

odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: **KARLO ŠTURA**

BROJ INDEKSA: **17-2-0379-2014**  
**0269087719**

xxx

Želim ustmeni kod (zaokružiti):

**prof. Uglešić**

asistent Kosor

1. Nađi koliko iznosi  $f(2.5)$  ako  $f$  zadovoljava  $\sin x dy = y \ln y dx$  i  $y(1) = 2$ . 15
2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete:  $y'' + 4y = 0$ , uz  $y(0) = 0$  i  $y'(0) = 2$ .  
Na kraju provjeri rješenje. 15
3. Skicirati razinske krivulje za  $f(x, y) = x^2 + y^2$ . Ima li ekstrema? Pronađi tangencijalnu ravninu u točki koju možeš sam odabrati. 15
4.  $\int_0^1 3x e^{x+1} dx = ?$  20
5.  $\int_1^3 \frac{dx}{x^2 - 2x + 4} = ?$  15
6. Neka je  $f(x) = \tan x$ . Skicirati graf funkcije  $f$  i površinu određenu integralom. Odrediti  $\int_0^{\pi/2} f(x) dx$ .  
Kolika je skicirana površina ispod grafa funkcije  $f$ ? 20

**15**

Ukupno:

**65**

$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x \sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$	



$$\textcircled{4} \int_0^1 3x e^{x+1} dx = \left[ \begin{array}{l} 3x = u \quad e^{x+1} dx = dv \\ 3dx = du \quad \int e^{x+1} dx = v \\ e^{x+1} = v \end{array} \right]$$

$$= uv - \int v du$$

$$= \left[ 3x \cdot e^{x+1} - \int_0^1 e^{x+1} \cdot 3 dx \right]$$

$$= \left[ 3x \cdot e^{x+1} - 3 \int_0^1 e^{x+1} dx \right]$$

$$= \left[ 3x \cdot e^{x+1} - 3e^{x+1} \right]_0^1$$

$$= 0 + 8,155 = \boxed{8,155} \quad \checkmark$$

$$\int e^{x+1} dx = \left[ \begin{array}{l} x+1 = t \\ dx = dt \end{array} \right]$$

$$\int e^t dt = e^{x+1} + c$$

$$= (3 \cdot 1 \cdot e^2 - 3e^2) - (-3e)$$

$$\begin{array}{r} 3 \cdot 2 \cdot 77 \\ 6 \\ 213 \\ \hline 8,13 \checkmark \end{array}$$

$$\textcircled{5} \int_1^3 \frac{dx}{x^2 - 2x + 4} = \int_1^3 \frac{dx}{x^2 - 2x + 1 + 3}$$

$$= \int_1^3 \frac{dx}{(x-1)^2 + \sqrt{3}^2} = \left[ \begin{array}{l} x-1 = t \\ dx = dt \end{array} \right] = \int_1^3 \frac{dt}{t^2 + \sqrt{3}^2}$$

$$= \left[ \frac{1}{\sqrt{3}} \arctan \frac{t}{\sqrt{3}} \right]_1^3 = \left[ \frac{1}{\sqrt{3}} \arctan \frac{x-1}{\sqrt{3}} \right]_1^3$$

$$= 0,495 - 0 = \boxed{0,495} \quad \checkmark$$

$$x^2 - 2x + 4 \neq 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{4 - 16}}{2} \text{ NIJE CELANI BROJ}$$

$$D_f = \mathbb{R}^2$$

$$\textcircled{3} \quad f(x,y) = x^2 + y^2$$

RA2, KRIV:

$$f(x,y) = c$$

$$c = 1$$

$$c = 2$$

$$c = 3$$

$$x^2 + y^2 = 1$$

$$x^2 + y^2 = 2$$

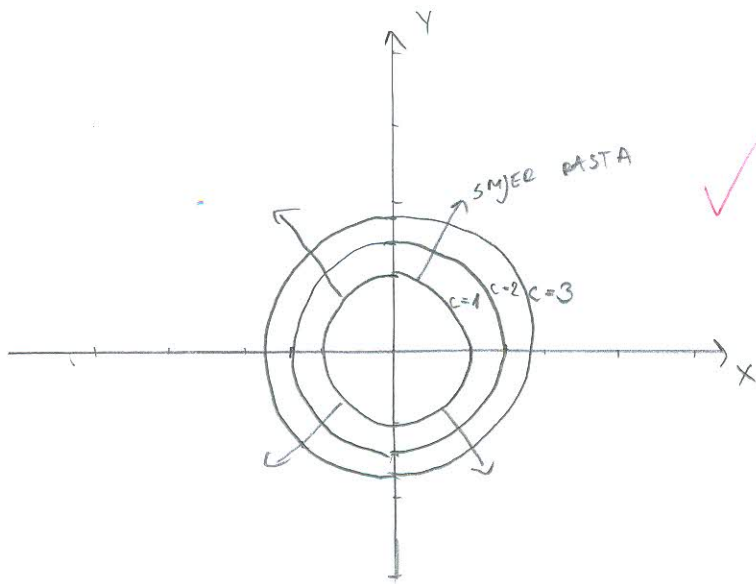
$$x^2 + y^2 = 3$$

$$x^2 + y^2 = r^2$$

$$r = \sqrt{1} = 1$$

$$r = \sqrt{2} = 1,41$$

$$r = \sqrt{3} = 1,73$$



EKSTREMI:

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

$$2x = 0$$

$$\boxed{x = 0}$$

$$2y = 0$$

$$\boxed{y = 0}$$

$$T_0(0,0)$$

$$\frac{\partial^2 f}{\partial x^2} = 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial y \partial x} = 0$$

$$\Delta = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2}(T_0) & \frac{\partial^2 f}{\partial x \partial y}(T_0) \\ \frac{\partial^2 f}{\partial y \partial x}(T_0) & \frac{\partial^2 f}{\partial y^2}(T_0) \end{vmatrix}$$

$$\frac{\partial^2 f}{\partial x^2} = 2 > 0$$

MINIMUM

$$\boxed{T(0,0,0)}$$

$$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 2 \cdot 2 - 0 \cdot 0 = 4 > 0$$

IMA EKS

3) TANG. PAVNINA

$$z = x^2 + y^2$$

$$T(1,1,2)$$

$$z - z_0 = f(x)(x - x_0) + f(y)(y - y_0)$$

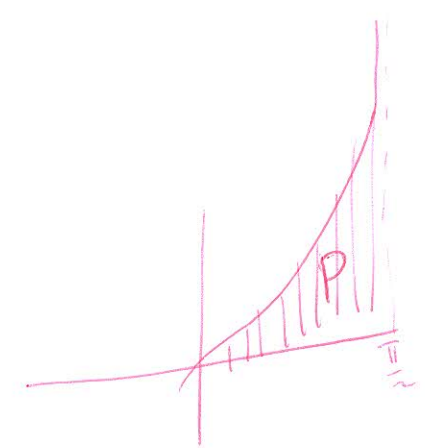
$$z - 2 = 2(x - 1) + 2(y - 1)$$

$$z - 2 = 2x - 2 + 2y - 2 \Rightarrow z - 2 = 2x + 2y - 4$$

$$\frac{\partial f}{\partial x} = 2x = 2 \cdot 1 = 2$$

$$\frac{\partial f}{\partial y} = 2y = 2 \cdot 1 = 2$$

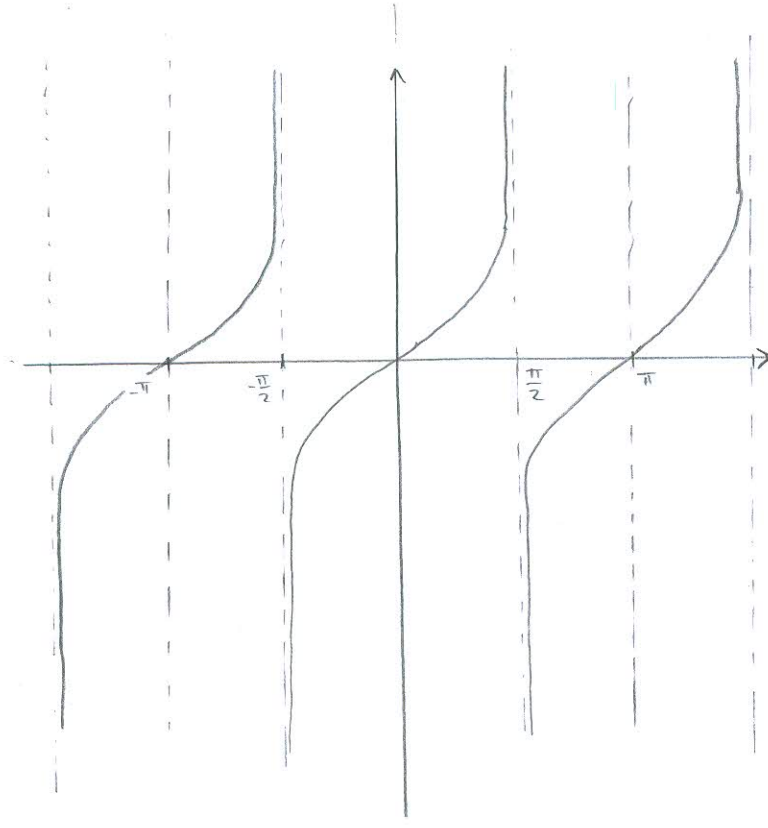
KARLO STURDA  
NAUTIKA  
0269 0877 19



1,571      15

6)  $f(x) = \tan x$

$$\int_0^{\frac{\pi}{2}} f(x) dx$$



$$\int_0^{\frac{\pi}{2}} \tan x dx = \left[ -\ln |\cos x| \right]_0^{\frac{\pi}{2}}$$

$$= -\ln |0| - (-\ln |1|)$$

$\downarrow$   
 $\Rightarrow (-\infty)$   
 $\Rightarrow +\infty$

$$= +\infty + 0 = +\infty \checkmark$$

$$P = \lim_{\substack{a \rightarrow -\frac{\pi}{2} \\ b \rightarrow \frac{\pi}{2}}} \int_a^b \tan x = \lim_{\substack{a \rightarrow -\frac{\pi}{2} \\ b \rightarrow \frac{\pi}{2}}} \left[ -\ln |\cos x| \right]_a^b$$

$$= -\ln |\cos \frac{\pi}{2}| - (-\ln |\cos \frac{\pi}{2}|)$$

$$= -\ln |0| + \ln |0| = \frac{-(-\infty) + (-\infty)}{+\infty - \infty} \text{ N/P}$$



