

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **MARKO MARASVIĆ**

BROJ INDEKSA: **17-1-0242-2017**

Želim ustmeni kod (zaokružiti):

prof. Uglešić

asistent Kosor

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' + 2y' = 1$, uz $y(0) = 0$ i $y'(0) = 0$.
Na kraju provjeri rješenje.

15

2. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$.

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3. Za funkciju $f(x, y) = \ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji).

20

4. $\int_0^2 \frac{x-1}{x^2+3x+2} dx = ?$

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5. Zadana je funkcija $f(x) = \sqrt{x}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[0, 4]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka.

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6. Integriranjem izračunati površinu trokuta zadanog točkama $A(1, -2)$, $B(2, 0)$, $C(-1, 1)$.

15

Ukupno:

23

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
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Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
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$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
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$$(2) \frac{y'}{x} = \frac{\sin x}{y} \cdot x$$

$$y' = \frac{1}{y} \cdot x \sin x \cdot y$$

$$y \frac{dy}{dx} = x \sin x \cdot dx$$

$$y dy = x \sin x dx / 5$$

$$\int y dy = \int x \sin x dx$$

$$\frac{y^2}{2} = -x \cos x + \sin x + c / 2 \quad \checkmark$$

$$y^2 = -2x \cos x + 2 \sin x + c \Rightarrow y = \sqrt{2 \sin x - 2x \cos x + c} //$$

$$\int x \sin x dx = \left| \begin{array}{l} u=x \quad dv=\sin x \\ du=dx \quad v=-\cos x \end{array} \right|$$

$$= -x \cos x + \int \cos x dx$$

$$= -x \cos x + \sin x + c$$

$$(3) f(x, y) = \ln\left(\frac{y}{x}\right)$$

$$D(f) = \{ \mathbb{R}^2 / \frac{y}{x} > 0 \} \quad \text{KOD} \rightarrow \langle 0, +\infty \rangle$$

$$c=1 \rightarrow \ln\left(\frac{y}{x}\right) = 1 \rightarrow \frac{y}{x} = e \rightarrow x = \frac{y}{e}$$

$$c=2 \rightarrow \ln\left(\frac{y}{x}\right) = 2 \rightarrow \frac{y}{x} = e^2 \rightarrow x = \frac{y}{e^2}$$

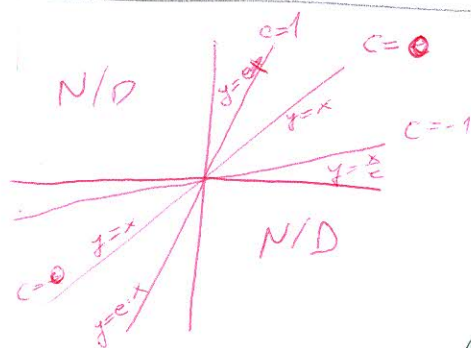
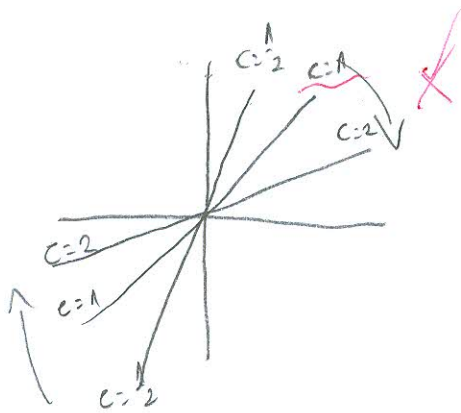
↑ DETAJNIJE GDJE JE $\frac{y}{x} > 0$?

LIMES NE POSTOJI JER SE VIŠE

RAZ. KRIVULJA

SJEČE U

JEDNOJ TOČKI



$$f(x, y) = \ln\left(\frac{y}{x}\right)$$

$$\frac{\partial f}{\partial x} = \frac{1}{\frac{y}{x}} \cdot \frac{1}{y} = \frac{1}{x}$$

$$\frac{1}{x} = 0$$

$$\frac{\partial f}{\partial y} = \frac{1}{\frac{y}{x}} \cdot \frac{-x}{y^2} = -\frac{1}{y}$$

$$-\frac{1}{y} = 0 \quad x=0 \quad y=0 \quad T(0,0)$$

$$A) \frac{\partial^2 f}{\partial^2 x} = \frac{x^{-2}}{-2}$$

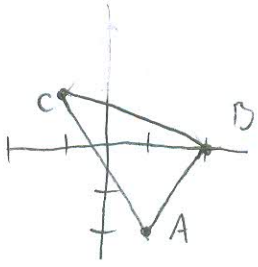
$$D) \frac{\partial^2 f}{\partial y^2} = \frac{y^{-2}}{2}$$

FKSTREMI NE POSTOJE

⑥ $A(1, -2)$ $B(2, 0)$ $C(-1, 1)$

MARJO MARASOVIC

17-1-0242-2019



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

AB

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - (-2) = \frac{0 - (-2)}{2 - 1} (x - 1)$$

$$y + 2 = \frac{2}{1} (x - 1)$$

$$y + 2 = 2x - 2$$

$$y = 2x - 4$$

BC

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 1 = \frac{-1 - 1}{2 - (-1)} (x - (-1))$$

$$3y + 3 = 1(3x + 3)$$

$$3y = 3x - 6/3$$

$$y = x - 2$$



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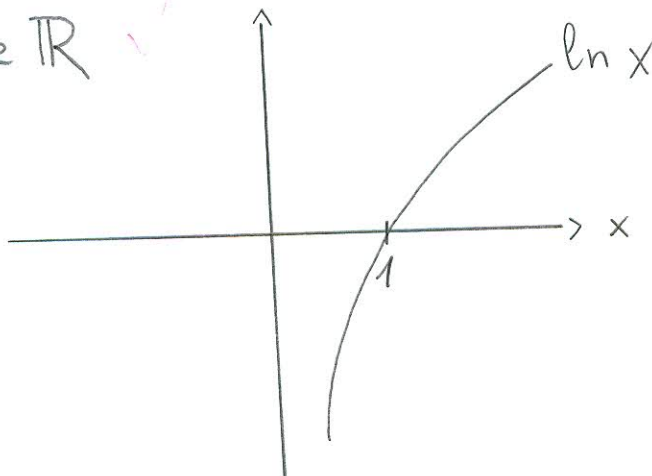
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3. $f(x, y) = \ln\left(\frac{y}{x}\right) \Rightarrow$ uvjeti $y \neq 0$ $\frac{x}{y} > 0$

$D = \{ [0, \infty), < 0, \infty) \} \cup \{ < -\infty, 0], < -\infty, 0) \}$

kodomena je \mathbb{R}



nastavak

