

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

odgovornosti studenata. **PIŠITE DVOSTRANO!**

xxo

IME I PREZIME: JAKOV ŽUBČIĆ

BROJ INDEKSA: 0269092107

Želim ustmeni kod (zaokružiti):

prof. Uglešić

asistent Kosor

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

- Nadi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$.
- Odrediti lokalne ekstreme funkcije $f(x, y) = e^x - x + y^2 + 2y$.
- Skicirati razinske krivulje za funkciju $f(x, y) = x^2 - y$.
- $\int_0^\pi \frac{\sin x}{\cos x + 5} dx$?
- Izračunaj površinu lika omeđenog krivuljama $f(x) = x^2 - 3x - 4$ i $g(x) = -x^2 + 3x + 4$.
- Izračunaj $\int_1^4 \frac{1 + \sqrt{x}}{x^2} dx$ i napravi provjeru izračunom aproksimacije integrala Simpsonomovom metodom.

15

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~~10+10~~

Ukupno:

50

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

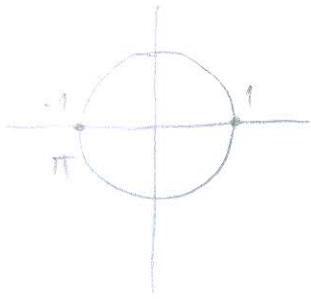
Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

$$4.) \int_0^{\pi} \frac{\sin x}{\cos x + 5} dx = \left[\begin{array}{l} \cos x + 5 = t \\ dt = -\sin x dx \end{array} \right] = \int \frac{\cancel{\sin x} \cdot \frac{-dt}{\cancel{\sin x}}}{t} = -\int \frac{dt}{t}$$

$$-\frac{dt}{\sin x} = dx \qquad = -\ln|\cos x + 5|$$

$$\left[-\ln|\cos x + 5| \right]_0^{\pi} = \left[-\ln|\cos \pi + 5| + \ln|\cos 0 + 5| \right]$$

$$= -\ln 4 + \ln 6 = 0,41 \quad \checkmark$$



$$6.1) \int_1^4 \frac{1+\sqrt{x}}{x^2} dx = \int \frac{1}{x^2} dx + \int \frac{\sqrt{x}}{x^2} dx = \dots ?$$

$$\int \frac{1}{x^2} dx = \int x^{-2} dx = -\frac{1}{x}$$

2.)

x	0	1	2
x_k	1	2	4
f_k	2	0,85	0,19

$$S = \frac{d}{6} (f_0 + 4f_1 + f_2)$$

$$S = \frac{3}{6} (2 + 4 \cdot 0,85 + 0,19)$$

$$\boxed{S = 2,8} \quad \times$$

5.) $f(x) = x^2 - 3x - 4$

$g(x) = -x^2 + 3x + 4$

$-x^2 + 3x + 4 = x^2 - 3x - 4$

$-2x^2 + 6x + 8 = 0$

$x = \frac{-6 \pm \sqrt{36 + 64}}{-4}$

$x = \frac{-6 \pm 10}{-4}$

$x_1 = 4$

$x_2 = -1$

$x^2 - 3x - 4 = 0$

$x = \frac{3 \pm \sqrt{9 + 16}}{2}$

$x = \frac{3 \pm 5}{2}$

$x_1 = 4$

$x_2 = -1$

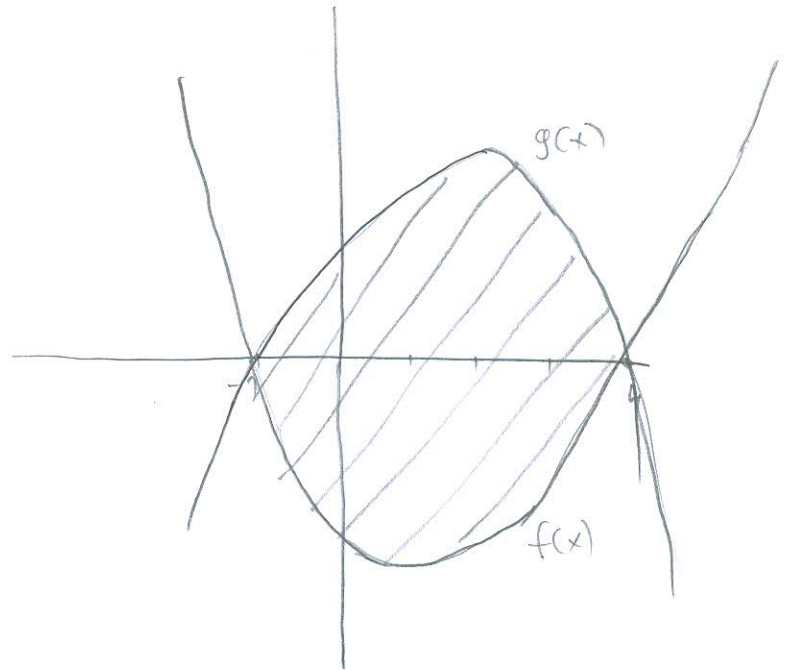
$-x^2 + 3x + 4 = 0$

$x = \frac{-3 \pm \sqrt{9 + 16}}{-2}$

$x = \frac{-3 \pm 5}{-2}$

$x_1 = 4$

$x_2 = -1$



$\int (x^2 - 3x - 4) dx = \frac{x^3}{3} - \frac{3x^2}{2} - 4x$

$\int (-x^2 + 3x + 4) dx = -\frac{x^3}{3} + \frac{3x^2}{2} + 4x$

$\left[-\frac{x^3}{3} + \frac{3x^2}{2} + 4x - \frac{x^3}{3} + \frac{3x^2}{2} + 4x \right]_{-1}^4 =$

$\left[-\frac{2}{3}x^3 + 3x^2 + 8x \right]_{-1}^4 =$

$\left[-\frac{2}{3} \cdot 4^3 + 3 \cdot 16 + 8 \cdot 4 - \left(\frac{2}{3} + 3 - 8 \right) \right] =$

$-42,7 + 48 + 32 + 4,33 = 41,63 \checkmark$

1. $xy' + y - e^x = 0, y(1) = 1$

$$xy' + y = e^x \quad | \cdot \frac{1}{x}$$

$$\frac{dy}{dx} + \frac{1}{x} \cdot y = \frac{e^x}{x}$$

$$\frac{dy}{dx} + \frac{1}{x} \cdot y = 0$$

$$\frac{dy}{dx} = -\frac{1}{x} \cdot y$$

$$\frac{dy}{y} = -\frac{dx}{x} \quad | \int$$

$$\int \frac{dy}{y} = - \int \frac{dx}{x}$$

$$\ln|y| = -\ln|x| + C$$

$$\ln|y| = \ln C - \ln|x|$$

$$y = \frac{C}{x}$$

$$C = U(x)$$

$$y = \frac{U(x)}{x}$$

$$x \cdot \left(\frac{u}{x}\right)' + \left(\frac{u}{x}\right) - e^x = 0$$

$$x \cdot \left(\frac{u' \cdot x - u}{x^2}\right) + \frac{u}{x} - e^x = 0$$

$$\frac{x^2 \cdot u' - x \cdot u}{x^2} + \frac{u}{x} - e^x =$$

$$u' - \frac{u}{x} + \frac{u}{x} = e^x$$

$$du = e^x dx \quad | \int$$

$$u = e^x + C$$

PARTIKULARNO RJEŠENJE:

$$y = \frac{e^x + 1 - e}{x}$$



$$y = \frac{e^x + C}{x}$$

$$1 = e^1 + C$$

$$1 - e = C$$

PROVERA:

$$1 = \cancel{e^1} + 1 - \cancel{e^1}$$

$$1 = 1 \quad \checkmark$$

$$x \cdot \left(\frac{e^x}{x} + \frac{c}{x}\right)' + \frac{e^x + c}{x} - e^x = 0$$

$$x \cdot \left(\frac{e^x \cdot x - e^x}{x^2} + \frac{x - c}{x^2}\right) + \frac{e^x + c}{x} - e^x = 0$$

$$\cancel{e^x} - \frac{e^x}{x} - \frac{c}{x} + \frac{e^x}{x} + \frac{c}{x} - e^x = 0$$

$$0 = 0$$

