MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ I1

IME I PREZIME: KREŠIMIR ANTOLOVIĆ **BROJ INDEKSA:** 17-2-0102-2014 0111117055

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + x} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 80

(1.)

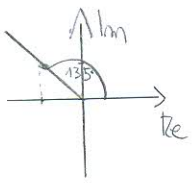
$z^3 + \overline{1+i} = 0$

$z^3 + 1 - i = 0$

$z^3 = -1 + i$

$z = \sqrt[3]{-1+i}$

$|w| = r = \sqrt{x^2+y^2} = \sqrt{(-1)^2+1^2} = \sqrt{2}$



$\operatorname{tg}\varphi = \frac{y}{x} = \frac{1}{-1} = -1$

$\arctan(-1) = -45°$

$\varphi = -45° + 180° = 135°$

$\varphi = \frac{3}{4}\pi$

$k=0$

$w_1 = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{\frac{3}{4}\pi + 0\cdot 2\pi}{3} + i\sin\frac{\frac{3}{4}\pi + 0\cdot 2\pi}{3}\right) = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{3\pi}{12} + i\sin\frac{3}{12}\pi\right)$

$= 0.79370 + 0.79370\,i$

$k=1$

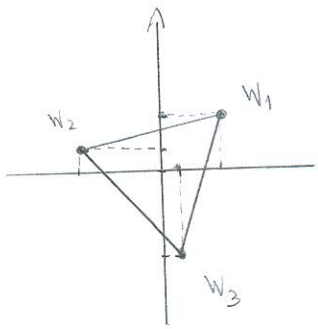
$w_2 = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{\frac{3}{4}\pi + 1\cdot 2\pi}{3} + i\sin\frac{\frac{3}{4}\pi + 1\cdot 2\pi}{3}\right) = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{11}{12}\pi + i\sin\frac{11}{12}\pi\right)$

$= -1.08422 + 0.29052\,i$

$k=2$

$w_3 = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{\frac{3}{4}\pi + 2\cdot 2\pi}{3} + i\sin\frac{\frac{3}{4}\pi + 1\cdot 2\pi}{3}\right) = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{19}{12}\pi + i\sin\frac{19}{12}\pi\right)$

$= 0.29052 - 1.08422\,i$



✓

2.

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{matrix} \\ /+1R\cdot(-2) \\ /+1R\cdot(-3) \\ /+1R\cdot(-1) \end{matrix} \approx \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right] \begin{matrix} /+2R\cdot(-2) \\ \\ /+2R\cdot 7 \\ /+2R\cdot 3 \end{matrix} \approx \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] /:8$$

$$\approx \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{2}{8} & 1 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \begin{matrix} /+3R\cdot 3 \\ /+3R\cdot(-1) \\ \\ /+3R\cdot(-7) \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{2}{8} & -1 \\ 0 & 0 & 1 & -\frac{2}{8} & 1 \\ 0 & 0 & 0 & -\frac{31}{4} & 0 \end{array}\right] /:\left(-\frac{31}{4}\right) \approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{2}{8} & -1 \\ 0 & 0 & 1 & -\frac{2}{8} & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{matrix} +4R\cdot\left(-\frac{1}{4}\right) \\ /+4R\cdot\left(-\frac{2}{8}\right) \\ /+4R\cdot\frac{2}{8} \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{aligned} x+2y-z+u &= 1 \\ 2x+5y-z+2u &= -2 \\ 3x-y-2z+u &= 5 \\ x-y+3z-5u &= 6 \end{aligned}$$

$$\begin{aligned} 2+2\cdot(-1)-1+0 &= -1 \\ 4+5\cdot(-1)-1+0 &= -2 \\ 6-(-1)-2+0 &= 5 \\ 2-(-1)+3-0 &= 6 \end{aligned}$$

$$\begin{aligned} -1 &= -1\ \surd \\ -2 &= -2\ \surd \\ 5 &= 5\ \surd \\ 6 &= 6\ \surd \end{aligned}$$

✓

$$\begin{aligned} x &= 2 \\ y &= -1 \\ z &= 1 \\ u &= 0 \end{aligned}$$

✓

$$\left[\begin{array}{cccc} 1 & 2 & -1 & 1 \\ 2 & 5 & -1 & 2 \\ 3 & -1 & -2 & 1 \\ 1 & -1 & 3 & -5 \end{array}\right] \left[\begin{array}{c} x \\ y \\ z \\ u \end{array}\right] = \left[\begin{array}{c} 2 \\ -1 \\ 1 \\ 0 \end{array}\right]$$

3.

$g(x)=\sqrt{x^2+x}-x$

$\sqrt{x^2+x} \geq 0$

$\sqrt{x^2+x} \geq 0 \ /^2$

$x^2+x \geq 0$

$x_1=-1 \quad x_2=0$

$D(g)=\mathbb{R}$

✗

H.A.

$y=\lim\limits_{x\to\infty} \sqrt{x^2+x}-x =$

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5.

$$f(x)=\ln(\sin(4x-2))$$

$$f'(x)=\frac{1}{\sin(4x-2)}\cdot\cos(4x-2)\cdot 4=\frac{1}{\sin(4x-2)}\cdot\frac{\cos(4x-2)}{1}\cdot 4=\frac{\cos(4x-2)}{\sin(4x-2)}\cdot 4=\frac{4\cos(4x-2)}{\sin(4x-2)}$$ ✓

6.

$$\left[\begin{array}{cccc|c}2&3&0&-2&0\\0&1&4&-2&1\\1&1&0&4&-2\\0&1&0&2&4\end{array}\right]\approx\left[\begin{array}{cccc|c}1&1&0&4&-2\\0&1&4&-2&1\\2&3&0&-2&0\\0&1&0&2&4\end{array}\right]/+1R\cdot(-2)\approx\left[\begin{array}{cccc|c}1&1&0&4&-2\\0&1&4&-2&1\\0&1&0&-10&4\\0&1&0&2&4\end{array}\right]/+2R\cdot(-1)\ /+2R\cdot(-1)\ /+2R\cdot(-1)\approx\left[\begin{array}{cccc|c}1&0&-4&6&-3\\0&1&4&-2&1\\0&0&-4&-8&3\\0&0&-4&4&3\end{array}\right]/:(-4)$$

$$\approx\left[\begin{array}{cccc|c}1&0&-4&6&-3\\0&1&4&-2&1\\0&0&1&2&-\frac{3}{4}\\0&0&-4&4&3\end{array}\right]/+3R\cdot 4\ /+3R\cdot(-4)\ /+3R\cdot 4\approx\left[\begin{array}{cccc|c}1&0&0&14&-6\\0&1&0&-10&4\\0&0&1&2&-\frac{3}{4}\\0&0&0&12&0\end{array}\right]/:12\approx\left[\begin{array}{cccc|c}1&0&0&14&-6\\0&1&0&-10&4\\0&0&1&2&-\frac{3}{4}\\0&0&0&1&0\end{array}\right]/+4R\cdot(-14)\ /+4R\cdot 10\ /+4R\cdot(-2)\approx\left[\begin{array}{cccc|c}1&0&0&0&-6\\0&1&0&0&4\\0&0&1&0&-\frac{3}{4}\\0&0&0&1&0\end{array}\right]$$

$r=4$ ✓

$x_1=-6$
$x_2=4$
$x_3=-\frac{3}{4}$
$x_4=0$

4.

$$h(x)=\frac{x^2-4}{x^2+2}$$

$x^2+2\neq 0$
$x^2\neq -2$ //

$D(h)=\mathbb{R}$

NULTOČKE:
$x^2-4=0$
$x^2=4/\sqrt{}$
$x_1=2\quad x_2=-2$

$$h'(x)=\frac{2x\cdot(x^2+2)-2x\cdot(x^2-4)}{(x^2+2)^2}$$

$$=\frac{2x^3+4x-2x^3+8x}{(x^2+2)^2}$$

$$h'(x)=\frac{12x}{(x^2+2)^2}$$

$\frac{12x}{(x^2+2)^2}=0\ /\ (x^2+2)^2$
$12x=0\ /:12$
$x=0$

$h(0)=\frac{0-4}{0+2}=-\frac{4}{2}=-2$

EKSTREM $\Rightarrow (0,-2)$

H.A.

$$\lim_{x\to\infty}\frac{x^2-4}{x^2+2}\ /:(x^2)$$

$$\lim_{x\to\infty}\frac{1-\frac{4}{x^2}}{1+\frac{2}{x^2}}=1$$ ($\frac{4}{x^2}\to 0$, $\frac{2}{x^2}\to 0$)

$y=1$ je HORIZONTALNA ASIMPTOTA

NEMA KOSIH ASIMPTOTA

$D(h): \mathbb{R}$

NULTOČKE:

$x_1 = 2 \quad (2,0)$

$x_2 = -2 \quad (-2,0)$

EKSTREMI:

$(0,-2)$ m

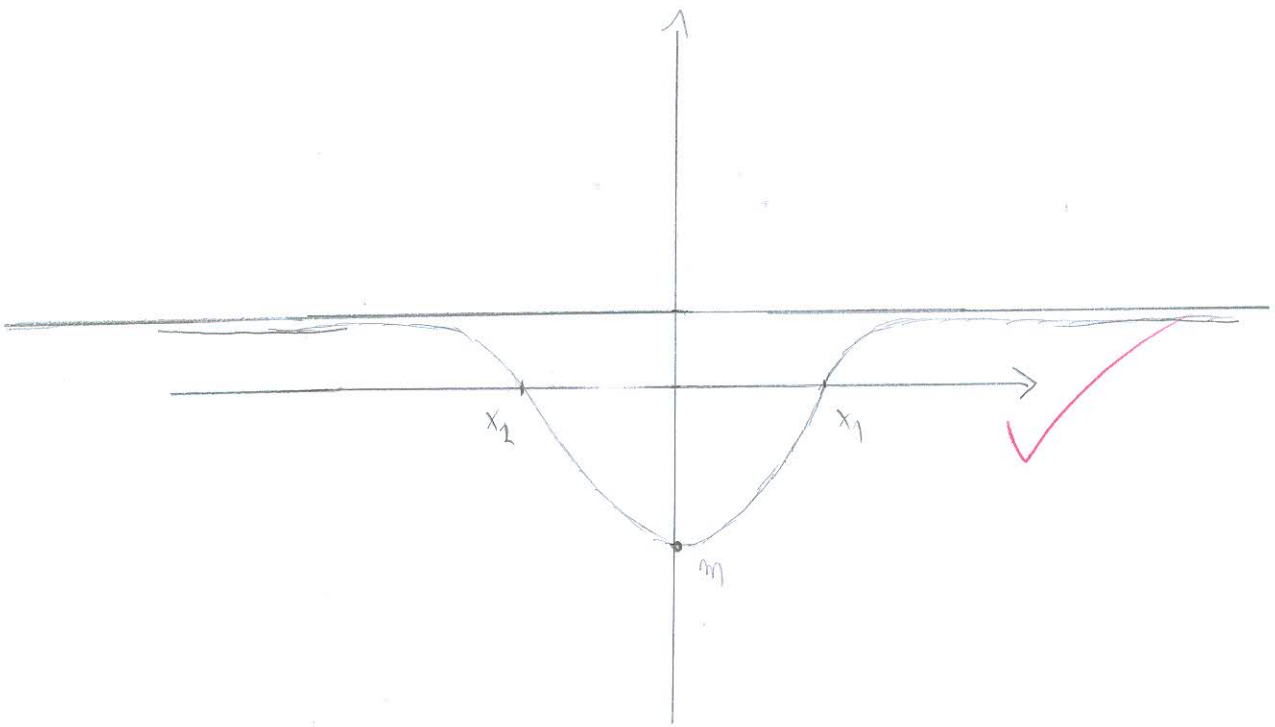
	$-\infty$		0		∞
$h'(x)$		−		+	
h		↘		↗	

$$h''(x) = \frac{12 \cdot (x^2+4x+4) - 12x \cdot (2x+4)}{(x^2+4x+4)^2}$$

$$= \frac{12x^2 + 48x + 48 - 24x^2 - 48x}{(x^2+4x+4)^2}$$

$$h''(x) = \frac{-12x^2 + 48}{(x^2+4x+4)^2}$$

$h(x) = h(-x) \Rightarrow$ FUNKCIJA JE PARNA (SIMETRIČNA S OBZIROM NA y OS)



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BROJ INDEKSA: 17-2-0219-2012

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Broj ↓ bodova

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Ukupno: 62

1.

$z^3 + 1 - i = 0$

$z^3 = -1 + i$

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2} = 1{,}41$

$\varphi = \pi + \operatorname{arctg}\frac{y}{x} = \pi + \operatorname{arctg}\frac{1}{-1}$

$k=0 \quad w_1 = 1{,}12\left(\cos\frac{\frac{3\pi}{4}}{3} + i\sin\frac{\frac{3\pi}{4}}{3}\right) = 1{,}12\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 0{,}79$

$k=1 \quad w_2 = 1{,}12\left(\cos\frac{\frac{3\pi}{4} + 2\pi}{3} + i\sin\frac{\frac{3\pi}{4} + 2\pi}{3}\right) = 1{,}12(-0{,}97 + 0{,}26i)$

$= -1{,}09 + 0{,}29i$

$k=2 \quad w_3 = 1{,}12\left(\cos\frac{\frac{3\pi}{4} + 4\pi}{3} + i\sin\frac{\frac{3\pi}{4} + 4\pi}{3}\right) = 1{,}12(0{,}26 - 0{,}97i) =$

$= 0{,}29 - 1{,}09i$