MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** XOO

IME I PREZIME: BEGIĆ ANTONIO **BROJ INDEKSA:** 17-2-0374-14

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi diferencijalnu jednadžbu $xyy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
2. Odredi ekstreme funkcije $f(x, y) = x^2 - y^2$. 15
3. Za funkciju $f(x, y) = \dfrac{x}{y}$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). 15 10
4. $\displaystyle\int_0^{\pi} \sin^2 x \cos^3 x \, \mathrm{d}x = ?$ 20
5. $\int_0^3 x^2 \ln x \, \mathrm{d}x = ?$ 15
6. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. 20

Ukupno: 60

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
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$\sin x$	$\cos x$
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$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \, \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln \lvert \cos x \rvert + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, \mathrm{d}x = \ln \lvert \sin x \rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \mathrm{e}^x \, \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

Begić
Antonio

① $xyy' = 1 - x^2 \;/ : x$ UVJET $y(1) = 1$

$yy' = \frac{1-x^2}{x}$

$y\frac{dy}{dx} = \frac{1}{x} - \frac{x^2}{x} \;/ \cdot dx$

$\int y\,dy = \int \frac{1}{x}dx - \int x\,dx$

$\frac{y^2}{2} = \ln|x| - \frac{x^2}{2} + C \;/ \cdot 2$ ✓

$y^2 = 2\ln|x| - x^2 + 2C$

$1^2 = 2\cdot\ln|1| - 1^2 + 2C$

$1 = 2\ln|1| - 1 + 2C \;/ : 2$

$\frac{1}{2} = \ln|1| - \frac{1}{2} + C$

$\frac{1}{2} - \ln|1| + \frac{1}{2} = C$

$\frac{1}{2} - 0 + \frac{1}{2} = C$

$1 = C$

Rj: $y^2 = 2\ln|x| - x^2 + 2\cdot 1$ ✓

$y = \sqrt{2\ln|x| - x^2 + 2}$

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② EKSTREMI

$f(x,y) = x^2 - y^2$

$\frac{\partial f}{\partial x} = 2x$ $\qquad$ $\frac{\partial f}{\partial y} = -2y$

$2x = 0$ $\qquad$ $-2y = 0$

$x = 0$ $\qquad$ $y = 0$

$T(0,0)$

(A) $\frac{\partial^2 f}{\partial^2 x} = 2$

(C) $\frac{\partial^2 f}{\partial^2 y} = -2$

(B) $\frac{\partial f}{\partial x \partial y} = 0$

$AC - B^2 = 2\cdot(-2) - 0^2 = -4 - 0 = -4 < 0$

$A > 0$

Točka $T(0,0)$ je sedlasta ili stacionarna točka ✓

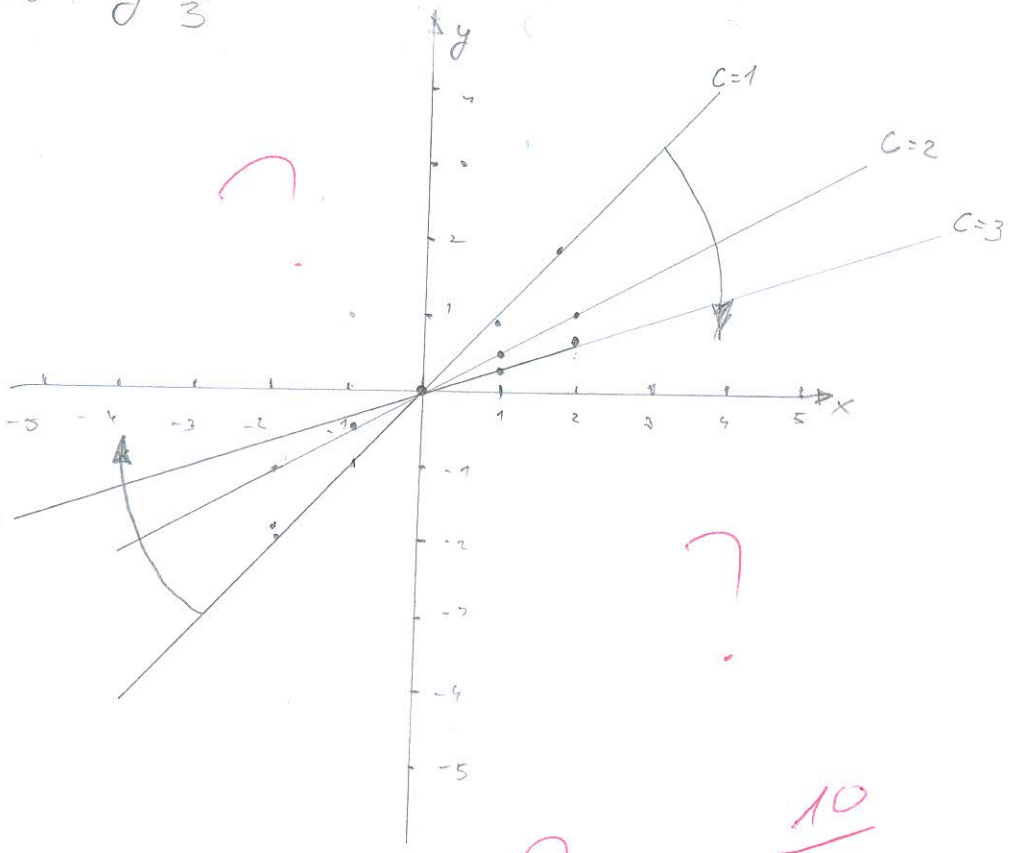
③ $f(x,y) = \frac{x}{y}$ $D(f) = \mathbb{R}^2 \setminus \{y \neq 0\}$ ✓

Kodomena = $\mathbb{R}$ ✓

$C=1 \rightarrow \frac{x}{y} = 1 \,/\cdot y \rightarrow x = y$ $C=2 \rightarrow \frac{x}{y} = 2 \,/\cdot y \rightarrow x = 2y \,/:2 \rightarrow y = \frac{x}{2}$

$C=3 \rightarrow \frac{x}{y} = 3 \,/\cdot y \rightarrow x = 3y \,/:3 \rightarrow y = \frac{x}{3}$

$C=0 \rightarrow \frac{x}{y} = 0 \,/\cdot y \rightarrow x = 0$



$y=x$	-3	-2	1	0	1	2	3
$y=-x$							

?

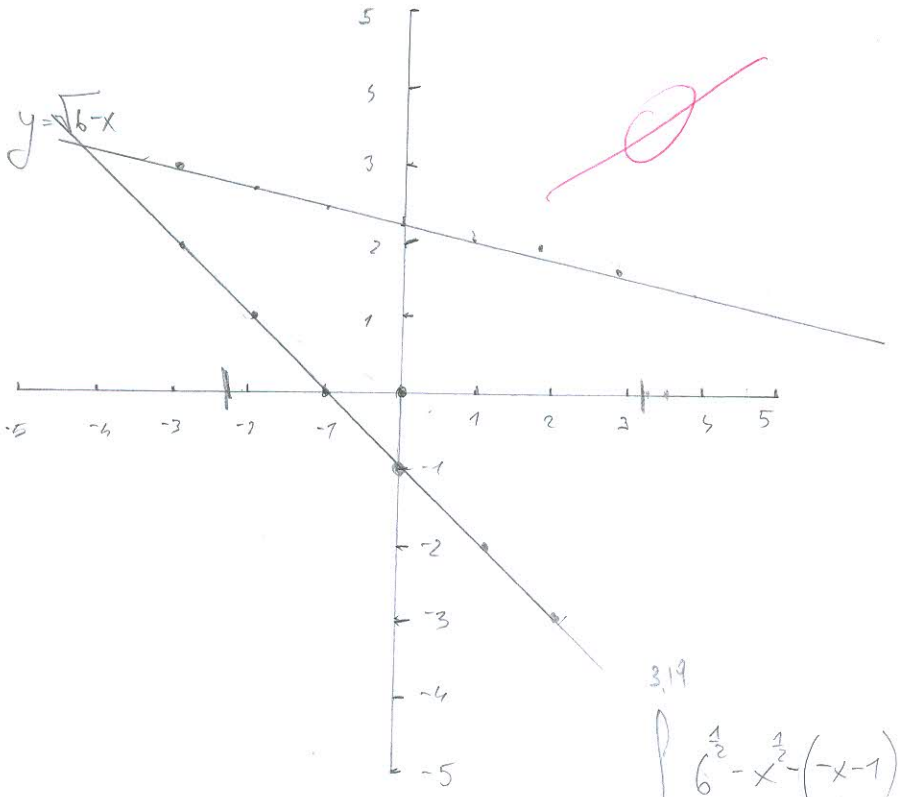
?

?

10

LIMES U ISHODIŠTU.

6. $x+y^2=6$ $x+y+1=0$ Begić Antonio



$x+y^2-6=x+y+1$ ✗

$x+y^2-6-x-y-1=0$

$y^2-y-7=0$

$a=1 \quad b=-1 \quad c=-7$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_1=-2,19 \quad x_2=3,19$

$\int_{-2,19}^{3,19} 6^{\frac{1}{2}}-x^{\frac{1}{2}}-(-x-1)=\int_{-2,19}^{3,19} 6^{\frac{1}{2}}-x^{\frac{1}{2}}+x+1$

$x+y^2=6$

$y^2=6-x$

$y=\sqrt{6-x}$

$x+y+1=0$

$y=-x-1$

$\int_{-2,19}^{3,19} 3,5+x^{\frac{1}{2}}=3,5x-\frac{1}{2}\frac{x^{-\frac{1}{2}}}{\frac{1}{2}}\Big|_{-2,19}^{3,19}$

$=3,5x-\frac{1}{2}\cdot\frac{1}{\frac{x}{2}}\Big|_{-2,19}^{3,19} = 10,85+7,20=\boxed{18,05}$

(5) $\int_0^3 x^2 \cdot \ln x \, dx = \left| \begin{matrix} \ln x = u & dv = x^2 \\ \frac{1}{x} = du & v = \frac{x^3}{3} \end{matrix} \right| = \ln x \cdot \frac{x^3}{3} \Big|_0^3 - \int_0^3 \frac{x^3}{3} \cdot \frac{1}{x} \, dx$

$= \ln x \cdot \frac{x^3}{3} \Big|_0^3 - \int_0^3 x^2 \cdot \frac{1}{3} = \ln|x| \cdot \frac{x^3}{3} \Big|_0^3 - \left(\frac{1}{3} \cdot \frac{x^3}{3} \right) \Big|_0^3 = \ln|x| \cdot \frac{x^3}{3} \Big|_0^3 - \left(\frac{x^3}{9} \right) \Big|_0^3$

$= \left[\ln|3| \cdot \frac{3^3}{3} \right] - \left[\ln|0| \cdot \frac{0^3}{0} \right] - \left[\frac{3^3}{9} - 0 \right] = 9{,}89 + 3 = 12{,}89$ ✗

(4) $\int_0^{\pi} \sin^2 x \cdot \cos^3 x \, dx = \int_0^{\pi} \sin^2 x \cdot \cos^2 x \cdot \cos x \, dx = \left| \begin{matrix} \sin x = t \\ \cos x \, dx = dt \\ \cos^2 + \sin^2 = 1 \\ \cos^2 = 1 - t^2 \end{matrix} \right| = \int_0^0 t^2 \cdot (1 - t^2) \cdot dt =$

$\int_0^0 t^2 - t^4 = 0 - 0 = \boxed{0}$ ✓

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xoo

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