MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** OOX

IME I PREZIME: MARKO MARASOVIĆ **BROJ INDEKSA:** 17-1-0242-2014

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y''+2y'=1$, uz $y(0)=0$ i $y'(0)=0$. Na kraju provjeri rješenje. 15

2. Nađi implicitno rješenje jednadžbe $\frac{y'}{x}=\frac{\sin x}{y}$. 15

3. Za funkciju $f(x,y)=\ln\left(\frac{y}{x}\right)$ odrediti domenu, kodomenu, razinske krivulje i limes u ishodištu (ako postoji). 20

4. $\int_0^2 \frac{x-1}{x^2+3x+2}\,\mathrm{d}x = ?$ 15

5. Zadana je funkcija $f(x)=\sqrt{x}$. Traži se površina ispod grafa funkcije (do osi apcise) na segmentu $[0,4]$. Podijeliti segment na nekoliko dijelova i preko trapezne formule procijeniti traženu površinu. Skicirati graf funkcije, površinu koja je dobivena procjenom i vizualno ocijeniti grešku numeričkog postupka. 20

6. Integriranjem izračunati površinu trokuta zadanog točkama $A(1,-2)$, $B(2,0)$, $C(-1,1)$. 15

Ukupno:

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
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$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int \frac{\mathrm{d}x}{a^2-x^2}=\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$
$\int x^\alpha\,\mathrm{d}x=\frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,\mathrm{d}x=-\ln\lvert\cos x\rvert+C$	$\int \frac{\mathrm{d}x}{x^2-a^2}=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int \frac{\mathrm{d}x}{x}=\ln\lvert x\rvert+C$	$\int \cot x\,\mathrm{d}x=\ln\lvert\sin x\rvert+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}}=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int \mathrm{e}^x\,\mathrm{d}x=\mathrm{e}^x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x}=\tan x+C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C$	$\int \frac{\mathrm{d}x}{\sin^2 x}=-\cot x+C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
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$\int \cos x\,\mathrm{d}x=\sin x+C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

② $\frac{y'}{x} = \frac{\sin x}{y} / \cdot x$

$y' = \frac{1}{y} \cdot x \sin x \cdot y$

$y \frac{dy}{dx} = x \sin x / \cdot dx$

$y\,dy = x \sin x\,dx / \int$

$\int y\,dy = \int x \sin x\,dx$

$\frac{y^2}{2} = -x \cos x + \sin x + c / \cdot 2$ ✓

$y^2 = -2x \cos x + 2 \sin x + c \Rightarrow y = \sqrt{2 \sin x - 2x \cos x + c}$ //

$\int x \sin x\,dx = \left| \begin{matrix} u = x & du = dx \\ dv = \sin x & v = -\cos x \end{matrix} \right|$

$= -x \cos x + \int \cos x + c$

$= -x \cos x + \sin x + c$

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③ $f(x,y) = \ln\left(\frac{y}{x}\right)$

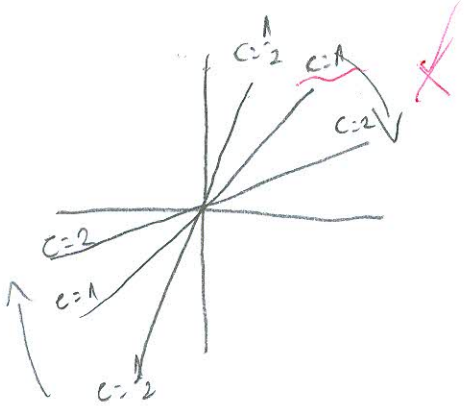
$D(f) = \{\mathbb{R}^2 / \frac{y}{x} > 0\}$ KOD $\to \langle 0, +\infty \rangle$ ✗

DETALJNIJE GDJE JE $\frac{y}{x} > 0$?

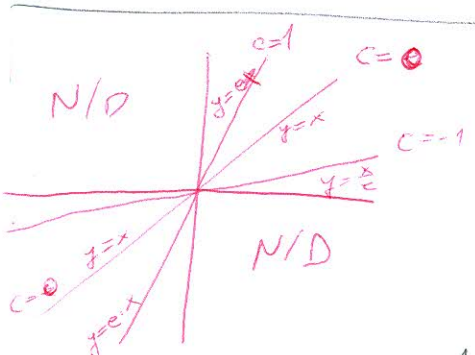
$c = 1 \to \ln\left(\frac{y}{x}\right) = 1 \to \frac{y}{x} = e \to x = \frac{y}{e}$

$c = 2 \to \ln\left(\frac{y}{x}\right) = 2 \to \frac{y}{x} \to \frac{y}{x} = e^2 \to x = \frac{y}{e^2}$

LIMES NE POSTOJI JER SE VIŠE RAZ. KRIVULJA SIJEČE U JEDNOJ TOČKI ✓



✗



8

$f(x,y) = \ln\left(\frac{y}{x}\right)$

$\frac{\partial f}{\partial x} = \frac{1}{\frac{y}{x}} \cdot \frac{1}{y} = \frac{1}{x}$

$\frac{1}{x} = 0$

$\frac{\partial f}{\partial y} = \frac{1}{\frac{y}{x}} \cdot \frac{-x}{y^2} = -\frac{1}{y}$

$-\frac{1}{y} = 0 \quad x = 0 \quad y = 0 \quad T(0,0)$

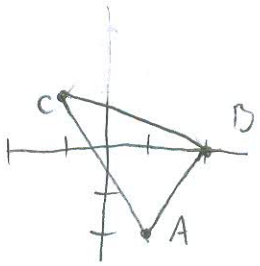
A! $\frac{\partial^2 f}{\partial^2 x} = \frac{x^{-2}}{-2}$

A! $\frac{\partial f}{\partial y} = \frac{y-2}{2}$

EKSTREMI NE POSTOJE.

MARKO MARASOVIĆ
17-1-0242-2014

(6) $A(1,-2)$ $B(2,0)$ $C(-1,1)$



$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

AB

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - (-2) = \frac{0 - (-2)}{2 - 1}(x - 1)$

$y + 2 = \frac{2}{1}(x - 1)$

$y + 2 = 2x + 3$

$y = 2x + 1$ //

BC

$y - 1 = \frac{0 - 1}{2 + 1}(x + 1)$

$y - 1 = \frac{-1}{3}(x + 1) \mid \cdot \frac{3}{1}$

$3y + 3 = 1(3x + 3)$

$3y = 3x - 6 \mid :3$

$y = x - 2$ //

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IME I PREZIME: MATIJA BULJAN **BROJ INDEKSA:** 17-2-0221-2012

Želim ustmeni kod (zaokružiti): prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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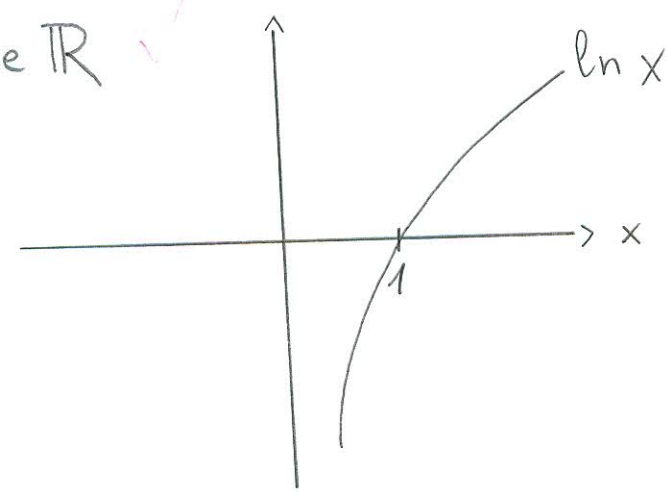
3. $f(x,y)=\ln\left(\frac{y}{x}\right)$ ⇒ uvjeti $y\neq 0$ $\frac{x}{y}>0$

$D=\{[0,\infty\rangle\,,\langle 0,\infty\rangle\}\cup\{\langle -\infty,0]\,,\langle -\infty,0\rangle]\}$

kodomena je $\mathbb{R}$



nastavak

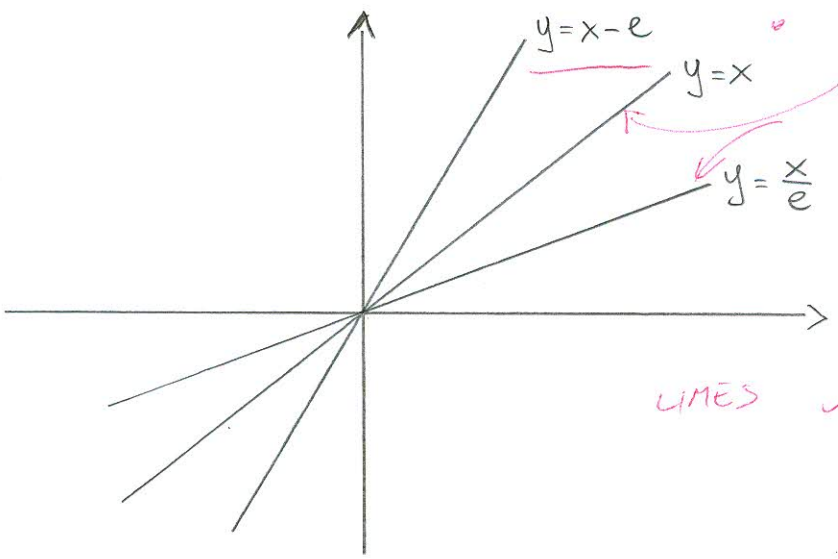
3.

razinske krivulje:

$\ln \frac{x}{y} = 0 \Rightarrow x = y$

$\ln \frac{x}{y} = 1 \Rightarrow \frac{x}{y} = e \Rightarrow y = \frac{x}{e}$

$\ln \frac{x}{y} = -1 \Rightarrow \frac{x}{e} = \frac{1}{e} \Rightarrow y = x - e$



KOJE VRIJEDNOSTI, ?

LIMES ✓ ISHODIŠTU

2. $\frac{y'}{x} = \frac{\sin x}{y}$

$y \frac{dy}{dx} = x \sin x \Rightarrow y\,dy = x \sin x\,dx \,/ \int$

$\frac{y^2}{2} = \int x \sin x\,dx = -x \cos x + \sin x + c$ ✓

$\Rightarrow y = \pm 2\sqrt{\sin x - x \cos x + c}$

c je proizvoljna konstanta

$I = \int x \sin x\,dx$

$\sin x\,dx = dv \Rightarrow v = -\cos x \; ; \; x = u$

$\int u\,dv = uv - \int v\,du$ — općenita formula

$\Rightarrow I = -x \cos x + \int \cos x\,dx$

$= -x \cos x + \sin x$