MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ I1

IME I PREZIME: KREŠIMIR ANTOLOVIĆ **BROJ INDEKSA:** 17-2-0102-2014 0111117055

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + x} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 80

(1.)

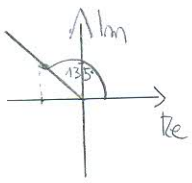
$z^3 + \overline{1+i} = 0$

$z^3 + 1 - i = 0$

$z^3 = -1 + i$

$z = \sqrt[3]{-1+i}$

$|w| = r = \sqrt{x^2+y^2} = \sqrt{(-1)^2+1^2} = \sqrt{2}$



$\operatorname{tg}\varphi = \frac{y}{x} = \frac{1}{-1} = -1$

$\arctan(-1) = -45°$

$\varphi = -45° + 180° = 135°$

$\varphi = \frac{3}{4}\pi$

$k=0$

$w_1 = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{\frac{3}{4}\pi + 0\cdot 2\pi}{3} + i\sin\frac{\frac{3}{4}\pi + 0\cdot 2\pi}{3}\right) = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{3\pi}{12} + i\sin\frac{3}{12}\pi\right)$

$= 0.79370 + 0.79370\,i$

$k=1$

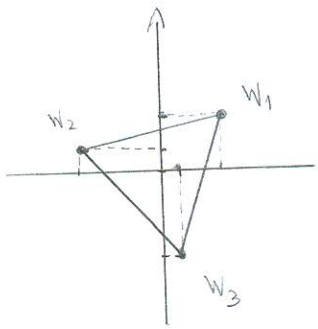
$w_2 = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{\frac{3}{4}\pi + 1\cdot 2\pi}{3} + i\sin\frac{\frac{3}{4}\pi + 1\cdot 2\pi}{3}\right) = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{11}{12}\pi + i\sin\frac{11}{12}\pi\right)$

$= -1.08422 + 0.29052\,i$

$k=2$

$w_3 = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{\frac{3}{4}\pi + 2\cdot 2\pi}{3} + i\sin\frac{\frac{3}{4}\pi + 1\cdot 2\pi}{3}\right) = \sqrt[3]{\sqrt{2}} \cdot \left(\cos\frac{19}{12}\pi + i\sin\frac{19}{12}\pi\right)$

$= 0.29052 - 1.08422\,i$



✓

2.

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{matrix} \\ /+1R\cdot(-2) \\ /+1R\cdot(-3) \\ /+1R\cdot(-1) \end{matrix} \approx \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right] \begin{matrix} /+2R\cdot(-2) \\ \\ /+2R\cdot 7 \\ /+2R\cdot 3 \end{matrix} \approx \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] /:8$$

$$\approx \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{2}{8} & 1 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \begin{matrix} /+3R\cdot 3 \\ /+3R\cdot(-1) \\ \\ /+3R\cdot(-7) \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{2}{8} & -1 \\ 0 & 0 & 1 & -\frac{2}{8} & 1 \\ 0 & 0 & 0 & -\frac{31}{4} & 0 \end{array}\right] /:\left(-\frac{31}{4}\right) \approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{2}{8} & -1 \\ 0 & 0 & 1 & -\frac{2}{8} & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{matrix} +4R\cdot\left(-\frac{1}{4}\right) \\ /+4R\cdot\left(-\frac{2}{8}\right) \\ /+4R\cdot\frac{2}{8} \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{aligned} x+2y-z+u &= 1 \\ 2x+5y-z+2u &= -2 \\ 3x-y-2z+u &= 5 \\ x-y+3z-5u &= 6 \end{aligned}$$

$$\begin{aligned} 2+2\cdot(-1)-1+0 &= -1 \\ 4+5\cdot(-1)-1+0 &= -2 \\ 6-(-1)-2+0 &= 5 \\ 2-(-1)+3-0 &= 6 \end{aligned}$$

$$\begin{aligned} -1 &= -1\ \surd \\ -2 &= -2\ \surd \\ 5 &= 5\ \surd \\ 6 &= 6\ \surd \end{aligned}$$

✓

$$\begin{aligned} x &= 2 \\ y &= -1 \\ z &= 1 \\ u &= 0 \end{aligned}$$

✓

$$\left[\begin{array}{cccc} 1 & 2 & -1 & 1 \\ 2 & 5 & -1 & 2 \\ 3 & -1 & -2 & 1 \\ 1 & -1 & 3 & -5 \end{array}\right] \left[\begin{array}{c} x \\ y \\ z \\ u \end{array}\right] = \left[\begin{array}{c} 2 \\ -1 \\ 1 \\ 0 \end{array}\right]$$

3.

$g(x)=\sqrt{x^2+x}-x$

$\sqrt{x^2+x} \geq 0$

$\sqrt{x^2+x} \geq 0 \ /^2$

$x^2+x \geq 0$

$x_1=-1 \quad x_2=0$

$D(g)=\mathbb{R}$

✗

H.A.

$y=\lim\limits_{x\to\infty} \sqrt{x^2+x}-x =$

KRESIMIR ANTOLOVIC 0111117055

5.

$$f(x)=\ln(\sin(4x-2))$$

$$f'(x)=\frac{1}{\sin(4x-2)}\cdot\cos(4x-2)\cdot 4=\frac{1}{\sin(4x-2)}\cdot\frac{\cos(4x-2)}{1}\cdot 4=\frac{\cos(4x-2)}{\sin(4x-2)}\cdot 4=\frac{4\cos(4x-2)}{\sin(4x-2)}$$ ✓

6.

$$\left[\begin{array}{cccc|c}2&3&0&-2&0\\0&1&4&-2&1\\1&1&0&4&-2\\0&1&0&2&4\end{array}\right]\approx\left[\begin{array}{cccc|c}1&1&0&4&-2\\0&1&4&-2&1\\2&3&0&-2&0\\0&1&0&2&4\end{array}\right]/+1R\cdot(-2)\approx\left[\begin{array}{cccc|c}1&1&0&4&-2\\0&1&4&-2&1\\0&1&0&-10&4\\0&1&0&2&4\end{array}\right]/+2R\cdot(-1)\ /+2R\cdot(-1)\ /+2R\cdot(-1)\approx\left[\begin{array}{cccc|c}1&0&-4&6&-3\\0&1&4&-2&1\\0&0&-4&-8&3\\0&0&-4&4&3\end{array}\right]/:(-4)$$

$$\approx\left[\begin{array}{cccc|c}1&0&-4&6&-3\\0&1&4&-2&1\\0&0&1&2&-\frac{3}{4}\\0&0&-4&4&3\end{array}\right]/+3R\cdot 4\ /+3R\cdot(-4)\ /+3R\cdot 4\approx\left[\begin{array}{cccc|c}1&0&0&14&-6\\0&1&0&-10&4\\0&0&1&2&-\frac{3}{4}\\0&0&0&12&0\end{array}\right]/:12\approx\left[\begin{array}{cccc|c}1&0&0&14&-6\\0&1&0&-10&4\\0&0&1&2&-\frac{3}{4}\\0&0&0&1&0\end{array}\right]/+4R\cdot(-14)\ /+4R\cdot 10\ /+4R\cdot(-2)\approx\left[\begin{array}{cccc|c}1&0&0&0&-6\\0&1&0&0&4\\0&0&1&0&-\frac{3}{4}\\0&0&0&1&0\end{array}\right]$$

$r=4$ ✓

$x_1=-6$
$x_2=4$
$x_3=-\frac{3}{4}$
$x_4=0$

4.

$$h(x)=\frac{x^2-4}{x^2+2}$$

$x^2+2\neq 0$
$x^2\neq -2$ //

$D(h)=\mathbb{R}$

NULTOČKE:
$x^2-4=0$
$x^2=4/\sqrt{}$
$x_1=2\quad x_2=-2$

$$h'(x)=\frac{2x\cdot(x^2+2)-2x\cdot(x^2-4)}{(x^2+2)^2}$$

$$=\frac{2x^3+4x-2x^3+8x}{(x^2+2)^2}$$

$$h'(x)=\frac{12x}{(x^2+2)^2}$$

$\frac{12x}{(x^2+2)^2}=0\ /\ (x^2+2)^2$
$12x=0\ /:12$
$x=0$

$h(0)=\frac{0-4}{0+2}=-\frac{4}{2}=-2$

EKSTREM $\Rightarrow (0,-2)$

H.A.

$$\lim_{x\to\infty}\frac{x^2-4}{x^2+2}\ /:(x^2)$$

$$\lim_{x\to\infty}\frac{1-\frac{4}{x^2}}{1+\frac{2}{x^2}}=1$$ ($\frac{4}{x^2}\to 0$, $\frac{2}{x^2}\to 0$)

$y=1$ je HORIZONTALNA ASIMPTOTA

NEMA KOSIH ASIMPTOTA

$D(h): \mathbb{R}$

NULTOČKE:

$x_1 = 2 \quad (2,0)$

$x_2 = -2 \quad (-2,0)$

EKSTREMI:

$(0,-2)$ m

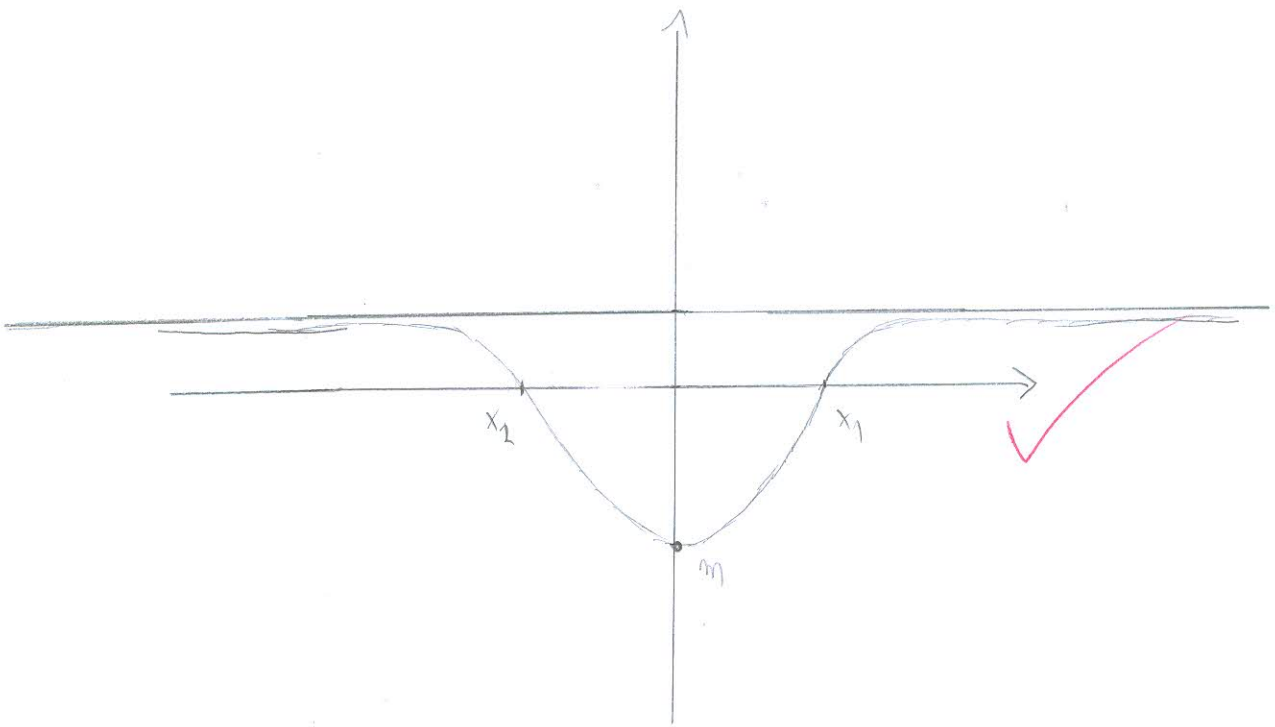
	$-\infty$		0		∞
$h'(x)$		−		+	
h		↘		↗	

$$h''(x) = \frac{12 \cdot (x^2+4x+4) - 12x \cdot (2x+4)}{(x^2+4x+4)^2}$$

$$= \frac{12x^2 + 48x + 48 - 24x^2 - 48x}{(x^2+4x+4)^2}$$

$$h''(x) = \frac{-12x^2 + 48}{(x^2+4x+4)^2}$$

$h(x) = h(-x) \Rightarrow$ FUNKCIJA JE PARNA (SIMETRIČNA S OBZIROM NA y OS)



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{I}1$

IME I PREZIME: LUKA STIPANOV

BROJ INDEKSA: 17-2-0219-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

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Ukupno: 62

1.

$z^3 + 1 - i = 0$

$z^3 = -1 + i$

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2} = 1{,}41$

$\varphi = \pi + \operatorname{arctg}\frac{y}{x} = \pi + \operatorname{arctg}\frac{1}{-1}$

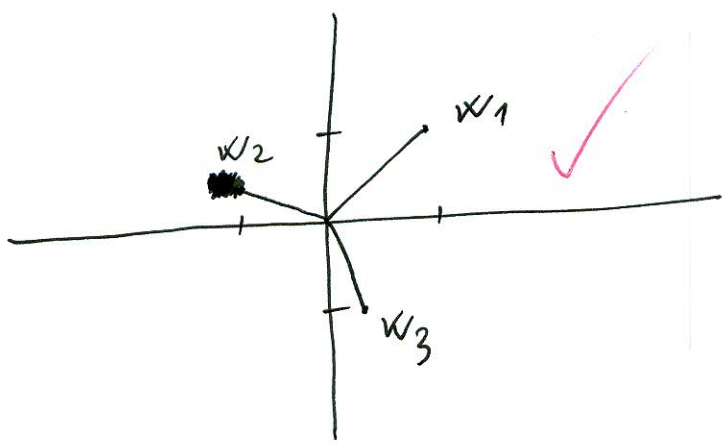
$k=0 \quad w_1 = 1{,}12\left(\cos\frac{\frac{3\pi}{4}}{3} + i\sin\frac{\frac{3\pi}{4}}{3}\right) = 1{,}12\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 0{,}79$

$k=1 \quad w_2 = 1{,}12\left(\cos\frac{\frac{3\pi}{4} + 2\pi}{3} + i\sin\frac{\frac{3\pi}{4} + 2\pi}{3}\right) = 1{,}12(-0{,}97 + 0{,}26i)$

$= -1{,}09 + 0{,}29i$

$k=2 \quad w_3 = 1{,}12\left(\cos\frac{\frac{3\pi}{4} + 4\pi}{3} + i\sin\frac{\frac{3\pi}{4} + 4\pi}{3}\right) = 1{,}12(0{,}26 - 0{,}97i) =$

$= 0{,}29 - 1{,}09i$



5. $f(x) = \ln(\sin(4x-2))$

$$f'(x) = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$$

$$= \frac{4\cos(4x-2)}{\sin(4x-2)}$$ ✓

3. $g(x) = \sqrt{x^2+x} - x$

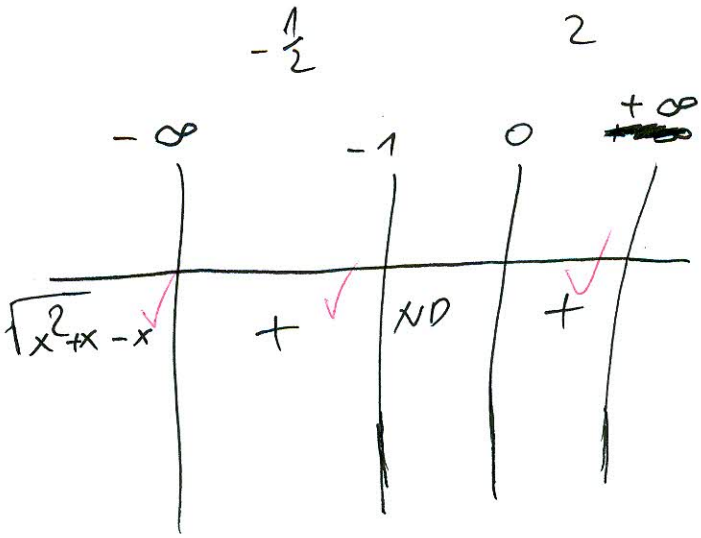
$x^2 + x \geq 0$

$x^2 + x = 0$

$x(x+1) = 0$

$x_1 = 0$

$x_2 = -1$



$\lim_{x \to 0_1} \sqrt{x^2+x} - x = \sqrt{0_1^2 + 0_1} - 0_1 = 0$ NEMA V.A.

$\lim_{x \to -1} \sqrt{x^2+x} - x = \sqrt{1-1} - 1 = 1$ NEMA V.A.

$\lim_{x \to +\infty} \sqrt{x^2+x} - x = +\infty + \infty = +\infty$ NEMA ✗

$\lim_{x \to -\infty} \frac{\sqrt{x^2+x} - x}{x} = \lim_{x \to +\infty} \frac{\sqrt{x^2+x} - x}{x}$ ✗

6.

$$\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \overset{\frac{1}{2}R_1}{\sim} \begin{bmatrix} 1 & \frac{3}{2} & 0 & -1 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} R3-R1$$

$$\begin{bmatrix} 1 & \frac{3}{2} & 0 & -1 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & -\frac{1}{2} & 0 & 5 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \begin{matrix} R_1-\frac{3}{2}R2 \\ \\ R3+\frac{1}{2}R_2 \\ R4-R2 \end{matrix} \sim \begin{bmatrix} 1 & 0 & -6 & 2 & -\frac{3}{2} \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 2 & 4 & -\frac{3}{2} \\ 0 & 0 & -4 & 4 & 3 \end{bmatrix} \cdot \frac{1}{2} R_3$$

$$\sim \begin{bmatrix} 1 & 0 & -6 & 2 & -\frac{3}{2} \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 1 & 2 & -\frac{3}{4} \\ 0 & 0 & -4 & 4 & 3 \end{bmatrix} \begin{matrix} R_1+6R3 \\ R_2-4R3 \\ \\ R4+4R3 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 14 & -6 \\ 0 & 1 & 0 & -10 & 2 \\ 0 & 0 & 1 & 2 & -\frac{3}{4} \\ 0 & 0 & 0 & -4 & 0 \end{bmatrix} \cdot \left(\frac{-1}{4}\right) R4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 14 & -6 \\ 0 & 1 & 0 & -10 & -2 \\ 0 & 0 & 1 & 2 & -\frac{3}{4} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} R_1-14R4 \\ R_2+10R4 \\ R3-2R4 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & -6 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & -\frac{3}{4} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

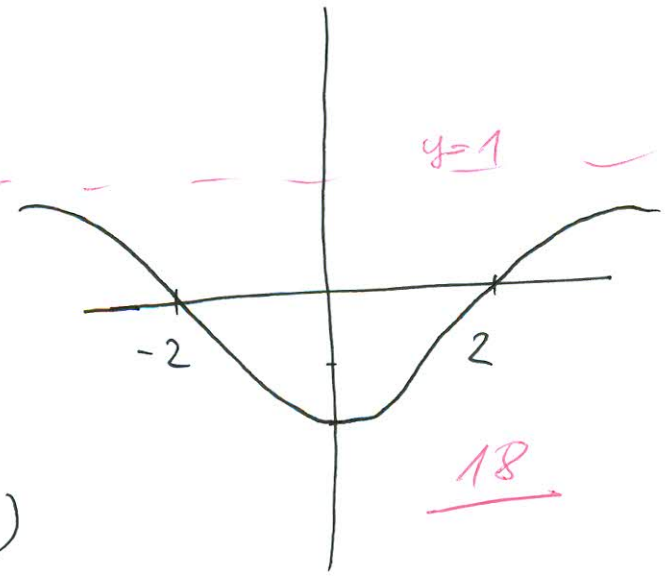
$V(M) = 4$

Luka Stanić

4. $h(x) = \frac{x^2-4}{x^2+2}$

NEMA V. A

PARNOST



18

$x^2+2 \neq 0$

$x^2 \neq -2$

$D_f = R$

$f(x) = \frac{(-x)^2-4}{(-x)^2+2}$

$\frac{x^2-4}{x^2+2} = f(x)$

$x^2-4=0$

$x^2=4$

$x = \pm 2$

FUNKCIJA JE PARNA

NIJE PERIODIČNA

$f(0) = \frac{-4}{2} = -2$

$\lim_{x \to +\infty} \frac{x^2-4 \,/\cdot x^2}{x^2+2 \,/\cdot x^2} = 1$

$\lim_{x \to +\infty} \frac{x^2-4}{x^2+2} = \left\{ \begin{matrix} x \to -x \\ -\infty \to +\infty \end{matrix} \right\} = 1$

$h'(x) = \frac{2x(x^2+2)-(x^2-4)2x}{(x^2+2)^2}$

$= \frac{12x}{(x^2+2)^2}$

$h'(x)=0$

$12x=0$

$x=0$

LUKA STIPANOV

2.

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IME I PREZIME: Stjepan Štefančić **BROJ INDEKSA:** 0269083933

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Ukupno: 41

1) $z^3 + \overline{1+i} = 0$

$z^3 + 1 - i = 0$

$z^3 = -1 + i$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-1)^2 + 1^2}$

$r = \sqrt{1+1} = \sqrt{2}$

$n = 3$

$x = -1$

$y = 1$

$\varphi = \arctan \frac{y}{x}$

$\varphi = \arctan \frac{1}{-1} = -\frac{1}{4}\pi - \pi = -\frac{5}{4}\pi$

$k = 0$

$k = 1$

$k = 2$

$w_1 = \sqrt[n]{r}\left[\cos \frac{\varphi + 2k\pi}{n} + i \sin \frac{\varphi + 2k\pi}{n}\right]$ $\sqrt[3]{\sqrt{2}} = 1.122$

$w_1 = \sqrt[3]{\sqrt{2}}\left[\cos \frac{-\frac{5}{4}\pi + 2 \cdot 0 \cdot \pi}{3} + i \sin \frac{-\frac{5}{4}\pi + 2 \cdot 0 \cdot \pi}{3}\right] =$

$= 1.122 \cdot [0.259 - 0.966i] = 0.291 - 1.084i$

$w_2 = 1.122 \cdot \left[\cos \frac{-\frac{5}{4}\pi + 2\pi}{3} + i \sin \frac{-\frac{5}{4}\pi + 2\pi}{3}\right] = 1.122 \cdot [0.707 + 0.707i] =$

$= 0.793 + 0.793i$

⑤ $f(x)=\ln(\sin(4x-2))$

$f'(x)=\frac{1}{\sin(4x-2)}\cdot\cos(4x-2)\cdot 4$

$\ln'(\sin(4x-2))=\frac{\ln}{\sin(4x-2)}$

$\sin'(4x-2)=\cos(4x-2)$

$(4x-2)'=4$

④ $h(x)=\frac{x^2-4}{x^2+2}$

Domena

$x^2+2\neq 0$

$x^2\neq -2\ /\sqrt{}$

$x\neq$

rešenja D

Nultočke

$x^2-4=0$

$x^2=4\ /\sqrt{}$

$x=\pm 2$

$x_1=2 \quad x_2=-2$

$y_1=0 \quad y_2=0$

$\frac{2^2-4}{2^2+2}=0$

$\frac{(-2)^2-4}{(-2)^2+2}=0$

$N_1(2,0) \quad N_2(-2,0)$

Ekstremi, rast-pad

$h'(x)=\frac{(x^2-4)'\cdot(x^2+2)-(x^2-4)\cdot(x^2+2)'}{(x^2+2)^2}=$

$=\frac{2x\cdot(x^2+2)-(x^2-4)\cdot(2x)}{(x^2+2)^2}=\frac{2x^3+4x-(2x^3-8x)}{(x^2+2)^2}=$

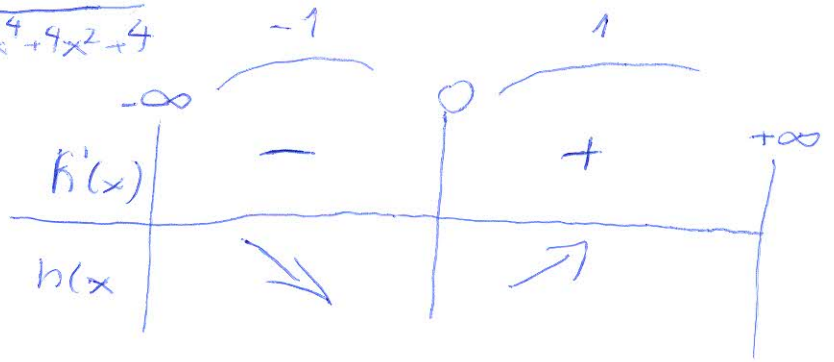
$=\frac{2x^3+4x-2x^3+8x}{(x^2+2)^2}=\frac{12x}{(x^2+2)^2}=\frac{12x}{x^4+4x^2+4}$

$12x=0\ /:12$

$x=0$ $\quad E(0,-2)$ min

$\frac{0^2-4}{0^2+2}=\frac{-4}{2}=-2$

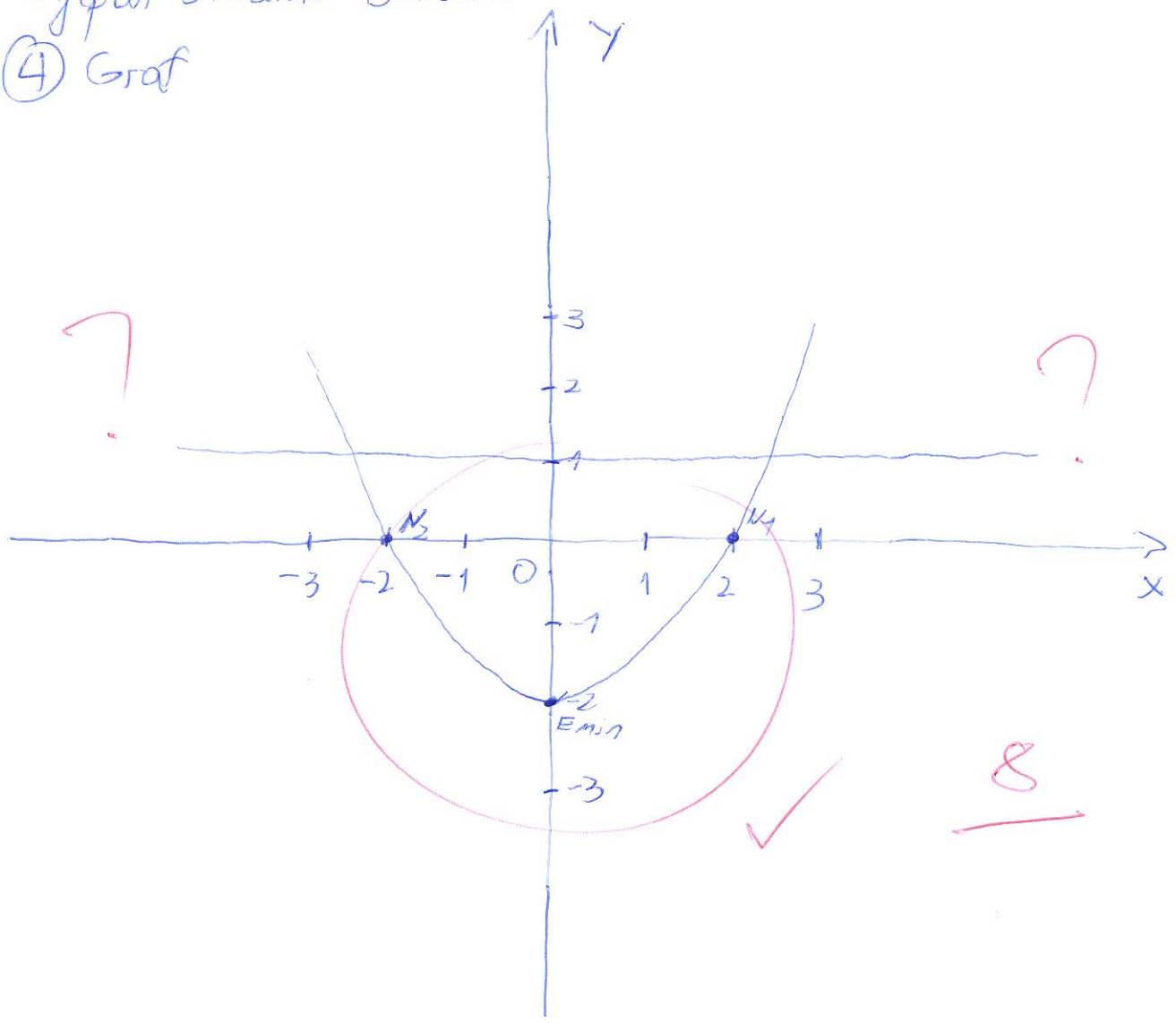
$y=-2$



$\frac{12x}{x^4+4x^2+4}$ $\quad -\ +$

Stjepan Štefančić 0268083933

④ Graf



?

?

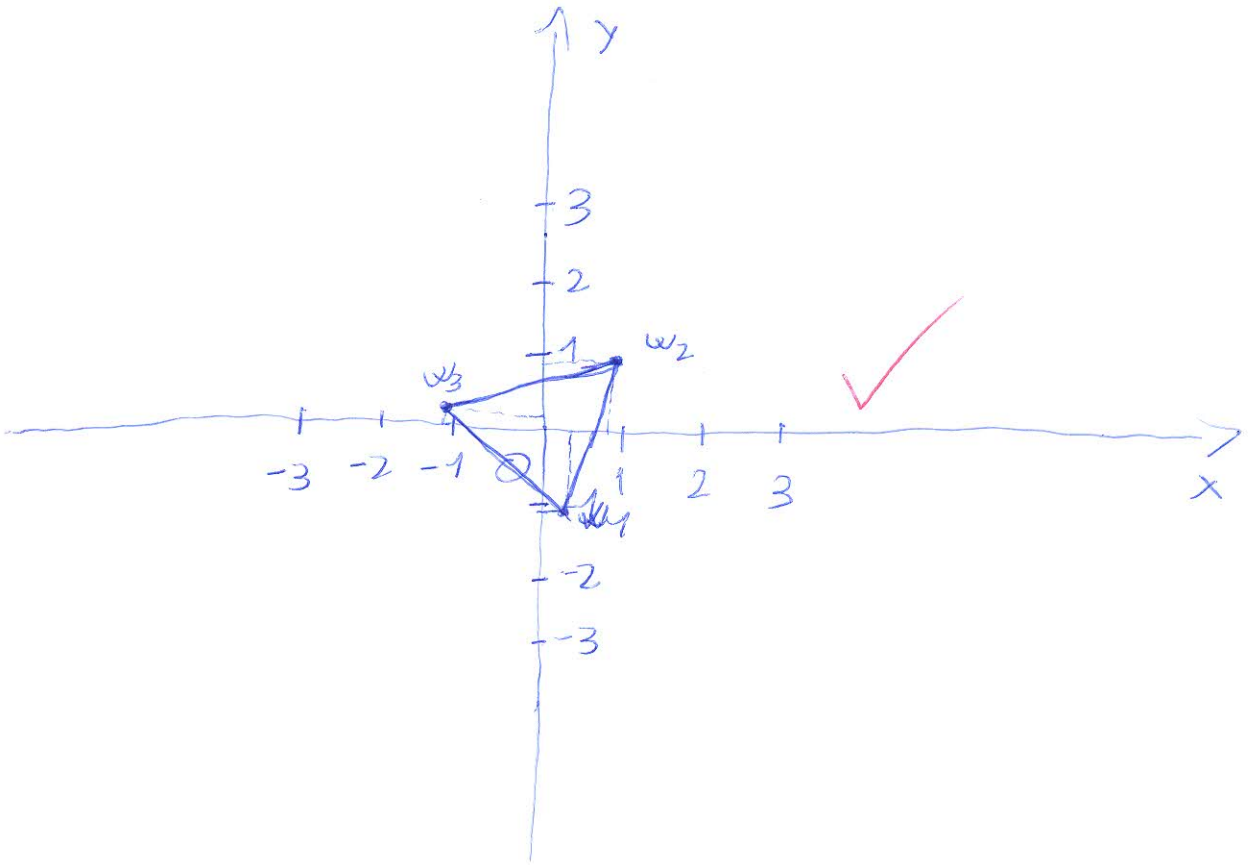
✓

8

(1)

$$w_3 = 1.122 \cdot \left[\cos \frac{-\frac{5}{4}\pi + 2 \cdot 2 \cdot \pi}{3} + i \sin \frac{-\frac{5}{4}\pi + 2 \cdot 2\pi}{3} \right] =$$

$$= 1.122 \cdot [-0.966 + 0.259i] = \boxed{-1.084 + 0.291}$$



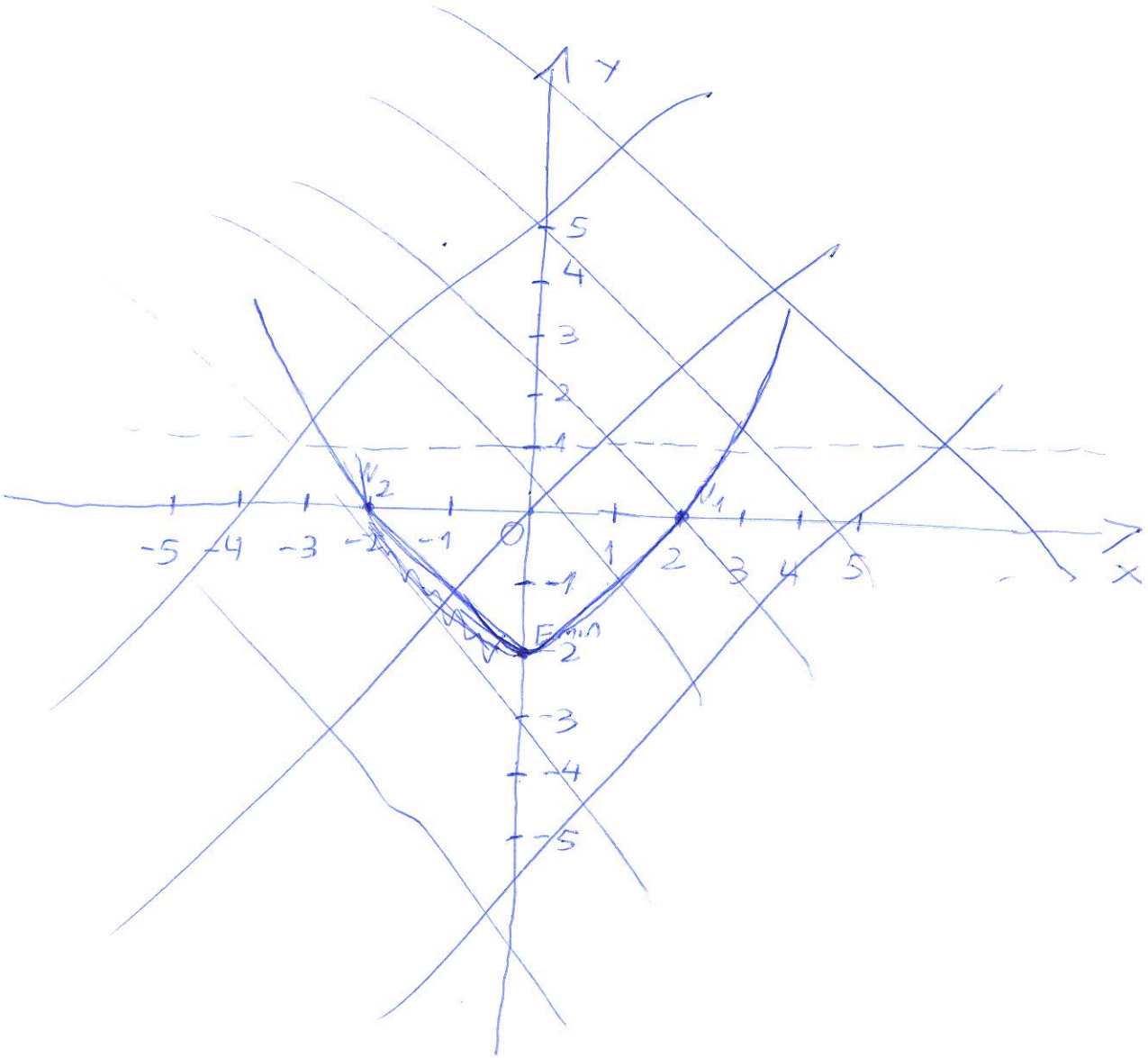
Stjepan Štefančić 0269083933

④

V.A. nema

H.A.

$$\lim_{x\to\pm\infty} \frac{x^2-4}{x^2+2} \;\Big|:x^2 = \lim_{x\to\pm\infty} \frac{1-\frac{4}{x^2}}{1+\frac{2}{x^2}} \;\Big|\; \frac{4}{x^2}\to 0,\ \frac{2}{x^2}\to 0 \quad = \frac{1}{1} \quad \underline{y=1}$$



PROF. UGLEŠIĆ

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{I}1$

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: LUKA ŠATALIĆ

BROJ INDEKSA: 17-2-0266-2013 0269025417

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + x} - x$. 5+15 ~~0~~ 8

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x-2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: ~~28~~

5. $f(x) = \ln(\sin(4x-2))$

$f'(x) = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$? ✗

$= 4 \cdot \operatorname{ctg}(4x-2)$ ✓

3. $g(x) = \sqrt{x^2 + x} - x$

$x^2 + x \geq 0$

$x(x+1) \geq 0$

$x_1 = 0, \quad x_2 = -1$

$\Rightarrow D_g = \langle -\infty, -1] \cup [0, +\infty \rangle$ ✓ nema točke prekida

$\lim_{x \to -\infty} g(x) = \lim_{x \to -\infty} (\sqrt{x^2 + x} - x) = \infty + \infty = \infty$ ✗ nema horizontalne asimptote u lijevoj strani

$$\lim_{x \to +\infty} g(x) = \lim_{x \to +\infty} \left(\sqrt{x^2+x} - x\right) =$$

$$= \lim_{x \to +\infty} \left(\left(\sqrt{x^2+x} - x\right) \cdot \frac{\sqrt{x^2+x} + x}{\sqrt{x^2+x} + x}\right) = \lim_{x \to +\infty} \left(\frac{x^2 + x - x^2}{\sqrt{x^2 - x} + x}\right) =$$

$$\lim_{x \to +\infty} \left(\frac{x}{\sqrt{x^2+x} + x}\right) = \lim_{x \to +\infty} \left(\frac{1}{\frac{\sqrt{x^2+x} + x}{x}}\right) =$$

$$= \lim_{x \to +\infty} \left(\frac{1}{\sqrt{1 - \frac{1}{x}} + 1}\right) = \frac{1}{1+1} = \frac{1}{2}$$

horizontalna asimptota u desnoj strani je

$y = \frac{1}{2}$ ✓

$$\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \overset{\text{VI}+2\text{II}}{\sim} \begin{bmatrix} 1 & 5 & 0 & 1 & -3 \\ 0 & 6 & 0 & 2 & -1 \\ 4 & 3 & 2 & 1 & 2 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \overset{\text{III}-2\text{I}}{\sim} \begin{bmatrix} 1 & 1 & 4 & 3 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 3 & 2 & 2 & 0 & -3 \\ 2 & -2 & 0 & 4 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 3 & -1 & 2 & 1 \\ 2 & 2 & -2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 1 & 4 & 3 & 4 & 1 \end{bmatrix} \overset{\text{IV}-\text{III}}{\sim} \begin{bmatrix} 0 & 3 & -2 & 2 & 1 \\ 1 & 0 & -1 & 1 & 2 \\ 2 & 4 & 0 & 1 & 2 \\ 0 & -1 & 4 & 0 & 0 \end{bmatrix} \overset{\text{I}+\text{II}}{\sim} \begin{bmatrix} 1 & 3 & -3 & 3 & 3 \\ 1 & 0 & -1 & 1 & 2 \\ 2 & 4 & 0 & 1 & 2 \\ 0 & -1 & 4 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 1 & 0 & -1 \\ 2 & 1 & 1 & 3 & -3 \\ 0 & 0 & 2 & 1 & 2 \\ 2 & 2 & 1 & 2 & 0 \end{bmatrix} \overset{\text{II}-2\text{I}}{\sim} \begin{bmatrix} 1 & 3 & 1 & 0 & -1 \\ 0 & -5 & -1 & 3 & -5 \\ 0 & 0 & 2 & 1 & 2 \\ 2 & 2 & 1 & 2 & 0 \end{bmatrix} \overset{\text{IV}-\text{I}}{\sim} \begin{bmatrix} 1 & 3 & 1 & 0 & -1 \\ 0 & -5 & -1 & 3 & -5 \\ 0 & 0 & 2 & 1 & 2 \\ 1 & -1 & 0 & 2 & -1 \end{bmatrix} \sim$$

$$\begin{bmatrix} \end{bmatrix}$$

$$x + 2y - z + u = -1$$

$$2x + 5y - z + 2u = -2$$

$$3x - y - 2z + u = 5$$

$$x - y + 3z - 5u = 6$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓ $\mathcal{I}1$

IME I PREZIME: FILIP HERCEGVIĆ **BROJ INDEKSA:** 17-1-0263-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

2. Gaussovom metodom riješi sustav linearnih jednadžbi, a zatim provjeri uvrštavanjem:

$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

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5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 36

(2) $x + 2y - z + u = -1$
$2x + 5y - z + 2u = -2$
$3x - y - 2z + u = 5$
$x - y + 3z - 5u = 6$

PROVJERA: $2 + 2\cdot(-1) - 1 + 0 = -1 \to -1 = -1$ //
$2\cdot 2 + 5\cdot(-1) - 1 + 2\cdot 0 = -2 \to -2 = -2$ //
$3\cdot 2 + 1 - 2\cdot 1 + 0 = 5 \to 5 = 5$ //
$2 + 1 + 3\cdot 1 - 5\cdot 0 = 6 \to 6 = 6$ //

$\begin{bmatrix} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{bmatrix} \cdot(-2), \cdot(-3), \cdot(-1) \approx \begin{bmatrix} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{bmatrix} \cdot(-2) \cdot 7 \cdot 3 \approx$

$\begin{bmatrix} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{bmatrix} :8 \approx \begin{bmatrix} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 7 & -6 & 7 \end{bmatrix} \cdot 3 \cdot(-1) \cdot(-7) \approx \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & -\frac{17}{4} & 0 \end{bmatrix} :\left(-\frac{17}{4}\right) \approx$

$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \cdot\left(-\frac{1}{4}\right) \cdot\left(-\frac{1}{4}\right) \cdot\frac{1}{4} \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$x = 2$
$y = -1$
$z = 1$
$u = 0$

HERLJEVIĆ

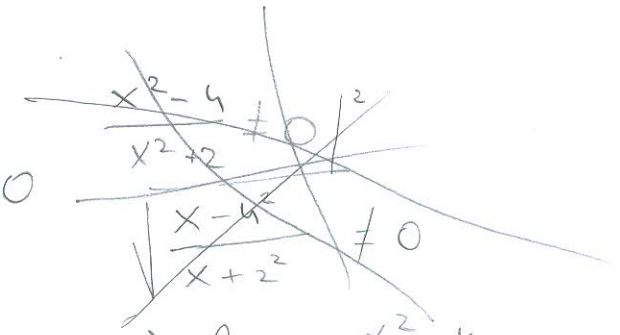
④ $h(x) = \frac{x^2-4}{x^2+2}$

$x^2+2 \neq 0$ N.T

$x^2 \neq -2$ $x^2-4 \neq 0$

$x \neq \sqrt{-2}$ $x^2 \neq 4$

$D_f = \mathbb{R}$ $x \neq \pm 2$



b) $\lim_{x \to \pm\infty} \frac{x^2-4}{x^2+2} - 0x = \frac{(x^2-4)-0}{x^2+2}$

NEMA

V.A. NEMA

H.A. $\lim_{x \to \pm\infty} \frac{x^2-4 \,:x^2}{x^2+2 \,:x^2} = \frac{1 - \frac{4}{x^2}}{1 + \frac{2}{x^2}} = 1$

K.A. a) $\lim_{x \to \pm\infty} \frac{\frac{x^2-4}{x^2+2}}{\frac{x}{1}} = \frac{x^2-4}{x^3+2x} \,:x^3 = \frac{\frac{1}{x} - \frac{4}{x^3}}{1 + \frac{2}{x^2}} = \frac{0}{1} = 0$

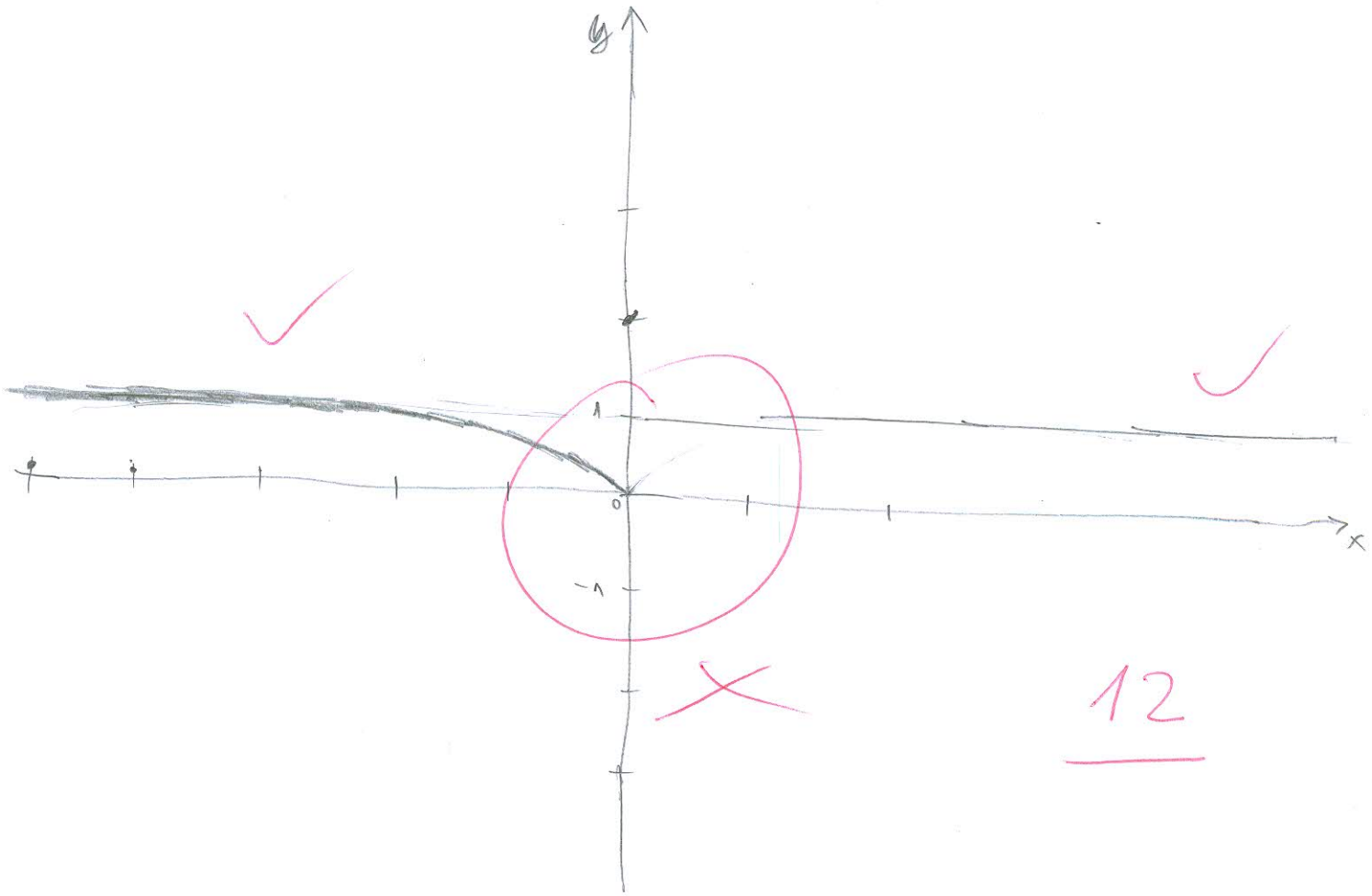
$f'_{(h)} = \frac{(2x)\cdot(x^2+2) - (x^2-4)\cdot(2x)}{(x^2+2)^2} = \frac{2x^3+4x-2x^3+8x}{(x^2+2)^2}$

$= \frac{12x}{(x^2+2)^2}$

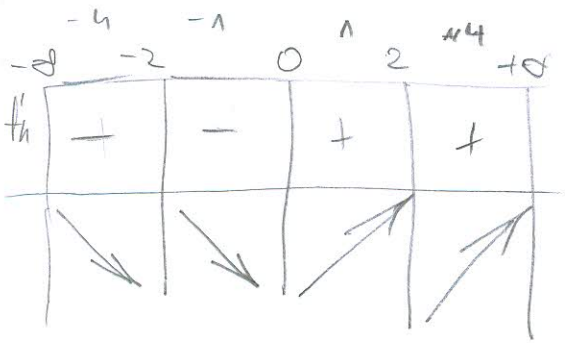
$f''_h = \frac{12\cdot(x^2+2)^2 - (2\cdot(x^2+2)\cdot 2x)\cdot 12x}{(x^2+2)^4} = \frac{12\cdot(x^4+\overbrace{2\cdot x^2}^{4x^2}\cdot 2+4) - (4x\,(x^2+2)\cdot 12x)}{(x^2+2)^4}$

$= \frac{12x^4+48x^2+48-((4x^3+8x)\cdot 12)}{(x^2+2)^4} = \frac{12x^4+48x^2+48-48x^3-96x}{(x^2+2)^4}$

2. DER. SE NE MOŽE DOBITI



12



$f'_h = 12x \neq 0$

$x_{1,2} = \frac{-12x \pm \sqrt{12^2 - 4 \cdot 0 \cdot c}}{0}$

$x_{1,2} = \frac{-12 \pm \sqrt{144 - 0}}{0}$

$x_{1,2} = \frac{-12 \pm 12}{0}$

$x_1 = 0$

$x_2 = 0$

HELJEVIC

⑤ $g(x) = \sqrt{x^2+x} - x$

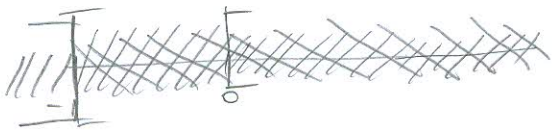
$x^2 + x \geq 0$

$x(x+1) \geq 0$

$x \geq 0 \quad x+1 \neq 0$

$x \neq -1$

$D_f: \langle -\infty, -1] \cup [0, +\infty \rangle$ ✓



VA. ~~V_1~~ $x_1 = 0$

~~V_2~~ $x_2 = -1$ ✗

H.A. $\lim_{x \to \pm\infty} \sqrt{x^2+x} - x$ ✗

NEMA H.A.

K.A. a) $\lim_{x \to \pm\infty} \frac{\sqrt{x^2+x} - x}{\frac{x}{1}} \Big| = \frac{x^2+x-x^2}{x^2} = \frac{x : x^2}{x^2 : x^2} = \frac{\not{1}^{0}}{\not{x}\ 1} = \frac{0}{1} = 0$ ✗

NEMA K. A.

⑤ $f(x) = \ln(\sin(4x-2))$

$f'(x) = \frac{1}{x} \cdot (\sin(4x-2)) + \ln \cdot (\cos(4x-2) \cdot 4)$

$f'(x) = \frac{1}{x} \cdot (\sin(4x-2)) + \ln \cdot (4\cos(4x-2))$ ✗

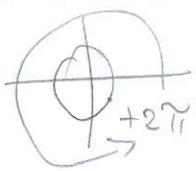
① $z^3 + \overline{1+i} = 0$ $\quad |w| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ $\quad tg\,\varphi = -\frac{1}{1} =$

$z^3 = -\overline{1-i}$

$z^3 = -1+i$ $\quad |W| = \sqrt{2} \cdot (\cos\varphi + i\sin\varphi)$

$|W| =$

$\varphi = -0{,}7853981634 + 2\pi$



$\varphi = 5{,}497787144$

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IME I PREZIME: MARKO VUKELIĆ **BROJ INDEKSA:** 17-2-0283-13 0269080097

POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 20

3.

$g(x) = \sqrt{x^2+x} - x$

$D_f : \langle -\infty, -1 \rangle \cup \langle 0, +\infty \rangle$ ✓

$x^2 + x \geq 0$

$x(x+1) \geq 0$

$x \neq 0 \quad x \neq -1$

$-\infty \; -1 \; 0 \; +\infty$

$+ \; \| \; - \; \| \; +$

V.A 0 i −1 JER IMAMO PREKIDE U DOMENI ✗

$\lim_{x \to 0} \frac{\sqrt{x^2+x} - x}{1} \cdot \frac{\sqrt{x^2+x} + x}{\sqrt{x^2+x} + x} = \lim_{x \to 0} \frac{\left(\sqrt{x^2+x} - x\right)^2}{\sqrt{x^2+x} + x}$

$\lim_{x \to 0} \frac{x^2 + x + x^2}{\sqrt{x^2+x} - x} \quad | : x^2$

$= \frac{\frac{1}{1} + \frac{1}{x} + 1}{\sqrt{\frac{1}{x^2} + \frac{1}{x^3}} - \frac{1}{x}} = \frac{2}{\sqrt{0} - \infty}$

$\lim_{x\to 1} \sqrt{x^2+x}-x \cdot \frac{\sqrt{x^2+x}-x}{\sqrt{x^2+x}-x} = \lim_{x\to 1} \frac{(\sqrt{x^2+x}-x)^2}{\sqrt{x^2+x}-x}$

$\lim_{x\to 1} \frac{x^2+x+x^2}{\sqrt{x^2+x}-x} \quad \lim_{x\to 1} \frac{x^2+x}{\sqrt{x^2+x}-x} \quad /:x^2$

$\frac{\frac{1}{1}+\frac{1}{x^3}}{\sqrt{\frac{1}{x^6}+\frac{1}{x^4}}-\frac{x}{x^3}}$

$\frac{1+0}{\sqrt{0}-0} = \frac{1}{0} = \infty$

$\lim_{x\to\infty} (x^4+x)$ D.H.A

$\lim_{x\to\infty} \sqrt{x^2+x}-x \cdot \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x} \quad \lim_{x\to\infty} \frac{(\sqrt{x^2+x}-x)^2}{\sqrt{x^2+x}+x}$

D.H.A

$\lim_{x\to\infty} \frac{x^2+x+x^2}{\sqrt{x^2+x}+x} \quad /:x^2$

$\frac{\frac{1}{1}+\frac{1}{x}+1}{\sqrt{\frac{1}{x^2}+\frac{1}{x}}+\frac{1}{x}} = \frac{2}{\sqrt{0}+0} =$

NEMA ✗

L.H.A

$\lim_{x\to-\infty} \frac{x^2+x+x^2}{\sqrt{x^2+x}+x} \quad /:x^2$

$= \frac{\frac{1}{1}-\frac{1}{x}+1}{\sqrt{\frac{1}{x^2}+\frac{1}{x}}+\frac{1}{x}} \quad \ldots$

NEMA ✓

D.H.A

5.

$f(x) = \ln(\sin(4x-2))$

$(4x-2)' = 4-0$

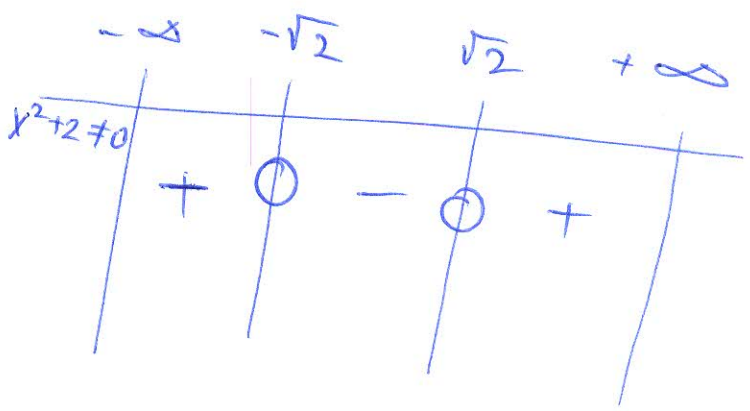
$f'(x) = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$ ✓

4.

$h(x) = \frac{x^2-4}{x^2+2}$

$x^2+2 \neq 0$

$x_{1,2} = \pm\sqrt{2}$



$D_f: x \in \mathbb{R} \setminus \{-\sqrt{2}, \sqrt{2}\}$

V.A $-\sqrt{2}$ i $\sqrt{2}$

D.M.A

$$\lim_{x\to+\infty} \frac{x^2-4}{x^2+2} \begin{matrix} /:x^2 \\ /:x^2 \end{matrix} = \lim_{x\to+\infty} \frac{\frac{1}{1} - \frac{4}{x^2}}{\frac{1}{1} + \frac{2}{x^2}} \quad \frac{1}{1} = 1$$

L.M.A

$$x\to-\infty \quad \frac{x^2-4}{x^2+2} \begin{matrix} /:-x^2 \\ /:x^2 \end{matrix} = \lim_{x\to-\infty} \frac{-1}{-1} = 1$$

D.K.A

$$k = \lim_{x \to \infty} \frac{f(x)}{x}$$

$$k = \lim_{x \to \infty} \frac{\frac{x^2-4}{x^2+2}}{\frac{x}{1}} \qquad \frac{x^2-4}{x(x^2+2)}$$

$$k = \lim_{x \to \infty} \frac{x^2-4}{x^3+2x} \;\Big|: x^3 = \frac{\frac{1}{x} - \frac{4}{x^2}}{1 + \frac{2}{x^2}}$$

$$= \frac{0-0}{1+0} = \frac{0}{1}$$

$$k = 0 \qquad = 0$$

9.

DERIVACIJA I EKSTREMI

$(I+II)^2 = I^2 + 2III - II^2$

$$h'(x) = \frac{(x^2-4)' \cdot (x^2+2) - (x^2+2)'(x^2-4)}{(x^2+2)^2}$$

$$= \frac{2x \cdot (x^2+2) - 2x(x^2-4)}{x^4 + 2 \cdot 2x^2 - 4}$$

$$= \frac{2x^3 + 4x - 2x^3 + 4x}{(x^2+2)\cdot(x^2-4)}$$

$$= \frac{8x}{x^4 + 4x^2 - 4}$$

$$= \frac{x^4 - 4x^2 + 2x^2 - 6}{x^4 + 4x^2 - 4}$$

$$= \frac{-2x^2 - 6}{4x^2 - 4}$$

$h'(x) = 0$

$$\frac{-2x^2-6}{4x^2-4} = 0 \quad / \cdot (4x^2-4)$$

$$-2x^2 - 6 = 0$$

$$2x^2 = 6$$

$$x^2 = 3 \quad / \sqrt{\ }$$

$$x = \sqrt{3}$$

1.73

NEMA GRAFA

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{I}1$

IME I PREZIME: TOMISLAV PERKOVIĆ **BROJ INDEKSA:** 17-2-0229-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

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$$\begin{aligned} x + 2y - z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

Provjeri uvrštavanjem! 16+3

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + x} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 19

2. $\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{matrix} \\ \cdot(-2) \\ \cdot(-3) \\ \cdot(-1) \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & 1 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right] \begin{matrix} \leftarrow 1 \cdot(-2) \\ \\ \cdot 7 \\ \cdot 3 \end{matrix} \sim$

$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \begin{matrix} \\ \\ :8 \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \begin{matrix} \leftarrow \cdot 3 \\ \leftarrow \cdot(-1) \\ \\ \leftarrow \cdot(-7) \end{matrix}$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & -\frac{17}{4} & 0 \end{array}\right] \begin{matrix} \\ \\ \\ :\left(-\frac{17}{4}\right) \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & 0 & \frac{1}{4} & -1 \\ 0 & 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{matrix} \leftarrow \cdot\left(-\frac{1}{4}\right) \\ \leftarrow \cdot\left(-\frac{1}{4}\right) \\ \leftarrow \cdot\frac{1}{4} \\ \\ \end{matrix} \sim$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$ $x=2 \quad y=-1 \quad z=1 \quad u=0$

$2+2\cdot(-1)-1+0=-1$ $\quad 2\cdot 2+5\cdot(-1)-1+2\cdot 0=-2$

$2-2-1+0=-1$ $\quad 4-5-1+0=-2$

$-1=-1$ $\quad -2=-2$

$3x - y - 2z + u = 5$ $\quad$ $x - y + 3z - 5u = 6$

$3 \cdot 2 - (-1) - 2 \cdot 1 + 0 = 5$ $\quad$ $2 - (-1) + 3 \cdot 1 - 5 \cdot 0 = 6$

$6 + 1 - 2 + 0 = 5$ $\quad$ $2 + 1 + 3 - 0 = 6$

$5 = 5$ $\quad$ $6 = 6$

3. $g(x) = \sqrt{x^2 + x} - x = 0$

Domeniu

$\sqrt{x^2 + x} \neq 0 \,/^2$

$x^2 + x \neq 0$

$x(x+1) \neq 0$

$D_f : \mathbb{R}$ ✗

5. $f(x) = \ln(\sin(4x-2))$

$f'(x) = \frac{1}{(\sin(4x-2))} \cdot \cos(4x-2)) \cdot 4 = \frac{4\cos}{(\sin(4x-2))}$ ✗

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ I1

IME I PREZIME: LUCIJA IVANDIĆ **BROJ INDEKSA:** 17-2-0109-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^3 + \overline{1+i} = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3

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Provjeri uvrštavanjem! 16+3

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4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+2}$. 20(graf)

5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x - 2))$. 15

6. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -2 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 8

Ukupno: 15

(5) $f(x) = \ln(\sin(4x-2))$

$f(x)' = \frac{1}{\sin(4x-2)} \cdot (\sin(4x-2))'$

$f(x)' = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot (4x-2)'$

$f(x)' = \frac{1}{\sin(4x-2)} \cdot \cos(4x-2) \cdot 4$

$f(x)' = \frac{4\cos(4x-2)}{\sin(4x-2)}$ ✓

④ $h(x) = \frac{x^2-4}{x^2+2}$

DOMENA

$x^2+2 \neq 0$

$x^2 \neq -2 / \sqrt{}$

$x \neq \sqrt{-2}$

$D_f \in \mathbb{R}$

- nema vertikalne asimptote, nema dockе presjeka!

PERIODIČNOST

Funkcija nije periodična!

PARNOST / NEPARNOST

$\frac{(x)^2-4}{(-x)^2+2} = \frac{x^2-4}{x^2+2}$ Funkcija je parna!

NUL TOČKE

$f(0) = \frac{0-4}{0+2} = -2$

ASIMPTOTE

Hor. d:

$\lim_{x \to -\infty} \frac{x^2-4 \,/ x^2}{x^2+2 \,/ x^2} = \lim_{x \to -\infty} \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{x^2}{x^2} + \frac{2}{x^2}} = 1$

$\lim_{x \to +\infty} \frac{x^2-4 \,/ x^2}{x^2+2 \,/ x^2} = \lim_{x \to +\infty} \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{x^2}{x^2} + \frac{2}{x^2}} = 1$

$y = 1$ osobrzna hor. asimptota

- nema kose ni vertikalne!

STACIONARNE TOČKE

$f(x) = \frac{x^2-4}{x^2+2}$

$f(x)' = \frac{(x^2-4)'(x^2+2) - (x^2-4)(x^2+2)'}{(x^2+2)^2}$

$f(x)' = \frac{2x(x^2+2) - (x^2-4) \cdot 2x}{(x^2+2)^2}$

$f(x)' = \frac{2x^3 + 4x - 2x^3 + 8x}{(x^2+2)^2} = \frac{12x}{(x^2+2)^2}$

nema stacionarnih točaka

TOČKE INFLEKSIJE

LUCIJA IVANDIĆ

$$f(x)'' = \frac{12x}{(x^2+2)^2}$$

$$f(x)'' = \frac{(12x)'(x^2+2)^2 - 12x((x^2+2)^2)'}{(x^2+2)^4}$$

$$f(x)'' = \frac{12(x^2+2)^2 - 12x \cdot 2(x^2+2)(x^2+2)'}{(x^2+2)^4}$$

$$f(x)'' = \frac{12(x^2+2)^2 - 24x(x^2+2)\cdot 2x}{(x^2+2)^4}$$

$$f(x)'' = \frac{(x^2+2)(12(x^2+2) - 48x)}{(x^2+2)^{4/3}}$$

$$f(x)'' = \frac{12x^2+24-48x}{(x^2+2)^3}$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{16-8}}{2} = -2 \pm 2\sqrt{2}$$

BODUJE SE SAMO GRAF $\varnothing$

③ $g(x) = \sqrt{x^2+x} - x$

DOMENA

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IME I PREZIME: Bojana Kolesta **BROJ INDEKSA:** 1710089011

POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno:

(4) $h(x) = \frac{x^2 - 4}{x^2 + 2}$

$x_1 = 2$

$x_1 = x^2 - 4$

$x_1 = 2^2 - 4$

$x_1 = 4 - 4$

$x_1 = 0$

$x_2 = 2$

$x_2 = x^2 + 2$

$x_2 = 2^2 + 2$

$x_2 = 4 + 2$

$x_2 = 6$

∅

[illegible]

④ $h(x) = \frac{x^2-4}{x^2+2}$ $x=2$

KOLEGA

[illegible] ∅

=

⑤ $f(x) = \ln(\sin(4x-2))$

$f'(x) =$ ∅

$f(x) = \sqrt{4+3x}$

1 COLEGA

$f(x)' = \frac{1}{2\sqrt{4+3x}} \cdot (4+3x)'$

$f(x)' = \frac{3}{2\sqrt{4+3x}}$

[redacted]

BOŽENA KOLEGA
17-1-0089-2011

1) /

2) /

3) /

4) -

5) $f(x) = \sqrt{4+3x}$

$f'(x) = \frac{1}{2\cdot\sqrt{4+3x}} \cdot (4+3x)'$

$f'(x) = \frac{3}{2\sqrt{4+3x}}$

6) /

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POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: JOSIP MATEŠIĆ

BROJ INDEKSA: 0269075368

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Ukupno: 0

(3.)

$g(x) = \sqrt{x^2+x} - x$

Domena:

$\sqrt{x^2+x} \geq 0 \,/^2$ $D = \mathbb{R}$ ✗

$x^2 + x \geq 0$

$x^2 \geq -x \,/\sqrt{}$

$x \geqslant \sqrt{-x}$

$x = \pm\sqrt{x}$

$g(x) = \sqrt{x^2+x} - x$

V.A.

$\lim_{x\to 1} \sqrt{x^2+x} - x$

$\lim_{x\to 1} \sqrt{1^2+1} - 1 = \lim_{x\to 1} -1+\sqrt{2}$

$\lim_{x\to -1} \sqrt{x^2+x} - x$

$\lim_{x\to -1} \sqrt{-1^2-1} - 1 = \lim_{x\to -1} -\infty$

$x = -1+\sqrt{2}$ ✗

4) A.

$$\lim_{x\to\infty} \sqrt{x^2+x} - x \quad (:x^2) \quad (:x^2)$$

$$= \lim_{x\to\infty} \sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}} - \frac{x^2}{x} = \lim_{x\to\infty} \sqrt{1+\frac{x}{x^2}} - \frac{x^2}{x} = ?$$

$$\lim_{x\to-\infty} \sqrt{x^2+x} - x = \lim_{x\to-\infty} \sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}} + \frac{x^2}{x} = \lim_{x\to-\infty} \sqrt{1-\frac{x}{x^2}} + \frac{x^2}{x} = ?$$

$\boxed{y=0}$? KOSA ? $\emptyset$

5) $f(x) = \ln(\sin(4x-2))$

$$f'(x) = \frac{1}{\sin(4x-2)} \quad X$$

$$f'(x) = \frac{1}{-\cos(4x-2)}$$

$$f'(x) = \frac{1}{-\cos(4\cdot 0)}$$

$$f'(x) = \frac{1}{-\cos 4}$$

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$\mathcal{I}1$

IME I PREZIME: Ivan Paleka

BROJ INDEKSA: 17-2-0305-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 0

Ivan Paleka 17-2-0505-2015

(4) $x+2y-z+u=-1$
$2x+5y-z+2u=-2$
$3x-y-2z+u=5$
$x-y+3z-5u=6$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{matrix} \\ \cdot(-2) \\ \cdot(-3) \\ \cdot(-1) \end{matrix} \approx \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -7 & -5 & -2 & 8 \\ 0 & -3 & 4 & -6 & 7 \end{array}\right] \begin{matrix} \\ \cdot(-2) \\ \cdot(7) \\ \cdot(3) \end{matrix}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \approx \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & -1 \\ 0 & 0 & 2 & -2 & 8 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \cdot(1) \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & 1 \\ 0 & 1 & 3 & -2 & 8 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \cdot(-1) \approx \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & 1 \\ 0 & 1 & 3 & -2 & 8 \\ 0 & 0 & -2 & 2 & 8 \\ 0 & 0 & 7 & -6 & 7 \end{array}\right] \begin{matrix} \\ \\ \\ \cdot(-1)(-1) \end{matrix}$$

(1) $h(x) = \frac{x^2-4}{x^2+2}$

0 DOMENA

$D(x) = \frac{x^2-4}{x^2+2}$

$x \neq 0$

$x^2+2 \neq 0$

$x^2 = 2$

$x = \sqrt{2}$

(2) $x = 0$ $x = -2$

$y = 0$

$y = \frac{x^2-4}{x^2+2} = 0$

(3)