MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	oxx	POPUNJAVA NASTAVNIK
IME I PREZIME: Roko Šimurina	**BROJ INDEKSA:** 17-1-0028-2010	Broj ↓ bodova
Želim ustmeni kod (zaokružiti): prof. Uglešić	asistent Kosor	

1. Riješi diferencijalnu jednadžbu $(1+e^x)\,yy' = e^x$ uz početni uvjet $y(0)=1$. 20
2. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x\sin x$. 15
3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x,y) = x + 2y + 1$. 5+5+5
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5. $$\int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx = ?$$ 15
6. Integriranjem izračunati površinu između krivulja $x=0$ i $y^2 = x+4$. 15

Ukupno: 65

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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6. $x = 0 \qquad y^2 = x + 4$

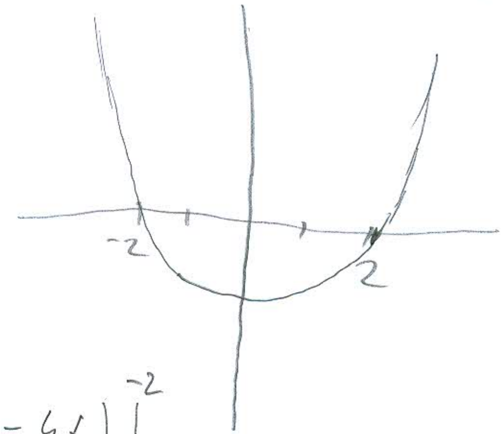
$(x \longleftrightarrow y)$

$y = 0$

$x^2 = y + 4$

$y = x^2 - 4$

$x_{1,2} = \pm 2$



$$P = \int_2^{-2} (x^2 - 4)\,dx = \left(\frac{x^3}{3} - 4x\right)\Bigg|_2^{-2}$$

$$= \frac{-8}{3} + 8 - \left(\frac{8}{3} - 8\right) = 16 - \frac{16}{3} = \frac{32}{3}$$

!!

① $(1+e^x)\, y y' = e^x$

$1+e^x = t$
$e^x dx = dt$

$y\,dy = \frac{e^x dx}{1+e^x}$

$\int y\,dy = \int \frac{dt}{t}$

$\frac{y^2}{2} = \ln|t| + c$

$y^2 = 2\ln|1+e^x| + 2c$

$y(0) = 1 \longrightarrow 2\ln 2 + 2c = 1$

$2c = 1 - 2\ln 2$

$c = \frac{1}{2} - \ln 2$ ✓

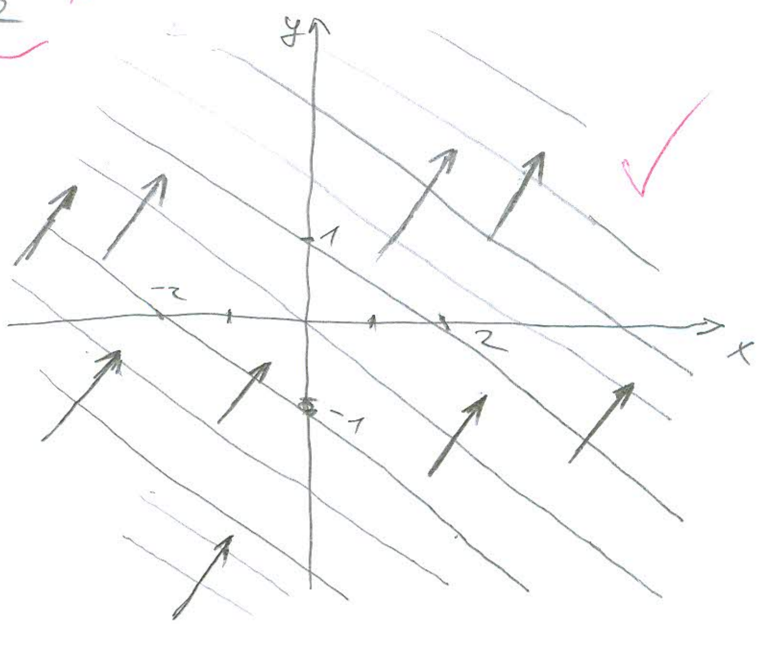
$y_p = \sqrt{2\ln|1+e^x| + 1 - 2\ln 2}$ ✓

③

$f(x,y) = x + 2y + 1$

$D(f) = \mathbb{R}^2$ ✓ $\quad K(f) = \mathbb{R}^2$ ✗

$x + 2y + 1 = 0$
$y = -\frac{1}{2}x - \frac{1}{2}$



✓

Koko Simurina

(4) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

k	0	1	2	3	4
x_k	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
f_k	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
S		$\boxed{1{,}002}$		$\boxed{1{,}002}$	

z broj = 2,004

$$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$$

$$S_1 = \frac{\pi}{12}(0 + 2\sqrt{2} + 1) = 1{,}002$$

$$S_2 = \frac{\pi}{2}(1 + 2\sqrt{2} + 1) = 1{,}002$$

$$S = 2{,}004$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx = 2{,}004$$

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IME I PREZIME: FILIP GORŠEK	**BROJ INDEKSA:**
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Ukupno: ~~0~~ 30 Kos

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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FILIP GURSEK

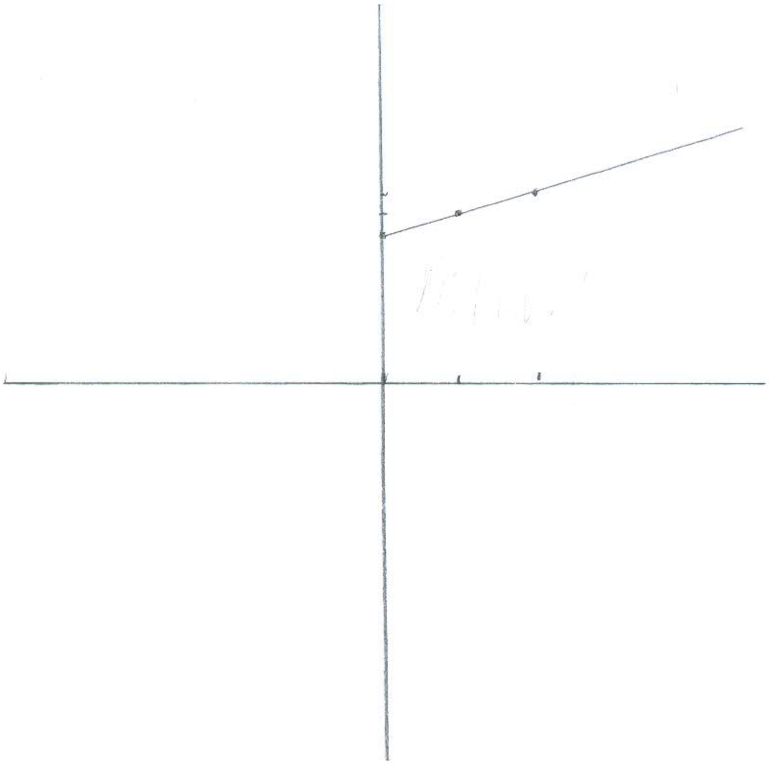
(6.) $X = 0 \qquad y^2 = x + 4$

X	1	2	0
$y^2 = x+4$	2.24	2.46	2

$y^2 = 0 + 4$

$y^2 = 4 / \sqrt{\ }$

$y_{1,2} = \pm 2$



FILIP GORŠEK

4.
$$\int_{-\pi/2}^{\pi/2} \cos x \, dx$$

$-\pi/2 \quad -\frac{\pi}{4} \quad 0 \quad \frac{\pi}{4} \quad \pi/2$

$-\pi/2$	$-\pi/4$	0
0	$0 \; \frac{\sqrt{2}}{2}$	1

$S_1 = \frac{d}{6}\left(f_0 + 4 \cdot f_1 + f_2\right)$

$= \frac{\frac{\pi}{2}}{6}\left(0 + 4 \cdot \left(\frac{\sqrt{2}}{2}\right) + 1\right)$

$= 1.002$

0	$\pi/4$	$\pi/2$
1	$\frac{\sqrt{2}}{2}$	0

$S_2 = \frac{\frac{\pi}{2}}{6} \cdot \left(1 + 4 \cdot \left(\frac{\sqrt{2}}{2}\right) + 1\right)$

$S_2 = 1.014$

$S_1 + S_2 = 1.002 + 1.014 = 2.016$ //

3. $f(x,y) = x + 2y + 1$ $\quad D(f)\{\mathbb{R}^2\}$ $\quad$ KODOMENA $\mathbb{R}$

10

$c = 1, \quad x + 2y + 1 = 0,$

$2y = -1 - x \;/:2$

$y = -\frac{1}{2} - \frac{x}{2}$

$y =$

NACRTATI RAZINSKE KRIVULJE

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IME I PREZIME: LEINA ADUM **BROJ INDEKSA:**

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK

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$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
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LEINA ADUM

(1)

$$(1+e^x)\,y\,y' = e^x$$

$$y \cdot y' = \frac{e^x}{1+e^x}$$

$$y \cdot \frac{dy}{dx} = \frac{e^x}{1+e^x} \quad / \cdot dx$$

$$y\,dy = \frac{e^x}{1+e^x}\,dx$$

$$\int \frac{e^x}{1+e^x}\,dx = \left\{ \begin{matrix} 1+e^x = t \\ e^x dx = dt \end{matrix} \right\} = \int \frac{dt}{t} = \ln|t| + c = \ln|1+e^x| + c$$

$$\int y\,dy = \int \frac{e^x}{1+e^x}\,dx$$

$$\frac{y^2}{2} = \ln|1+e^x| + C$$

$$\cancel{y^2 = 2\ln|1+e^x| + 2C}$$

$$\cancel{y = \sqrt{2\ln|1+e^x| + 2C}}$$

$$\cancel{y' = \frac{1}{2}\left(2\ln|1+e^x| + 2C\right)^{-\frac{1}{2}} \cdot \left(2\,\frac{1}{1+e^x} \cdot e^x + 0\right)}$$

$$\cancel{y' = \frac{1}{2}\,\frac{\frac{2e^x}{1+e^x}}{\sqrt{2\ln|1+e^x| + 2C}}}$$

$$\sqrt{2\ln|(1+e^x)\cdot c} \cdot \frac{Ce^x}{\sqrt{2\ln|c+ce^x|}\cdot(C+Ce^x)} = \frac{e^x}{1+e^x}$$

$$\frac{y^2}{2} = \ln|1+e^x| + \ln|C|$$

$$\frac{Ce^x}{C+Ce^x} = \frac{e^x}{1+e^x}$$

$$\frac{y^2}{2} = \ln\left(|1+e^x|\cdot c\right)$$

$$C \cdot \frac{e^x}{1+e^x} = \frac{e^x}{1+e^x}$$

$$y^2 = 2\ln\left(|1+e^x|\cdot c\right)$$

$$C = 1$$

$$y = \sqrt{2\ln\left(|1+e^x|\cdot c\right)}$$

VIDI ŠIMURINA.

$$y' = \left(2\ln|1+e^x|\right)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}\left(2\ln|(1+e^x)\cdot c|\right)^{-\frac{1}{2}} \cdot$$

$$y' = \frac{1}{2}\left(2\ln|c+ce^x|\right)^{-\frac{1}{2}} = \frac{1}{2}\,\frac{1}{\sqrt{2\ln|c+ce^x}} \cdot 2\,\frac{1}{c+ce^x} \cdot Ce^x$$

$$y' = \frac{ce^x}{\sqrt{2\ln|c+ce^x|}\cdot(c+ce^x)}$$

2) $4y'' - y = 2x \sin x$

$4r^2 - 1 = 0$

$4r^2 = 1$

$r^2 = \frac{1}{4}$

$r_1 = -\frac{1}{2}$

$r_2 = \frac{1}{2}$

$y_H = C_1 \cdot e^{-\frac{1}{2}x} + C_2 \cdot e^{\frac{1}{2}x}$ ✓

$y_P = e^{\alpha x}\left(P_N(x)\cos\beta x + Q_n(x)\sin\beta x\right)$

$\alpha = 0 \quad \rightarrow 1$

$P_N = 0 \rightarrow$

$Q_H = 2x \rightarrow Ax + B$

$y_P = (Ax + B)\sin x$

$y_P = Ax\sin x + B\sin x$

$y_P' = Ax' \cdot \sin x + Ax \cdot \cos x + B\cos x$

$y_P' = A \cdot \sin x + \cos x\,(Ax + B)$

$y_P'' = A\cos x + \sin x\,(Ax + B) + \cos x \cdot A$

$y_P'' = 2A\cos x + \sin x\,(Ax + B)$

$4\left(2A\cos x + \sin x\,(Ax + B) - (Ax\sin x + B\sin x)\right) = 2x\sin x$

$8A\cos x + 4\sin x\,(Ax + B) - \sin x\,(Ax + B) = 2x\sin x$

$8A\cos x + 3\sin x\,(Ax + B) = 2x\sin x \quad /:\sin$ ✗

$8A\frac{\cos x}{\sin x} + 3(Ax + B) = 2x$ ✗

$8A\frac{\cos x}{\sin x} + 3Ax + 3B = 2x \qquad 3A = 2$

$8A\,\mathrm{ctg}\,x + 3B = 0$

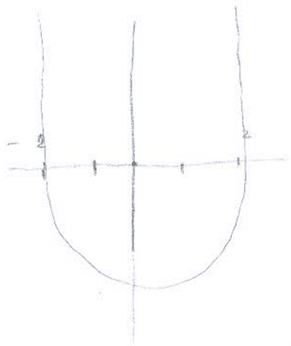
(6) $x = 0$ $\qquad y \leftrightarrow x$

$y^2 = x + 4$

$y = 0$

$x^2 = y + 4$

$y = x^2 - 4$



$y' = 2x$

$0 = x^2 - 4$

$x_{1,2} = \pm 2$ $\qquad \int_{-2}^{2} 0 - (x^2 - 4) = \int_{-2}^{2} 0 - x^2 + 4 =$

$$\int_{-2}^{2} -x^2 + 4\,dx = -\frac{x^3}{3} + 4x \Big|_{-2}^{2} = -\frac{8}{3} + 8 - \left(-\frac{-8}{3} - 8\right) =$$

$$= \frac{-8 + 24 - 8 + 24}{3} = \frac{32}{3}$$ ✓

LEINA ADUM

(3) Domena je $\mathbb{R}$ ✓

$$f(x,y) = x + 2y + 1$$

$$x + 2y + 1 = c$$

$$2y = 1 + c + x$$

$$y = \frac{1+x}{2} + \frac{c}{2}$$

SKICA ?

7

(5) $$\int_0^2 \frac{2x^2+x+2}{x^2-1}\,dx \Rightarrow \quad 2x^2+x+2 : x^2-1 = 2 + \frac{x+4}{x^2-1}$$

$$\frac{-2x^2+2}{x+4}$$

Ø

$$2\int_0^2 dx + \int_0^2 \frac{x+4}{x^2-1}\,dx = 2\int_0^2 dx + \int_0^2 \frac{x}{x^2-1}\,dx + \int_0^2 \frac{4}{x^2-1} =$$

(1)

$$= 2\cdot x + \frac{1}{2}\ln|x^2-1| + 4\cdot\frac{1}{2}\ln|x^2-1| \Big|_0^2 \Rightarrow ♡$$

(1) $$\int \frac{x}{x^2-1}\,dx = \left\{ \begin{matrix} x^2-1 = t \\ 2x\,dx = dt \end{matrix} \right\} = \frac{1}{2}\int \frac{2x\,dx}{x^2-1} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|x|$$

$$= \frac{1}{2}\ln|x^2-1|$$

♡ $$\Rightarrow 2\cdot 2 + \frac{1}{2}\ln|4-1| + 2\ln|4-1| - \left(0 + \frac{1}{2}\ln|-1| + 2\ln|-1|\right) =$$

$$= 4 + \frac{1}{2}\ln 3 + 2\ln 3 - \frac{1}{2}\ln|-1|^{0} + 2\cancel{\ln}|-1|^{0} =$$

$$= 4 + 0{,}55 + 2{,}2 = 6{,}75$$

✗

INTEGRAL JE NEPRAVI ZBOG SINGULARITETA ZA $x=1$