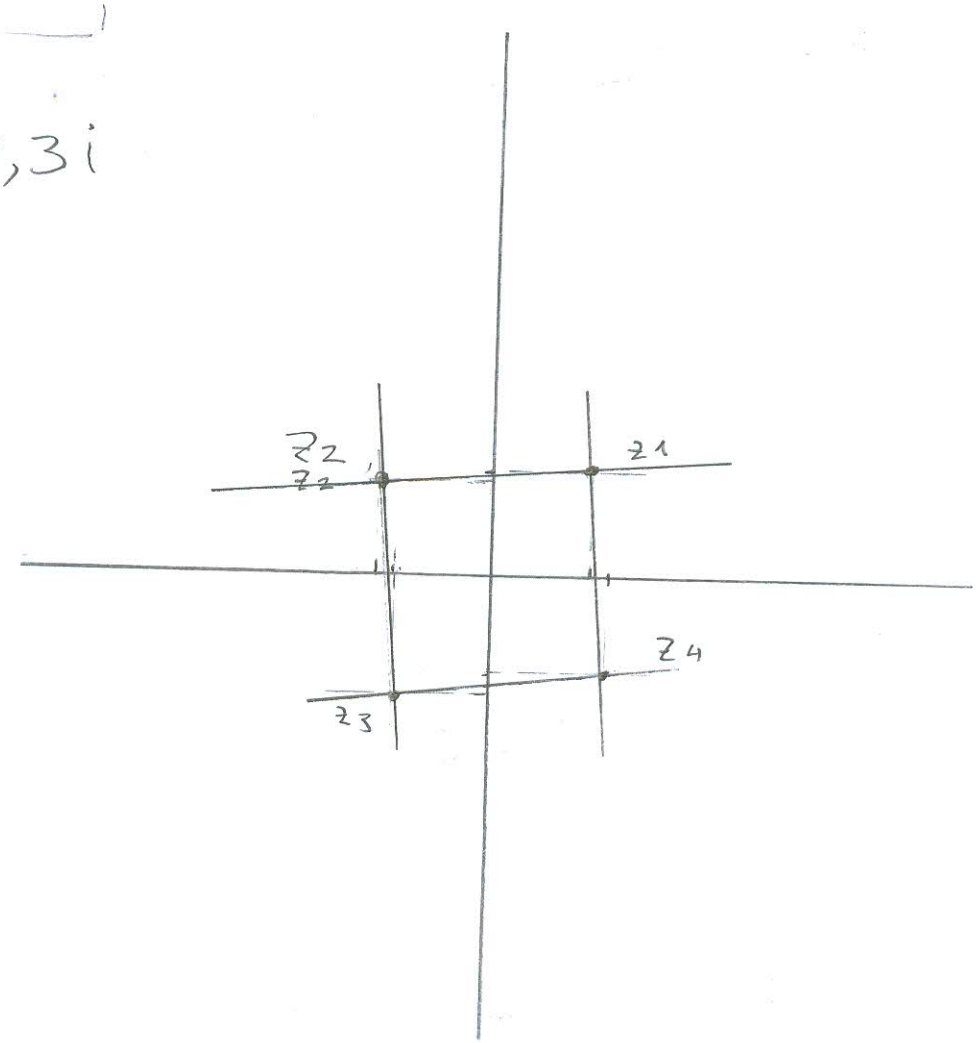
$z_2 = -1{,}5 + 1{,}3i$

$k = 2$

$z_3 = -1{,}3 - 1{,}5i$

$k = 3$

$z_4 = 1{,}5 - 1{,}3i$



5.

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \begin{array}{l} \\ +\,\mathrm{I}\cdot(-2) \\ +\,\mathrm{I}\cdot(-3) \\ +\,\mathrm{I}\cdot(-1) \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 5 & 1 & -2 & 7 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \begin{array}{l} +\,\mathrm{II}\cdot(-2) \\ +\,\mathrm{III} \\ +\,\mathrm{II}\cdot \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$$

③ $g(x) = \ln(4 - x^2)$ STIPE KATALINIĆ

$4 - x^2 > 0$ $D(f) = \langle 2, +\infty \rangle$ ✗

$-x^2 > -4$

$x^2 > 4$

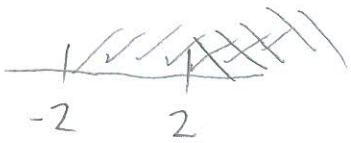
$x_1 = 2$

$x_2 = -2$

$g(x) = \ln(4 - (-x)^2)$

$g(-x) = \ln(4 - x^2)$

funkcija je parna ✓



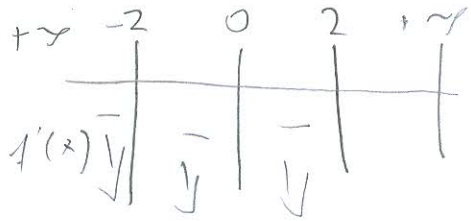
$$f'(x) = \frac{1}{(4 - x^2)} \cdot (-2x)$$

$$f'(x) = \frac{-2x}{(4 - x^2)}$$

$-2x = 0 \,/\cdot(-2)$

$x = 0$ stacionarna točka $(0,0)$

$$f'(0) = \frac{-2 \cdot 0}{(4 - 0^2)^{2-}} = \frac{0}{4} = +\infty \; 0$$



V. ASIM. nema

H. ASIM

Funkcija je konkovna ✓

STIPE KATALINIĆ

(4) 2AD $h(x) = \frac{x^2 - 2x - (2+1)}{x^2+1}$

(1) $x^2 + 1 \neq 0$

$x^2 \neq -1$ $D(f) = \mathbb{R}$

nema domene

(2) nema vertik. asimpto

H. A

$\lim_{x\to+\infty} \frac{x^2-2x-(2+1)}{x^2+1} \,/\, x^2 = \lim_{x\to+\infty} \frac{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{(2+1)}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} = \frac{1}{1} = 1$

$y = 1$

H. ORIZ. ASIMP

$y = 1$

$\lim_{x\to-\infty} \frac{x^2-2x-(2+1)}{x^2+1} \left(\begin{matrix} x\to -x \\ +\infty \to -\infty \end{matrix}\right) = \lim_{x\to+\infty} \frac{(-x^2) -}{}$

(3.) nije periodično

(4.) $h(-x) = \frac{(-x)^2 - (2\cdot(-x) - (2+1)}{(-x^2)+1}$

$h(-x) = \frac{x^2 + 2x - (2+1)}{x^2+1}$ ni parno ni neparno

$h(-x) = -\frac{(x^2 - 2x + (2-1)}{x^2+1}$

5. $x^2 - 2x - (2+1) = 0$

$x^2 - 2x - 3 = 0$

$x_1 = 1 \quad b = -2 \quad c = -3$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2}$

$x_{1,2} = \frac{2 \pm 4}{2}$

$x_1 = 3 \quad (3, 0)$ → nul točke

$x_2 = -1 \quad (-1, 0)$

6. STACIONARNE TOČKE

$h(x) = \frac{x^2 - 2x - (2+1)}{x^2+1}$

$h'(x) = \frac{(2x-2)(x^2+1) - (x^2-2x-(2+1)\cdot 2x}{(x^2+1)^2}$

$h'(x) = \frac{2x^3 + 2x - 2x^2 - 2 - 2x^3 + 4x^2 + 6x}{(x^2+1)^2}$

$h'(x) = \frac{-4x - 6x^2 - 2}{(x^2+1)^2}$

$h'(x) = \frac{-6x^2 - 4x - 2}{(x^2+1)^2}$

$-6x^2 - 4x - 2 = 0$

$x_1 = -6 \quad b = -4 \quad c = -2$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{4 \pm \sqrt{16 - 48}}{-12}$

$x_1 = -\frac{1}{3}$] stacionarna točka

GRAF

nastavak ⇒

4 ZAD

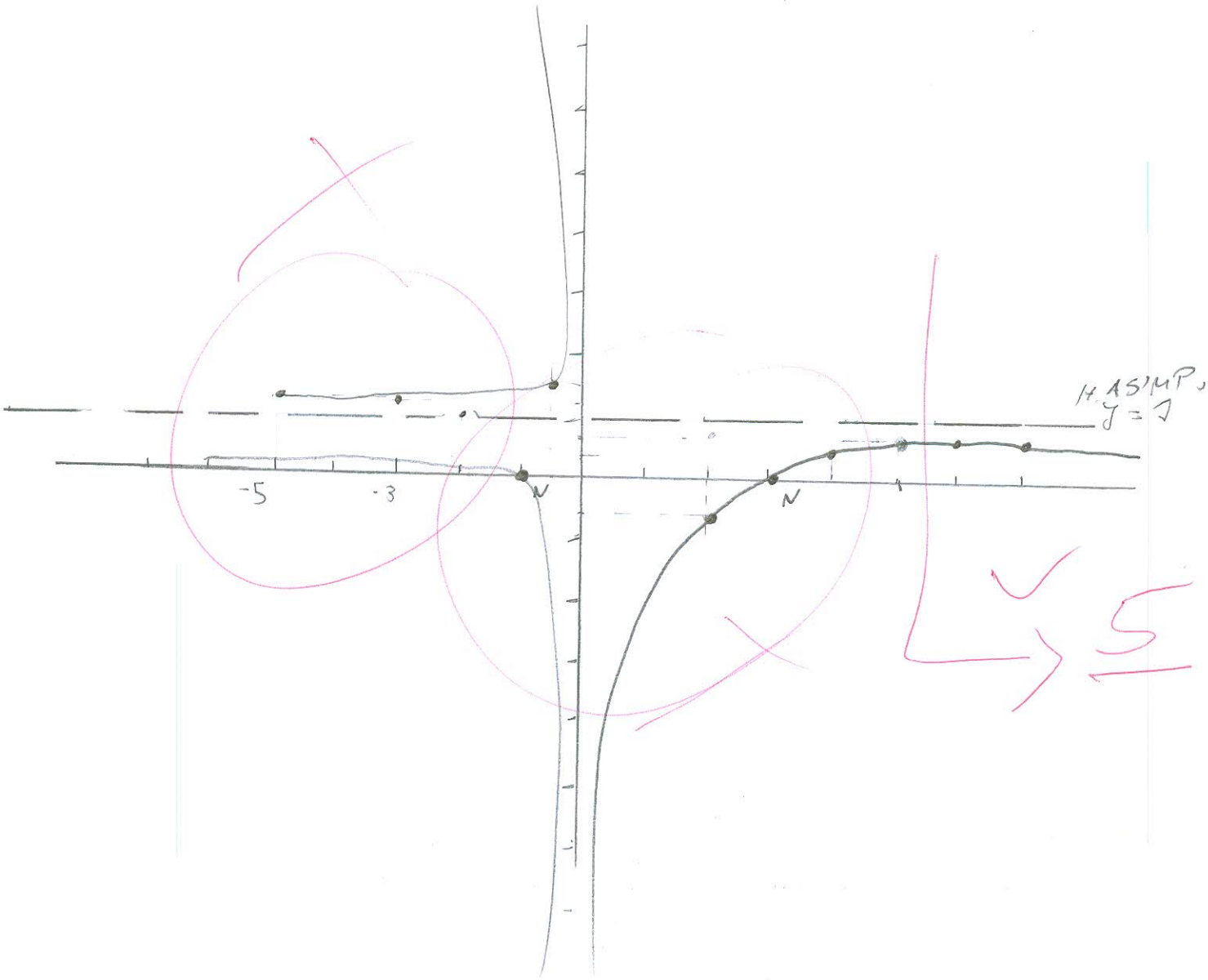
STIPE KATALINIĆ

	$+\infty$		-1		$-\frac{1}{3}$		1		3		$+\infty$
$f'(x)$	$-$	$\|$	$-$	$\|$	$-$	$\|$	$-$	$\|$	$-$	$\|$	$-$
$f(x)$	$\searrow$	$\|$	$\searrow$	$\|$	$\searrow$	$\|$	$\searrow$	$\|$	$\searrow$	$\|$	$\searrow$

Nema ni lokalnog minimuma ni lokalnog maksimuma

Pomoćne točke

x	-5	-3	-2	-0,5	2	4	5	6	7
y	1,2	1,2	1	-1,4	-0,6	0,29	0,5	0,5	0,6



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IME I PREZIME: MARKO MARASOVIĆ **BROJ INDEKSA:** 17-1-0242-2014

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$3x-y-2z+u=2$
$x-y+3z-5u=3$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \cdot(-2)\cdot(-3)\cdot(-1)$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -7 & 1 & -2 & -7 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \cdot(-2)\cdot(7)\cdot(3)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -28 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \cdot 1:8$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \cdot(-1)\cdot(3)\cdot(7)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{5}{4} & -\frac{3}{2} \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 0 & -\frac{17}{4} & \frac{31}{2} \end{array}\right] \cdot 1:\left(-\frac{17}{4}\right)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{5}{4} & -\frac{3}{2} \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 0 & 1 & -\frac{62}{13} \end{array}\right] \cdot\left(-\frac{5}{4}\right)\cdot\left(\frac{1}{4}\right)\cdot\left(\frac{1}{4}\right)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{52}{17} \\ 0 & 1 & 0 & 0 & \frac{24}{17} \\ 0 & 0 & 1 & 0 & -\frac{75}{17} \\ 0 & 0 & 0 & 1 & -\frac{248}{17} \end{array}\right]$$

OSTATAK NA SLJ. STRANICI →

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{52}{17} \\ 0 & 1 & 0 & 0 & \frac{24}{17} \\ 0 & 0 & 1 & 0 & -\frac{75}{17} \\ 0 & 0 & 0 & 1 & -\frac{248}{17} \end{array}\right] \qquad \left[\begin{array}{l} x_1 = \frac{52}{17} \\ y = \frac{24}{17} \\ z = -\frac{75}{17} \\ u = \frac{-248}{17} \end{array}\right]$$

$\frac{-146}{17}$

PROVJERA:

$$\frac{52}{17} + \left(2 \cdot \frac{24}{17}\right) - \left(-\frac{75}{17}\right) + \left(-\frac{248}{17}\right) = 3$$

$$\left(2 \cdot \frac{52}{17}\right) + \left(5\left(\frac{24}{17}\right)\right) - \left(-\frac{75}{17}\right) + \left(2 \cdot \frac{-248}{17}\right) = 3$$

$$\left(3 \cdot \frac{52}{17}\right) - \left(\frac{24}{17}\right) - \left(2 \cdot \frac{-75}{17}\right) + \left(-\frac{240}{17}\right) = 2$$

$$\frac{52}{17} - \frac{24}{17} + \left(3 \cdot \frac{-75}{17}\right) - \left(5 \cdot \frac{-248}{17}\right) = 3 \quad \sqrt{} \sqrt{} \times$$

$$-\frac{197}{17} - x = 3$$

$$-\frac{197}{17} - 3 = x$$

$$\frac{52 - 24 - 225 + 1240}{17} = \frac{1292 - 249}{17} =$$

$$= \frac{1043}{17} \neq 3$$

$h(x) = f(x)$

MARKO MARASOVIĆ
17-1-0242-2014

4) $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$

$x^2+1 \neq 0$

$x^2 \neq -1 \sqrt{}$

$x = \sqrt{-1}$

$D(f) = \mathbb{R}$

2° NGMA VA

HA $\lim_{x\to\pm\infty} \dfrac{x^2-2x-3}{x^2+1} \left(\dfrac{:x^2}{:x^2}\right) = \left[\dfrac{\infty}{\infty}\right] = \dfrac{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} = \dfrac{1-0-0}{1+0} = \dfrac{1}{1} = 1$

OHA = $y_{\text{rac}1}$

NEMA VA

3° $f(x)=0 \Rightarrow x^2-2x-3=0$

$x_{12} = \dfrac{2 \pm \sqrt{4+12}}{2}$

$x_1 = -1$ → $N_{x1}(-1, 0)$

$x_2 = 3$ → $N_{x2}(3, 0)$

$f(0) = \dfrac{-3}{1} = -3$

$S(0, -3)$

$f'(x) = \dfrac{x^2-2x-3}{x^2+1} = \dfrac{(2x-2)(x^2+1) - (x^2-2x-3)(2x)}{(x^2+1)^2} = \dfrac{(2x^3+2x-2x^2-2) - (2x^3-4x^2-6x)}{(x^2+1)^2}$

$= \dfrac{2x^3+2x-2x^2-2-2x^3+4x^2+6x}{(x^2+1)^2} = \dfrac{2x^2+8x-2}{(x^2+1)^2}$

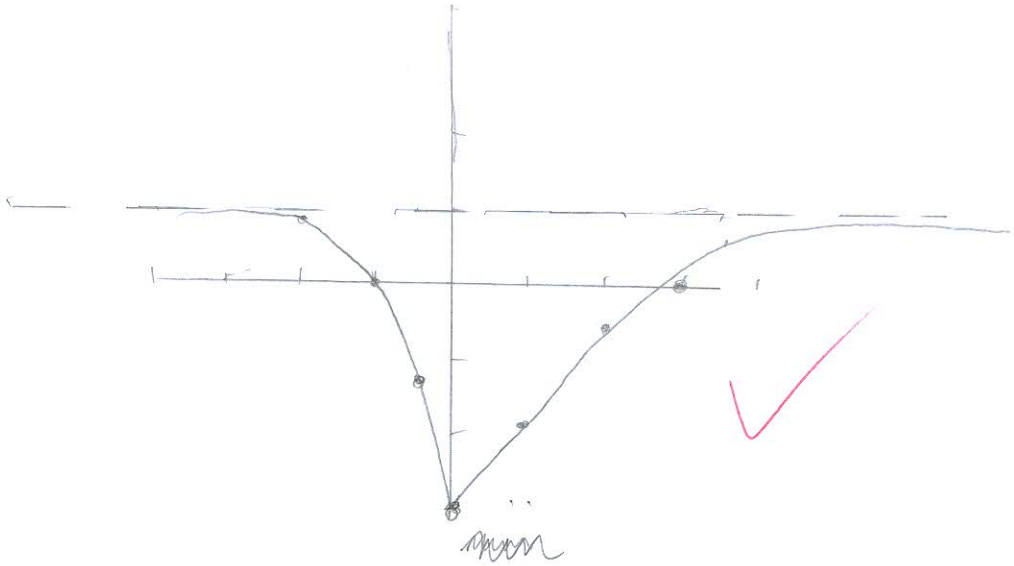
$f'(x)$	$\nearrow$	-1	$\searrow$	0	$\searrow$	3	$\nearrow$
$f(x)$				min.			

SS: SPAMA →

$f'(x) = 0 \Rightarrow 2x^2+8x-2=0$

$x_1 = -4{,}2 \qquad x_2 = 0{,}2$

(4) NASTAVAK



MARKO MARKOVIĆ 17-1-0242-2014

(2) $f(x)=x-\sqrt{x^2-2}$

$x^2-2\geq 0$

$x^2\geq 2$

$x\geq \pm\sqrt{2}$

$D\langle -\infty, +\infty\rangle$

$D(f)=R$ ✗ NEMA VA

$f=0 \Rightarrow x-\sqrt{x^2-x}=0$

$\sqrt{x^2-x}=-x\ /^2$

$x^2-x=-x^2$

$2x^2=0$

HA $\lim_{x\to\infty} x-\sqrt{x^2-2}=+\infty$ ✗

⑥ $\lim_{x \to +\infty} \frac{e^x}{x} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=}$

③

$g(x) = \ln(4 - x^2)$

DOMENA

$4 - x^2 \neq 0$

$4 = x^2$ √

$x \neq \pm 2$

~~$D(g) = R\{-2, 2\}$~~ ✗

$D_g = \langle -\infty, -2] \cup [2, +\infty \rangle$ ✗

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{H}2$

IME I PREZIME: Bruno Stipanov **BROJ INDEKSA:** 17-1-9189-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3

2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5

3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10

4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)

5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 5

④ $f(x) = \frac{x^2-2x-(2+1)}{x^2+1} = \frac{x^2-2x-3}{x^2+1}$

domena:

$x^2+1 \neq 0$

$x^2 \neq -1 \ /\sqrt{\ }$ $\quad D_f: \mathbb{R}$

$x \neq \pm\sqrt{-1}$

asimptote:

V.A. $\lim\limits_{x\to+\infty} \frac{x^2-2x-3 \ /:x^2}{x^2+1 \ /:x^2} = \frac{1}{1} = 1$ $\quad \boxed{x=1}$ V.A

H.A. $\lim\limits_{x\to\infty} \frac{x^2-2x-3 \ /:x}{x^2+1 \ /:x} = \frac{1}{1} = 1$ $\quad \boxed{y=1}$ H.A.

K.A. $y = kx + l$

$k = \frac{\frac{x^2-2x-3}{x^2+1}}{x} = \frac{x^2-2x-3 \ /:x^3}{x^3+x \ /:x^3} = \frac{0}{1} = 0$ nema K.A.

derivacija:

$f(x) = \frac{x^2-2x-3}{x^2+1}$

$f'(x) = \frac{(x^2-2x-3)'\cdot(x^2+1) - (x^2-2x-3)(x^2+1)'}{(x^2+1)^2}$

$= \frac{2x-2\cdot(x^2+1) - (x^2-2x-3)2x}{(x^2+1)^2}$

$= \frac{2x^3+2x-2x^2-2-2x^3+4x^2+6x}{(x^2+1)^2}$

$f'(x) = \boxed{\frac{2x^2+8x-2}{(x^2+1)}}$

ekstremi: $f'(x)=0$

$\frac{2x^2+8x-2}{(x^2+1)} = 0 \ / \cdot (x^2+1)$

$2x^2+8x-2=0$

$x_{1,2} = \frac{-8\pm\sqrt{64-4\cdot2\cdot(-2)}}{4}$

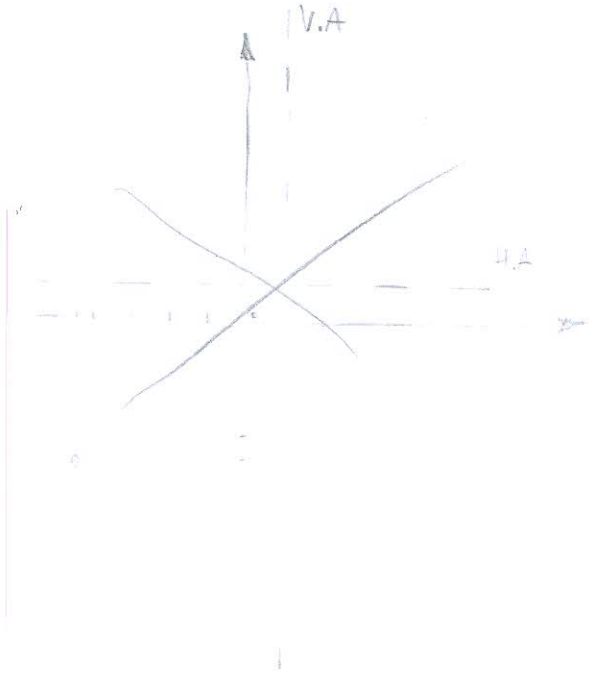
$x_{1,2} = \frac{-8\pm\sqrt{80}}{4}$

$x_1 = 0{,}24$, $x_2 = -4{,}24$

	$-\infty$		$-4{,}24$		$0{,}24$		1		$+\infty$
		-5		0		$0{,}5$		2	
$f'(x)$		$+$		$-$		$+$			
$f(x)$		↗		↘		↗		↗	

(0,84 , 0,033)

(-4,24 , 3,58)



4. zadatak

derivacija:

$$f(x)\ \frac{x^2-2x-(2+1)}{x^2+1}$$

$$f'(x)=\frac{(x^2-2x-(2+1))'\cdot(x^2+1)-(x^2-2x-(2+1))\cdot(x^2+1)'}{(x^2+1)}$$

$$f'(x)=\frac{(2x-2)\cdot(x^2+1)-(x^2-2x-(2+1))\cdot 2x}{(x^2+1)}$$

$$=\frac{2x^3+2x-x^2-2-(2x^3-4x^2-(4x+2x))}{(x^2+1)}=\frac{2x^3+2x-x^2-2-2x^3+4x^2+4x+2x}{(x^2+1)}$$

$$=\boxed{\frac{-5x^2+8x-2}{(x^2+1)}}$$

ekstremi: $f'(x)=0$

$$\frac{-5x^2+8x-2}{(x^2+1)}=0\ /\cdot(x^2+1)$$

$$-5x^2+8x-2=0$$

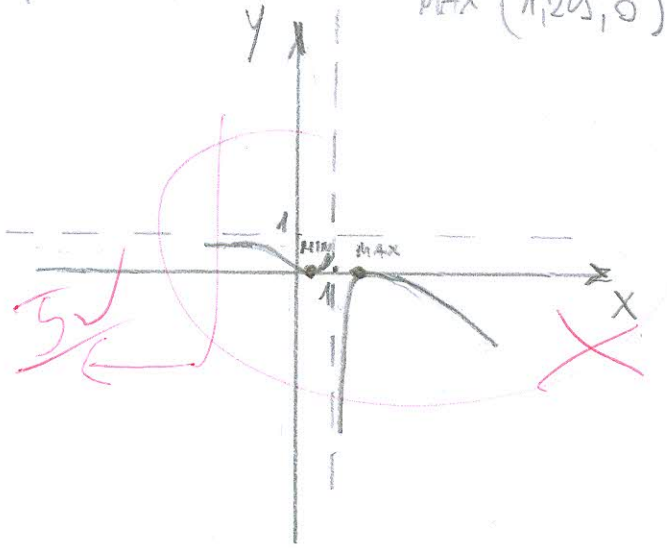
$$x_{1,2}=\frac{-8\pm\sqrt{64-40}}{-10}=\frac{-8\pm\sqrt{24}}{-10}$$

$x_1=0{,}31 \qquad x_2=1{,}29$

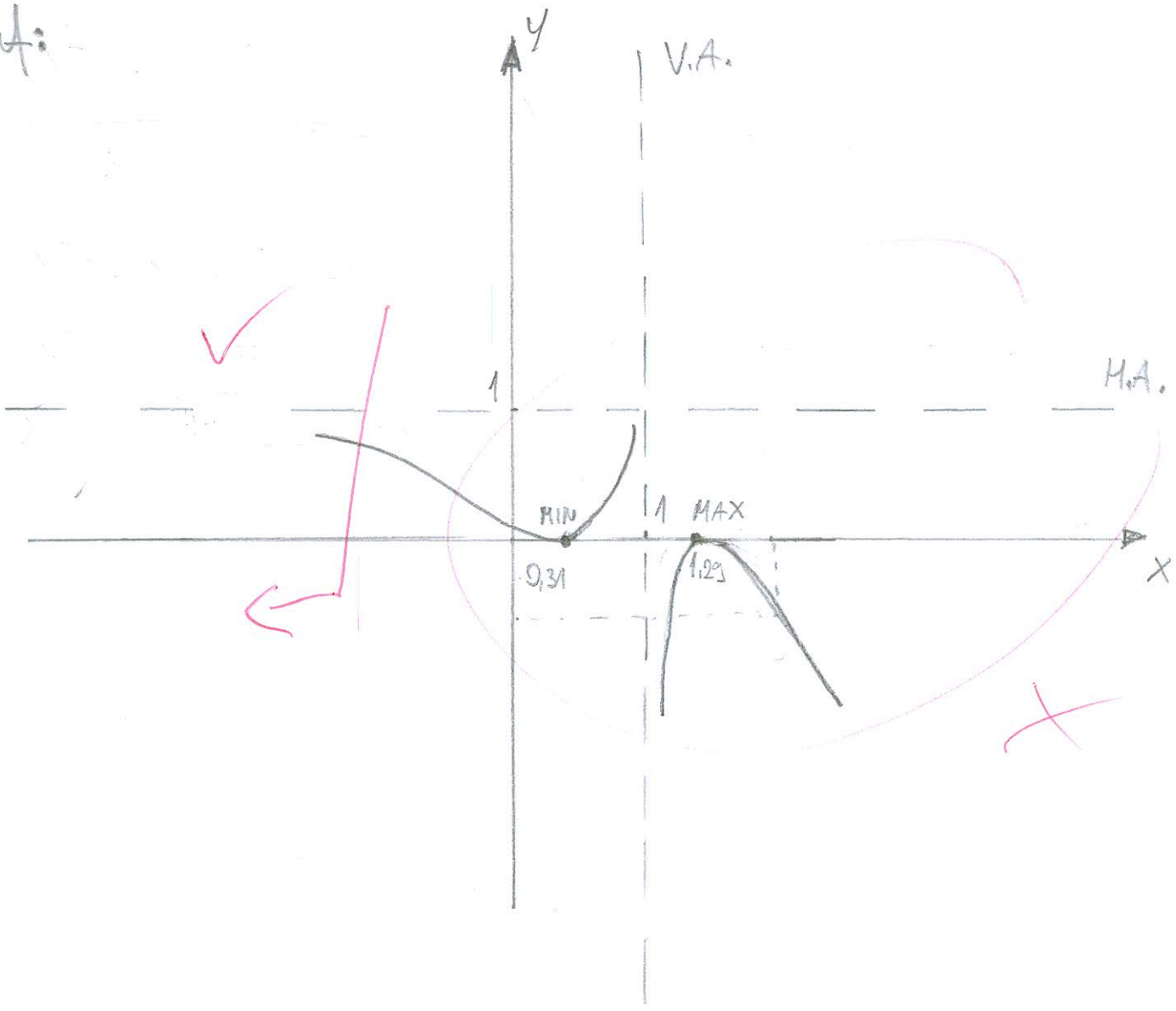
	$-\infty$		0,31		1		1,29	$+\infty$
	-1		0,5		1,2		2	
$f(x)$	−		+					
$f'(x)$	↘		↗		↗		↘	

MIN (0,31 , 0)

MAX (1,29 , 0)



graf:



③ $\ln(4-x^2)$

$4-x^2>0$

$4-x^2=0$

$-x^2=-4 \;/\cdot(-1)$

$x^2=4 \;/\sqrt{}$

$x=\pm 2$

	$-\infty$		-2		2		$+\infty$
		-3		1		3	
$4-x^2=0$		$-$	0	$+$	0	$+$	

$D_f: \langle -2, +\infty \rangle$ ✗

$$x+2y-z+u=3$$
$$2x+5y-z+2u=3$$
$$3x-y-2z+u=2$$
$$x-y+3z-5u=3$$

$$\begin{bmatrix}1&2&-1&1&3\\2&5&-1&2&3\\3&-1&-2&1&2\\1&-1&3&-5&3\end{bmatrix} \begin{matrix}\cdot(-2)-II\\ \\ \cdot(-3)-III\\ -I\end{matrix} \sim \begin{bmatrix}1&2&-1&1&3\\0&-1&-1&0&3\\0&-7&-1&2&-7\\0&1&-4&6&0\end{bmatrix} \cdot(-1)$$

$$\sim \begin{bmatrix}1&2&-1&1&3\\0&1&1&0&-3\\0&-7&-1&2&-7\\0&1&-4&6&0\end{bmatrix} \begin{matrix}\cdot(-2)-I\\ \cdot 7+III\\ -IV\\ \\ \end{matrix} \sim \begin{bmatrix}1&0&3&1&3\\0&1&1&0&-3\\0&0&8&2&-14\\0&0&6&6&3\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&3&1&3\\0&1&1&0&-3\\0&0&2&-4&-11\\0&0&6&6&3\end{bmatrix} :2$$

$$\sim \begin{bmatrix}1&0&3&1&3\\0&1&1&0&-3\\0&0&1&-2&-\frac{11}{2}\\0&0&6&6&3\end{bmatrix} \begin{matrix}\cdot(-3)-I\\ -II\\ \cdot 6-IV\\ \\ \end{matrix} \sim \begin{bmatrix}1&0&0&-7&\frac{27}{2}\\0&1&0&0&4\\0&0&1&-2&-\frac{11}{2}\\0&0&0&-18&30\end{bmatrix} :(-18)$$

$$\sim \begin{bmatrix}1&0&0&-7&\frac{27}{2}\\0&1&0&0&4\\0&0&1&-2&-\frac{11}{2}\\0&0&0&1&-\frac{15}{9}\end{bmatrix} \begin{matrix}\cdot 2-III\\ \cdot 7-I\\ \\ \\ \end{matrix} \sim \begin{bmatrix}1&0&0&0&-\frac{31}{3}\\0&1&0&0&4\\0&0&1&0&-\frac{11}{6}\\0&0&0&1&-\frac{15}{9}\end{bmatrix}$$

$$x=-\frac{31}{3}$$
$$y=4$$
$$z=-\frac{11}{6}$$
$$u=-\frac{15}{9}$$

PROVJERA:

$$-\frac{30}{9}-\frac{11}{2}=\frac{-99}{18}$$

$$-\frac{15}{9}-\frac{27}{2}=\frac{30-}{18}=\frac{-186}{18}$$

DALJE...?

$\emptyset$

$$-\frac{31}{3}-4-\frac{33}{6}+\frac{75}{9}=3$$

$$\frac{?}{18}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ H2

IME I PREZIME: Josip Jukić **BROJ INDEKSA:** 17-1-0256-2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 5

1. $z^4 - (4-i)^2 = 0$

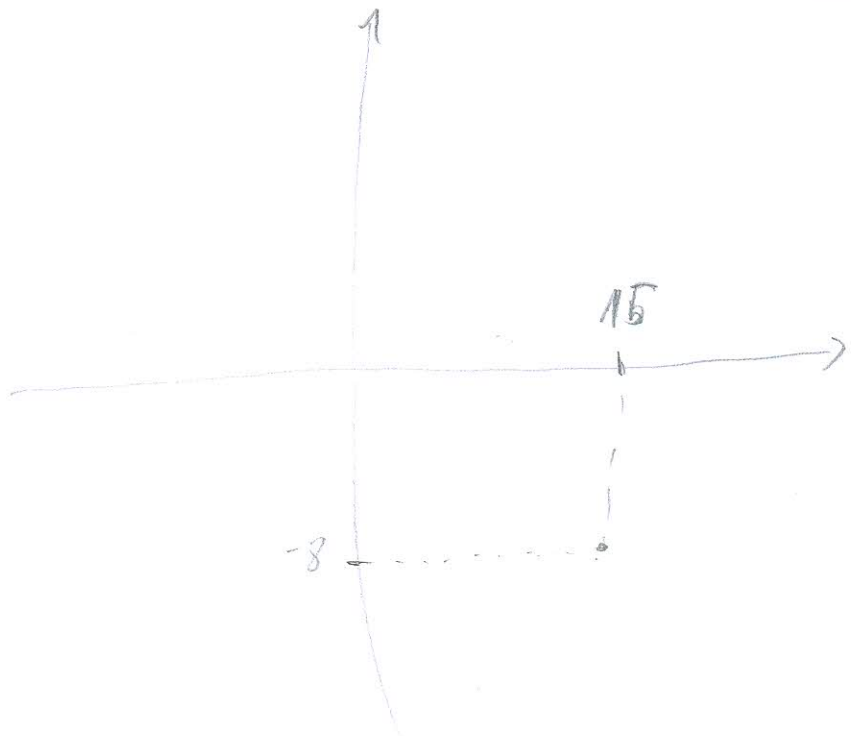
$z^4 = (4-i)^2$

$z^4 = 16 - 8i - 1$

$z^4 = 15 - 8i \,/\sqrt[4]{\ }$

$z = \sqrt[4]{15-8i}$

$|z| = \sqrt{x^2+y^2} = 17$



$\operatorname{tg} f = \dfrac{r}{x}$

$\operatorname{tg} f = \dfrac{-8}{15}$

$f = -28^\circ 07' + 360^\circ$

$f = 331^\circ 53' 39''$

$k=0\ldots \sqrt[n]{|z|}\cdot\left(\cos\frac{\varphi+2k\pi}{n}+i\sin\frac{\varphi+2k\pi}{n}\right)$

$k=0\ldots \sqrt[4]{17}\cdot\left(\cos\frac{330°51'39''}{4}+i\sin\frac{330°51'39''}{4}\right)$

$k=0\ldots \sqrt[4]{17}\cdot(-0.121+i\,0.87)$

$k=0\ldots -0.25+i\,1.76$

$k=1\ldots \sqrt[4]{17}\cdot\left(\cos\frac{330°51'39''+2\cdot1\cdot\pi}{4}+i\sin\frac{330°51'39''+2\cdot1\cdot\pi}{4}\right)$

$k=1\ldots \sqrt[4]{17}\cdot(0.995+i\,0.099)$

$k=1\ldots 2.01+i\,0.133$

$k=2\ldots \sqrt[4]{17}\cdot\left(\cos\frac{330°51'39''+2\cdot2\cdot\pi}{4}+i\sin\frac{330°51'39''+2\cdot2\cdot\pi}{4}\right)$

$k=2\ldots \sqrt[4]{17}\cdot(0.997+i\,0.072)$

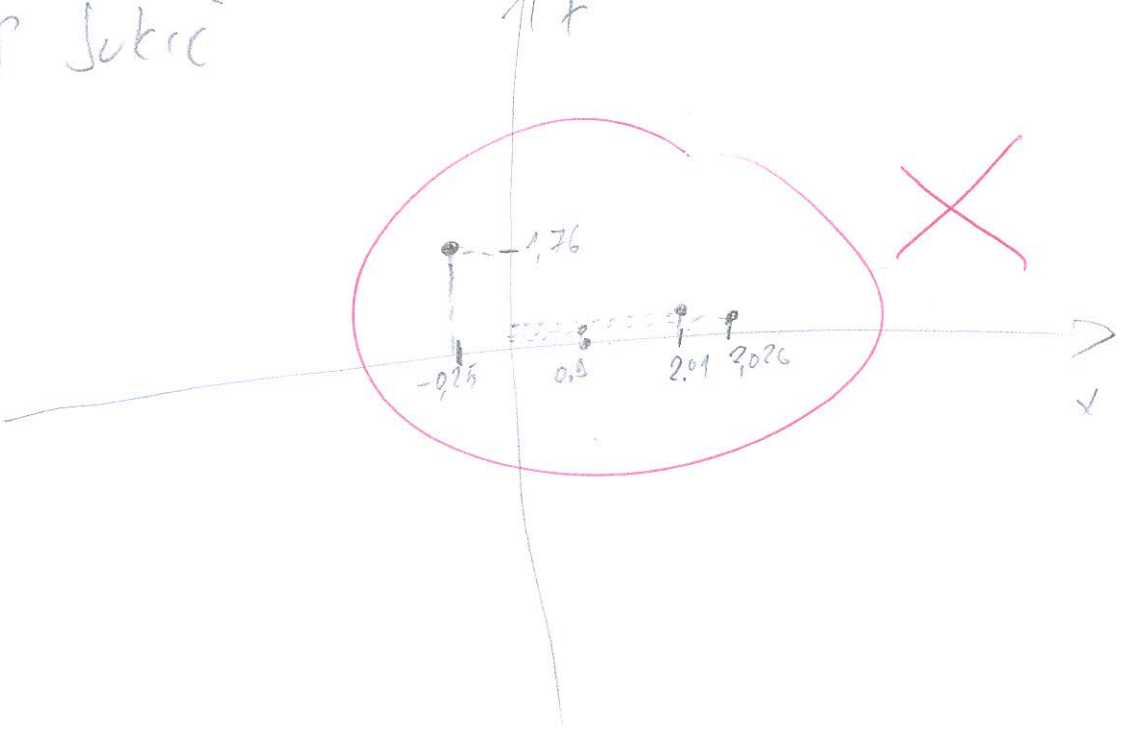
$k=2\ldots 2.024+i\,0.146$

$k=3\ldots \sqrt[4]{17}\cdot\left(\cos\frac{330°51'39''+2\cdot3\cdot\pi}{4}+i\sin\frac{330°51'39''+2\cdot3\cdot\pi}{4}\right)$

$k=3\ldots \sqrt[4]{17}\cdot(0.998+i\,0.044)$

$k=3\ldots 2.026+i\,0.089$

Josip Jukić



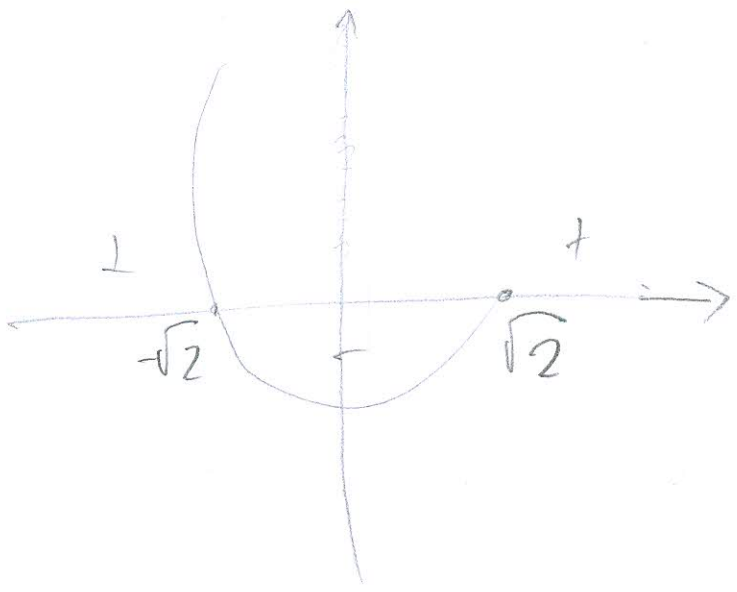
② $f(x) = x - \sqrt{x^2-2}$

$\sqrt{x^2-2} \dots$

$x^2 - 2 > 0$

$x^2 > 2 \quad / \sqrt{}$

$x = \pm\sqrt{2}$



$x \in \langle -\infty, -\sqrt{2} \rangle \cup \langle \sqrt{2}, +\infty \rangle$

Josip Sokić

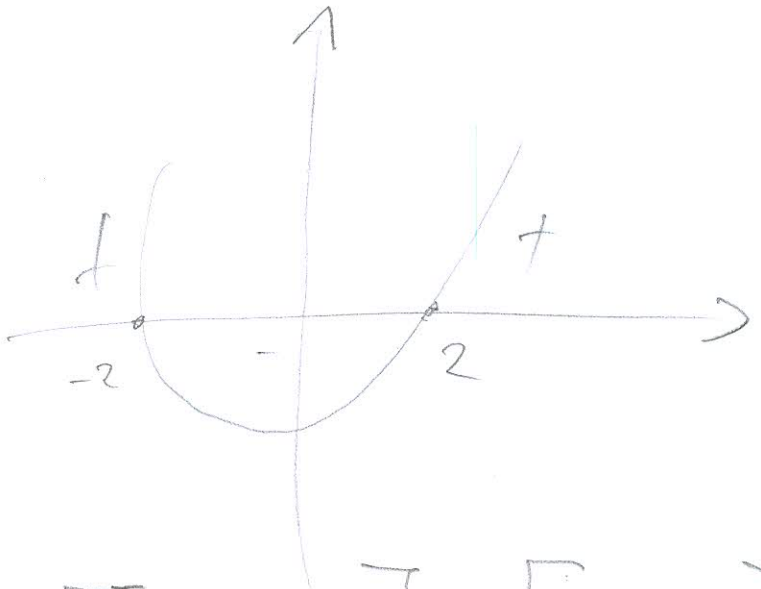
(3.) $y(x) = \ln(4 - x^2)$

$4 - x^2 \geq 0$

$-x^2 \geq -4 \quad / \cdot (-1)$

$x^2 \geq 4 \quad / \sqrt{\ }$

$x = \pm 2$



$x \in \langle -\infty, -2] \cup [2, +\infty \rangle$

Sosip Sekić

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \quad \cdot(-2) \quad \cdot(-3) \quad \cdot(-1)$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & -7 & 1 & -2 & -7 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \quad \cdot(-2) \quad \cdot(7) \quad \cdot(3)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -28 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \quad :8$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \quad \cdot(-1) \quad \cdot(-3) \quad \cdot(-7)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & -\frac{3}{2} \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 0 & -\frac{17}{4} & \frac{31}{5} \end{array}\right] \quad \cdot\left(-\frac{17}{4}\right)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & -\frac{3}{2} \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 0 & 1 & \frac{124}{85} \end{array}\right] \quad \cdot\left(\frac{1}{4}\right) \cdot\left(-\frac{1}{4}\right) \cdot\left(-\frac{1}{4}\right)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{317}{170} \\ 0 & 1 & 0 & 0 & \frac{23}{170} \\ 0 & 0 & 1 & 0 & -\frac{533}{170} \\ 0 & 0 & 0 & 1 & \frac{124}{85} \end{array}\right] \quad \begin{matrix} x \\ r \\ z \\ 0 \end{matrix}$$

$$\begin{bmatrix} x \\ r \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{317}{170} \\ \frac{23}{170} \\ -\frac{533}{170} \\ \frac{124}{85} \end{bmatrix}$$

Josip Jukić

$x + 2y - 2 + 0 = 3$

$-\frac{317}{170} + 2 \cdot \frac{23}{170} - \left(-\frac{533}{170}\right) + \frac{124}{85} = 3$

$3x - y - 2z + 0 = 2$

$3 \cdot \left(-\frac{317}{170}\right) - \frac{23}{170} - 2 \cdot \left(-\frac{533}{170}\right) + \frac{124}{85} = 2$

PROVJERA NIJE IZVEDENA DO KRAJA. —

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: Josip Ergotić **BROJ INDEKSA:** 17-2-0387-2014

Prof. Uglešić

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 0

(5)

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \cdot(-2) \cdot(-3) \cdot(-1)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & \frac{39}{2} \\ 0 & 1 & 0 & \frac{1}{4} & -\frac{13}{2} \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{7}{2} \\ 0 & 0 & 0 & -\frac{17}{4} & -\frac{67}{2} \end{array}\right] /\left(-\frac{17}{4}\right)$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -7 & 1 & -2 & -7 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \cdot(-2) \cdot 7 \cdot 3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & \frac{39}{2} \\ 0 & 1 & 0 & \frac{1}{4} & -\frac{13}{2} \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{7}{2} \\ 0 & 0 & 0 & 1 & \frac{67}{2} \end{array}\right] \cdot(-\tfrac{1}{4}) \cdot(-\tfrac{1}{4}) \cdot(\tfrac{1}{4})$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -28 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] /:8$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{223}{8} \\ 0 & 1 & 0 & 0 & \frac{15}{8} \\ 0 & 0 & 1 & 0 & -\frac{39}{8} \\ 0 & 0 & 0 & 1 & -\frac{67}{2} \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{7}{2} \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \cdot 3 \cdot(-1) \cdot(-7)$$

$x = \frac{223}{8}$

$y = \frac{15}{8}$

$z = -\frac{39}{8}$

$u = -\frac{67}{2}$

PROVJERA:

$$\frac{223}{8} + 2\cdot\left(\frac{15}{8}\right) - \left(-\frac{39}{8}\right) + \left(-\frac{67}{2}\right) = 3 \quad \checkmark\checkmark$$

DALJE...?

Josip Energetic

① $z^4 - (4-i)^2 = 0$

$z^4 - 16 - 4i + i^2 = 0$

$z^4 - 16 - 4i - 1 = 0$

$z^4 - 17 - 4i = 0$

⑥ $\lim_{x \to \infty} \frac{e^x}{x} = \frac{e^\infty}{\infty} =$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{H}2$

IME I PREZIME: Mateo Bilaver **BROJ INDEKSA:** 0269081720

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 0

1.) $z^4-(4-i)^2=0$

$z^4=4(4+i)^2 \,/^2$

$z^2=4+i$

$r=\sqrt{x^2+y^2}$

$r=\sqrt{4^2+1^2}$

$r=\sqrt{17}$

$tg\varphi=\frac{y}{x}$

$tg\varphi=\frac{1}{4}$

$tg\varphi=0.249$

$z^n=r^n(\cos(n\cdot\varphi)+i\sin(n\cdot\varphi))$

$z^2=\sqrt{17}^2(\cos(2\cdot0.249)+i\sin(2\cdot0.249)$

$z^2=17(\cos(2\cdot0.249)+i\sin(2\cdot0.249)$

$z^2=17(\cos(0.4898)+i\sin(0.4898)$

$z=15+7.99i$

$r=\sqrt{x^2+y^2}$

$r=\sqrt{15^2+7.99^2}$

$r=16.99$

$tg\varphi=\frac{y}{x}$

$tg\varphi=\frac{7.99}{15}$

$\varphi=0.4894$

$$z = 2\sqrt{5}\left(\cos\frac{\theta+2\pi}{2} + i\sin\left(\frac{\theta-2\pi}{2}\right)\right)$$

$$= 2\sqrt{17}\left(\cos\frac{0.4894}{2} + i\sin\left(\frac{0.4894}{2}\right)\right)$$

$$= 8.2462\{0.9702\} + 0.2422i$$

$$z_1 = 8.00 + 2.01i$$

$$z_2 = 2\sqrt{17}\left(\cos\frac{0.4894+2\pi}{2} + i\sin\left(\frac{0.4894+2\pi}{2}\right)\right)$$

$$z_2 = 8.2462\left(-0.9702\right) + \left(-0.2423i\right)$$

$$= -8.00 + 1.998i$$

4 RJEŠENJA

2.) $f(x) = x - \sqrt{x^2-2}$

Domena

$$x - \sqrt{x^2-2} \geq 0 \quad / \cdot 2$$

$$2x - \sqrt{x^2-2} = 0$$

$$2x - x^2 - 2 = 0$$

$$x^2 - 2 = 0 \quad /^2$$

$$x - 4 = 0$$

$$x \neq 4$$

$D(f) = \mathbb{R}\setminus\{4\}$

Asimptote

$$\lim_{x\to 4} x - \sqrt{x^2-2}$$

4) $h(x) = \frac{x^2 - 2x - (2+1)}{x^2+1}$

Domena

$x^2 + 1 \neq 0$

$x^2 \neq -1$

Ničlo čke

$\frac{0^2 - 2\cdot 0 - (2+1)}{0^2+1} = -3 \quad T(1,-3)$

Asimptote

V.A $\lim_{x \to -3} \frac{x^2 - 2x - (2+1)}{x^2+1} = \frac{0}{1}$ = neodrečen oblik

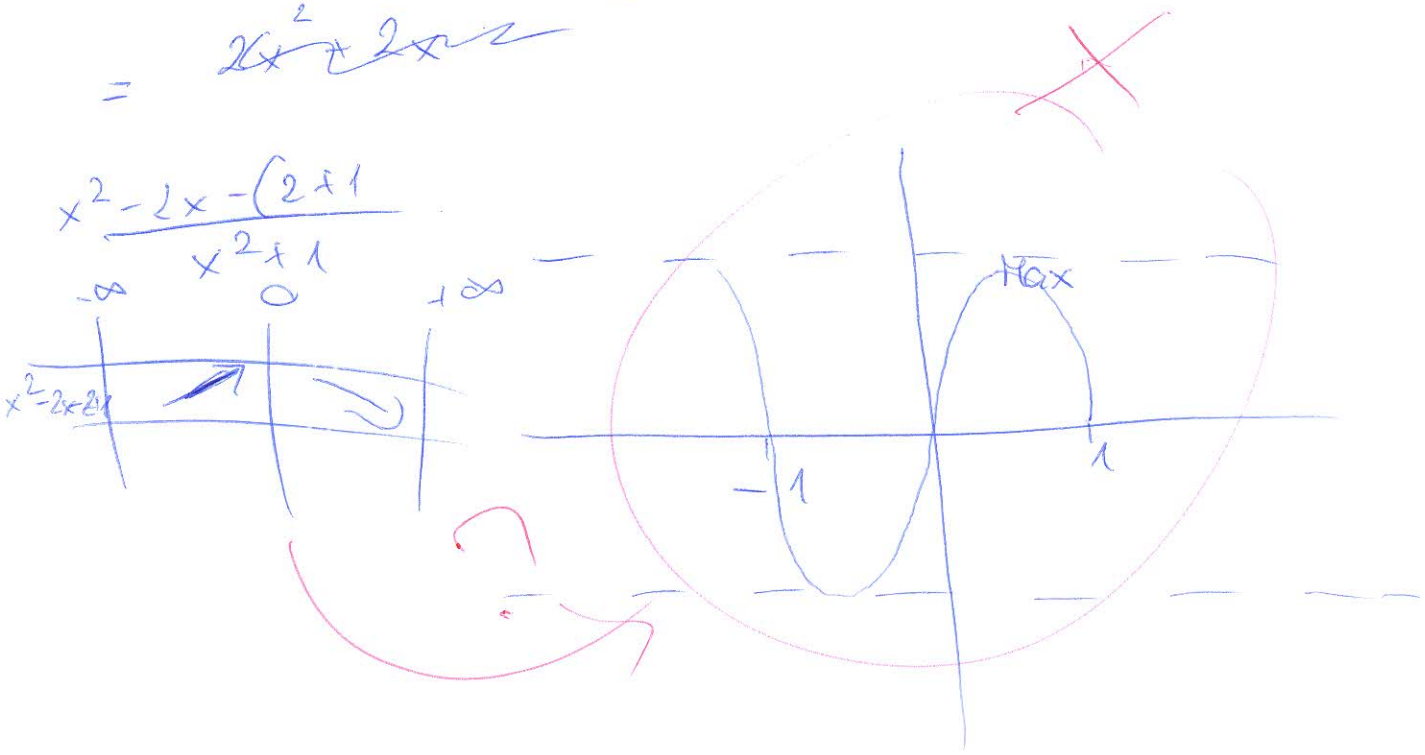
$\frac{(-3)^2 + 1}{\neq 0}$

nema kose ni horizontalne

$\frac{2x - (x+1) - 2x - (2x+1) - (2x+1)}{(x^2+1)^2}$

$= 2x^2 + 2x - 2$

$\frac{x^2 - 2x - (2+1)}{x^2+1}$



6.) $\lim_{x \to \infty} = \frac{e^x}{x} = 1.35600 \cdot 10^{37}$ ✗

Mateo Bikaver

2.) Asimptote

$\lim_{x \to 4} 4 - \sqrt{4^2 - 2} = 4 - \sqrt{14} = +\infty$ X

$\lim_{x \to \infty} = \infty - \sqrt{x^2 - 2} = 0 = -\infty$

3.) $g(x) = \ln(4 - x^2)$

$\ln(4 - x^2) = 0$

$\log 4 - x^2 = 0$

$4 - x^2 = e^0$

$x^2 = -e^0 - 4$

$x^2 = -5$

funkcija je neparna X

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{14}{81} & -\frac{35}{27} \\ 0 & 1 & 0 & -\frac{2}{81} & -\frac{5}{27} \\ 0 & 0 & 1 & -\frac{11}{5} & -\frac{14}{3} \\ 0 & 0 & 0 & 1 & \frac{141}{197} \end{array}\right] \quad / \cdot \frac{14}{81} + I \;/ \cdot \frac{2}{81} + II \;/ \cdot \frac{11}{5} + III$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{231}{197} \\ 0 & 1 & 0 & 0 & -\frac{13}{81} \\ 0 & 0 & 1 & 0 & -\frac{308}{5} \\ 0 & 0 & 0 & 1 & \frac{141}{197} \end{array}\right]$$

$$\begin{bmatrix} x \\ y \\ z \\ a \end{bmatrix} = \begin{bmatrix} -1.17 \\ -0.16 \\ -61.6 \\ 0.71 \end{bmatrix}$$

PROVJERA ?

5.) $x + 2y - z + u = 3$

$2x - 5y - z + 2u = 3$

$3x - y - 2z + u = 2$

$x - y + 3z - 5u = 3$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & -5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \begin{array}{l} /\cdot(-2)+II \\ /\cdot(-3)+III \\ /\cdot(-1)+IV \end{array} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & -9 & 1 & -1 & -3 \\ 0 & -7 & 1 & -2 & -7 \\ 0 & -3 & -1 & -6 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & -9 & 1 & -1 & -3 \\ 0 & -7 & 1 & -2 & -7 \\ 0 & -3 & -1 & -6 & 0 \end{array}\right] /:(-9) \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & -\frac{1}{9} & \frac{1}{9} & \frac{3}{9} \\ 0 & -7 & 1 & -2 & -7 \\ 0 & -3 & -1 & -6 & 0 \end{array}\right] \begin{array}{l} /\cdot(-2)+I \\ /\cdot 7+III \\ /\cdot 3+IV \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -\frac{7}{9} & \frac{7}{9} & \frac{7}{3} \\ 0 & 1 & -\frac{1}{9} & \frac{1}{9} & \frac{3}{9} \\ 0 & 0 & \frac{2}{9} & -\frac{11}{9} & -\frac{14}{3} \\ 0 & 0 & -\frac{4}{3} & -\frac{17}{3} & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & -\frac{7}{9} & \frac{7}{9} & \frac{7}{3} \\ 0 & 1 & -\frac{1}{9} & \frac{1}{9} & \frac{3}{9} \\ 0 & 0 & \frac{2}{9} & -\frac{11}{9} & -\frac{14}{3} \\ 0 & 0 & -\frac{4}{3} & -\frac{17}{3} & 1 \end{array}\right] /:\frac{2}{9}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -\frac{7}{9} & \frac{7}{9} & \frac{7}{3} \\ 0 & 1 & -\frac{1}{9} & \frac{1}{9} & \frac{3}{9} \\ 0 & 0 & 1 & -\frac{11}{9} & -\frac{14}{3} \\ 0 & 0 & -\frac{4}{3} & -\frac{17}{3} & 1 \end{array}\right] \begin{array}{l} /\cdot\frac{7}{9}+I \\ /\cdot\frac{1}{9}+II \\ /\cdot\frac{4}{3}+IV \end{array}$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{14}{81} & -\frac{35}{27} \\ 0 & 1 & 0 & -\frac{2}{81} & -\frac{5}{27} \\ 0 & 0 & 1 & -\frac{11}{9} & -\frac{14}{3} \\ 0 & 0 & 0 & 1 & \frac{141}{197} \end{array}\right]$$

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IME I PREZIME: TOMISLAV LUCIN

BROJ INDEKSA: 0269085366

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno:

2. $f(x) = x - \sqrt{x^2-2}$ DOMENA

$x^2-2 \geq 0$

$x^2 \geq 2$

$x \geq \sqrt{2}$

$D_f \in [\sqrt{2}, +\infty\rangle$

EKSTREMI

$f'(x) = 0$

$f'(x) = 1 - \dfrac{1}{2\sqrt{x^2-2}} \cdot 2x = 1 - \dfrac{2x}{2\sqrt{x^2-2}} \quad /^2$

$= 1 - \dfrac{4x^2}{4(x^2-2)} = 1 - \dfrac{4x^2}{4x^2-8} = \dfrac{4x^2-8-4x^2}{4x^2-8} = \dfrac{-8}{4x^2-8}$

NULTOČKE

$f(x) = 0$

$x - \sqrt{x^2-2} = 0 \quad /^2$

$x^2 - x - 2 = 0$

$a=1, b=-1, c=-2$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{1 \pm \sqrt{1-4\cdot 1\cdot(-2)}}{2} = \dfrac{1 \pm \sqrt{1+8}}{2} = \dfrac{1\pm 3}{2}$

$x_1 = -1$

$x_2 = 2$

ASIMPTOTE

$\frac{-8}{4x^2-8} = 0$

$-8 \neq 0$

NEMA EKSTREMA

V.A.

$\lim_{x \to 2} x - \sqrt{x^2-2} = 2 - \sqrt{2}$

$\lim_{x \to -1} x - \sqrt{x^2-2} = -1 - (\sqrt{-1}) = -1 + \sqrt{-1}$

NEMA VERTIKALNIH ASIMPTOTA ✓

H.A.

$\lim_{x \to +\infty} x - \sqrt{x^2-2} \cdot \frac{x+\sqrt{x^2-2}}{x+\sqrt{x^2-2}} = \lim_{x \to +\infty} \frac{(x-\sqrt{x^2-2})^2}{x+\sqrt{x^2-2}}$

$\lim_{x \to +\infty} \frac{x-x^2-2 \;/:x^2}{x+\sqrt{x^2-2} \;/:x^2} = \lim_{x \to +\infty} \frac{\frac{x}{x^2} - \frac{x^2}{x^2} - \frac{2}{x^2}}{\frac{x}{x^2} + \sqrt{\frac{x^2}{x^4} - \frac{2}{x^4}}}$

$\lim_{x \to +\infty} \frac{\frac{1}{x} - 1 - \frac{2}{x^2}}{\frac{1}{x} + \sqrt{\frac{1}{x^2} - \frac{2}{x^4}}} = 0$

NEMA HORIZONTALNIH ASIMPTOTA ✗

3. $g(x) = \ln(4-x^2)$

$4-x^2 > 0$

$-x^2 > -4 \;/\cdot(-1)$

$x^2 < 4$

$x < 2$

$Df \in \langle -\infty, 2 \rangle$ ✗

$\emptyset$

4. $h(x)=\frac{x^2-2x-(2+1)}{x^2+1}$

DOMENA

$x^2+1\neq 0$ $\quad D_f=[\mathbb{R}]$

$x^2\neq -1$

$x\neq\sqrt{-1}$

$x\neq i$

EKSTREMI

$f'(x)=0$

↓

NULTOCKE

$\frac{x^2-2x-(2+1)}{x^2+1}=0$

$\frac{x^2-2x-2-1}{x^2+1}=0$

$\frac{x^2-2x-3}{x^2+1}=0 \;/\cdot(x^2+1)$

$x^2-2x-3=0$

$a=1,\ b=-2,\ c=-3$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{2\pm\sqrt{4-4\cdot1\cdot(-3)}}{2}=\frac{2\pm\sqrt{16}}{2}$

$x=\frac{2\pm4}{2}$ ↗ $x_1=-1$, $x_2=3$

$\frac{(x^2-2x-(2+1))'\cdot(x^2+1)-(x^2-2x-(2+1))\cdot(x^2+1)'}{(x^2+1)^2}=\frac{(2x-2)\cdot(x^2+1)-(x^2-2x-3)\cdot 2x}{(x^2+1)^2}$

$=\frac{(2x-2)-(x^2-2x-3)\cdot 2x}{x^2+1}=\frac{2x-2-2x^3+4x^2+6x}{x^2+1}=\frac{-2x^3+4x^2+8x-2}{x^2+1}$

$\frac{-2x^3+4x^2+8x-2}{x^2+1}=0\;/\cdot(x^2+1)$

$-2x^3+4x^2+8x-2=0$

GRAF ?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{H}2$

IME I PREZIME: TOMISLAV BOLONJA **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 0

2) $f(x) = x - \sqrt{x^2-2}$

$x = \sqrt{x^2-2}$

$x = x-2$

$x - x = -2$

$-x = -2 \mid (-1)$

$x = 2$

$D = R\{2\}$ ✗

5)
$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \begin{matrix} \\ \text{II}-2\cdot\text{I} \\ \\ \\ \end{matrix} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \begin{matrix} \\ \text{III}-3\cdot\text{I} \\ \\ \\ \end{matrix} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -1 & -2 & 0 & -6 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \begin{matrix} \\ \\ \\ \\ \text{IV}-\text{II} \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -1 & -2 & 0 & -6 \\ 0 & -1 & 3 & 0 & -9 \end{array}\right] \begin{matrix} \text{I}+2\cdot\text{II} \\ \\ \\ \\ \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -1 & -2 & 0 & -6 \\ 0 & -1 & 3 & 0 & -9 \end{array}\right] \begin{matrix} \\ \text{III}+\text{II} \\ \\ \\ \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & -2 & 0 & -9 \\ 0 & -1 & 3 & 0 & -9 \end{array}\right] \begin{matrix} \\ \\ \\ \\ \text{IV}+\text{II} \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & -2 & 0 & -9 \\ 0 & 0 & 4 & 1 & -12 \end{array}\right] \begin{matrix} \text{I}-\text{III} \\ \\ \\ \\ \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 18 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & -2 & 0 & -9 \\ 0 & 0 & 4 & 1 & -12 \end{array}\right] \begin{matrix} \\ \\ \\ \\ \text{IV}+2\cdot\text{III} \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 18 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & -2 & 0 & -9 \\ 0 & 0 & 0 & 1 & -30 \end{array}\right] \begin{matrix} \\ \\ \\ \\ \text{III}+3\cdot\text{II} \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 18 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 3 & 1 & 0 & -18 \\ 0 & 0 & 0 & 1 & -30 \end{array}\right] \begin{matrix} \text{I}+\text{III} \\ \\ \\ \\ \end{matrix} \left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 3 & 1 & 0 & -18 \\ 0 & 0 & 0 & 1 & -30 \end{array}\right] \begin{matrix} \\ \\ \cdot(-1) \\ \\ \end{matrix} \left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -3 & -1 & 0 & 18 \\ 0 & 0 & 0 & 1 & -30 \end{array}\right] \begin{matrix} \text{I}+\text{III} \\ \\ \\ \\ \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ & & & & \\ & & & & \\ & & & & \end{array}\right] \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 18 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -3 & -1 & 0 & 18 \\ 0 & 0 & 0 & 1 & -30 \end{array}\right] \begin{matrix} \\ \\ \text{III}\cdot\text{I} \\ \\ \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 18 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 & 324 \\ 0 & 0 & 0 & 1 & -30 \end{array}\right]$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: IVAN BILAVER

BROJ INDEKSA: 0143 mbs: 17-2-0078-2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 0

(1.) $z^4 - (4-i)^2 = 0$ ✗

(2.) $f(x) = x - \sqrt{x^2-2}$

$f'(x) = 1 - 2 \cdot 2$ ✗

$f'(x) = 1 - 4$

$f'(x) = -3$

(3.) $g(x) = \ln(4-x^2)$

$g'(x) = \dfrac{1}{\ln(4-x^2)} + \ln(2x+1)$

$g'(x) = \dfrac{1}{\frac{1}{\ln}}(x-2x) + \ln(2x+1)$

$g'(x) = \dfrac{1}{\frac{\ln}{1}}(1-2x) + \ln(2x+1)$

0

IVAN BILAVER

(5) $x+2y-z+v=3$ | $\cdot(-2)$

$2x+5y-z+2v=3$

$3x-y-2z+v=2$

$x-y+3z-5v=3$

$-2x+(-4)y+2z-2v=-6$ | $+2$

$2x+5y-z+2v=3$

$3x-y-2z+v=2$

$x-y+3z-5v=3$

$0+2y+4z-0=-4$

$2x+5y-z+2v=3$ | $\cdot(-5)$

$3x-y-2z+v=2$ | $\cdot(+11)$

$x-y+3z-5v=3$ | $\cdot(+11)$

$x-y+3z-5v=3$ | $+1$

$0+2y+4z-0=-4$

$-3x+0+5z-3v=-2$

$4x-0-1z+2v=3$

$2x-0+4z-4v=4$ | $\cdot(-1)$

$2x-0+4z-4v=4$

$0+2y+4z-0=-4$

$-3x+0+5z-3v=-2$

$4x-0-1z+2v=3$

$-2x-0-4z+4v=-4$ | $\cdot 3$

$2x-0+4z-4v=4$ | $\cdot 4$

$0+2y+4z-0=-4$ | $\cdot 3$

$-3x+0+5z-3v=-2$ | $\cdot 2$

$4x-0-1z+2v=3$ | $\cdot 3$

$0x-0-0+0=0$

$2x-0+4z-4v=4$ | $\cdot 3$

$0+0y+0z-0=0$

$0+0+0z-0v=0$

$0x-0-0+0=0$

$0x-0-0+0=0$

$0x-0+0z-0=0$

0

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: MARKO MILOŠ **BROJ INDEKSA:** 17-1-0170-2013

Prof. dr. sc. N. Uglešić

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 2 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno:

(5)

$x+2y-z+u=3$

$2x+5y-z+2u=3$

$3x-y-2z+u=2$

$x-y+3z-5u=3$

$$\left|\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right| \begin{array}{l} / \cdot(-2) \\ / \cdot(-3) \\ / \cdot(-1) \end{array} \sim \left|\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -7 & 1 & -2 & -7 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right| \begin{array}{l} + \\ / \cdot(-2) \\ / \cdot(-7) \\ / \cdot(-3) \\ + \\ + \end{array}$$

$$\sim \left|\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -28 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right| /:8 \sim \left|\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 7 & -6 & -9 \end{array}\right| \begin{array}{l} + \\ + \\ / \cdot(3) \\ / \cdot(-1) \\ / \cdot(-7) \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & -\frac{3}{2} \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 0 & -\frac{17}{4} & \frac{31}{2} \end{array}\right| \quad /:\left(-\frac{17}{4}\right)$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & -\frac{3}{2} \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 0 & 1 & -\frac{62}{17} \end{array}\right| \quad \begin{array}{l} \leftarrow \\ \leftarrow \\ + \\ + \end{array} \quad \cdot\frac{1}{4} \quad / \cdot\left(-\frac{1}{4}\right)$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{35}{34} \\ 0 & 1 & 0 & 0 & \frac{33}{34} \\ 0 & 0 & 1 & 0 & \frac{75}{17} \\ 0 & 0 & 0 & 1 & -\frac{62}{17} \end{array}\right|$$

$\frac{5}{11} + 2 \cdot \frac{33}{34} + \frac{75}{17} - \frac{62}{17} = 3$

$\left(\frac{35}{34}\right) + 5 \cdot \frac{33}{34} - \frac{75}{17} + 2 \cdot \left(\frac{62}{17}\right) 3$

$\cdot\left(\frac{35}{34}\right) - \frac{33}{34} - 2 \cdot \left(-\frac{75}{17}\right) - \frac{62}{17}$

$\frac{5}{4} - \frac{33}{34} + 3 \cdot \frac{75}{17} - 5 \cdot \left(-\frac{65}{17}\right) = \frac{-35-33+450+650}{34} = \frac{832}{34} \neq 3$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & -\frac{3}{2} \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{7}{2} \\ 0 & 0 & 0 & -\frac{17}{4} & \frac{31}{2} \end{array}\right|$$

$$x + 2y - z + u = 3$$
$$2x + 5y - z + 2u = 3$$
$$3x - y - 2z + u = 2$$
$$x - y + 3z - 5u = 3$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{H}2$

IME I PREZIME: JURE BRKLJAČIĆ **BROJ INDEKSA:** 17-2-0224-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-2}$. 5+15+5
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5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 3 \\ 2x + 5y - z + 2u &= 3 \\ 3x - y - 2z + u &= 2 \\ x - y + 3z - 5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 0

(2.) $f(x) = x - \sqrt{x^2-2}$

$f(x) = 1 - \sqrt{x}$

(4.) $h(x) = \dfrac{x^2-2x-(2+1)}{x^2+1}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: Josip Rašov **BROJ INDEKSA:** 0269093396

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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6. Izračunati i provjeriti uvrštavanjem: $\lim\limits_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: 0

2) $f(x) = x - \sqrt{x^2-2}$

$f'(x) = x \cdot x^{1-1} - x \cdot x^{2-1} - 0$ ✗

$f'(x) = x \cdot x^0 - x \cdot x - 0$

$f'(x) = x - x^2$

$f''(x) = x \cdot x^{1-1} - x \cdot x^{2-1}$

$f''(x) = x \cdot x^0 - x \cdot x$

$\boxed{f''(x) = x^0 - x^2}$

0

5)

$$\left[\begin{array}{cccc|c} 3 & 1 & 2 & -1 & 1 & 6 \\ 3 & 2 & 5 & -1 & 2 & 11 \\ 2 & 3 & -1 & -2 & 1 & 3 \\ 3 & 1 & -1 & 3 & -5 & -1 \end{array}\right]$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: DIKO KOLEGA **BROJ INDEKSA:** 17-1-0248-2016

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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Ukupno: ∅

5. $x+2y-z+u=3$

$2x+5y-z+2u=3$

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$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] R_2-2R_1 \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] +3R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -4 & -5 & -2 & 1 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] -R_4 \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -4 & -5 & -2 & 1 \\ 0 & -2 & 2 & -6 & 2 \end{array}\right] \sim$$

∅

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IME I PREZIME: Šime Labus **BROJ INDEKSA:** 0269093263

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^2 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
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6. Izračunati i provjeriti uvrštavanjem: $\lim_{x\to\infty} \dfrac{e^x}{x}$. 5

Ukupno: N/D

NISAM ISPRAVLJAO JER JER SAM VIDIO DA JE STUDENT U DEPU IMA MOBITEL ZA KOJI SAM RANIJE NAGLASIO DA SE NE SMIJE UZA SE.

(2)

$f(x) = x - \sqrt{x^2-2}$

$x^2 - 2 \geq 0$

$(x-\sqrt{2})(x+\sqrt{2}) \geq 0$

$D: \langle -\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty \rangle$

$f'(x) = 1 - \dfrac{1}{2\sqrt{x^2-2}} \cdot (x^2-2)'$

$f'(x) = 1 - \dfrac{2x}{2\cdot\sqrt{x^2-2}}$

$f'(x) = 1 - \dfrac{x}{\sqrt{x^2-2}}$

$f''(x) = 0 - \left(\dfrac{\sqrt{x^2-2} - x \cdot \frac{1}{2\sqrt{x^2-2}} \cdot 2x}{x^2-2}\right)$

$f''(x) = -\dfrac{\sqrt{x^2-2} - \frac{x^2}{\sqrt{x^2-2}}}{x^2-2}$

$f''(x) = -\dfrac{x^2-2-x^2}{\sqrt{x^2-2}\cdot(x^2-2)}$

$f''(x) = \dfrac{2}{\sqrt{x^2-2}\cdot(x^2-2)}$

ASIMPTOTE:

$\lim_{x\to\infty} x - \sqrt{x^2-2} \cdot \dfrac{x+\sqrt{x^2-2}}{x+\sqrt{x^2-2}}$

$= \lim_{x\to\infty} \dfrac{x^2-x^2+2}{x+\sqrt{x^2+2}}$

$y = 0$ je H.A → nema U.A.

Nema K.A. jer nema otvorenih intervala u domeni!

⑥ $\lim_{x\to\infty} \frac{e^x}{x} = \lim_{x\to\infty} \frac{e^x}{1} = \frac{\infty}{1} = \frac{\infty}{\infty}$

↓

L'Hospitalovo pravilo $\left(\frac{\infty}{\infty}\right)$

$\frac{e^{100}}{100} = 2.688 \cdot 10^{41} \approx \infty$

①
$z^4 - (4 - i^2) = 0$

$z^4 = (4 - i)^2$

$z^4 = 16 - 8i + i^2$, $i^2 = -1$

$z^4 = 15 - 8i$

$z = \sqrt[2]{15 - 8i}$

$w = 15 - 8i$

$|w| = \sqrt{15^2 + (-8)^2} = 17$

$\operatorname{tg} w = \frac{-8}{15} \rightarrow w = -28° + 360° = 332°$

$w = 17(\cos 332° + i \sin 332°)$

$z_1 = \sqrt[4]{17}(\cos 83° + i \sin 83°) = 0.249540118 + 0.968364461 1i = 1.217904579i$

$z_2 = \sqrt[4]{17}(\cos 173° + i \sin 173°) = -0.9775269404 - 0.2108105329i = -1.1885374733i$

$z_3 = \sqrt[4]{17}($

$z_4 = \sqrt[4]{17}($

④