MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO **IME I PREZIME:** JAKOV ŽUBČIĆ **BROJ INDEKSA:** 0269032107 Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. 15

2. Odrediti lokalne ekstreme funkcije $f(x, y) = e^x - x + y^2 + 2y$. 15

3. Skicirati razinske krivulje za funkciju $f(x, y) = x^2 - y$. 15

4. $\int_0^\pi \frac{\sin x}{\cos x+5} dx$? 20

5. Izračunaj površinu lika omeđenog krivuljama $f(x) = x^2 - 3x - 4$ i $g(x) = -x^2 + 3x + 4$. 15

6. Izračunaj $\displaystyle\int_1^4 \frac{1+\sqrt{x}}{x^2} dx$ i napravi provjeru izračunom aproksimacije integrala Simsponovom metodom. ~~10+10~~

Ukupno: 50

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

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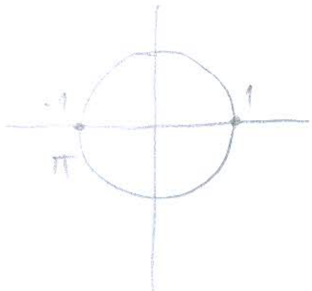
4.) $\int_0^{\pi} \frac{\sin x}{\cos x + 5} dx = \left[\begin{array}{l} \cos x + 5 = t \,/\,' \\ dt = -\sin x \, dx \end{array} \right] = \int \frac{\cancel{\sin x} \cdot \frac{-dt}{\cancel{\sin x}}}{t} = -\int \frac{dt}{t}$

$-\frac{dt}{\sin x} = dx$ $\qquad = -\ln|\cos x + 5|$

$\left[-\ln|\cos x + 5| \right]_0^{\pi} = \left[-\ln|\cos \pi + 5| + \ln|\cos 0 + 5| \right]$

$= -\ln 4 + \ln 6 = 0,41$ ✓



?

6.-1.) $\int_1^4 \frac{1+\sqrt{x}}{x^2} dx = \int \frac{1}{x^2} dx + \int \frac{\sqrt{x}}{x^2} dx = \ldots$?

$\int \frac{1}{x^2} dx = \int x^{-2} dx = -\frac{1}{x}$

2.)

x	0	1	2
x_k	1	2	4
f_k	2	0,85	0,19

×

$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$

$S = \frac{3}{6}(2 + 4 \cdot 0,85 + 0,19)$

$\boxed{S = 2,8}$ ×

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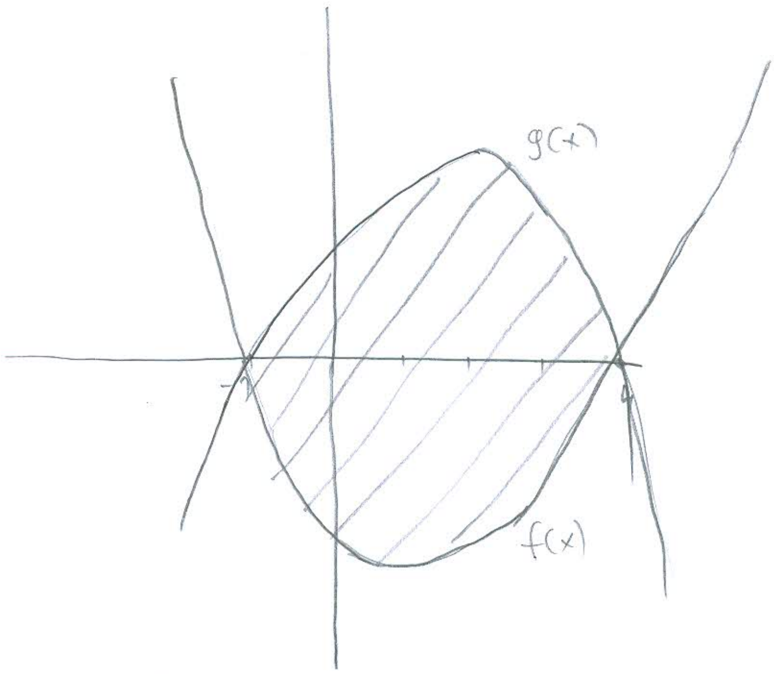
5.) $f(x) = x^2 - 3x - 4$

$g(x) = -x^2 + 3x + 4$



$-x^2 + 3x + 4 = x^2 - 3x - 4$

$-2x^2 + 6x + 8 = 0$

$x = \frac{-6 \pm \sqrt{36 + 64}}{-4}$

$x = \frac{-6 \pm 10}{-4}$

$x_1 = 4$

$x_2 = -1$

$x^2 - 3x - 4 = 0$

$x = \frac{3 \pm \sqrt{9 + 16}}{2}$

$x = \frac{3 \pm 5}{2}$

$x_1 = 4$

$x_2 = -1$

$-x^2 + 3x + 4 = 0$

$x = \frac{-3 \pm \sqrt{9 + 16}}{-2}$

$x = \frac{-3 \pm 5}{-2}$

$x_1 = 4$

$x_2 = -1$

$\int (x^2 - 3x - 4)\,dx = \frac{x^3}{3} - \frac{3x^2}{2} - 4x$

$\int (-x^2 + 3x + 4)\,dx = -\frac{x^3}{3} + \frac{3x^2}{2} + 4x$

$\left[-\frac{x^3}{3} + \frac{3x^2}{2} + 4x - \frac{x^3}{3} + \frac{3x^2}{2} + 4x \right]_{-1}^{4} =$

$\left[-\frac{2}{3}x^3 + 3x^2 + 8x \right]_{-1}^{4} =$

$\left[-\frac{2}{3} \cdot 4^3 + 3 \cdot 16 + 8 \cdot 4 - \left(\frac{2}{3} + 3 - 8 \right) \right] =$

$-42,7 + 48 + 32 + 4,33 = 41,63$ ✓

JAKOV ZUBCIC 0269092107

1. $xy' + y - e^x = 0 \quad , y(1) = 1$

$$xy' + y = e^x \;/\cdot \frac{1}{x}$$

$$\frac{dy}{dx} + \frac{1}{x}\cdot y = \frac{e^x}{x}$$

$$\frac{dy}{dx} + \frac{1}{x}\cdot y = 0$$

$$\frac{dy}{dx} = -\frac{1}{x}\cdot y$$

$$\frac{dy}{y} = -\frac{dx}{x} \;/\int$$

$$\int\frac{dy}{y} = -\int\frac{dx}{x}$$

$$\ln|y| = -\ln|x| + C$$

$$\ln|y| = \ln C - \ln|x|$$

$$y = \frac{C}{x}$$

$$C = U(x)$$

$$y = \frac{U(x)}{x}$$

$$x\cdot\left(\frac{U}{x}\right)' + \left(\frac{U}{x}\right) - e^x = 0$$

$$x\cdot\left(\frac{U'\cdot x - U}{x^2}\right) + \frac{U}{x} - e^x = 0$$

$$\frac{\cancel{x^2}\cdot U'}{\cancel{x^2}} - \frac{\cancel{x}\cdot U}{x^{\cancel{2}}} + \frac{U}{x} - e^x =$$

$$U' - \cancel{\frac{U}{x}} + \cancel{\frac{U}{x}} = e^x$$

$$du = e^x\,dx \;/\int$$

$$U = e^x + C$$

$$y = \frac{e^x + C}{x}$$

$$1 = e^1 + C$$

$$\boxed{1 - e = C}$$

PARTIKULARNO RJEŠENJE:

$$\boxed{y = \frac{e^x + 1 - e}{x}}$$

$$1 = \cancel{e^1} + 1 - \cancel{e^1}$$

$$1 = 1 \quad \checkmark$$

PROVJERA:

$$x\cdot\left(\frac{e^x}{x} + \frac{C}{x}\right)' + \frac{e^x + C}{x} - e^x = 0$$

$$x\cdot\left(\frac{e^x\cdot x - e^x}{x^2} + \frac{x - C}{x^2}\right) + \frac{e^x + C}{x} - e^x = 0$$

$$\cancel{e^x} - \cancel{\frac{e^x}{x}} - \cancel{\frac{C}{x}} + \cancel{\frac{e^x}{x}} + \cancel{\frac{C}{x}} - \cancel{e^x} = 0$$

$$0 = 0$$

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IME I PREZIME: IVAN VRLIKA **BROJ INDEKSA:**	
Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor	

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Ivan Welka

1.

$xy' + y - e^x = 0 \qquad y(1) = 1$

$xy' + y = e^x \quad \Big| \cdot \frac{1}{x}$ LINEARNA ODJ

$y' + \frac{1}{x} \cdot y = \frac{e^x}{x} \quad \Rightarrow \quad y' + g(x) \cdot h(y) = h(x)$

$y' + \frac{1}{x} \cdot y = 0$

$y' = -\frac{1}{x} \cdot y \quad \Rightarrow \quad y' = G(x) \cdot H(y)$

$\frac{dy}{dx} = -\frac{1}{x} \cdot y \quad \Big| \cdot dx \cdot \frac{1}{y}$

$\frac{1}{y} dy = -\frac{1}{x} dx$

$\int \frac{1}{y} dy = -\int \frac{1}{x} dx$

$\ln|y| = -\ln|x| + C$

$\ln|y| = -\ln(x \cdot C)$

$y = \frac{1}{x \cdot C}$

PROVJERA:

$\left(\frac{1}{x \cdot C}\right)' + \frac{1}{x \cdot C} \cdot \frac{1}{x} = 0$

$\frac{1' \cdot (x \cdot C) - 1 \cdot (x \cdot C)'}{(x \cdot C)^2} + \frac{1}{x^2 \cdot C} = 0$

$-\frac{C}{x^2 \cdot C} + \frac{1}{x^2 \cdot C} = 0$

$y = \frac{1}{x \cdot u(x)}$

$y' = \frac{1' \cdot (x \cdot u(x)) - 1 \cdot (x \cdot u(x))'}{(x \cdot u(x))^2}$

$y' = -\frac{(u(x) + x \cdot u'(x))}{(x \cdot u(x))^2}$

$\frac{-u(x) - x \cdot u'(x)}{x^2 u^2(x)} + \frac{1}{x \cdot u(x)} \cdot \frac{1}{x} = \frac{e^x}{x}$

$\frac{-\cancel{u(x)} - \cancel{x} \cdot u'(x) + \cancel{u(x)}}{x^{\cancel{2}} u(x)^2} = \frac{e^x}{x}$

$\frac{-u'(x)}{x \cdot u^2(x)} = \frac{e^x}{x}$

$\cancel{x} \cdot u^2(x) \cdot e^x = -u'(x) \cdot \cancel{x}$

$u^2(x) \cdot e^x = -u'(x)$

$\varnothing$

3. $f(x,y) = x^2 - y$ $\qquad$ $D(f) = \{(x,y) \in \mathbb{R}^2 : \mathbb{R}^2\}$

$f(x,y) = c$

$x^2 - y = c$

$-y = \frac{c}{x^2} \quad \Big| \cdot (-1)$

$y = -\frac{c}{x^2}$

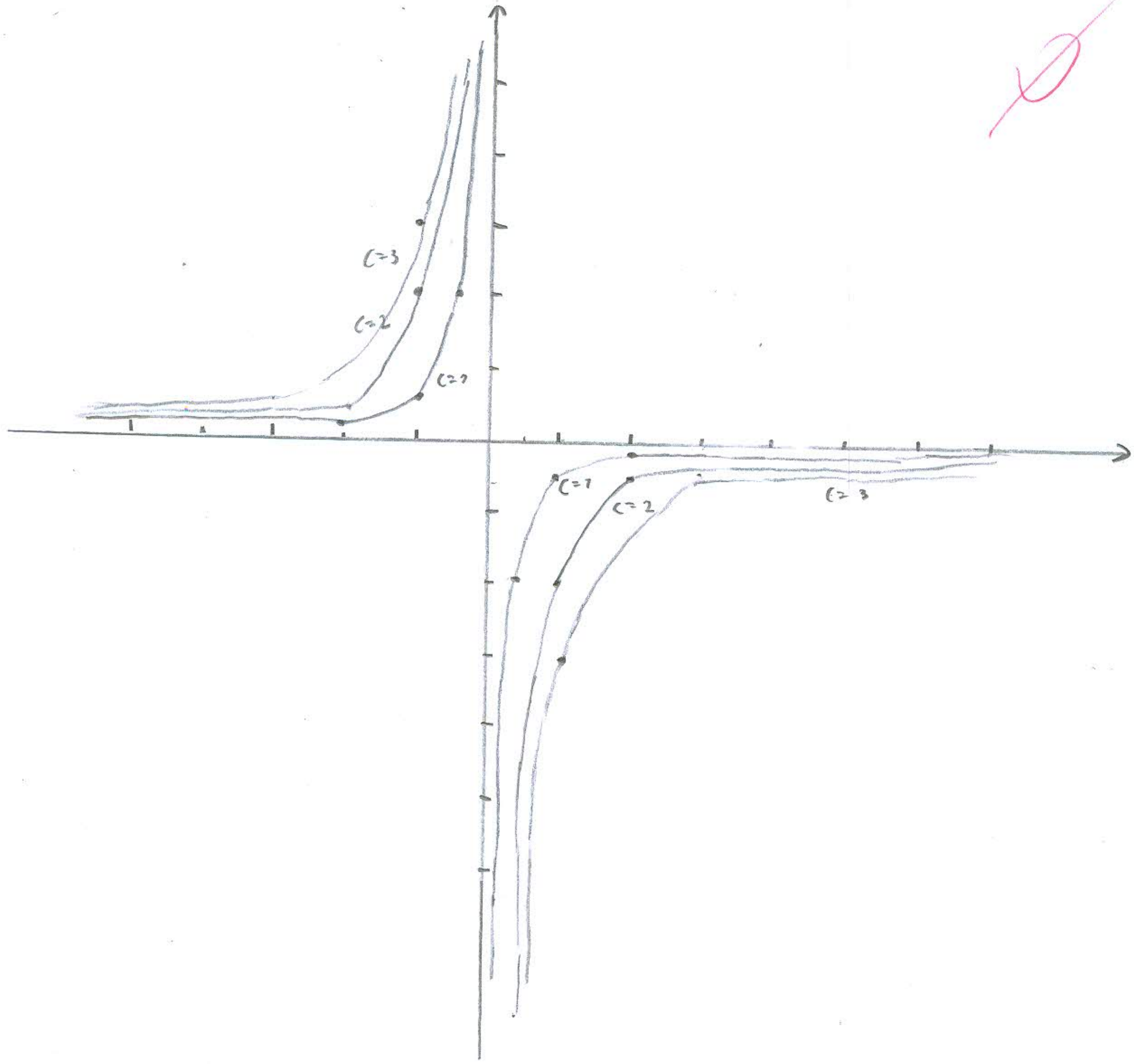
$c = \frac{1}{2} \Rightarrow y = -\frac{c}{x^2} \Rightarrow -\frac{1}{2} \cdot \frac{1}{x^2}$

$c = 2 \Rightarrow y = -\frac{c}{x^2} \Rightarrow -2 \cdot \frac{1}{x^2}$

$c = 3 \Rightarrow y = -\frac{c}{x^2} \Rightarrow -3 \cdot \frac{1}{x^2}$

RAZINSKE

$y = x^2 - c$



$\emptyset$

Leon Volika

4. $\int_0^{\pi} \frac{\sin x}{\cos x + 5}\, dx$

$$\int \frac{\sin x}{\cos x + 5}\, dx = \left\{ \begin{array}{ll} t = \tan\frac{x}{2} & \sin x = \frac{2t}{1+t^2} \\ dx = \frac{2dt}{1+t^2} & \cos x = \frac{1-t^2}{1+t^2} \end{array} \right\} = \int \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2} + 5} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{\frac{2t}{\cancel{(1+t^2)}}}{\frac{1-t^2+5+5t^2}{\cancel{(1+t^2)}}} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{2t}{4t^2+6} \cdot \frac{2dt}{1+t^2} = \int \frac{2t}{\cancel{2}(2t^2+3)} \cdot \frac{\cancel{2}dt}{1+t^2}$$

$$= \int \frac{2t}{(2t^2+3)(1+t^2)}\, dt = \int \frac{-4t}{2t^2+3}\, dt + \int \frac{2t}{1+t^2}\, dt$$

$$= -\ln|2t^2+3| + \ln|1+t^2| + C$$

$$= -\ln\left|2\tan^2\frac{x}{2} + 3\right| + \ln\left|1 + \tan^2\frac{x}{2}\right| + C$$

$$\frac{2t}{(2t^2+3)(1+t^2)} = \frac{At+B}{(2t^2+3)} + \frac{Ct+D}{(1+t^2)} \quad \Big| \cdot (2t^2+3)(1+t^2)$$

$$2t = (At+B)(1+t^2) + (Ct+D)(2t^2+3)$$

$$2t = At + At^3 + B + Bt^2 + 2Ct^3 + 3Ct + 2Dt^2 + 3D$$

$$2t = (A+2C)t^3 + (B+2D)t^2 + (A+3C)t + B + 3D$$

$A + 2C = 0$

$B + 2D = 0$

$A + 3C = 2$

$B + 3D = 0$

$A = 2 - 3C$

$A = 2 - 6$

$\boxed{A = -4}$

$2 - 3C + 2C = 0$

$2 - C = 0$

$-C = -2$

$\boxed{C = 2}$

$B = -2D$

$\boxed{B = 0}$

$-2D + 3D = 0$

$\boxed{D = 0}$

$$\int \frac{-4t}{2t^2+3}\,dt = -4\int \frac{t}{2t^2+3}\,dt = \begin{bmatrix} 2t^2+3 = u \,|' \\ 4t\,dt = du \\ t\,dt = \frac{du}{4} \end{bmatrix} = -4\int \frac{1}{u}\,\frac{du}{4}$$

$$= -4 \cdot \frac{1}{4}\int \frac{1}{u}\,du$$

$$= -\ln|u| + C$$

$$= -\ln|2t^2+3| + C$$

$$\int \frac{2t}{1+t^2}\,dt = 2\int \frac{t}{1+t^2}\,dt = \begin{bmatrix} 1+t^2 = u \,|' \\ 2t\,dt = du \\ t\,dt = \frac{du}{2} \end{bmatrix} = 2\int \frac{1}{u}\,\frac{du}{2}$$

$$= 2\,\frac{1}{2}\int \frac{1}{u}\,du$$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

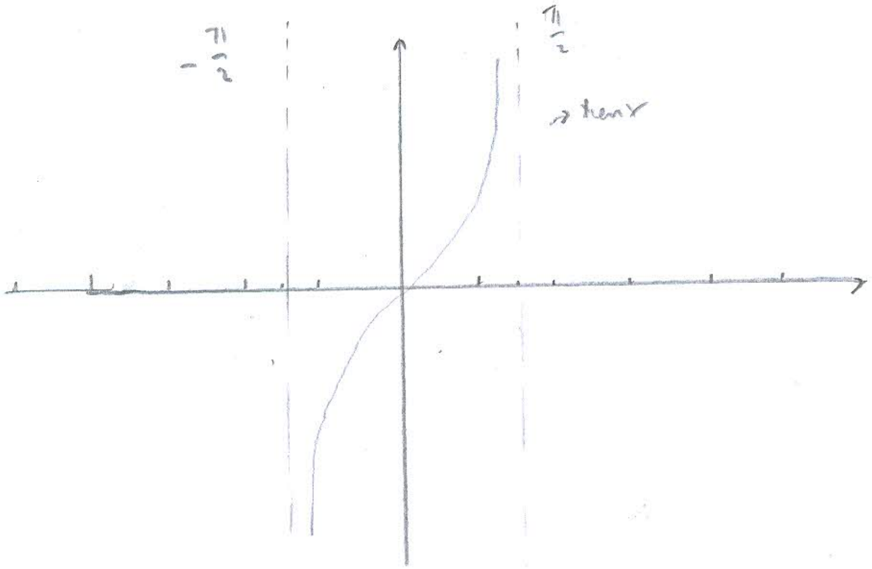
$$= \ln|1+t^2| + C$$

$$\lim_{b\to\pi-}\int_0^b \frac{\sin x}{\cos x+5}\,dx = \lim_{b\to\pi-}\left[-\ln\left|2\tan^2\frac{x}{2}+3\right| + \ln\left|1+\tan^2\frac{x}{2}\right|\right]_0^b$$

$\nearrow \frac{\pi}{2}$ -

$$= -\infty + \infty = NIP$$

$\cancel{}$



Ivan Velika

$$\int_0^{\pi} \frac{\sin x}{\cos x+5}\,dx = \Big[-\ln|\cos x+5|\Big]_0^{\pi} = -\ln|\cos\pi+5| + \ln|\cos 0+5|$$

$$= -\ln|-1+5| + \ln|1+5|$$

$$= \ln 6 - \ln 4 = \_\_$$

Ivan Vrčika

2.

$f(x,y) = e^x - x + y^2 + 2y$

$\frac{\partial f}{\partial x} = e^x - 1 \qquad \frac{\partial f}{\partial y} = 2y + 2$

$e^x - 1 = 0 \qquad 2y + 2 = 0$

$e^x = 1 \qquad 2y = -2$

$x = \ln 1 \qquad y = -\frac{2}{2}$

$x = 0 \qquad y = -1$

$T_0(0,-1)$

↓ stacionarne točke

$\frac{\partial^2 f}{\partial x^2} = e^x \qquad \frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial^2 f}{\partial x^2}(T_0) = e^0 = 1 \qquad \frac{\partial^2 f}{\partial x \partial y} = 0$

$\Delta = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 \cdot 1 - 0 = 2$

$\Delta > 0 \quad \frac{\partial^2 f}{\partial x^2} > 0 \quad T_0(0,-1) \Rightarrow$ lokalni minimum ✓

Leon Vrlika

6.

$$\int_1^4 \frac{1+\sqrt{x}}{x^2}\,dx = \left[-\frac{1}{x} - \frac{2}{\sqrt{x}}\right]_1^4 = -\frac{5}{4} + 3 = \frac{7}{4} = 1.75$$

$$\int \frac{1+\sqrt{x}}{x^2}\,dx = \int \frac{1}{x^2}\,dx + \int \frac{\sqrt{x}}{x^2}\,dx$$

$$= \int x^{-2}\,dx + \int \frac{x^{\frac{1}{2}}}{x^2}\,dx$$

$$= \frac{x^{-1}}{-1} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$= -\frac{1}{x} - 2x^{-\frac{1}{2}} + C$$

$$= -\frac{1}{x} - \frac{2}{\sqrt{x}} + C$$

PROVJERA SIMPSON

k	0	1	2
x_k	1	2.5	4
$y_k = f(x_k)$	2	0.41298	0.1875
$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$	1.919		

Leon Veika

5.

$f(x) = x^2 - 3x - 4$

$x_1x_2 = \dfrac{3 \pm \sqrt{9 - 4 \cdot -4}}{2}$

$x_1x_2 = \dfrac{3 \pm \sqrt{25}}{2}$

$x_1x_2 = \dfrac{3 \pm 5}{2}$

$x_1 = 4$
$x_2 = -1$

$x = -\dfrac{b}{2a}$ $y = -\dfrac{b^2 - 4ac}{4a}$

$x = \dfrac{3}{2}$ $y = -\dfrac{25}{4} = -6.25$

$T\left(\frac{3}{2}, -\frac{25}{4}\right)$

$a > 0$ $\cup$

↓

tjeme

$g(x) = -x^2 + 3x + 4$

$x_1x_2 = \dfrac{-3 \pm \sqrt{9 - 4 \cdot (-1) \cdot 4}}{-2}$

$x_1x_2 = \dfrac{-3 \pm \sqrt{25}}{-2}$

$x_1x_2 = \dfrac{-3 \pm 5}{-2}$

$x_1 = 4$
$x_2 = -1$

$a < 0$ $\cap$

$x = -\dfrac{b}{2a}$

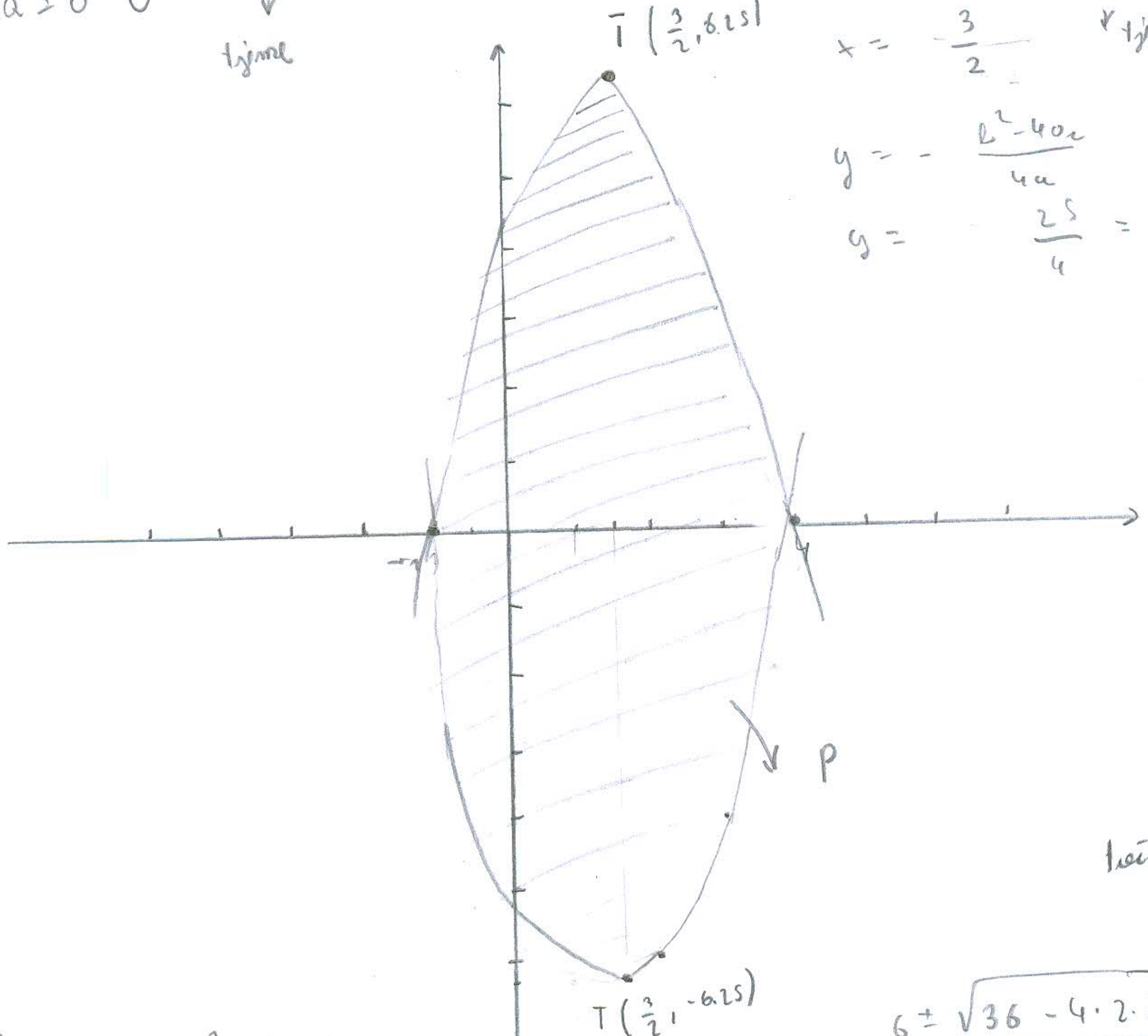
$x = \dfrac{3}{2}$

$T\left(\frac{3}{2}, 6.25\right)$

↓ tjeme

$y = -\dfrac{b^2 - 4ac}{4a}$

$y = \dfrac{25}{4} = 6.25$



$x^2 - 3x - 4 = -x^2 + 3x + 4$

$2x^2 - 6x - 8 = 0$

$x_1x_2 = \dfrac{6 \pm \sqrt{36 - 4 \cdot 2 \cdot -8}}{4}$

$x_1x_2 = \dfrac{6 \pm \sqrt{100}}{4}$

$x_{12} = \dfrac{6 \pm 10}{4}$

$\boxed{x_1 = 4 \quad x_2 = -1}$

↑ točke sjecišta

Ivan Vrlika

$$P = \int_{-1}^{4} -x^2+3x+4-(x^2-3x-4) = \int_{-1}^{4} -x^2+3x+4-x^2+3x+4$$

$$P = \int_{-1}^{4} -2x^2+6x+8\,dx = \left[-\frac{2}{3}x^3+3x^2+8x\right]_{-1}^{4}$$

$$= \frac{112}{3}+\frac{13}{3} = \frac{125}{3} = 41.6666$$

$$\int -2x^2+6x+8\,dx = -2\int x^2\,dx + 6\int x\,dx + 8\int dx$$

$$= -2\frac{x^3}{3} + 6\frac{x^2}{2} + 8x + C$$

$$= -\frac{2}{3}x^3 + 3x^2 + 8x + C$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XXO

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: Kristian Sorić **BROJ INDEKSA:**

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

1. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = e^x - x + y^2 + 2y$. 15
3. Skicirati razinske krivulje za funkciju $f(x, y) = x^2 - y$. 15
4. $\int_0^\pi \frac{\sin x}{\cos x + 5} dx$? 20
5. Izračunaj površinu lika omeđenog krivuljama $f(x) = x^2 - 3x - 4$ i $g(x) = -x^2 + 3x + 4$. 15
6. Izračunaj $\int_1^4 \frac{1+\sqrt{x}}{x^2} dx$ i napravi provjeru izračunom aproksimacije integrala Simsponovom metodom. 10+10

Ukupno: 40

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \|\cos x\| + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int \frac{dx}{x} = \ln \|x\| + C$	$\int \cot x \, dx = \ln \|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\| x + \sqrt{x^2 \pm a^2} \right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a}\right)\right] + C$	

binomni integral $x_1 n_1$

6. $$\int_1^4 \frac{1+\sqrt{x}}{x^2}dx = \int_1^4 \frac{1+x^{\frac{1}{2}}}{x^2}dx = \begin{bmatrix} x = t^2 \\ dx = 2t\,dt \\ \begin{array}{c|c|c} x & 1 & 4 \\ t & 1 & 2 \end{array} \end{bmatrix} =$$

$x = t^s$ $x = t^2$

$s = 2$

$d = 1$

n	1		2
t_k	1	1.5	2
$y_k(t_k)$	4	1.318	0.60355
$\frac{d}{6}(f_1+4f_2+f_3)$		1.66	

$$= \int_1^2 \frac{1+t}{t^4} 2t\,dt =$$

$$= 2\int_1^2 \frac{1+t}{t^{4\,3}} \cdot \cancel{t}\,dt =$$

$$= 2\int_1^2 \frac{1+t}{t^3}dt = 2\int_1^2 \frac{dt}{t^3} + 2\int_1^2 \frac{1}{t^2}dt = 2\int_1^2 t^{-3}dt + 2\int_1^2 t^{-2}dt =$$

$$= 2\left[\frac{t^{-2}}{-2}\right]_1^2 + 2\left[\frac{t^{-1}}{-1}\right]_1^2 = 2\left[-\frac{1}{2}\cdot\frac{1}{t^2}\right]_1^2 + 2\left[-\frac{1}{t}\right]_1^2 =$$

$$2\cdot\left(-\frac{1}{2}\cdot\frac{1}{2^2} - \left(-\frac{1}{2}\cdot\frac{1}{1^2}\right)\right) + 2\left(-\frac{1}{2} - \left(-\frac{1}{1}\right)\right) = \frac{7}{4} = \underline{\underline{1.75}}$$ ✓

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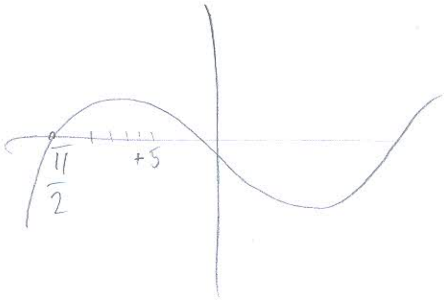
① $\int_0^{\pi} \frac{\sin x}{\cos x + 5}\, dx = 0{,}405$ $\quad \cos x = 0 \quad$ za $\cos\left(\frac{\pi}{2}\right)$

prekidnost

$\frac{10+\pi}{2}$



$\int_0 \frac{\sin x}{\cos x+5}\, dx =$

$t = \tan\frac{x}{2} \qquad dx = 2\arctan x$

x	0	π
t	0	1

$$\int \frac{\frac{2t}{1+t}}{\frac{1-t}{1+t}+5} \cdot \frac{2dt}{1+t} = \int \frac{\frac{2t}{1+t}}{\frac{1-t+5+5t}{1+t}} \cdot \frac{2dt}{1+t} = \int \left(\frac{\frac{2t}{1+t}}{\frac{6+4t}{1+t}}\right) \cdot \frac{2dt}{1+t} =$$

$$= \int \frac{2t(1+t)}{\cancel{1+t}(6+4t)} \frac{2dt}{\cancel{1+t}} = \int \frac{2t+2t^2}{6+4t} \cdot 2dt = \int \frac{\cancel{2}(t+t^2)}{\cancel{2}(3+2t)} \cdot 2dt =$$

$$2\int_0^{\frac{\pi}{2}} \frac{t+t^2}{3+2t} \cdot dt =$$

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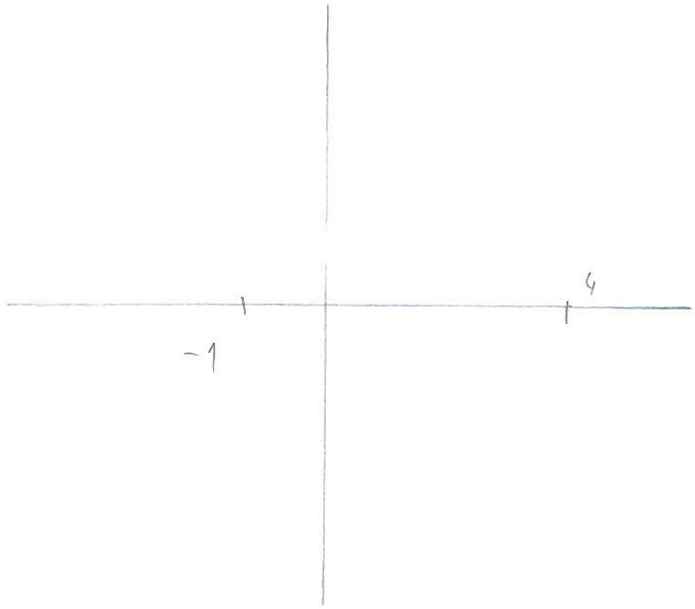
⑤ $f(x) = x^2 - 3x - 4 \qquad g(x) = -x^2 + 3x + 4$



$$x^2 - 3x - 4 = -x^2 + 3x + 4$$

$$2x^2 - 6x - 8 = 0$$

$$x_1 = 4 \qquad x_2 = -1$$

$$\int_{-1}^{4} -x^2 + 3x + 4 - x^2 + 3x + 4$$

$$\int_{-1}^{4} -2x^2 + 6x + 8 = \int_{-1}^{4} -2x^2\,dx + \int_{-1}^{4} 6x\,dx + \int_{-1}^{4} 8\,dx =$$

$$-2\left[x^3 \cdot \frac{1}{3}\right]_{-1}^{4} + 6\left[x^2 \cdot \frac{1}{2}\right]_{-1}^{4} + 8\left[x\right]_{-1}^{4} = -2 \cdot \left(\frac{1}{3} \cdot 4^3 - \frac{1}{3} \cdot (-1)^3\right) + 6\left(\frac{1}{2} \cdot 4^2 - \frac{1}{2} \cdot (-1)^2\right)$$

$$+ 8(4+1) = \frac{125}{3}$$

✓

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2. $f(x,y) = e^x - x + y^2 + 2y$

$\frac{\partial f}{\partial x} = e^x - 1 + 0 + 0$ $\qquad$ $\frac{\partial f}{\partial y} = 2y + 2$

$= e^x - 1$

$\frac{\partial f}{\partial x^2} = e^x$ $\qquad$ $\frac{\partial f}{\partial y^2} = 2$

$e^x = 0$ $\qquad$ $\cancel{2}y = 0$

$x = 0$

$T_0(0,0)$

$\Delta = [$

?

ϕ

Kristian Sorić Brodostrojarstvo redovni

(1)

$xy' + y - e^x = 0 \quad /:x \qquad \overset{x}{y}(1) = \overset{y}{1}$

$y' + \overset{z}{\left(\frac{y}{x}\right)} - \frac{e^x}{x} = 0 \qquad z = \frac{y}{x}$

$y = z \cdot x$

$y' = z' \cdot x + z$

$z' \cdot x + z = -z + \frac{e^x}{x}$

$z' \cdot x + 2z = \frac{e^x}{x}$

$\frac{dz}{dx} \cdot x + 2z = \frac{e^x}{x} \quad /:x$

$\frac{dz}{dx} + \frac{2z}{x} = \frac{e^x}{x^2}$

$\frac{dz}{dx} = \frac{e^x}{x^2} - \frac{2z}{x} \quad /\cdot dx$

$dz = \left(\frac{e^x}{x^2} - \frac{2z}{x}\right) dx$

$\int dz = \int \left(\frac{e^x}{x^2} - \frac{2z}{x}\right) dx$

$\int dz = \int \frac{e^x}{x^2} dx - 2 \int \frac{z}{x} dx \; ?$

$xy' + y - e^x = 0$

$xy' + y = e^x$

$\emptyset$

3. $f(x,y) = x^2 - y$ $\qquad$ $x^2 - y = 0$ $\quad$ $D_f \{\mathbb{R}\}$

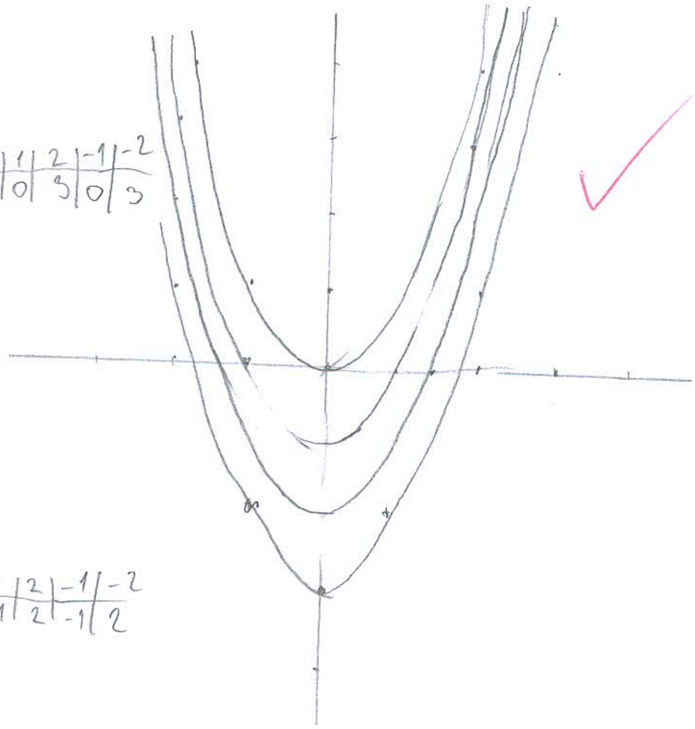
$C = 0$

$x^2 - y = 0$

x	0	1	2	-1	-2
y	0	1	4	1	4

$-y = -x^2$

$y = x^2$



$C = 1$

$x^2 - y = 1$

x	0	1	2	-1	-2
y	-1	0	3	0	3

$-y = 1 - x^2$

$y = x^2 - 1$

$C = 2$

$x^2 - y = 2$

x	0	1	2	-1	-2
y	-2	-1	2	-1	2

$-y = 2 - x^2$

$y = x^2 - 2$

$C = 3$

$x^2 - y = 3$

x	0	1	2	-1	-2
y	-3	-2	1	-2	1

$-y = 3 - x^2$

$y = x^2 - 3$