

odgovornosti studenata. **PIŠITE DVOSTRANO!**

OXO

IME I PREZIME: **FILIP ŠTARLEK**

BROJ INDEKSA: **17-2-0230-2012**

Želim ustmeni kod (zaokružiti):

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asistent Kosor

1. Riješiti diferencijalnu jednačbu:  $4y'' - y = x \sin x$  i na kraju provjeriti rješenje.

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2. Riješi diferencijalnu jednačbu  $x^2yy' = 1 - x^2$  uz rubni uvjet  $y(1) = 1$ .

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3. Skiciraj razinske krivulje funkcije  $f(x, y) = \ln(x + y)$ .

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4.  $\int_0^2 \frac{x+2}{3x^2-2x-5} dx = ?$

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5.  $\int_{-2}^0 3\sqrt{1-3x} dx = ?$

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6. Integriranjem odrediti površinu trokuta koji je zadan točkama  $A(2, 2)$ ,  $B(0, -4)$ ,  $C(4, 0)$ .

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Ukupno:

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$f$	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$	

7.  $4y'' - y = x \sin x$

$4z^2 - 1 = 0$

$z_{1/2} = \pm \frac{1}{2}$

$y_h = C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{1}{2}x}$

$y_p = Ax \cos x + Bx \sin x$

$y_p' = A(x' \cdot \cos x + x \cdot \cos x') + B(x' \cdot \sin x + x \cdot \sin x')$   
 $= A(\cos x - x \sin x) + B(\sin x + x \cos x)$   
 $= A \cos x - Ax \sin x + B \sin x + Bx \cos x$

$y_p'' = -A \sin x - A(x' \cdot \sin x + x \cdot \sin x') + B \cos x + B(x' \cos x + x \cdot \cos x')$   
 $= -A \sin x - A(\sin x + x \cos x) + B \cos x + B(\cos x - x \sin x)$   
 $= -2A \sin x - Ax \cos x + B \cos x + B \cos x - Bx \sin x$   
 $= -2A \sin x - Ax \cos x + 2B \cos x - Bx \sin x$

$$4y'' - y = x \sin x$$

$$4 \cdot (-2A \sin x - Ax \cos x + 2B \cos x - Bx \sin x) - Ax \cos x - Bx \sin x = x \sin x$$

$$-8A \sin x - 4Ax \cos x + 8B \cos x - 4Bx \sin x - Ax \cos x - Bx \sin x = x \sin x$$

$$-5B = 1$$

$$A = 0$$

$$B = -\frac{1}{5}$$

$$y_p = -\frac{1}{5} x \sin x$$

$$y = -\frac{1}{5} x \sin x + C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{1}{2}x}$$

PROWERA:

$$y' = -\frac{1}{5} (\sin x + x \cos x) - \frac{1}{2} C_1 e^{-\frac{1}{2}x} + \frac{1}{2} C_2 e^{\frac{1}{2}x}$$

$$y'' = -\frac{1}{5} \cos x - \frac{1}{5} x \sin x - \frac{1}{4} C_1 e^{-\frac{1}{2}x} + \frac{1}{4} C_2 e^{\frac{1}{2}x}$$

$$4y'' = -\frac{4}{5} \cos x - \frac{4}{5} x \sin x + C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{1}{2}x}$$

$$4 \cdot \left( -\frac{1}{5} x \sin x + C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{1}{2}x} \right) - \left( -\frac{4}{5} x \sin x + C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{1}{2}x} \right)$$

$$\frac{4}{5} x \sin x + \frac{4}{4} C_1 e^{-\frac{1}{2}x} + \frac{4}{4} C_2 e^{\frac{1}{2}x} + \frac{4}{5} x \sin x - C_1 e^{-\frac{1}{2}x} - C_2 e^{\frac{1}{2}x}$$

$$\frac{4}{5} x \sin x + \frac{4}{5} x \sin x + C_1 e^{-\frac{1}{2}x} + C_2 e^{\frac{1}{2}x} - C_1 e^{-\frac{1}{2}x} - C_2 e^{\frac{1}{2}x}$$

$$x \sin x = x \sin x$$

5)  $\int_{-2}^0 3\sqrt{1-3x} dx = \begin{cases} 1-3x = t^2 \\ -3 dx = 2t dt \cdot (-1) \\ 3 dx = -2t dt \end{cases}$  Substitution

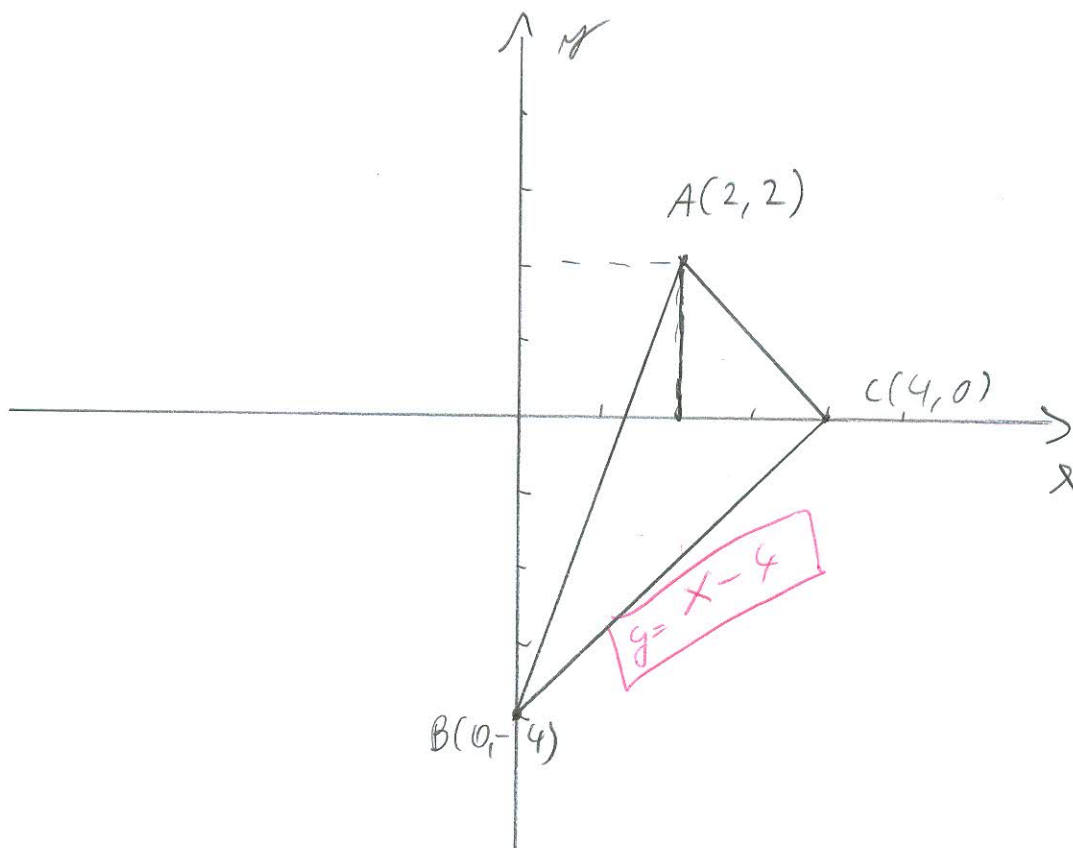
$= \int_{0}^{-2} -2t^2 dt = -2 \int_{-2}^0 t^2 dt = -\frac{2}{3} (1-3x)^{\frac{3}{2}}$

$= -\frac{2}{3} [(1-3 \cdot 0) - (1-3 \cdot (-2))]^{\frac{3}{2}} = -\frac{2}{3} +$

$\boxed{= -\frac{2}{3}}$



- (b.) A(2, 2)
- B(0, -4)
- C(4, 0)



$$\overline{AB} = (y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$(y - 2) = \frac{-4 - 2}{0 - 2} \cdot (x - 2)$$

$$y = 3(x - 2) + 2$$

$$\boxed{y = 3x - 4}$$

$$\overline{AC} = (y - 2) = \frac{0 - 2}{4 - 2} (x - 2)$$

$$y = -x + 2 + 2$$

$$\boxed{y = -x + 4}$$

$\int_{-4}^0 \int_0^2$

$$\overline{BC} = y + 4 = \frac{0 + 4}{4 - 0} (x - 4)$$

$$\boxed{y = x - 4}$$

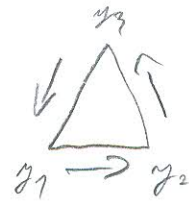
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$$P = \frac{1}{2} [2 \cdot (-4) + 0 \cdot (0-2) + 4 \cdot (2+4)]$$

OVO MI JE  
INTEGRACIJA

$$= \frac{1}{2} (-8 + 0 + 24) = \frac{16}{2} = 8$$

$$\boxed{P = 8}$$



$$\textcircled{4} \int_0^2 \frac{x+2}{3x^2-2x-4-1} = \int \frac{x}{3x} - \frac{-2(x+2)-1}{3x}$$

$$(x-x_1)(x-x_2) \quad (x+1)(x-\frac{5}{3})$$

$$3x^2-2x-5=0 \quad x^2$$

$$D = 4 + 4 \cdot 3 \cdot 5$$

$$D = 64$$

$$x_{1/2} = \frac{2 \pm 8}{6}$$

$$\boxed{x_1 = -1}$$

$$\boxed{x_2 = \frac{5}{3}}$$

NEPRAVI  
INTEGRAL

$$\int \frac{x+2}{(x+1)(x-\frac{5}{3})} = \frac{A}{x+1} + \frac{B}{x-\frac{5}{3}}$$

$$x+2 = \frac{Ax - A\frac{5}{3}}{3} + \frac{Bx+B}{3}$$

$$\boxed{A+B=1} \Rightarrow A=1-B$$

$$-\frac{5}{3}A+B=2 \quad | \cdot 3$$

$$\boxed{A = -\frac{3}{8}}$$

$$-5A+3B=6$$

$$-5(1-B)+3B=6$$

$$-5+5B+3B=6$$

$$8B=11$$

$$\boxed{B = \frac{11}{8}}$$

$$= 0,42 - 1,52$$

$$-\frac{3}{8} \ln|3| + \frac{11}{8} \ln|0,33| \approx -1,94$$

$$\frac{3}{8} \ln|11| + \frac{11}{8} \ln|1,67| = 0,27$$

0

$$\Rightarrow \boxed{\approx -1,24} \quad \phi$$

$$3x^2-2x-5 = A /$$

$$6x-2$$

$$\int -\frac{3}{8(x+1)} + \int \frac{11}{8(x-\frac{5}{3})}$$

$$\int -\frac{3}{8} \ln|x+1| + \frac{11}{8} \ln|x-\frac{5}{3}| + C$$

