MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** xoo IME I PREZIME: BEGIĆ ANTONIO BROJ INDEKSA: 17-2-0374-14 Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
2. Odredi ekstreme funkcije $f(x, y) = x^2 - y^2$. 15
3. Za funkciju $f(x, y) = \dfrac{x}{y}$ odrediti domenu, ✓ kodomenu, ✓ razinske krivulje $1/2$ i limes u ishodištu (ako postoji). ~~15~~ 10
4. $\displaystyle\int_0^{\pi} \sin^2 x \cos^3 x \, dx = ?$ 20
5. $\int_0^3 x^2 \ln x \, dx = ?$ ~~15~~
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. ~~20~~

Ukupno: 60

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
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$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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Begic Antonio

1\. $xyy' = 1 - x^2 \; /:x$ UVJET $y(1)=1$

$yy' = \frac{1-x^2}{x}$

$y\frac{dy}{dx} = \frac{1}{x} - \frac{x^2}{x} \; /\cdot dx$

$\int y\,dy = \int \frac{1}{x}dx - \int x\,dx$

$\frac{y^2}{2} = \ln|x| - \frac{x^2}{2} + C \; /\cdot 2$

$y^2 = 2\ln|x| - x^2 + 2C$

$1^2 = 2\cdot\ln|1| - 1^2 + 2C$

$1 = 2\ln|1| - 1 + 2C \; /:2$

$\frac{1}{2} = \ln|1| - \frac{1}{2} + C$

$\frac{1}{2} - \ln|1| + \frac{1}{2} = C$

$\frac{1}{2} - 0 + \frac{1}{2} = C$

$1 = C$

RJ: $y^2 = 2\ln|x| - x^2 + 2\cdot 1 \; /\sqrt{}$

$y = \sqrt{2\ln|x| - x^2 + 2}$

2\. EXTREMI

$f(x,y) = x^2 - y^2$

$\frac{\partial f}{\partial x} = 2x$ $\frac{\partial f}{\partial y} = -2y$ $T(0,0)$

$2x = 0$

$x = 0$

$-2y = 0$

$y = 0$

A $\frac{\partial^2 f}{\partial^2 x} = 2$

C $\frac{\partial^2 f}{\partial^2 y} = -2$

B $\frac{\partial f}{\partial x \partial y} = 0$

$AC - B^2 = 2\cdot(-2) - 0^2 = -4 - 0 = -4 < 0$

$A > 0$

Točka $T(0,0)$ je sedlasta ili stacionarna točka

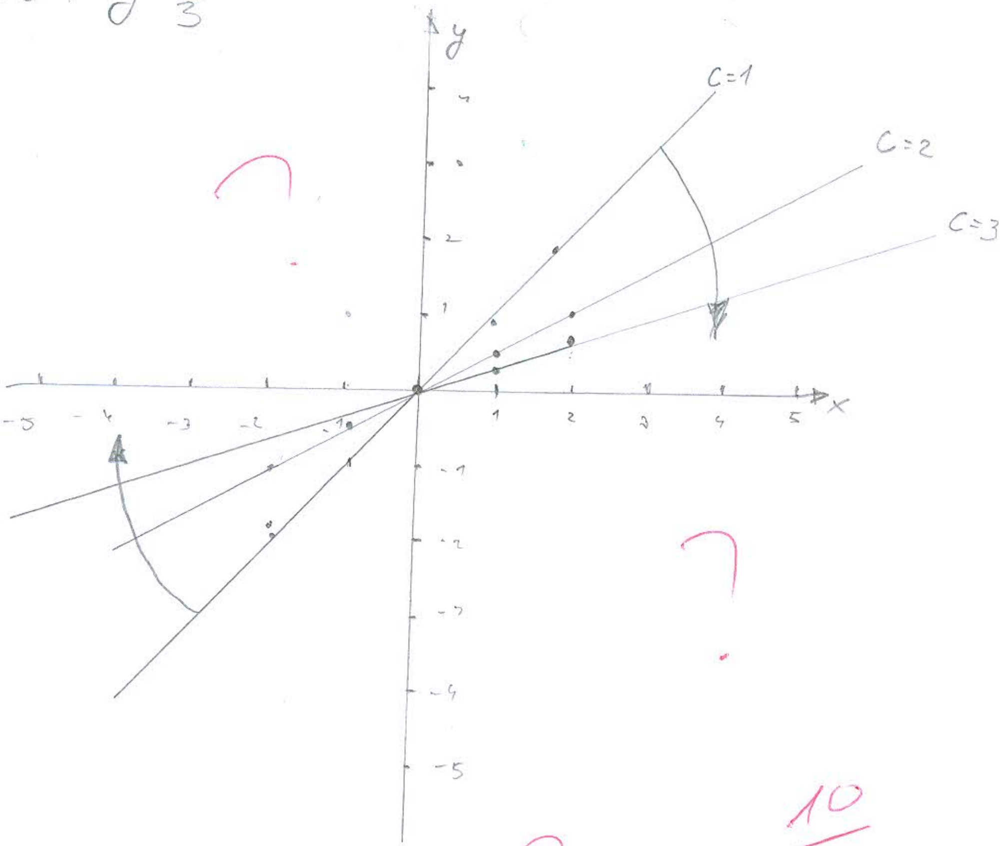
(3) $f(x,y) = \frac{x}{y}$ $D(f) = \mathbb{R}^2 \setminus \{y \neq 0\}$ ✓

Kodomena $= \mathbb{R}$ ✓

$C=1 \rightarrow \frac{x}{y} = 1 / \cdot y \rightarrow x = y$ $C=2 \rightarrow \frac{x}{y} = 2 \,_{\cdot y} \rightarrow x = 2y \xrightarrow{:2} y = \frac{x}{2}$

$C=3 \rightarrow \frac{x}{y} = 3 / \cdot y \rightarrow x = 3y / :3 \rightarrow y = \frac{x}{3}$

$C=0 \rightarrow \frac{x}{y} = 0 \cdot y \rightarrow x = 0$



?

$y=x$	-3	-2	1	0	1	2	3
$y=-x$							

⋮

?

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LIMES U ISHODIŠTU ?

6. $x+y^2=6$ $x+y+1=0$ Begić Antonio

$x+y^2-6=x+y+1$

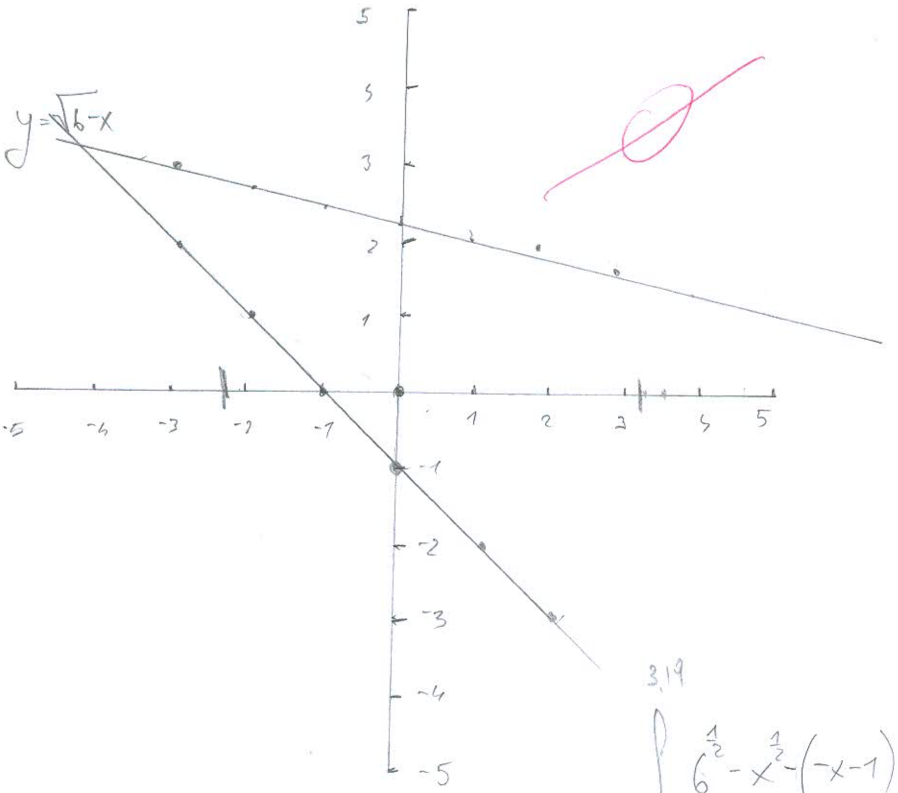
$x+y^2-6-\cancel{x}-y-1=0$

$y^2-y-7=0$

$a=1$ $b=-1$ $c=-7$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_1=-2,19$ $x_2=3,19$



$$\int_{-2,19}^{3,19} 6^{\frac{1}{2}}-x^{\frac{1}{2}}-(-x-1)=\int_{-2,19}^{3,19} 6^{\frac{1}{2}}-x^{\frac{1}{2}}+x+1$$

$x+y^2=6$

$y^2=6-x$

$y=\sqrt{6-x}$

$x+y+1=0$

$y=-x-1$

$$\int_{-2,19}^{3,19} 3,5+x^{\frac{1}{2}} = 3,5x-\frac{1}{2}\frac{x^{-\frac{1}{2}}}{\frac{1}{2}}\Bigg|_{-2,19}^{3,19}$$

$$=3,5x-\frac{1}{2}\cdot\frac{1}{\frac{x}{2}}\Bigg|_{-2,19}^{3,19} = 10,85+7,20=\boxed{18,05}$$

(5) $\int_0^3 x^2 \cdot \ln x \, dx = \left| \begin{matrix} \ln x = u & dv = \int x^2 \\ \frac{1}{x} = du & v = \frac{x^3}{3} \end{matrix} \right| = \ln x \, \frac{x^3}{3} \Big|_0^3 - \int_0^3 \frac{x^3}{3} \cdot \frac{1}{x} \, dx$

$= \ln x \, \frac{x^3}{3} \Big|_0^3 - \int_0^3 x^2 \cdot \frac{1}{3} = \ln|x| \cdot \frac{x^3}{3} \Big|_0^3 - \left(\frac{1}{3} \cdot \frac{x^3}{3} \right) \Big|_0^3 = \ln|x| \cdot \frac{x^3}{3} \Big|_0^3 - \left(\frac{x^3}{9} \right) \Big|_0^3$

$= \left[\ln|3| \cdot \frac{3^3}{3} \right] - \left[\ln|0| \cdot \frac{0^3}{0} \right]^0 - \left(\frac{3^3}{9} - 0 \right) = 9{,}89 + 3 = 12{,}89$ ✗

(4) $\int_0^{\pi} \sin^2 x \cdot \cos^3 x \, dx = \int_0^{\pi} \sin^2 x \cdot \cos^2 x \cdot \cos x \, dx = \left| \begin{matrix} \sin x = t \\ \cos x \, dx = dt \\ \cos^2 + \sin^2 = 1 \\ \cos^2 = 1 - t^2 \end{matrix} \right| = \int_0^0 t^2 \cdot (1 - t^2) \cdot dt =$

$\int_0^0 t^2 - t^4 = 0 - 0 = \boxed{0}$ ✓

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IME I PREZIME: SANDRO GRDOVIĆ **BROJ INDEKSA:** 17-2-0213-2012

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK

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2. $f(x,y) = x^2 - y^2$

$$\frac{df}{dx} = 2x$$

$$\frac{df}{dy} = 2y$$ ✗

$$\frac{d^2f}{dx^2} = 2$$

$$\frac{d^2f}{dy^2} = 2$$ ✗

$$\frac{d^2f}{dx\,dy} = \frac{d^2f}{dy\,dx} = 0$$

$$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0$$

$2 \to > 0$

$2x = 0 \qquad 2y = 0$

$x = 0 \qquad y = 0$

$T(0,0) \Rightarrow$ lokalni minimum ✗

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4. $$\int_0^{\pi} \sin^2 x \cdot \cos^3 x \, dx = \int_0^{\pi} \sin^3 x \,(1 - \sin^3 x) \cdot \cos dx = \begin{vmatrix} \sin x = t \\ \cos dx = dt \end{vmatrix}$$

? ?

$$= \int_{t_1}^{t_2} t^2 (1 - t^2)\, dt = \int_{t_1}^{t_2} t^2 dt - \int_{t_1}^{t_2} t^4 dt = \frac{t^3}{3} - \frac{t^5}{5} \Bigg|_{t_1}^{t_2}$$

$$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} \Bigg|_0^{\pi} = (0 - 0) - (0 - 0) = 0$$ ✓ //

SANDRA GRUBOVIC

5. $\int_0^3 x^2 \ln x \, dx = \left| \begin{array}{ll} u = \ln x & dv = x^2 \, dx \\ du = \frac{dx}{x} & v = \frac{x^3}{3} \end{array} \right.$

$= \ln x \cdot \frac{x^3}{3} \Big|_0^3 - \frac{1}{3} \int_0^3 x^2 \, dx$

$= \ln 3 \cdot 9 - 0 - \frac{1}{3} \frac{x^3}{3} \Big|_0^3$

$= 9 \ln 3 - \frac{1}{9} (3^3 - 0^3)$

$= 9 \ln 3 - 3$

$\approx 6,8875$ ✓

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3. $f(x, y) = \frac{x}{y}$

$y \neq 0$

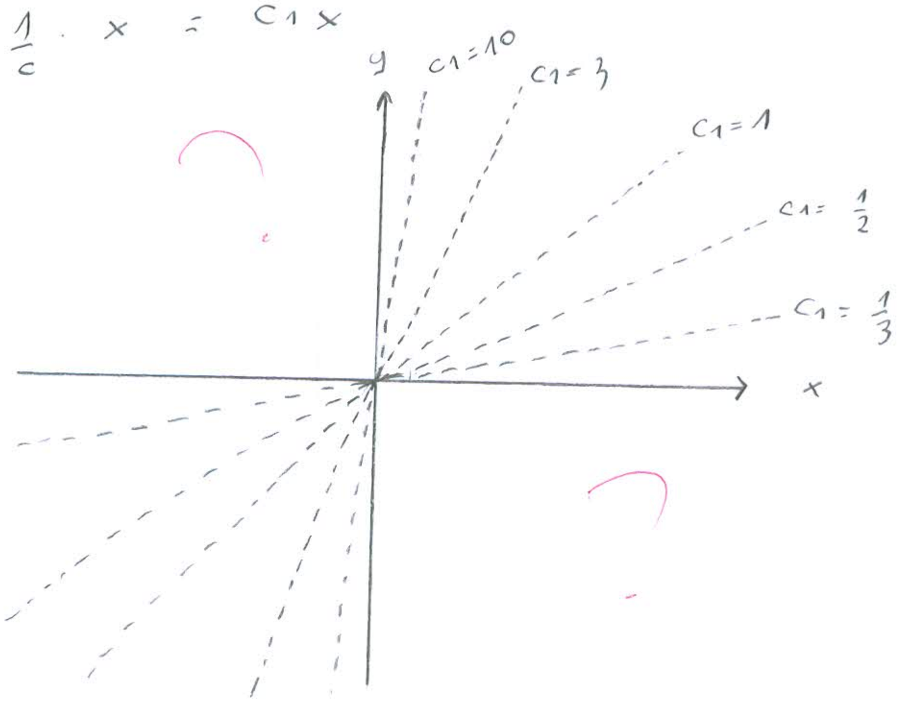
$D_f: (x, y) \in \mathbb{R}^2, \; y \neq 0$ ✓ (DOMENA)

$K_R: \mathbb{R}$ (KODOMENA) ✓

RAZINA KRIVULJE:

$C = \frac{x}{y}$

$y = \frac{x}{c} = \frac{1}{c} \cdot x = c_1 x$



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→

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x}{y} = \left(\frac{0}{0}\right) \overset{L'H.}{=} \lim_{\substack{x\to 0\\ y\to 0}} \frac{1}{1} = 1$$ ✗

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1. $x y y^{1} = 1 - x^2$? $y(1) = 1$

$$x y \cdot \frac{dy}{dx} = 1 - x^2 \quad \Big| \cdot \frac{dx}{x}$$

$$y\,dy = \frac{1-x^2}{x}\,dx \quad \Big| \int$$

$$\int y\,dy = \int \frac{1-x^2}{x}\,dx$$

$$\frac{y^2}{2} = \int \frac{1}{x} - \int x \; dx$$

$$\frac{y^2}{2} = \ln x - \frac{x^2}{2} + C$$

$$\frac{1}{2} = \ln 1 - \frac{1}{2} + c$$

$$1 + \ln 1 = c \quad \Rightarrow \quad c = 1$$ ✓

$$\frac{y^2}{2} = \ln x - \frac{x^2}{2} + 1 \quad \Big| \cdot 2$$ ✓

$$y^2 = 2\ln x - x^2 + 2$$

$$y^2 = 2(\ln x + 1) - x^2 \quad \Big| \sqrt{}$$

$$y = \sqrt{2(\ln x + 1) - x^2}$$