MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

XXX

IME I PREZIME: KARLO ŠTURA

BROJ INDEKSA: 17-2-0379-2014 0269087719

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $\sin x\, dy = y \ln y\, \mathrm{d}x$ i $y(1) = 2$. 15
2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 4y = 0$, uz $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje. 15
3. Skicirati razinske krivulje za $f(x, y) = x^2 + y^2$. Ima li ekstrema? Pronađi tangencijalnu ravninu u točki koju možeš sam odabrati. 15
4. $\displaystyle\int_0^1 3x\,\mathrm{e}^{x+1}\,\mathrm{d}x = ?$ 20
5. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ 15
6. Neka je $f(x) = \tan x$. Skicirati graf funkcije f i površinu određenu integralom. Odrediti $\int_0^{\pi/2} f(x)\,\mathrm{d}x$. Kolika je skicirana površina ispod grafa funkcije f? ~~20~~ 15

Ukupno: 65

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

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(4) $\int_0^1 3xe^{x+1}dx = \begin{bmatrix} 3x=u & e^{x+1}dx=dv \\ 3dx=du & \int e^{x+1}dx=v \\ & e^{x+1}=v \end{bmatrix}$

$\int e^{x+1}dx = \begin{bmatrix} x+1=t \\ dx=dt \end{bmatrix}$

$\int e^t dt = e^{x+1}+C$

$= uv - \int vdu$

$= \left[3x\cdot e^{x+1} - \int e^{x+1}\cdot 3\,dx\right]_0^1$

$= \left[3x\cdot e^{x+1} - 3\int e^{x+1}dx\right]_0^1$

$= \left[3x\cdot e^{x+1} - 3e^{x+1}\right]_0^1$

$= (3\cdot 1\cdot e^2 - 3e^2) - (-3e)$

$3.2.77$

6

2.13

8.13 ✓

$= 0 + 8{,}155 = \boxed{8{,}155}$ ✓

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(5) $\int_1^3 \frac{dx}{x^2-2x+4} = \int_1^3 \frac{dx}{x^2-2x+1+3}$

$x^2-2x+4 \neq 0$

$x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{2\pm\sqrt{(-2)^2-4\cdot1\cdot4}}{2\cdot1}$

$= \frac{2\pm\sqrt{4-16}}{2}$ NIJE REALAN BROJ

$D_f = \mathbb{R}$

$= \int_1^3 \frac{dx}{(x-1)^2+\sqrt{3}^2} = \begin{bmatrix} x-1=t \\ dx=dt \end{bmatrix} = \int_1^3 \frac{dt}{t^2+\sqrt{3}^2}$

$= \left[\frac{1}{\sqrt{3}} \arctan\frac{t}{\sqrt{3}}\right]_1^3 = \left[\frac{1}{\sqrt{3}}\arctan\frac{x-1}{\sqrt{3}}\right]_1^3$

$= 0{,}495 - 0 = \boxed{0{,}495}$ ✓

③ $f(x,y)=x^2+y^2$

RAZ. KRIV:

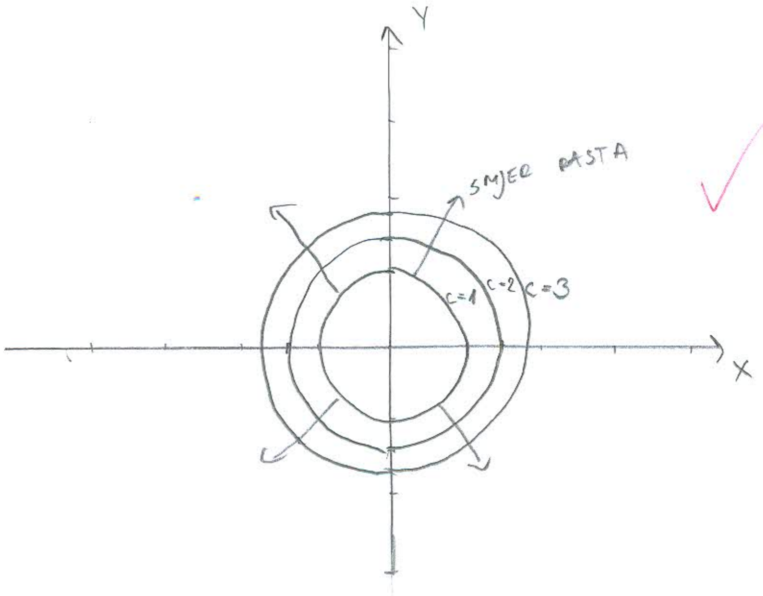
$f(x,y)=c$

$c=1$ $\quad$ $c=2$ $\quad$ $c=3$

$x^2+y^2=1$ $\quad$ $x^2+y^2=2$ $\quad$ $x^2+y^2=3$

$x^2+y^2=r^2$

$r=\sqrt{1}=1$ $\quad$ $r=\sqrt{2}=1,41$ $\quad$ $r=\sqrt{3}=1,73$



EKSTREMI:

$\frac{\partial f}{\partial x}=2x$

$\frac{\partial f}{\partial y}=2y$ $\quad$ $T_0(0,0)$

$2x=0$

$\boxed{x=0}$

$2y=0$

$\boxed{y=0}$

$\frac{\partial^2 f}{\partial x^2}=2$ $\quad$ $\frac{\partial^2 f}{\partial x \partial y}=0$

$\frac{\partial^2 f}{\partial y^2}=2$ $\quad$ $\frac{\partial^2 f}{\partial y \partial x}=0$

$$\Delta=\begin{vmatrix} \frac{\partial^2 f}{\partial x^2}(T_0) & \frac{\partial^2 f}{\partial x \partial y}(T_0) \\ \frac{\partial^2 f}{\partial y \partial x}(T_0) & \frac{\partial^2 f}{\partial y^2}(T_0) \end{vmatrix}$$

$\frac{\partial^2 f}{\partial x^2}=2>0$

MINIMUM

$\boxed{T(0,0,0)}$

$$\Delta=\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}=2\cdot 2-0\cdot 0=4>0$$

IMA EKS

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(3) TANG. RAVNINA

$T(1,1,2)$

$z = x^2 + y^2$

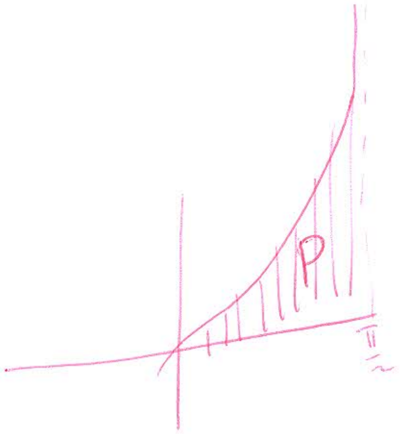
$z - z_0 = f(x)(x - x_0) + f(y)(y - y_0)$

$\boxed{z - 2 = 2(x-1) + 2(y-1)}$

$\boxed{z - 2 = 2x - 2 + 2y - 2 \Rightarrow z - 2 = 2x + 2y - 4}$ ✓

$\frac{\partial f}{\partial x} = 2x = 2 \cdot 1 = 2$

$\frac{\partial f}{\partial y} = 2y = 2 \cdot 1 = 2$

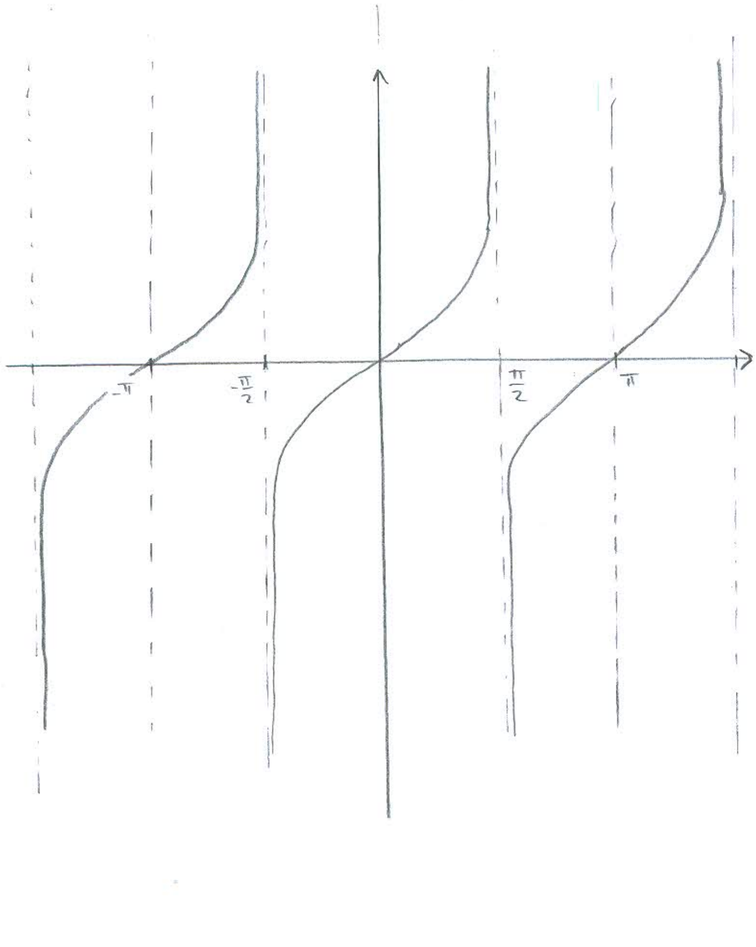


(6) $f(x) = \tan x$

$\int_0^{\frac{\pi}{2}} f(x)\,dx$

15

1,571



$\int_0^{\frac{\pi}{2}} \tan x\,dx = \Big[-\ln|\cos x|\Big]_0^{\frac{\pi}{2}}$

$= -\ln|0| - (-\ln|1|)$

$\quad\;\; (-\infty)$

$= \boxed{+\infty} + 0$

$= +\infty + 0 = +\infty$ ✓

$P = \lim_{\substack{a \to -\frac{\pi}{2} \\ b \to \frac{\pi}{2}}} \int_a^b \tan x - \lim_{\substack{a \to -\frac{\pi}{2} \\ b \to \frac{\pi}{2}}} \Big[-\ln|\cos x|\Big]_a^b$

$= -\ln|\cos \frac{\pi}{2}| - (-\ln|\cos \frac{\pi}{2}|)$

$= -\ln|0| + \ln|0| = -(-\infty) + (-\infty)$

$= \boxed{\infty - \infty \quad N/P}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx

POPUNJAVA NASTAVNIK
Broj ↓
bodova

IME I PREZIME: LOVRE BUBALO BROJ INDEKSA: 17-2-0389-2014

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

1. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $\sin x\,dy = y\ln y\,dx$ i $y(1) = 2$. 15
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Ukupno: 30

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
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$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
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$u\cdot v - \int v\,du$

(4) $$\int_0^1 3xe^{x+1}dx = \begin{bmatrix} 3x = u & \int e^{x+1} = \int dv \\ 3 = du & e^{x+1} = v \end{bmatrix} = \left[3x\cdot e^{x+1} + \int e^{x+1}\cdot 3\right]_0^1 = \left[3x\cdot e^{x+1} + 3e^{x+1}\right]_0^1 =$$

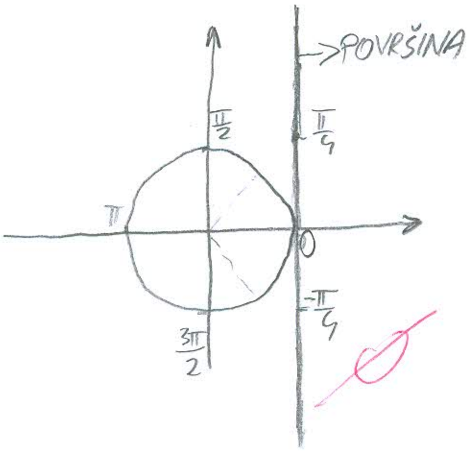
$$= \left(3\cdot 1\cdot e^{1+1} + 3\cdot e^{1+1}\right) - \left(3\cdot 0\cdot e^{0+1} + 3e^{0+1}\right) = 44{,}334 - 8{,}155 = \underline{36{,}179}$$

⑤ $$\int_1^3 \frac{dx}{x^2-2x+4} = \int_1^3 \frac{dx}{x^2-2x+1+3} = \int_1^3 \frac{dx}{(x-1)^2+(\sqrt{3})^2} = \left[\frac{1}{\sqrt{3}} \arctan \frac{x-1}{\sqrt{3}}\right]_1^3 =$$

$$= \left(\frac{\sqrt{3}}{3} \arctan \frac{3-1}{\sqrt{3}}\right) - \left(\frac{\sqrt{3}}{3} \arctan \frac{0}{\sqrt{3}}\right) = \left(\frac{\sqrt{3}}{3} \arctan \frac{2\sqrt{3}}{3}\right) - \left(\frac{\sqrt{3}}{3} \arctan 0\right) =$$

$$= 0{,}4948 - 0 = \underline{\underline{0{,}4948}}$$

⑥ $f(x) = \tan x$ $\qquad \int_0^{\frac{\pi}{2}} f(x)\,dx = ?$



x	y
0	0
π	0
$\frac{\pi}{4}$	1
$-\frac{\pi}{4}$	-1

$$\int_0^{\frac{\pi}{2}} \tan x\,dx = \lim_{a\to\frac{\pi}{2}} \int_0^a \tan x\,dx =$$

$$= \left(-\ln|\cos a|\right) - \left(-\ln|\cos 0|\right) =$$

NE POSTOJI ← $= -\ln 0 \qquad + 0 = 0$

UNUTAR ln strogo veće od nule

VIDI ŠTURA

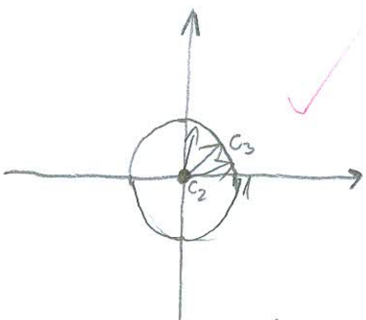
③ $f(x,y) = x^2+y^2$ $\qquad f(x,y) = C$ $\qquad (x-x_0)(y-y_0) = r^2$

$S(x_0, y_0)$

KRUŽNICE

$C_1 = -1$ $\qquad C_2 = 0$ $\qquad C_3 = 1$

$x^2+y^2 = -1 \Rightarrow N/P$ $\qquad x^2+y^2 = 0$ $\qquad x^2+y^2 = 1$



TANGENCIJALNA RAVNINA: $\qquad (1, 1, 2)$

$f(x,y) = x^2+y^2$

$z_0 = 1+1 = 2$

$$z - z_0 = f(T_x)(x-x_0) + f(T_y)(y-y_0)$$

$$z - 2 = 2(x-1) + 2(y-1)$$

$\frac{df}{dx} = 2x$ $\qquad f(T_x) = 2$

$\frac{df}{dy} = 2y$ $\qquad f(T_y) = 2$

EKSTREMI →

LOVRE BUBALO

$$\frac{\partial f}{\partial x^2} = 2 \qquad \frac{\partial f}{\partial x^2 \partial y^2} = 0$$

$$\frac{\partial f}{\partial y^2} = 2 \qquad \frac{\partial f}{\partial y^2 \partial x^2} = 0$$

$$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0$$

LOKALNI MINIMUM ✓

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xxx	POPUNJAVA NASTAVNIK
IME I PREZIME: JURE PERIĆ **BROJ INDEKSA**: 0269083660	Broj ↓ bodova
Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor	

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5. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ ~~15~~ 12

6. Neka je $f(x) = \tan x$. Skicirati graf funkcije f i površinu određenu integralom. Odrediti $\int_0^{\pi/2} f(x)\, dx$. Kolika je skicirana površina ispod grafa funkcije f? 20

Ukupno: 12

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
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(3.) $f(x,y) = x^2 + y^2$

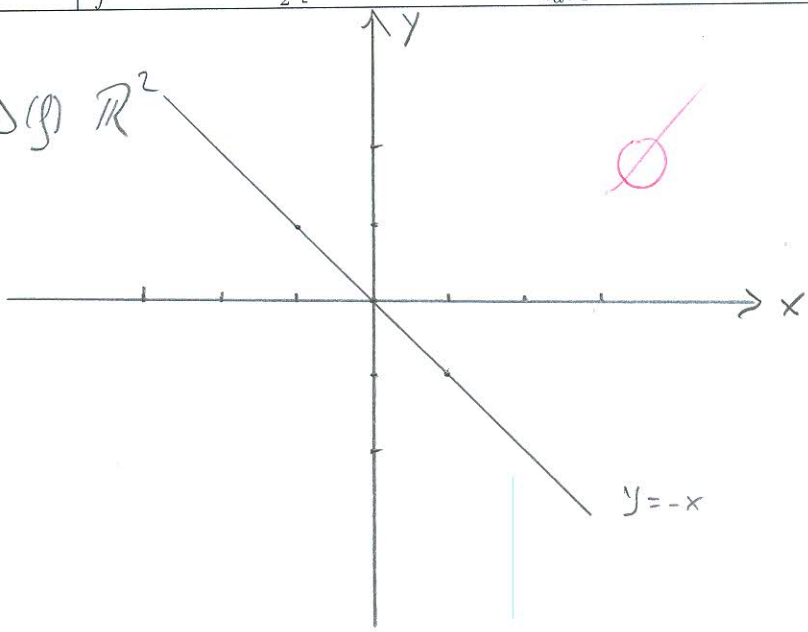
$D(f)\ \mathbb{R}^2$

$x^2 + y^2 = 0$

$y^2 = -x^2 \,/\sqrt{\ }$

$y = -x$

x	0	1	-1
y	0	-1	1



$\emptyset$

NEMA EKSTREMA

5. $\int_1^3 \frac{dx}{x^2-2x+4}$

$\int_1^2 \frac{dx}{x^2-2x+4}$ $d = 2-1 = 1$

$\int_2^3 \frac{dx}{x^2-2x+4}$ $d = 3-2 = 1$

0	1	2
1	0,5	2
0,333	0,307	0,25

0	1	2
2	2,5	3
0,25	0,1905	0,192

$S_1 = \frac{d}{6}(f_0 + f_1 \cdot 4 + f_2)$

$S_1 = \frac{1}{6}(0,333 + 4 \cdot 0,307 + 0,25) = 0,301$

$S_2 = \frac{d}{6}(f_0 + f_1 \cdot 4 + f_2)$

$S_2 = \frac{1}{6}(0,25 + 4 \cdot 0,1905 + 0,192)$

$S_2 = 0,192$

$S_{uk} = S_1 + S_2 = 0,301 + 0,192 = \underline{\underline{0,493}}$ ✓

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RJEŠENJE APROKSIMACIJOM NE DONOSI PUNI BROJ BODOVA

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IME I PREZIME: KRISTIAN ŽIVKOVIĆ **BROJ INDEKSA:**
Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

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Ukupno: 23

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_a x$ $(a > 0)$	$\frac{1}{x \ln a}$
e^x	e^x
a^x $(a > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

KRISTIAN ŽIVKOVIĆ

(4)

$$\int_0^1 3xe^{x+1}dx = 3\int_0^1 \underbrace{x}_{u}\,\underbrace{e^{x+1}dx}_{dv} \qquad \begin{bmatrix} u=x & dv=e^{x+1}dx \\ du=dx & v=e^{x+1} \end{bmatrix}$$

$$u\cdot v-\int v\cdot du = 3\left[x\cdot e^{x+1}-\int e^{x+1}dx\right]_0^1$$

$$=3\left[x\cdot e^{x+1}-e^{x+1}\right]_0^1$$

$$=3\cdot\left[1\cdot e^{1+1}-0\cdot e^{0+1}-e^{1+1}+e^{0+1}\right]$$

$$=3\cdot\left[7{,}389-0+7{,}389+2.71\right]$$

$$\boxed{=8{,}154}$$

(5)

$$\int_1^3 \frac{dx}{x^2-2x+4} = \int_1^3 \frac{1}{x^2-2x+4}\,dx = \frac{1}{(x-1)^2+3}$$

uvjet

$x^2-2x+4 \neq 0$

$D: \mathbb{R}$

$$=\frac{1}{(x-1)^2+3} = \frac{A}{(x-1)+3} + \frac{B}{(x+1)}$$

$$\int_1^3 \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$$

$$\boxed{=0{,}494}$$

$$1 = A(x+1)+B(x-1)+C$$

$$1 = Ax+A+Bx-B$$

$$1 = A - B$$
$$0 = Ax + Bx$$

$$1 = A + Ax$$

$$1 = 2A$$
$$2A = 1 \quad /:2 \qquad \boxed{A=\tfrac{1}{2}}$$

$$1 = \tfrac{1}{2} - B$$

$$B = \tfrac{1}{2} - 1$$

$$\boxed{B = -\tfrac{1}{2}}$$

$f = 2,5$

$\sin x \, dy = y \ln y \, dx$ i $y(1) = 2$

$\emptyset$

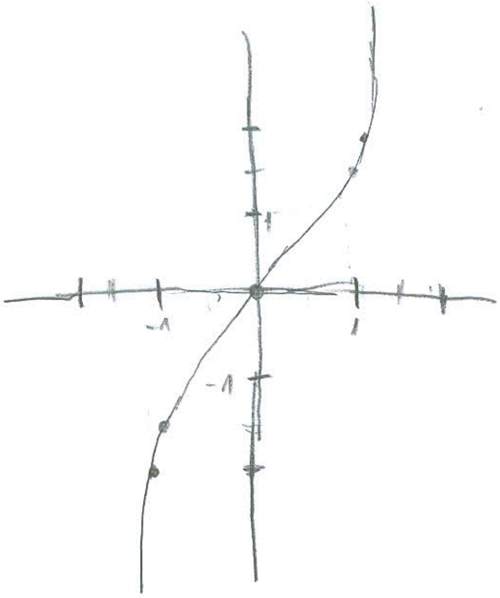
KRISTIAN ŽIVKOVIĆ

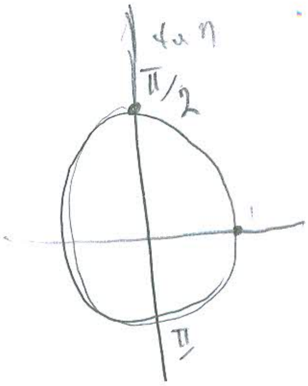
6

$f(x) = \tan x$

$f(x) = x$

x	-1	0	1	-1,1	1,1
y	-1,55	0	1,55	-1,96	1,96





$$\int_0^{\pi/2} f(x)\,dx$$

$$\int_0^{\pi/2} \tan x\,dx$$

$$\left[-\ln|\cos x|\right]_0^{\pi/2}$$

$$= \left[-\ln\left|\cos\frac{\pi}{2}\right| + \ln|\cos 0|\right]$$

$$-\infty + 0 \qquad = +\infty$$

3

N.P.

$f(x,y) = x^2 + y^2$

KRISTAN ŽIVKOVIĆ

③ $f(x,y) = x^2 + y^2$

$x = -1$ $x = 1$ $x = 0$

$y = -1$ $y = 1$ $y = 0$

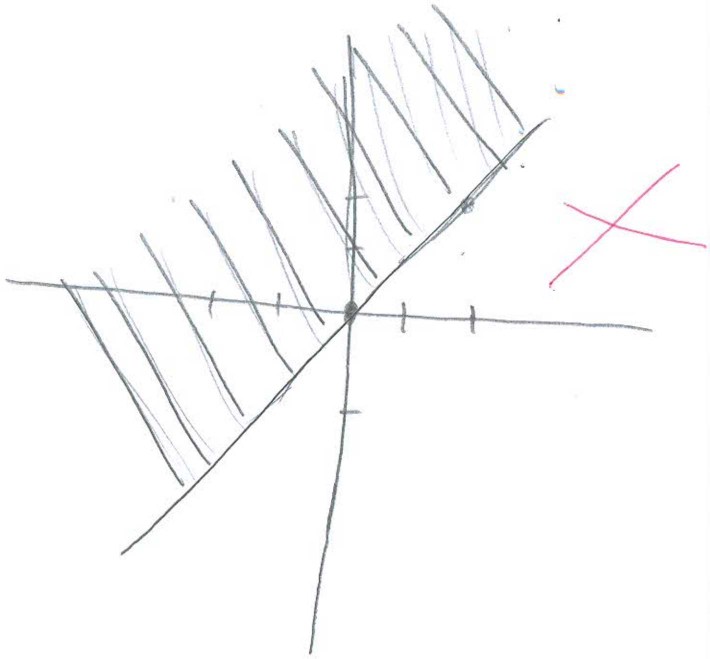
$f(x,y) = x^2 + y^2$

-1 0 1

$x = (-1)^2 + (-1)^2$ $x = (0)^2 + (0)^2$ $x = (1)^2 + (1)^2$

$x = 2$

$x = 2$ $x = 0$



KRISTIAN ŽIVKOVIĆ

(4)

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** XXX

Ime i prezime: Josip Ganta **Broj indeksa:** 17-2-0385-2014

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

Popunjava nastavnik Broj ↓ bodova

1. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $\sin x\, dy = y \ln y \, dx$ i $y(1) = 2$. 15
2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 4y = 0$, uz $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje. 15
3. Skicirati razinske krivulje za $f(x, y) = x^2 + y^2$. Ima li ekstrema? Pronađi tangencijalnu ravninu u točki koju možeš sam odabrati. 15
4. $\displaystyle\int_0^1 3x\, e^{x+1}\, dx = ?$ 20 16
5. $\int_1^3 \frac{dx}{x^2 - 2x + 4} = ?$ 15
6. Neka je $f(x) = \tan x$. Skicirati graf funkcije f i površinu određenu integralom. Odrediti $\int_0^{\pi/2} f(x)\, dx$. Kolika je skicirana površina ispod grafa funkcije f? 20

Ukupno: 16

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

(2) $y'' + 4y = 0$ $y(0) = 0$ $y'(0) = 2$ Josip Ganta

$r^2 + 4r = 0$

$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$r_{1,2} = \frac{4 \pm \sqrt{16}}{2} \Rightarrow$

$r_{1,2} = \frac{4 \pm 4}{2} \Rightarrow r_1 = 4$

$r_2 = 0$

$C_1 e^0 + C_2 e^{4y} \Rightarrow$

$C_1 e^0 + 4C_2 e^{4y} = 0$

$C_1 e^0 + 16C_2 e^{4y} = 0$

$C_1 e^0 + 4C_2 e^{4 \cdot 0} = 0$

$C_1 + 4C_2 = 0$

$C_1 e^0 + 16 C_2 e^{4y} = 0$

$C_1 e^0 + 16 C_2 e^{4 \cdot 2} = 0$

$C_1 + 16 C_2 e^8 = 0$

RJEŠENJE ?

$\emptyset$

(1)

$f(2,5)$

$f = \sin x \, dy = y \ln y \, dx$

$y(1) = 2$

$\int \sin 2 \, dy = 1 \ln 1 \, dx$

$0,9092 = 0$

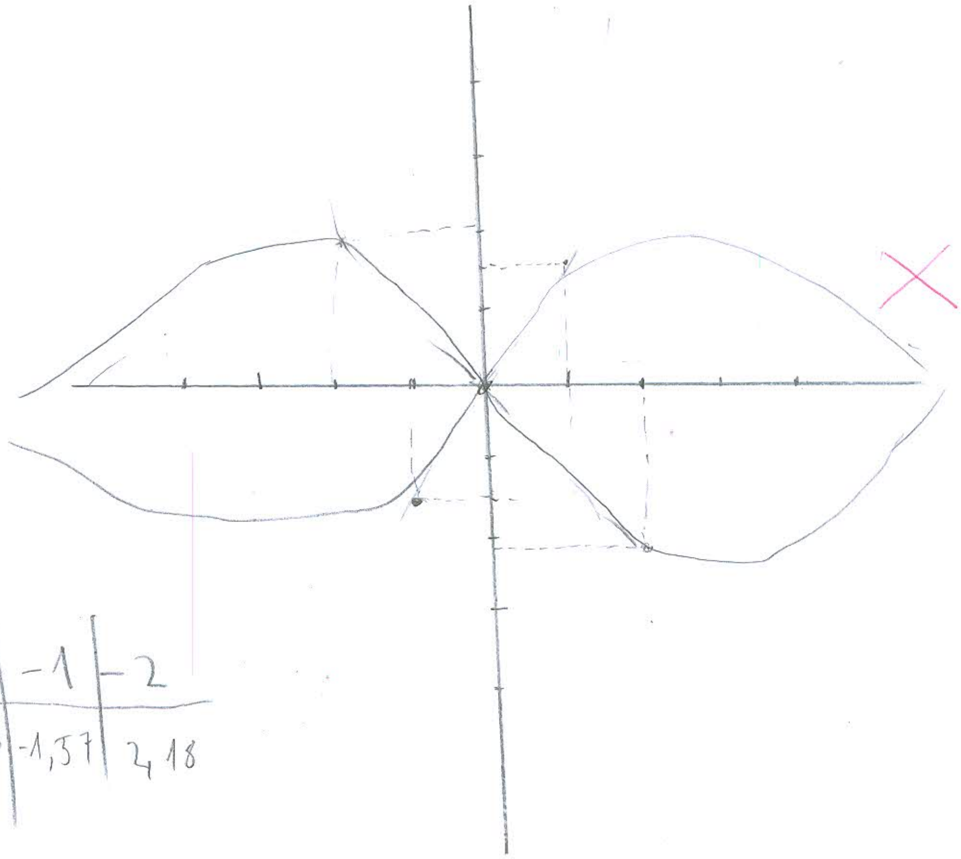
$\emptyset$

6

Josip Ganda

$f(x) = \tan x$

$\int_0^{\frac{\pi}{2}} f(x)\,dx$



X	0	1	2	-1	-2
Y	0	1,57	-2,18	-1,57	2,18

$\int_0^{\frac{\pi}{2}} \tan x \, dx = \left[-\ln |\cos x| \right]_0^{\frac{\pi}{2}} \Rightarrow$

$\tan x \neq 0$

$\tan x^{-1} = 0$

$x = 0$

$\Rightarrow \left(-\ln \left| \cos \frac{\pi}{2} \right| - \left| -\ln |\cos 0| \right| \right)$

$\Rightarrow 0$ $\varnothing$

$P = \pi$

③ $f(x,y) = x^2 + y^2$

$\frac{\partial f}{\partial x} = 2x$ $\quad \frac{\partial'' f}{\partial x''} = 2$

$x^2 + y^2 \geq 0$

$\frac{\partial f}{\partial y} = 2y$ $\quad \frac{\partial'' f}{\partial y} = 2$

$c = 0$ $\quad 0 = x^2 + y^2$

$\frac{\partial f}{\partial x \partial y} = 1$ $\quad \frac{\partial f}{\partial y \partial x} = 1$

$y^2 = -x^2 / \sqrt{}$

$y = -x$

x	0	1	2
y	0	-1	-2

$\Delta V > 0$

$\Delta V \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3$

$Df: \mathbb{R}$ $\quad KD: \mathbb{R}^2$

$c = 1$

$1 = x^2 + y^2$

$y^2 = 1^2 - x^2 / \sqrt{}$

$y = 1 - x$

x	0	1
y	1	0

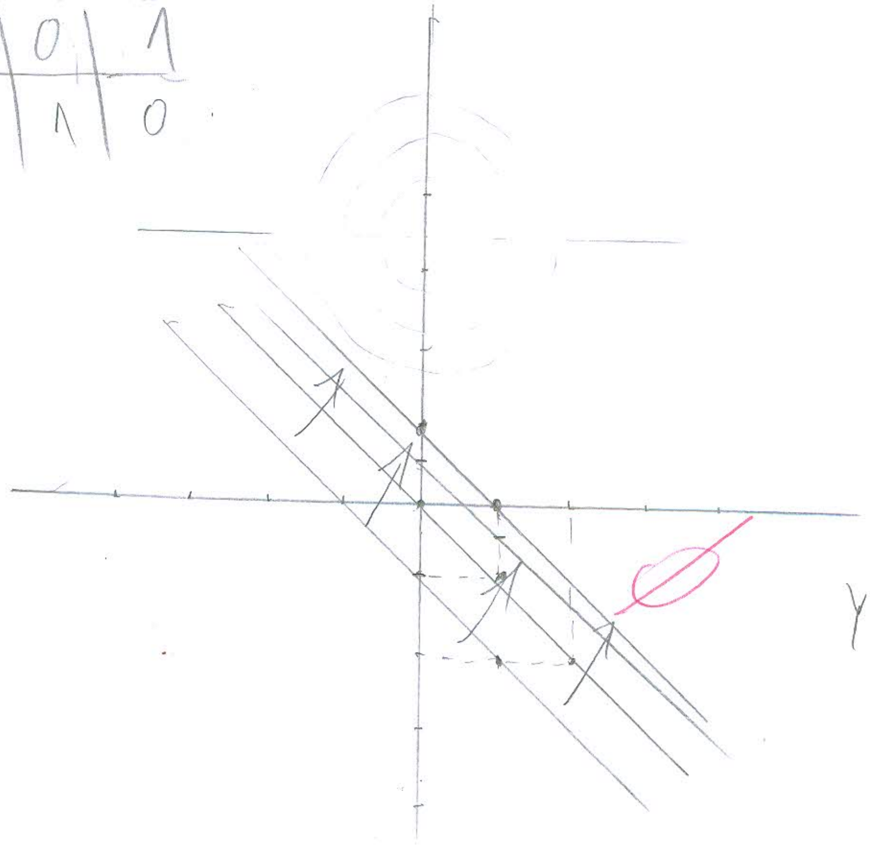
$c = -1$

$-1 = x^2 + y^2$

$y^2 = -x^2 - 1 / \sqrt{}$

$y = -x - 1$

x	0	1
y	-1	-2



$c = \frac{1}{2}$ $\quad \frac{1}{2} = x^2 + y^2$

$y = -x + 0{,}71$ $\quad y^2 = -x^2 + \frac{1}{2} / \sqrt{}$

x	0	1
y	0,7	-0,3

(5)

Josip Garita

$$\int_1^3 \frac{dx}{x^2-2x+4} = \left| \begin{matrix} x^2-2x+4 \Rightarrow (x-2)^2 \\ (x-2)^2 = t \end{matrix} \right|$$

$$2x-2dx=dt$$

$$\int_1^3 \frac{dx}{(x-2)^2} = \left| \begin{matrix} (x-2)^2 = t \\ dx = dt \end{matrix} \right|$$

$$\int_1^3 \frac{dt}{t} \Rightarrow \Big[\ln|t|\Big]_1^3 \Rightarrow \Big[\ln|(x-2)^2|\Big]_1^3 \Rightarrow$$

$$\Rightarrow \ln|(3-2)^2| - \ln|(1-2)^2| \Rightarrow$$

$$\Rightarrow \ln|1| - \ln|1|$$

$$\Rightarrow 0$$

$\emptyset$

4. $\int_0^1 3x\,e^{x+1}\,dx = \int u\,dv =$

$$\left| \begin{array}{ll} u = 3x & dv = e^{x+1} \\ du = 3dx /:\frac{1}{3} & v = e^{x+1} \\ dx = \frac{1}{3}du & \end{array} \right|$$

$uv - \int u\,du$

$3x\,e^{x+1} - \int_0^1 e^{x+1}\,\frac{1}{3}du \Rightarrow 3x\,e^{x+1} - 1{,}556$

$= 22{,}1672 - 1{,}556 = 20{,}611683$

$\int_0^1 e^{x+1}\,\frac{1}{3}du \Rightarrow \frac{1}{3}\int_0^1 e^{x+1} \Rightarrow \frac{1}{3}\left[e^{x+1}\right]_0^1 \Rightarrow$

$\frac{1}{3}\left[e^{1+1} - e^{0+1}\right] \Rightarrow$

$= \frac{1}{3}\left[7{,}389056099 - 2{,}718281828\right]$

$= \frac{1}{3}\left[4{,}670774271\right]$

$= 1.556924757$

$3x\,e^{x+1}$

$3\cdot 1\,e^{1+1} - 3\cdot 0\,e^{0+1}$

$$\int_0^1 3xe^{x+1}\,dx$$

Josip Ganta

4,4816

$$\int_0^1 3xe^{x+1}\,dx$$

$$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$$

k	0	1	2
x_k	0	$\frac{1}{2}$	1
f_k	0	6,7224	22,1672

$$S = \frac{1}{6}(0 + 4 \cdot 6{,}7224 + 22{,}1672)$$

$$S = \frac{1}{6}(26{,}8896 + 22{,}1672)$$

$$S = \frac{1}{6}(49{,}0568)$$

$$S = 8{,}17613333$$ ✓ 16

(3)

$$z - z_0 = f(x)(T)(x - x_0) + f_y(T)(y - y_0)$$

?

④

$\int_0^{\frac{1}{2}} 3x\, e^{x+1} = \frac{1}{3}\left[e^{x+1}\right]_0^1 \Rightarrow$ $\int_{\frac{1}{2}}^{1} 3x\, e^{x+1}$

k	0	1	2
x_k	0	0,25	0,5
f_k	0	2,62	6,72

k	0	1	2
x_k	0,5	0,75	1
f_k	6,72	12,94	2,079

$S = \frac{1}{3}(f_0 + 4f_1 + f_2)$

$S = \frac{1}{3}(0 + 4 \cdot 2{,}62 + 6{,}72)$

$S = \frac{1}{3}(17{,}2)$

$S = 5{,}73$

$S = \frac{1}{3}(f_0 + 4f_1 + f_2)$

$S = \frac{1}{3}(6{,}72 + 4 \cdot 12{,}94 + 2{,}079)$

$S = \frac{1}{3}(60{,}559)$

$S = 20{,}18$

$\Delta S = S_1 + S_2$

$\Delta S = 5{,}73 + 20{,}18$

$\Delta S = 23{,}91$