MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: FILIP ŠTRLEK **BROJ INDEKSA:** 17-2-0230-2012

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje. 20
2. Riješi diferencijalnu jednadžbu $x^2yy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
3. Skiciraj razinske krivulje funkcije $f(x,y) = \ln(x+y)$. 15
4. $\displaystyle\int_0^2 \frac{x+2}{3x^2-2x-5}\mathrm{d}x = ?$ ~~20~~
5. $\displaystyle\int_{-2}^0 3\sqrt{1-3x}\,\mathrm{d}x = ?$ ~~15~~
6. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(2,2)$, $B(0,-4)$, $C(4,0)$. ~~15~~

Ukupno: 20

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

1.

$$4y'' - y = x \sin x$$

$$4\lambda^2 - 1 = 0$$

$$\boxed{\lambda_{1/2} = \pm\frac{1}{2}}$$

$$\boxed{y_h = C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{1}{2}x}}$$

$$y_p = Ax\cos x + Bx\sin x$$

$$y_p' = A(x'\cdot\cos x + x\cdot\cos x') + B(x'\cdot\sin x + x\cdot\sin x')$$
$$= A(\cos x + (-x\sin x)) + B(\sin x + x\cos x)$$
$$= A\cos x - Ax\sin x + B\sin x + Bx\cos x$$

$$y_p'' = -A\sin x - A(x'\cdot\sin x + x\cdot\sin x') + B\cos x + B(x'\cos x + x\cdot\cos x')$$
$$= -A\sin x - A(\sin x + x\cos x) + B\cos x + B(\cos x - x\sin x)$$
$$= -A\sin x - A\sin x - Ax\cos x + B\cos x + B\cos x - Bx\sin x$$
$$= -2A\sin x - Ax\cos x + 2B\cos x - Bx\sin x$$

$$4y'' - y = x\sin x$$

$$4\cdot(-2A\sin x - Ax\cos x + 2B\cos x - Bx\sin x) - Ax\cos x - Bx\sin x = x\sin x$$

$$-8A\sin x - 4Ax\cos x + 8B\cos x \underline{- 4Bx\sin x} - Ax\cos x \underline{- Bx\sin x} = x\sin x$$

$$-5B = 1 \qquad \boxed{A = 0}$$

$$\boxed{B = -\frac{1}{5}}$$

$$\boxed{y_p = -\frac{1}{5}x\sin x}$$

$$\boxed{y = -\frac{1}{5}x\sin x + C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{1}{2}x}}$$

PROVJERA:

$$y' = -\frac{1}{5}(\sin x + x\cos x) - \frac{1}{2}C_1 e^{-\frac{1}{2}x} + \frac{1}{2}C_2 e^{\frac{1}{2}x}$$

$$= -\frac{1}{5}\sin x - \frac{1}{5}x\cos x - \frac{1}{2}C_1 e^{-\frac{1}{2}x} + \frac{1}{2}C_2 e^{\frac{1}{2}x}$$

$$y'' = -\frac{1}{5}\sin x - \frac{1}{5}(\cos x - x\sin x) + \frac{1}{4}C_1 e^{-\frac{1}{2}x} + \frac{1}{4}C_2 e^{\frac{1}{2}x}$$

$$4\cdot\left(\frac{1}{5}x\sin x + \frac{1}{4}C_1 e^{-\frac{1}{2}x} + \frac{1}{4}C_2 e^{\frac{1}{2}x}\right) - \left(-\frac{1}{5}x\sin x + C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{1}{2}x}\right)$$

$$\underline{\frac{4}{5}x\sin x} + \frac{4}{4}C_1 e^{-\frac{1}{2}x} + \frac{4}{4}C_2 e^{\frac{1}{2}x} + \underline{\frac{1}{5}x\sin x} - C_1 e^{-\frac{1}{2}x} - C_2 e^{\frac{1}{2}x}$$

$$\underbrace{\frac{4}{5}x\sin x + \frac{1}{5}x\sin x} + \cancel{C_1 e^{-\frac{1}{2}x}} + \cancel{C_2 e^{\frac{1}{2}x}} - \cancel{C_1 e^{-\frac{1}{2}x}} - \cancel{C_2 e^{\frac{1}{2}x}}$$

$$\underline{x\sin x = x\sin x}$$

Filip Pavlek

5.

$$\int_{-2}^{0} 3\sqrt{1-3x}\,dx = \left[\begin{array}{l} 1-3x = t^2 /' \\ -3\,dx = 2t\,dt /\cdot(-1) \\ 3\,dx = -2t\,dt \end{array}\right.$$

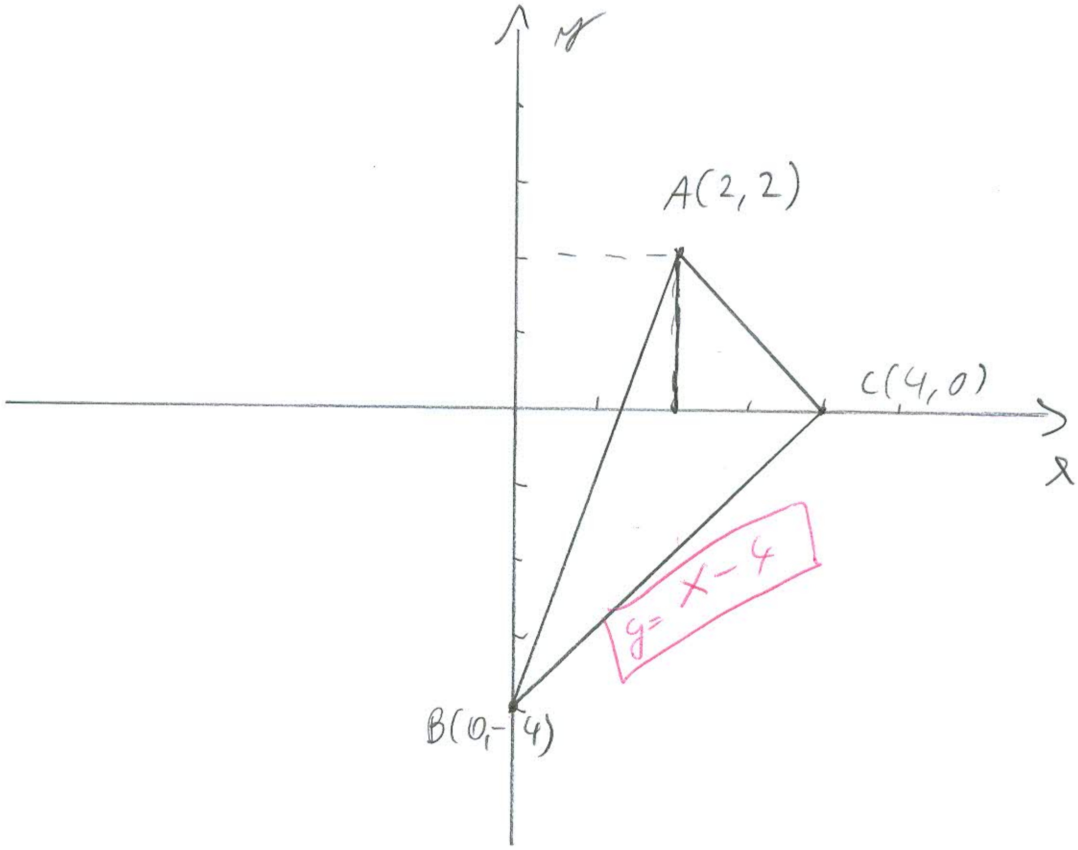
$$= \int_{-2}^{0} -2t^2\,dt = -2\int_{-2}^{0} t^2 = -\frac{2}{3}\cancel{t}(1-3x)^3 =$$

$$= -\frac{2}{3}\left[(1-3\cdot 0) - [1-3\cdot(-2)]^3\right] = -\frac{2}{3} +$$

$$\boxed{= -\frac{2}{3}}$$

Filip Ftrbr

6. $A(2,2)$
$B(0,-4)$
$C(4,0)$



$\overline{AB} = (y-y_1) = \frac{y_2-y_1}{x_2-x_1} \cdot (x-x_1)$

$(y-2) = \frac{-4-2}{0-2} \cdot (X-2)$

$y = 3(x-2)+2$

$\boxed{y = 3X - 4}$

$\overline{AC} = (y-2) = \frac{0-2}{4-2}(x-2)$

$y = -X+2+2$

$\boxed{y = -X+4}$

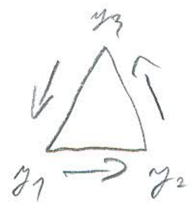
$\overline{BC} = y+4 = \frac{0+4}{4-0}(x-4)$

$\boxed{y = x-8}$

$$P = \frac{1}{2}\left[2\cdot(-4) + 0\cdot(0-2) + 4\cdot(2+4)\right]$$

$$= \frac{1}{2}(-8 + 0 + 24) = \frac{16}{2} = 8$$

OVO NIJE INTEGRACIJA



$$\boxed{P = 8}$$

---

4. $$\int_0^2 \frac{x+2}{3x^2 - 2x - 4 - 1} = \int \frac{x}{3x}$$

$$\underbrace{-2x-4}_{-2(x+2)-1}$$

$(x-x_1)(x-x_2)$ $\quad (x+1)(x-\frac{5}{3})$

$3x^2 - 2x - 5 = 0$ $\quad x^2$

$D = 4 + 4\cdot 3\cdot 5$

$D = 64$

$x_{1/2} = \frac{2 \pm 8}{6}$

$$\boxed{x_1 = -1}$$

$$\boxed{x_2 = \frac{5}{3}}$$

NEPRAVI INTEGRAL?

$$\int \frac{x+2}{(x+1)(x-\frac{5}{3})} = \frac{A}{x+1} + \frac{B}{x-\frac{5}{3}}$$

$$x+2 = Ax - A\frac{5}{3} + Bx + B$$

$\boxed{A+B=1} \Rightarrow A = 1-B$ $\quad 3x^2 - 2x - 5 = A \,/\,'$

$-\frac{5}{3}A + B = 2 \;/\cdot 3$ $\quad \boxed{A = -\frac{3}{8}}$ $\quad 6x - 2$

$-5A + 3B = 6$

$-5(1-B) + 3B = 6$

$-5 + 5B + 3B = 6$

$8B = 11$

$\boxed{B = \frac{11}{8}}$

$$\int -\frac{3}{8(x+1)} + \int \frac{11}{8(x-\frac{5}{3})}$$

$$-\frac{3}{8}\ln|x+1| + \frac{11}{8}\ln\left|x - \frac{5}{3}\right| + C$$

$-0{,}42 \quad -1{,}52$

$-\frac{3}{8}\ln|3| + \frac{11}{8}\ln|0{,}33| \approx -1{,}94$

$\underbrace{\frac{3}{8}\ln|1|}_{0} + \frac{11}{8}\ln|1{,}67| = 0{,}71$

$\Rightarrow \boxed{\approx 1{,}24}$ $\quad 0$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK

IME I PREZIME: DOMAGOJ GROZAJ **BROJ INDEKSA:** 17-1-0056-2011

Želim ustmeni kod (zaokružiti): $\boxed{\text{prof. Uglešić}}$ asistent Kosor

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x \sin x$ i na kraju provjeriti rješenje.	20
2. Riješi diferencijalnu jednadžbu $x^2yy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	15
3. Skiciraj razinske krivulje funkcije $f(x, y) = \ln(x + y)$.	15
4. $\int_0^2 \frac{x+2}{3x^2 - 2x - 5} \mathrm{d}x = ?$	20
5. $\int_{-2}^0 3\sqrt{1 - 3x} \mathrm{d}x = ?$	15
6. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(2, 2)$, $B(0, -4)$, $C(4, 0)$.	~~15~~
Ukupno:	15

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln \lvert \cos x \rvert + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, \mathrm{d}x = \ln \lvert \sin x \rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a}\right)\right] + C$	

$$P_1 = \int_0^2 AC - AB$$

$BC - AB$

$$= \int_0^2 (x-4) - (3x-4)\, dx$$

$$= \int_0^2 x - \cancel{4} - 3x + \cancel{4}\, dx$$

$$= \int_0^2 -2x\, dx = -2 \int_0^2 x\, dx = -2 \cdot \frac{x^2}{2} \Big|_0^2$$

$$= -x^2 \Big|_0^2 = -2^2 + 0^2 = \underline{4} \checkmark$$

$$P_2 = \int_2^4 \underline{(x-4) - (3x-4)}\, dx \times$$

$$= \int_2^4 x - 4 - 3x + 4\, dx = \int_2^4 -2x\, dx = -2 \frac{x^2}{2} \Big|_2^4$$

$$= -x^2 \Big|_2^4 = -4^2 - 2^2 = \underline{\underline{12}} \times$$

$$P_{uk} = 16 \times$$

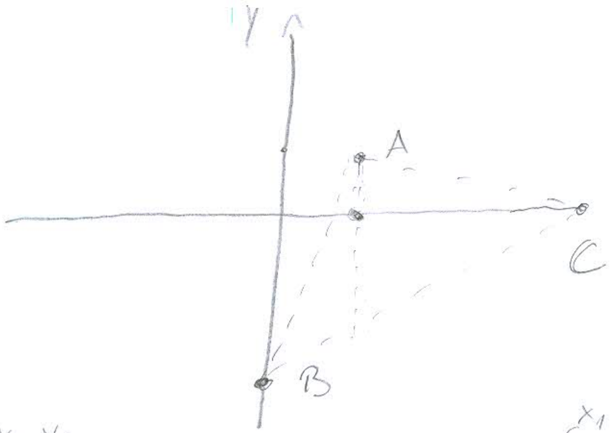
6 ZADATAK

$$(y-y_1)(x_2-x_1)=(y_2-y_1)(x-x_1)$$

A(2,2)

B(0,-4)

C(4,0)



$\begin{matrix} x_1 & y_1 \\ (2, & 2) \end{matrix}$ $\begin{matrix} x_2 & y_2 \\ (4, & 0) \end{matrix}$

$AC = (y-2)(4-2) = (0-2)(x-2)$

$= 2(y-2) = -2(x-2)$

$2y - 4 = -2x + 4$

$2y = -2x + 4 + 4$

$2y = -2x + 8 \quad /:2$

$\underline{y = -x + 4}$

$\begin{matrix} x_1 & y_1 \\ (2, & 2) \end{matrix}$ $\begin{matrix} x_2 & y_2 \\ (0, & -4) \end{matrix}$

$AB = (y-2)(0-2) = (-4-2)(x-2)$

$-2(y-2) = -6(x-2)$

$-2y + 4 = -6x + 12$

$-2y = -6x + 12 - 4$

$-2y = -6x + 8 \quad /:(-2)$

$y = 3x - 4$

$\begin{matrix} x_1 & y_1 \\ (0, & -4) \end{matrix}$ $\begin{matrix} x_2 & y_2 \\ (4, & 0) \end{matrix}$

$BC = (y+4)(4-0) = (0+4)(x-0)$

$4(y+4) = 4x$

$4y + 16 = 4x$

$4y = 4x - 16 \quad /:4$

$\underline{y = x - 4}$

$P_1 = \int_0^2 AC - A_B$

$= \int_0^2 (x-4) - (3x-4)\,dx$

$=$

⑤ ZADATAK

$$\int_{-2}^{0} 3\sqrt{1-3x}\, dx$$

$t = 1-3x$

$dt = -3\,dx$

$dx = \frac{dt}{-3}$

$1-(3\cdot 0)=1$

$1-(3\cdot(-2))$

$=7$

$$\int_{1}^{7} \cancel{3}\sqrt{t}\,\frac{dt}{\cancel{3}}$$

$\frac{1}{2}+1=\frac{3}{2}$

$$\int_{1}^{7} \sqrt{t}\, dt = \int t^{\frac{1}{2}}\, dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \Big|_{1}^{7} = \frac{2t^{\frac{3}{2}}}{3} = \left(\frac{2\cdot 7^{\frac{3}{2}}}{3}\right) - \left(\frac{2\cdot 1^{\frac{3}{2}}}{3}\right)$$

$= 12,3 - 0,66 = 11,63$ ✓

GROZDAJ

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OXO

POPUNJAVA NASTAVNIK

IME I PREZIME: LARA SJEPICA **BROJ INDEKSA:** 17-2-0382-2014

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

		Broj ↓ bodova
1.	Riješiti diferencijalnu jednadžbu: $4y'' - y = x\sin x$ i na kraju provjeriti rješenje.	20
2.	Riješi diferencijalnu jednadžbu $x^2yy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$.	15
3.	Skiciraj razinske krivulje funkcije $f(x,y) = \ln(x+y)$.	15
4.	$\int\limits_0^2 \frac{x+2}{3x^2-2x-5}\mathrm{d}x = ?$	20
5.	$\int\limits_{-2}^0 3\sqrt{1-3x}\mathrm{d}x = ?$	~~15~~
6.	Integriranjem odrediti površinu trokuta koji je zadan točkama $A(2,2)$, $B(0,-4)$, $C(4,0)$.	~~15~~ $\emptyset$

Ukupno: ~~15~~ $\emptyset$

Kos

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

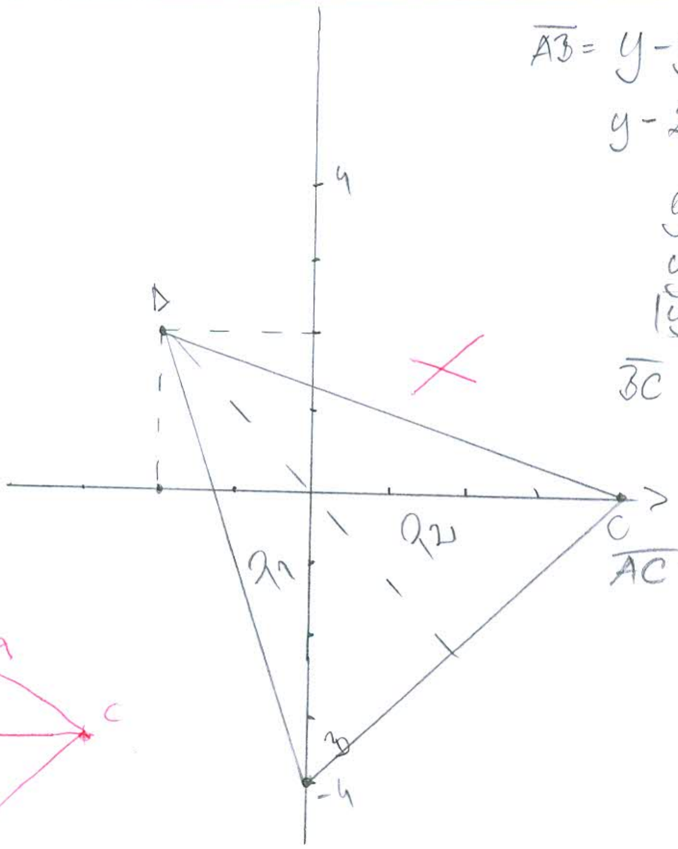
Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

6.) $A(\underset{x_1}{2},\underset{y_1}{2})$

$B(\underset{x_2}{0},\underset{y_2}{-4})$

$C(\underset{x_1}{4},\underset{y_2}{0})$



$\overline{AB} = y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y - 2 = \frac{-4-2}{0-2}(x-2)$

$y = 3(x-2)+2$

$y = 3x - 6 + 2$

$\boxed{y = 3x - 4}$

$\overline{BC} = y + 4 = \frac{0+4}{4-0}(x-0)$

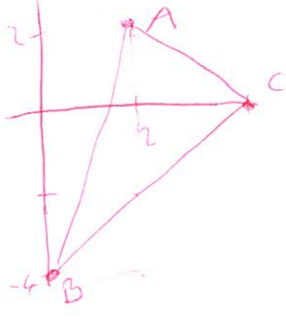
$\boxed{y = 1x - 4}$

$\overline{AC} = y - 2 = \frac{0-2}{4-2}(x-2)$

$y = -1(x-2)+2$

$y = -x + 2 + 2$

$\boxed{y = -x + 4}$



$$P_1 = \int_0^2 AB - AC = \int_0^2 \cancel{3x-4} = \int_0^2 -x+4$$

$$\cancel{= (0-4+0+4) - (6-4+2+4)}$$

$$\cancel{= -8 - 8}$$

$$= 4$$

$$P_1 = \int_2^4 AC - \int_0^2 AB = \int_2^4 (-x+4) - \int_0^2 3x-4 \quad \times$$

$$= (-2+4-4+4) - (0-4+6-4)$$

$$= 2+2$$

$$= 4 \,//$$

$$\cancel{P_2 = \int_0^4 BC - \int_0^2 AB = \int_0^4 (x-4) - \int_0^2 (3x-4)}$$

$$\cancel{= (0-4) - (0-4)}$$

$$= (0-4+4-4) - (0-4+6-4)$$

$$\cancel{-4+2}$$

$$P_2 = \int_0^2 AB - \int BC = \int_0^2 (3x-4) - \int_0^4 (x-4) \quad \times$$

$$= (0-4+6-4) - (0-4+4-4)$$

$$= -2+4$$

$$= \boxed{2}$$

$$P_{UKUPNO} = P_1 + P_2 = \boxed{6} \,//$$

✓

LARA ŠTEFICA

5. $\int_{-2}^{0} 3\sqrt{1-3x}\,dx = \left\{ \begin{array}{c} 1-3x = t^2 \\ dt = 3dx \\ dx = \frac{dt}{3} \end{array} \right\} = t \cdot \frac{dt}{3}$

$\varnothing$

$= (1-3x)^{\frac{1}{2}} \cdot \frac{3dx}{3} \Big|_{-2}^{0} = \cancel{\frac{1}{2}(3x)^{\frac{1}{2}}}$

$= \dfrac{(1-3x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} = 12,35$ VIDI GROZDAS

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** OXO

POPUNJAVA NASTAVNIK

Broj ↓ bodova

Ime i prezime: TOMISLAV BULIĆ **Broj indeksa:** 17-2-0271-2013

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

1. Riješiti diferencijalnu jednadžbu: $4y'' - y = x\sin x$ i na kraju provjeriti rješenje. 20
2. Riješi diferencijalnu jednadžbu $x^2yy' = 1 - x^2$ uz rubni uvjet $y(1) = 1$. 15
3. Skiciraj razinske krivulje funkcije $f(x, y) = \ln(x + y)$. 15
4. $\displaystyle\int_0^2 \frac{x+2}{3x^2-2x-5}\,\mathrm{d}x = ?$ 20
5. $\displaystyle\int_{-2}^0 3\sqrt{1-3x}\,\mathrm{d}x = ?$ 15
6. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(2,2)$, $B(0,-4)$, $C(4,0)$. 15

Ukupno: ∅

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

⑥ $A(\overset{x_1}{2},\overset{y_1}{2})$

$B(\overset{x_2}{0},\overset{y_2}{-4})$

$C(4,0)$

AB

$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$

$y - 2 = \frac{-4-2}{0-2}(x-2)$

$y - 2 = 3x - 6$

$y = 3x - 4$ ✓

AC

$y - 2 = \frac{0-2}{4-2}(x-2)$

$y - 2 = -x + 2$

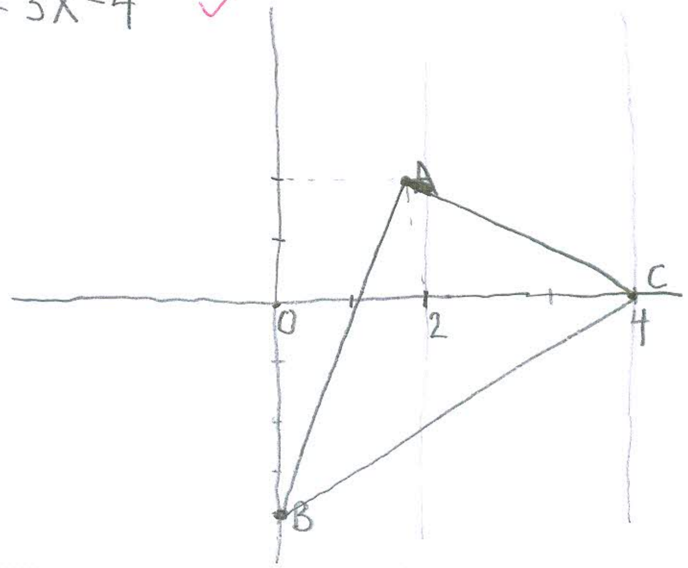
$y = -x + 4$ ✓

BC

$y + 4 = \frac{0+4}{4-0}(x - 0)$

$y + 4 = x$

$y = x - 4$ ✓



→

$$\int_0^2 AB - BC\,dx + \int_2^4 AC - BC\,dx$$

$$= \int_0^2 (3x-4)-(x-4)\,dx + \int_2^4 -x+4-x-4\,dx$$

$$= \int_0^2 2x-8\,dx + \int_2^4 -2x\,dx$$

$$= -8\int_0^2 2x\,dx + -\int_2^4 2x\,dx$$

$$= -16\int_0^2 x\,dx - 2\int_2^4 x\,dx$$

$$= -16 \cdot \int_0^2 \frac{x^2}{2}\,dx - 2\int_2^4 \frac{x^2}{2}\,dx$$

$$= -16 \cdot (2-0) - 2 \cdot \left(\frac{16}{2} - 2\right)$$

$$= -32 - 12$$

×

TOMISLAV BULIĆ

$x^2yy' = 1 - x^2 \,/\, : x^2 \qquad y(1) = 1$

$\cancel{yy' = \frac{1}{x^2} - \frac{1}{1} = \frac{1-}{x^2}}$

(1)

$4y'' - y = x\sin x \,/\, : 4$

$y'' - \frac{y}{4} = \frac{x\sin x}{4}$

$y'' - \frac{y}{4} = 0$

$r^2 + 0r - \frac{1}{4} = 0$

$r^2 = \frac{1}{4} \,/\, \sqrt{}$

$r_1 = \frac{1}{2} \quad r_2 = -\frac{1}{2}$

$y(x) = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x}$

$y'(x) = C_1 \frac{1}{2} e^{\frac{1}{2}x} + C_2 -\frac{1}{2} e^{-\frac{1}{2}x}$ ✓

$u = x \qquad dv = \sin x$

$du = 1 \qquad v = -\cos x$

?

$x\sin x - \int -\cos x$

$x\sin x + \int \cos x \, dx$

$x\sin x + \sin x + C$

$C_1 \frac{1}{2} e^{\frac{1}{2}x} + C_2 -\frac{1}{2} e^{-\frac{1}{2}x} = x\sin x + \sin x + C$ ✗

$x^2 y y' = 1 - x^2$ $\qquad y(1) = 1$

$x^2 y \frac{dy}{dx} = 1 - x^2 \quad | \cdot dx$

$x^2 y\, dy = (1 - x^2)\, dx \quad | : x^2$

$\int y\, dy = -\int dx$ $\quad$ X

$\frac{y^2}{2} \cancel{dy} = -x + C$

$\frac{y^2}{2} = -1 \quad | \cdot 2$

$y^2 = -2$ $\quad \emptyset$