MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

oxx

IME I PREZIME: Roko Šimurina **BROJ INDEKSA:** 17-1-0028-2010

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $(1+e^x)\,yy' = e^x$ uz početni uvjet $y(0)=1$. — 20
2. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x\sin x$. — 15
3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x,y) = x + 2y + 1$. — 5+5+5
4. Numeričkom integracijom odrediti vrijednost $\int_{-\pi/2}^{\pi/2} \cos x\,dx$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) — 20
5. $\int_0^2 \frac{2x^2+x+2}{x^2-1}dx = ?$ — 15
6. Integriranjem izračunati površinu između krivulja $x=0$ i $y^2 = x+4$. — 15

Ukupno: ~~75~~ 65

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

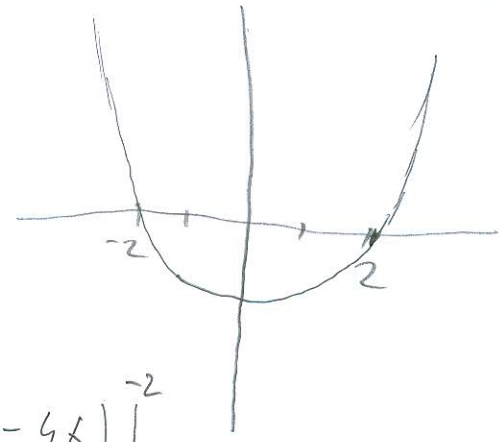
6. $x = 0$ $\quad y^2 = x+4$

$(x \leftrightarrow y)$

$y = 0$ $\quad x^2 = y+4$

$y = x^2 - 4$

$x_{1,2} = \pm 2$



$$P = \int_{-2}^{2} |x^2 - 4|\,dx = \left(\frac{x^3}{3} - 4x\right)\Big|_{-2}^{2}$$

$$= \frac{-8}{3} + 8 - \left(\frac{8}{3} - 8\right) = 16 - \frac{16}{3} = \frac{32}{3}$$

!!!

① $(1+e^x)\,y\,y' = e^x$

$1+e^x = t$

$e^x dx = dt$

$y\,dy = \frac{e^x dx}{1+e^x}$

$\int y\,dy = \int \frac{dt}{t}$

$\frac{y^2}{2} = \ln|t| + c$

$y^2 = 2\ln|1+e^x| + 2c$

$y(0) = 1 \longrightarrow 2\ln 2 + 2c = 1$

$2c = 1 - 2\ln 2$

$c = \frac{1}{2} - \ln 2$ ✓

$y_p = \sqrt{2\ln|1+e^x| + 1 - 2\ln 2}$ ✓

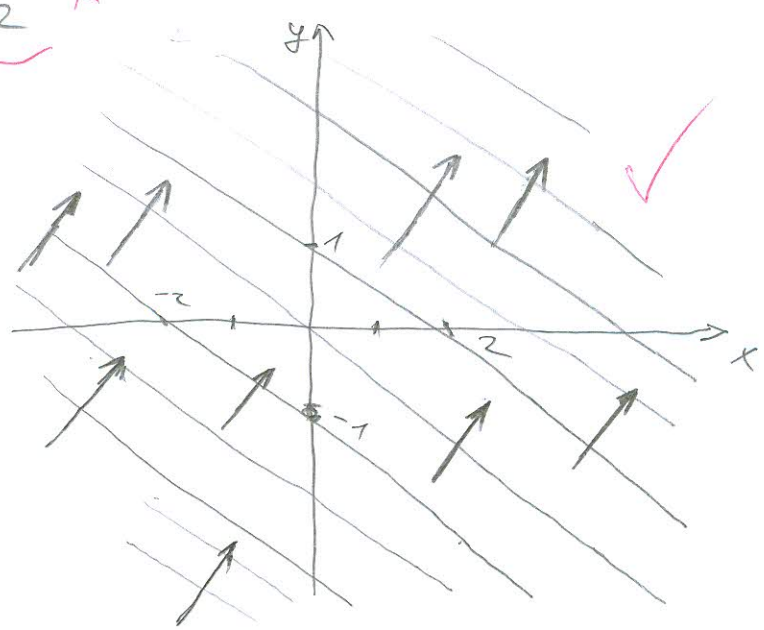
③

$f(x,y) = x + 2y + 1$

$D(f) = \mathbb{R}^2$ ✓ $\quad k(f) = \mathbb{R}^2$ ✗

$x + 2y + 1 = 0$

$y = -\frac{1}{2}x - \frac{1}{2}$



✓

Roko Simurina

(4) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

k	0	1	2	3	4
x_k	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
f_k	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
S		1,002		1,002	

Σ bioj = 2,004 ✓

$$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$$

$$S_1 = \frac{\pi}{12}(0 + 2\sqrt{2} + 1) = 1{,}002$$

$$S_2 = \frac{\pi}{2}(1 + 2\sqrt{2} + 1) = 1{,}002$$

$$S = 2{,}004 \quad ✓$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx = 2{,}004 \quad ✓$$

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IME I PREZIME: FILIP GOREK **BROJ INDEKSA:**

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

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5. $\displaystyle\int_0^2 \frac{2x^2+x+2}{x^2-1}\mathrm{d}x = ?$ 15
6. Integriranjem izračunati površinu između krivulja $x=0$ i $y^2 = x+4$. 15

Ukupno: 30 Kosor

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
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$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert+C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert+C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x+C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
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(6.) $x = 0 \quad y^2 = x + 4$

x	1	2	0
$y^2 = x+4$	2.24	2.46	2

$y^2 = 0 + 4$

$y^2 = 4 / \sqrt{}$

$y_{1,2} = \pm 2$