MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: ANTUN ŽANETIĆ

BROJ INDEKSA: 17-2-0169-2012

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

2. Zadan je P paraboloid $x^2 + y^2 = 5z$, $z \leq 4$. Izračunati $\iint_P \mathrm{d}S$? 20

3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y \,\mathrm{d}x\mathrm{d}y$. 20

4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y \,\mathrm{d}x$. ~~20~~

5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. ~~20~~

Ukupno:

40

NASTAVAK 3. ZAD

$$\int_0^1 \left[\cancel{\frac{y^2}{2}} - \cancel{3y} + \frac{9}{2} - \cancel{y^2} + \cancel{3y} - \cancel{\frac{y^2}{2}} - y + \frac{1}{2} + \cancel{y^2} - y\right] dy =$$

$$= \int_0^1 (-2y + 10)\,dy = \left(-2 \cdot \frac{y^2}{2} + 10y\right)\Big|_0^1 = -1 + 10 = 9$$

Antun Zanetic

(4.) $\iint\limits_{\partial S} x+y \, dx$

$\Downarrow$

$\int\limits_{\partial S}(x+y)ds \Rightarrow$ KRIVULJNI INTEGRAL 1. VRSTE NA SKALARNOM ϕ.

$$r(t)=\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

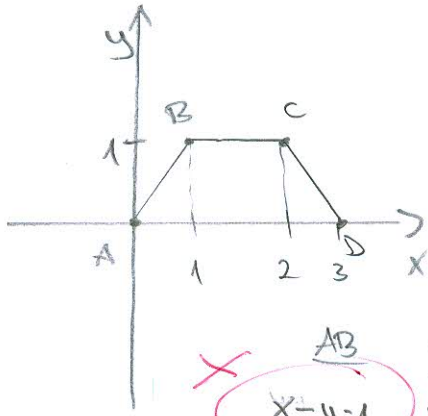
$\phi(t)=x \quad , \quad \psi(t)=y \quad , \quad \chi(t)=0$

$\phi'(t)=1 \quad , \quad \psi'(t)=1 \quad , \quad \chi(t)=0$

$\|\vec{n}\| = \sqrt{1^2+1^2+0} = \sqrt{2}$

$f\circ r = f(x,y,0) = x+y$

$\iint\limits_{\partial S} x+y \, dx \Rightarrow \int\limits_{\partial S} x+y \, dx$



$P = x+y$

$\frac{\partial Q}{\partial x}=0 \qquad \frac{\partial P}{\partial y}=1$

AB: $x=y-1$ BC: $y=1$

CD: $x=-y+3$ AD: $y=0$

$\iint\limits_{\partial S}(0-1)dx\,dy = \iint\limits_{\partial S} -1 \, dx\,dy$

$\int_{0}^{1}\int_{y-1}^{-y+3} -1 \, dx\,dy = \int_{0}^{1} -x\Big|_{y-1}^{-y+3} dy = \int_{0}^{1} -\left[(-y+3)-(y-1)\right] dy = \int_{0}^{1}(y-3+y-1)\,dy =$

$= \int_{0}^{1} 2y-4 \, dy = 2\cdot\frac{y^2}{2} - 4y\Big|_{0}^{1} = 1-4 = -3$ //

Antun Zanetić

(1) $y'''(t) + y''(t) = \cos t$, $y(0)=0$, $y'(0)=0$, $y''(0)=0$

$$y'''(t) + y''(t) = \cos t \quad / \mathcal{L}$$

$$\mathcal{L}(y'''(t)) + \mathcal{L}(y''(t)) = \mathcal{L}(\cos t)$$

$$s^3 \cdot Y(s) - s^2 \cdot y(0) - s \cdot y'(0) - y''(0) + s^2 \cdot Y(s) - s \cdot y(0) - y'(0) = \frac{s}{s^2+1}$$

$$Y(s)(s^3+s^2) - s^2 \cdot 0 - s \cdot 0 - 0 - s \cdot 0 - 0 = \frac{s}{s^2+1}$$

$$Y(s)(s^3+s^2) = \frac{s}{s^2+1} \quad / : (s^3+s^2)$$

$$Y(s) = \frac{s}{(s^3+s^2)(s^2+1)}$$

$$Y(s) = \frac{s}{s^2(s+1)(s^2+1)} \Rightarrow \text{RASTAV NA PARCIJALNE RAZLOMKE}$$

$$\frac{s}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} \quad / \cdot \left[s^2(s+1)(s^2+1)\right]$$

$$s = A(\underbrace{s(s+1)}_{(s^2+s)}(s^2+1)) + B(s+1)(s^2+1) + C(s^2(s^2+1)) + (Ds+E)(s^2(s+1))$$

$$s = As^4 + As^2 + As^3 + As + Bs^3 + Bs + Bs^2 + B + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$$

$$s = s^4(A+C+D) + s^3(A+B+D+E) + s^2(A+B+C+E) + s(A+B) + B$$

$$\left.\begin{aligned} A+C+D&=0 \\ A+B+D+E&=0 \\ A+B+C+E&=0 \\ A+B&=1 \end{aligned}\right\}$$

$\boxed{A=1}$

$\boxed{B=0}$

$1+C+D=0$

$1+D+E=0$

$1+C+E=0$

$C+D=-1 \Rightarrow C=-1-D$

$D+E=-1 \rightarrow D+\left(-\frac{1}{2}\right)=-1$

$C+E=-1$

$\boxed{D=-\frac{1}{2}}$

$D+E=-1$

$-1-D+E=-1$

$D+E=-1 \quad / +$

$-D+E=0$

$2E=-1$

$\boxed{E=-\frac{1}{2}}$

$C=-1-\left(-\frac{1}{2}\right)$

$\boxed{C=-\frac{1}{2}}$

$$Y(s) = \frac{1}{s} + 0 - \frac{1}{2} \cdot \frac{1}{s+1} - \frac{1}{2} \cdot \frac{s}{s^2+1} - \frac{1}{2} \cdot \frac{1}{s^2+1} \quad / \mathcal{L}^{-1}$$

$$y(t) = 1 - \frac{1}{2} \cdot e^{-t} - \frac{1}{2} \cdot \cos t - \frac{1}{2} \sin t$$

2. paraboloid $x^2 + y^2 = 5z$, $z \leq 4$.

$$x^2+y^2 = 5z \ /:5$$

$$z = \frac{x^2}{5} + \frac{y^2}{5}$$

$$r(x,y) = \begin{bmatrix} x \\ y \\ \frac{x^2}{5} + \frac{y^2}{5} \end{bmatrix}$$

$$\frac{\partial r}{\partial x} = \begin{bmatrix} 1 \\ 0 \\ \frac{2}{5}x + 0 \end{bmatrix}, \qquad \frac{\partial r}{\partial y} = \begin{bmatrix} 0 \\ 1 \\ 0 + \frac{2}{5}y \end{bmatrix}$$

$$\left\| \frac{\partial r}{\partial x} \times \frac{\partial r}{\partial y} \right\| = \begin{vmatrix} i & j & k \\ 1 & 0 & \frac{2}{5}x \\ 0 & 1 & \frac{2}{5}y \end{vmatrix} = i \cdot \begin{vmatrix} 0 & \frac{2}{5}x \\ 1 & \frac{2}{5}y \end{vmatrix} - j \cdot \begin{vmatrix} 1 & \frac{2}{5}x \\ 0 & \frac{2}{5}y \end{vmatrix}$$

$$+ k \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -\frac{2}{5}xi - \frac{2}{5}yj + k$$

$$\vec{n} = \begin{bmatrix} -\frac{2}{5}x \\ -\frac{2}{5}y \\ 1 \end{bmatrix}$$

$$\|\vec{n}\| = \sqrt{\left(-\frac{2}{5}x\right)^2 + \left(-\frac{2}{5}y\right)^2 + 1} = \sqrt{\frac{4}{25}x^2 + \frac{4}{25}y^2 + 1} = \sqrt{\frac{4x^2+4y^2+25}{25}}$$

$$= \frac{\sqrt{4x^2+4y^2+25}}{5}$$

$$\text{for } f\left(x, y, \frac{x^2}{5} + \frac{y^2}{5}\right) = 1$$

PRELAZAK U POLARNE KOORDINATE

$x = r\cos\varphi$
$y = r\sin\varphi$

$$(r\cos\varphi)^2 + (r\sin\varphi)^2 = 5 \cdot 4$$
$$r^2(\cos^2\varphi + \sin^2\varphi) = 20$$
$$r^2 = 20$$
$$r = \sqrt{20}$$

Granice

$r \in [0, \sqrt{20}]$
$\varphi \in [0, 2\pi]$

$$\iint_P dS = \int_0^{2\pi}\int_0^{\sqrt{20}} \frac{\sqrt{4[(r\cos\varphi)^2+(r\sin\varphi)^2]+25}}{5}\, r\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^{\sqrt{20}} \frac{\sqrt{4[r^2\cos^2\varphi+r^2\sin^2\varphi]+25}}{5}\, r\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{20}} \frac{\sqrt{4r^2(\cos^2\varphi+\sin^2\varphi)+25}}{5}\, r\,dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^{\sqrt{20}} \frac{\sqrt{4r^2+25}}{5}\, r\,dr\,d\varphi = \left| \begin{array}{l} 4r^2+25 = u \;/' \\ 8r\,dr = du \;/:8 \\ r\,dr = \frac{du}{8} \end{array} \right| =$$

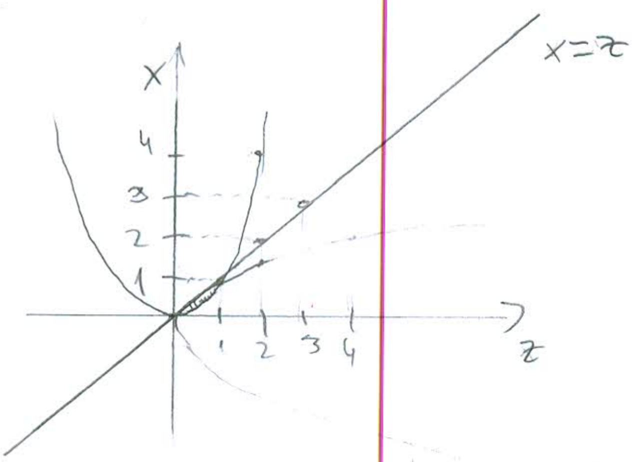
$$= \int_0^{2\pi}\int_0^{\sqrt{20}} \frac{1}{5}\cdot\frac{1}{8}\cdot\sqrt{u}\,du\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{20}} \frac{1}{40}\, u^{\frac{1}{2}}\,du\,d\varphi = \int_0^{2\pi} \frac{1}{40}\cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^{\sqrt{20}} d\varphi =$$

$$= \int_0^{2\pi} \frac{1}{40}\cdot\frac{2u^{\frac{3}{2}}}{3}\Bigg|_0^{\sqrt{20}} d\varphi = \int_0^{2\pi} \frac{1}{40}\cdot\frac{2(4r^2+25)^{\frac{3}{2}}}{3}\Bigg|_0^{\sqrt{20}} d\varphi = \cancel{\int_0^{2\pi}\frac{1}{40}\left(\frac{2\cdot 105^{\frac{3}{2}}}{3}\right)-\left(\frac{2\cdot 25^{\frac{3}{2}}}{3}\right)} d\varphi$$

$$= \int_0^{2\pi}\left[\left(\frac{\cancel{2}\cdot 105^{\frac{3}{2}}}{\cancel{120}_{60}}\right)-\left(\frac{\cancel{2}\cdot 25^{\frac{3}{2}}}{\cancel{120}_{60}}\right)\right] d\varphi = \int_0^{2\pi} \frac{80^{\frac{3}{2}}}{60}\,d\varphi = \frac{80^{\frac{3}{2}}}{60}\varphi\Bigg|_0^{2\pi} = \frac{80^{\frac{3}{2}}}{\cancel{60}}\cdot\cancel{2}\pi = \frac{80^{\frac{3}{2}}}{30}\pi =$$

$$= \frac{\sqrt{80^3}}{30}\pi \;//$$

(5.) $f(x,y,z) = y$, $x = z^2$, $z = x$, $y = -1$, $y = 1$



$x = z^2$

z	x
0	0
1	1
2	4

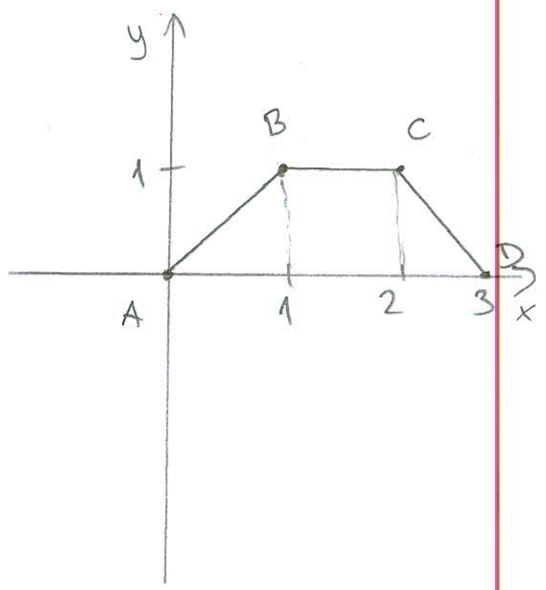
$$\int_0^1\int_0^1\int_{z^2}^{z} y\,dx\,dy\,dz = \int_0^1\int_0^1 x\cdot y\Big|_{z^2}^{z} dy\,dz = \int_0^1\int_0^1 \left[(z\cdot y)-(z^2\cdot y)\right] dy\,dz =$$

$$= \int_0^1 z\cdot\frac{y^2}{2} - z^2\cdot\frac{y^2}{2}\Bigg|_0^1 dz = \int_0^1\left(\frac{1}{2}z - \frac{1}{2}z^2\right)dz = \int_0^1 \frac{1}{2}(z - z^2)\,dz = \frac{1}{2}\cdot\frac{z^2}{2} - \frac{1}{2}\cdot\frac{z^3}{3}\Bigg|_0^1$$

$$= \frac{z^2}{4} - \frac{z^3}{6}\Bigg|_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{1}{12} \;//$$

③ $\iint_S x+y\,dx\,dy$

Anton Žanetić



A(0,0)
B(1,1)
C(2,1)
D(3,0)

AB

$$\boxed{y-y_1=\frac{y_2-y_1}{x_2-x_1}\cdot(x-x_1)}$$

$$y-1=\frac{1-0}{1-0}(x-0)$$

$$y-1=x$$

$$y=x+1 \Rightarrow x=y-1$$

BC

$$y-1=\frac{1-1}{2-1}(x-1)$$

$$y=1$$

CD

$$y-1=\frac{0-1}{3-2}(x-2)$$

$$y-1=-x+2$$

$$y=-x+3 \Rightarrow$$

$$\cancel{-}x=y-3$$
$$x=-y+3$$

AD

$$y-0=\frac{0-0}{3-0}(x-0)$$

$$y=0$$

∫ Granice

$$x\in[y-1,-y+3]$$
$$y\in[0,1]$$

$$\int_0^1\int_{y-1}^{-y+3} x+y\,dx\,dy=\int_0^1\left(\frac{x^2}{2}+x\cdot y\right)\Big|_{y-1}^{-y+3}dy=\int_0^1\left[\left(\frac{(-y+3)^2}{2}+(-y+3)\cdot y\right)-\left(\frac{(y-1)^2}{2}+(y-1)\cdot y\right)\right]dy$$

$$=\int_0^1\left[\frac{y^2-6y+9}{2}-y^2+3y-\frac{y^2+2y-1}{2}+y^2-y\right]dy=\int_0^1\left[\frac{y^2}{2}-3y+\frac{9}{2}-y^2+3y-\frac{y^2}{2}-y+\frac{1}{2}\ldots\right]dy$$

NASTAVAK NA 1 STR.

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: Filip Bačinić

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Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20
$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$
2. Zadan je P paraboloid $x^2 + y^2 = 5z$, $z \leq 4$. Izračunati $\iint_P \mathrm{d}S$? 20
3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y \,\mathrm{d}x\mathrm{d}y$. 20
4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y \,\mathrm{d}x$. 20
5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. 20

Ukupno: 40

(2)

$x^2 + y^2 = 5z$, $z \leq 4$ $\qquad \iint_P ds = ?$

$x^2 + y^2 = \sqrt{5z}^2$ $\qquad z \in [0, 4]$

$ds = dxdy = r dr d\varphi$

$x = r\cos\varphi$ $\qquad r^2 = 5z \,/\sqrt{\ }$

$y = r\sin\varphi$ $\qquad r = \sqrt{5z}$

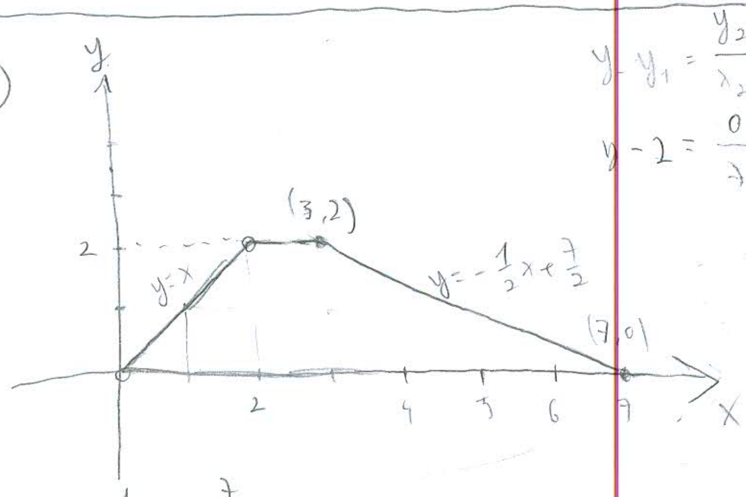
$z = z$ $\qquad r = \sqrt{5 \cdot 4}$

$r = \sqrt{20}$

$$\boxed{\int_0^{\sqrt{20}} \int_0^{2\pi} r\, dr\, d\varphi} =$$

$$= r \int_0^{\sqrt{20}} \left(\varphi \Big|_0^{2\pi} \right) dr = r \int_0^{\sqrt{20}} 2\pi \, dr = r\, 2\pi\, r \Big|_0^{\sqrt{20}} = \sqrt{20}\, 2\pi \sqrt{20} = \boxed{40\pi}$$

(3)



$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 2 = \frac{0 - 2}{7 - 3}(x - 3)$ $\qquad y = -\frac{1}{2}(x - 3) + 2 = -\frac{1}{2}x + \frac{7}{2}$

$$\boxed{\int_0^2 \int_y^{-2y+7} x + y \, dx\, dy}$$

$y = -\frac{1}{2}x + \frac{7}{2}$

$-\frac{1}{2}x = y - \frac{7}{2}$ $\qquad -x = 2y - 7$

$\boxed{x = -2y + 7}$

$$\int_0^2 \left[\int_y^{-2y+7} x+y \, dx \right] dy = \int_0^2 \frac{x^2}{2} + yx \Big|_y^{-2y+7} dy$$

$$= \int_0^2 \left(\frac{(7-2y)^2}{2} + y(-2y+7) \right) - \left(\frac{y^2}{2} + y^2 \right) dy =$$

$$= \int_0^2 \frac{49}{2} - \frac{\cancel{28}^{14} y}{\cancel{2}} + \frac{\cancel{4}^{2} y^2}{\cancel{2}} \cancel{- 2y^2} + 7y - \frac{3}{2} y^2 \, dy =$$

$$= \int_0^2 - \frac{3}{2} y^2 - 7y + \frac{49}{2} \, dy = -\frac{3}{2} \frac{y^3}{3} - 7 \frac{y^2}{2} + \frac{49}{2} y \Big|_0^2$$

$$= -\frac{\cancel{3}}{2} \cdot \frac{2^3}{\cancel{3}} - 7 \frac{2^2}{2} + \frac{49}{\cancel{2}} \cancel{2} =$$

$$= -4 - 14 + 49 = 31$$

Filip Bačinić

1. $y'''(t) + y''(t) = \cos t \qquad y(0) = y'(0) = y''(0) = 0$

$$s^3F(s) - \cancel{s^2 f(0)} - \cancel{s f'(0)} - \cancel{f''(0)} + s^2F(s) - \cancel{s f(0)} - \cancel{f'(0)} = \frac{s}{s^2+1}$$

$$F_s(s^3+s^2) = \frac{s}{s^2+1} \qquad F(s) = \frac{\cancel{s}}{s^{\cancel{2}}(s+1)(s^2+1)}$$

$$\frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} \quad / \cdot s^2(s+1)(s^2+1)$$

$$(As+A)(s^2+1) + (Bs^2+Bs)(s^2+1) + Cs^2(s^2+1) + (Ds^3+Es^2)(s+1) =$$

$$= As^3 + A + As + As^2 + Bs^4 + Bs^3 + Bs^2 + Bs + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2 =$$

$$= s^4(B+C+D) + s^3(A+B+D+E) + s^2(A+B+C+E) + s(A+B) + A$$

$B+C+D=0 \qquad C+D=-1 \qquad C+D=-1 \qquad -1-E-1-E=1 \qquad -2E=3$

$A+B+D+E=0 \qquad D+E=-1 \qquad D=-1-E \qquad \boxed{E=-\frac{3}{2}}$

$A+B+C+E=0 \qquad C+E=-1 \qquad C=-1-E \qquad \boxed{C=\frac{1}{2}}$

$A+B=1 \qquad \boxed{D=\frac{1}{2}}$

$\boxed{A=0} \quad \boxed{B=1}$

$$\frac{\frac{1}{2}s - \frac{3}{2}}{s^2+1} = \frac{\frac{s-3}{2}}{s^2+1} = \frac{1}{2}\cdot\frac{s-3}{2(s^2+1)}$$

$$F(s) = \frac{1}{s} + \frac{1}{2}\cdot\frac{1}{s+1} + \frac{1}{2}\cdot\frac{s-3}{s^2+1}$$

$$= \frac{1}{s} + \frac{1}{2}\cdot\frac{1}{s+1} + \frac{1}{2}\cdot\frac{s}{s^2+1} - \frac{3}{2}\cdot\frac{1}{s^2+1}$$

$$f(t) = 1 + \frac{e^{t}}{2} + \frac{1}{2}\cos t - \frac{3}{2}\sin t \qquad f(0) = 1 + \frac{1}{2} + \frac{1}{2} = 2$$

① NASTAVAK

$$F(s)=\frac{1}{s(s+1)(s^2+1)}$$

$$\frac{A}{s}+\frac{B}{s+1}+\frac{Cs+D}{s^2+1} \quad \Big/ \cdot \; s(s+1)(s^2+1)$$

$$(As+A)(s^2+1)+Bs(s^2+1)+(Cs^2+Ds)(s+1)$$

$$As^3+As^2+As+A+Bs^3+Bs+Cs^3+Cs^2+Ds^2+Ds$$

$$s^3(A+B+C)+s^2(A+C+D)+s(A+B+D)+A$$

$A+B+C=0 \qquad B+C=-1 \quad \to \quad B=-1-C \qquad \boxed{B=-\frac{1}{2}}$

$A+C+D=0 \qquad C+D=-1 \quad \to \quad D=-1-C \qquad \boxed{D=-\frac{1}{2}}$

$A+B+D=0 \qquad B+D=-1 \qquad -1-C-1-C=-1$

$\boxed{A=1} \qquad -2C=1$

$\boxed{C=-\frac{1}{2}}$

$$F(s)=\frac{1}{s}-\frac{1}{2}\frac{1}{s+1}-\frac{1}{2}\left(\frac{s+1}{s^2+1}\right)$$

$$=\frac{1}{s}-\frac{1}{2}\cdot\frac{1}{s+1}-\frac{1}{2}\cdot\frac{s}{s^2+1}-\frac{1}{2}\cdot\frac{1}{s^2+1}$$

$$\boxed{f(t)=1-\frac{e^{-t}}{2}-\frac{1}{2}\cos t-\frac{1}{2}\sin t}$$ ✓

$$f(0)=1-\frac{1}{2}-\frac{1}{2}-0=0 \; ✓$$

Filip Bačinić

(5) $f(x,y,z) = y$

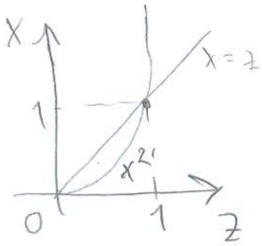
$x = z^2 \quad z = x$

$y = -1$

$y = 1$

$x = z^2 = z$

$z^2 = z$



$$\int_0^1 \int_x^{x^2} \int_{-1}^{1} y \, dy \, dz \, dx$$

$$\int_0^1 \int_x^{x^2} \frac{y^2}{2} \Big|_{-1}^{1} dz \, dx = \int_0^1 \int_x^{x^2} \cancel{\frac{1}{2} - \frac{1}{2}} \, dz \, dx = \int_0^1 z \Big|_x^{x^2} dx =$$

$$= \int_0^1 x^2 - x \, dx = \frac{x^3}{3} - \frac{x^2}{2} \Big|_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$

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1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 20

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5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. — ~~20~~

Ukupno: 35

1. $f'''(t)\ s^3F(s) - s^2 f(0) - sf'(0) - f''(0)$

$y''(t) - y''(t) = \cos t$

$s^3 Y(s) - \cancel{s^2 y(0)} - s\cancel{y(0)} - \cancel{y(0)} + s^2 Y(s) - s\cancel{y(s)} - \cancel{y(0)} = \frac{s}{s^2+1}$

$y(s)(s^3 + s^2) = \frac{s}{s^2+1} \quad / \frac{1}{s^3+s^2}$

$y(s) = \frac{\cancel{s}}{(s^2+1)s^2(s+1)} = \frac{1}{s(s^2+1)(s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} + \frac{D}{s+1}$

$1 = A\cdot(s^3+s^2+s+1) + (Bs+C)(s^2+s) + D(s^3+s)$

$1 = As^3 + As^2 + As + A + Bc^3 + Bs^2 + Cs^2 + Cs + Ds^3 + Ds$

$A + B + D = 0 \Rightarrow A + B = -D$
$A + B + C = 0 \Rightarrow A + B = -C$
$-D = -C \Rightarrow C = D \Rightarrow \boxed{D = -\frac{1}{2}}$

$A + C + D = 0 \Rightarrow 1 + 2C = 0 \Rightarrow \boxed{C = -\frac{1}{2}}$

$\boxed{A = 1}$

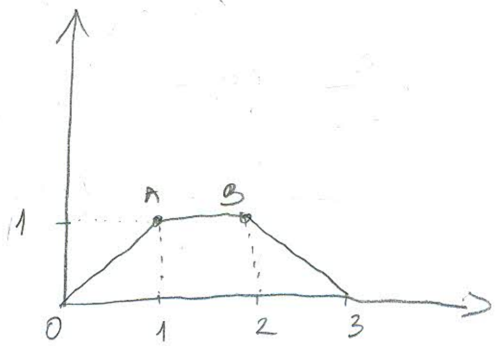
$A + B = -C$

$B = \frac{1}{2} - 1 = \boxed{-\frac{1}{2}}$

$y(s) = -\frac{1}{s} - \frac{1}{2}\frac{s+1}{s^2+1} - \frac{1}{2}\frac{1}{s+1} = \frac{1}{s} - \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{1}{s+1}$

$L^{-1}[y(s)] = 1 - \frac{1}{2}\cos t - \frac{1}{2}\sin t - \frac{1}{2}e^{-t}$

③ a) $\iint_S (x+y)\,dx\,dy$



$O(0,0)$
$A(1,1)$
$B(2,1)$
$C(3,0)$

$\overline{AB} \ldots y=1$
$\overline{OC} \ldots y=0$
$\overline{OA} \ldots y=x$

$\overline{BC} \ldots (x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$
$\overline{BC} \ldots (3-2)(y-1) = (0-1)(x-2)$
$\overline{BC} \ldots y-1 = -x+2$
$\overline{BC} \ldots y = -x+3 \Rightarrow x = 3-y$

$$\iint_S (x+y)\,dx\,dy = \int_0^1 \int_y^{3-y} (x+y)\,dx\,dy = \int_0^1 \left(\frac{x^2}{2} + yx\right)\Big|_y^{3-y} dy$$ 15

$=$

$$= \int_0^1 \left[\frac{1}{2}\left((3-y)^2 - y^2\right) + y(3-y-y)\right]dy = \int_0^1 \left[\frac{1}{2}(9-6y+y^2-y^2)+3y-2y^2\right]dy$$

$$= \int_0^1 \left(\frac{9}{2} - 12y + 3y - 2y^2\right)dy = \frac{9}{2}\int_0^1 dy - 9\int_0^1 y\,dy - 2\int_0^1 y^2\,dy$$

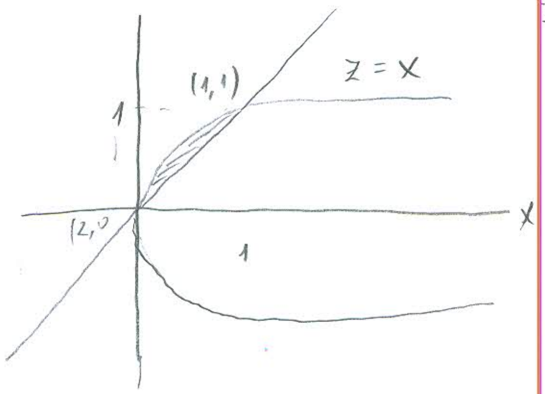
$$= \frac{9}{2}y\Big|_0^1 - \frac{9}{2}y^2\Big|_0^1 - \frac{2}{3}y^3\Big|_0^1 = \cancel{\frac{9}{2}} - \cancel{\frac{9}{2}} - \frac{2}{3} = \underline{\underline{-\frac{2}{3}}}$$ ✗

Anamaria Jozić

5. $f(x,y,z) = y$; $x = z^2$, $z = x$, $y = -1$ $y = 1$

$z = \sqrt{x}$ $x = 0 \to z = 0$

$x = 1 \to z = \pm 1$



$$\iiint f(x,y,z)\,dx\,dy\,dz = \iiint y\,dx\,dy\,dz$$?

$$\int_0^1 dx \int_0^1 y\,dy \cdot z \int_x^{\sqrt{x}} dz$$

$= \frac{1}{2}\left[x \right]_2^1$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: KRISTIAN MARTINOVIĆ **BROJ INDEKSA:** 17-2-0110-2011	Broj ↓ bodova
Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

2. Zadan je P paraboloid $x^2 + y^2 = 5z$, $z \leq 4$. Izračunati $\iint_P \mathrm{d}S$? 20

3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y \,\mathrm{d}x\mathrm{d}y$. ~~20~~

4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y \,\mathrm{d}x$. 20

5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. ~~20~~

Ukupno: 20

(1) $y'''(t) + y''(t) = \cos t, \quad y(0)=0,\ y'(0)=0,\ y''(0)=0$

$$s^3Y(s) - s^2Y(0) - sy'(0) - y''(0) + s^2Y(s) - sy(0) - y'(0) = \frac{s}{s^2+1}$$

$$s^3Y(s) - 0 - 0 - 0 + s^2Y(s) - 0 - 0 = \frac{s}{s^2+1}$$

$$s^3Y(s) + s^2Y(s) = \frac{s}{s^2+1}$$

$$Y(s)(s^3+s^2) = \frac{s}{s^2+1}$$

$$Y(s)\,s^2(s+1) = \frac{s}{s^2+1}$$

$$Y(s) = \frac{\frac{s}{s^2+1}}{\frac{s^2(s+1)}{1}} = \frac{s}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} \quad / \cdot s^2(s+1)(s^2+1)$$

$$s = As(s+1)(s^2+1) + B(s+1)(s^2+1) + Cs^2(s^2+1) + (Ds+E)s^2(s+1)$$

$$s = As(s^3+s+s^2+1) + B(s^3+s+s^2+1) + Cs^4 + Cs^2 + (Ds+E)(s^3+s^2)$$

$$s = As^4 + As^2 + As^3 + As + Bs^3 + Bs + Bs^2 + B + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$$

$A + C + D = 0$

$A + B + D + E = 0$

$A + B + C + E = 0$

$A + B = 1$

$\boxed{B = 0}$

$A + 0 = 1$

$\boxed{A = 1}$

$A+C+D=0$ $\quad$ $A+B+D+E=0$ $\quad$ $A+B+C+E=0$

$1+C+D=0$ $\quad$ $1+0+D+E=0$ $\quad$ $1+0+C+E=0$

$C+D=-1$ $\quad$ $D+E=-1$ $\quad$ $C+E=-1$

$D=-1-E$ $\quad$ $C=-1-E$

$C+D=-1$

$(-1-E)+(-1-E)=-1$

$-1-E-1-E=-1$

$-2-2E=-1$

$-2E=-1+2$

$-2E=1$

$\boxed{E=-\frac{1}{2}}$

$D=-1-\left(-\frac{1}{2}\right)$ $\quad$ $C=-1-\left(-\frac{1}{2}\right)$

$D=-1+\frac{1}{2}$ $\quad$ $C=-1+\frac{1}{2}$

$\boxed{D=-\frac{1}{2}}$ $\quad$ $\boxed{C=-\frac{1}{2}}$

$\boxed{A=1}$ $\quad$ $\boxed{B=0}$

$$Y(s)=\frac{1}{s}+\cancel{\frac{0}{s^2}}+\frac{\left(-\frac{1}{2}\right)}{s+1}+\frac{\left(-\frac{1}{2}\right)s+\left(-\frac{1}{2}\right)}{s^2+1}$$

$$Y(s)=\frac{1}{s}-\frac{1}{2}\cdot\frac{1}{s+1}-\frac{1}{2}\cdot\frac{s}{s^2+1}-\frac{1}{2}\cdot\frac{1}{s^2+1}$$

$$\boxed{y(t)=1-\frac{1}{2}e^{-t}-\frac{1}{2}\cos(t)-\frac{1}{2}\sin(t)}$$ ✓

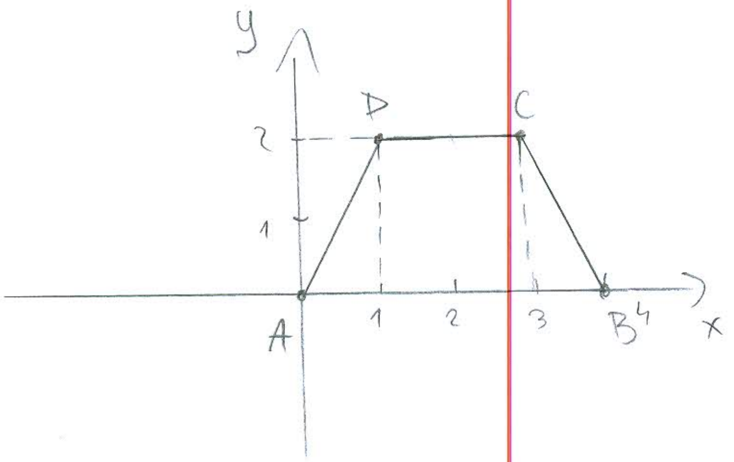
$$y(0)=1-\frac{1}{2}\underbrace{e^{-0}}_{=1}-\frac{1}{2}\cdot 1-\frac{1}{2}\cdot 0=1-\frac{1}{2}-\frac{1}{2}-0=0$$ ✓

$$y'(t)=0-\frac{1}{2}e^{-t}+\frac{1}{2}\sin(t)-\frac{1}{2}\cos(t)$$

$$y'(0)=0-\frac{1}{2}\cdot 1+\frac{1}{2}\cdot 0+\frac{1}{2}\cdot 1=-\frac{1}{2}+\frac{1}{2}=0$$ ✓

$$y''(t)=-\frac{1}{2}e^{-t}+\frac{1}{2}\cos(t)+\frac{1}{2}\sin(t)=-\frac{1}{2}+\frac{1}{2}=0$$ ✓

3. KRISTIAN MARTINOVIC



$\overline{AB} \dots y=0$

$\overline{CD} \dots y=2$

$\overset{x_1\,y_1}{A(0,0)} \quad \overset{x_2\,y_2}{D(1,2)}$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(1-0)(y-0)=(2-0)(x-0)$

$\overline{AD} \dots y=2x$

$\overset{x_1\,y_1}{B(4,0)} \quad \overset{x_2\,y_2}{C(3,2)}$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(3-4)(y-0)=(2-0)(x-4)$

$-1(y-0)=2x-8$

$-y=2x-8 \quad /:(-1)$

$\overline{BC} \dots y=-2x+8$

$$\iint_S (x+y)\,dxdy = \int_0^1\int_1^3\int_3^4 (x+y)\,dxdydz = \int_0^1\int_1^3 \left(\frac{x^2}{2}+xy\right)\Big|_3^4 dydz$$

$$= \int_0^1\int_1^3 \left(\frac{4^2}{2}+4y\right)-\left(\frac{3^2}{2}+3y\right) dydz = \int_0^1\int_1^3 \left(8+4y-\frac{9}{2}-3y\right)dydz$$

$$= \int_0^1\int_1^3 \left(\frac{7}{2}+y\right)dydz = \int_0^1 \left(\frac{7}{2}y+\frac{y^2}{2}\right)\Big|_1^3 dz = \int_0^1 \left(\frac{7}{2}\cdot 3+\frac{3^2}{2}\right)-\left(\frac{7}{2}\cdot 1+\frac{1}{2}\right)dz$$

$$= \int_0^1 \left(\frac{21}{2}+\frac{9}{2}-\frac{7}{2}-\frac{1}{2}\right) = \int_0^1 11\,dz = (11z)\Big|_0^1 = 11\cdot 1-11\cdot 0$$

$$= 11 \checkmark$$

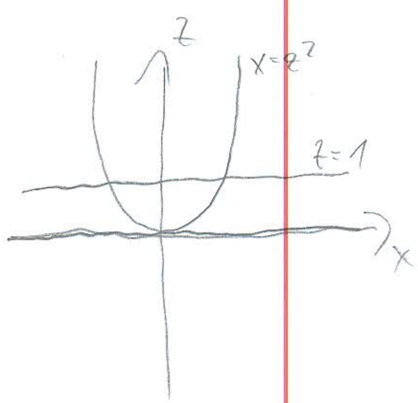
(5.) $f(x,y,z)=y$

$x=z^2$
$z=x$

$y=-1$
$y=1$

$z^2-z=0$
$z(z-1)=0$
$\boxed{z=0}$
$z-1=1$
$\boxed{z=1}$

$y\in[-1,1]$
$x\in[0,1]$
$z\in[0,1]$



$$\iiint_X y = \int_0^1\int_0^1\int_{-1}^1 y\cdot dy\,dx\,dz = \int_0^1\int_0^1 \left(\frac{y^2}{2}\right)\Bigg|_{-1}^{1} dx\,dz =$$

$$= \int_0^1\int_0^1 \left(\frac{1}{2}-\frac{1}{2}\right) dx\,dz = 0$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: ANĐELA UHODA   BROJ INDEKSA: 17-2-0106-2011

Zaokružiti nastavnika za ustmeni: prof. Uglešić   asistent Kosor

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 20

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

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3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y \,\mathrm{d}x\mathrm{d}y$. — 20

4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y \,\mathrm{d}x$. — 20

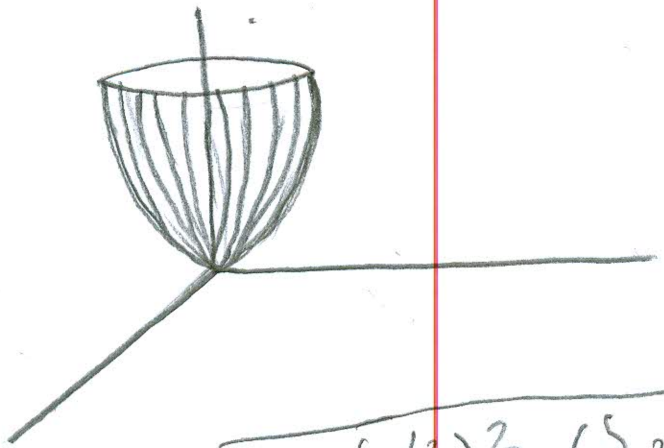
5. Izračunati integral funkcije $f(x,y,z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. — 20

Ukupno: 20

2. $x^2+y^2=5z$, $z\le 4$

$\iint_P dS = ?$?

$S \ldots \dfrac{x^2+y^2}{5} = z$



$\dfrac{\delta z}{\delta x} = \dfrac{2}{5}x$

$dS \ldots = \sqrt{1+\left(\dfrac{dz}{\delta x}\right)^2+\left(\dfrac{\delta z}{\delta y}\right)^2}$

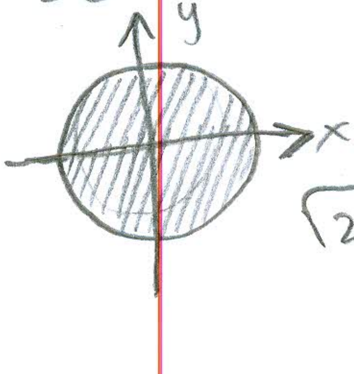
$\dfrac{\delta z}{\delta y} = \dfrac{2}{5}y$

$dS = \sqrt{1+\dfrac{4x^2}{25}+\dfrac{4y^2}{25}}\,dxdy$

$\Downarrow$

$\sqrt{\dfrac{25+4x^2+4y^2}{25}}$

$\underline{xoy}$ $x^2+y^2=20$



$\sqrt{20} = 2\sqrt{5}$

→

$x = r\cos\varphi$
$y = r\sin\varphi$
$J = r$

$r \Big|_0^{2\sqrt{5}} \quad \varphi \Big|_0^{2\pi} \quad \sqrt{\frac{25+4x^2+4y^2}{25}}$

$$\int_0^{2\pi} d\varphi \int_0^{2\sqrt{5}} r \cdot \sqrt{\frac{25+4(r\cos\varphi)^2+4(r\sin\varphi)^2}{25}}\, dr$$

$$\int_0^{2\pi} d\varphi \int_0^{2\sqrt{5}} r \cdot \sqrt{\frac{25+8r^2}{25}}\, dr =$$

$$\int_0^{2\pi} d\varphi \int_0^{2\sqrt{5}} r \cdot \frac{\sqrt{8}\, r}{5}\, dr = \frac{\sqrt{8}}{5} \int_0^{2\pi} d\varphi \int_0^{2\sqrt{5}} r^2\, dr =$$

$$\frac{\sqrt{8}}{5} \int_0^{2\pi} d\varphi \left(r^3 \Big|_0^{2\sqrt{5}} \right) =$$

$$= \frac{\sqrt{8}}{5} \int_0^{2\pi} d\varphi \left[40\sqrt{5} \right] = 40\sqrt{5} \cdot \frac{\sqrt{8}}{5} \cdot \left(\varphi \Big|_0^{2\pi} \right)$$

$$= 16\sqrt{10} \cdot 2\pi = 32\sqrt{10}\pi$$

MATEMATIKA 3 Hudela Ulloda BROJ INDEKSA: 17-2-0106-2011

5. Izračunaj integral funkcije $f(x,y,z)=y$ u dijelu prostora omeđenog plohama $x=z^2$, $z=x$, $y=-1$ i $y=1$.

$x=z^2$

$z=x$

$y=-1$

$y=1$

MATEMATIKA 3 Anđela Ukoda BROJ INDEKSA: 17-2-0106-2011

3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral:

$$\iint_S x + y \, dx dy$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARIO IVANAC **BROJ INDEKSA:** 17-1-0096-2011

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

2. Zadan je P paraboloid $x^2 + y^2 = 5z$, $z \le 4$. Izračunati $\iint_P \mathrm{d}S$? 20

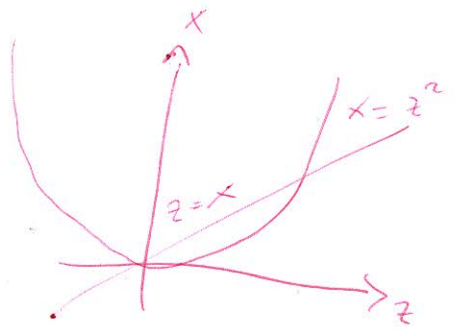
3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y\, \mathrm{d}x\mathrm{d}y$. 20

4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y\, \mathrm{d}x$. 20

5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. 20

Ukupno: 20

5) $x = z^2 \quad z = x \quad y = -1 \quad y = 1$



$$\left.\begin{aligned} x &= z \\ x &= z^2 \end{aligned}\right\} \quad z^2 = z$$

$$z^2 - z = 0$$

$$z(z-1) = 0$$

$$x \in [z^2, z] \qquad z_1 = 0 \quad z - 1 = 0$$

$$z_2 = 1$$

$$y \in [-1, 1] \qquad z \in [0, 1]$$

$$\int_{-1}^{1}\int_{0}^{1}\int_{z^2}^{z} y\, dx\, dz\, dy = \int_{-1}^{1}\int_{0}^{1} y x \Big|_{z^2}^{z} dz\, dy = \int_{-1}^{1}\int_{0}^{1} (yz) - (yz^2)\, dz\, dy$$

$$\int_{-1}^{1} y \cdot \frac{z^2}{2} - y \cdot \frac{z^3}{3} \Big|_0^1 dy = \int_{-1}^{1} y \cdot \frac{1^2}{2} - y \cdot \frac{1^3}{3}\, dz = \int_{-1}^{1} \frac{1}{2} y - \frac{1}{3} y\, dy$$

$$= \frac{1}{2} \cdot \frac{y^2}{2} - \frac{1}{3} \cdot \frac{y^2}{2} \Big|_{-1}^{1} = \frac{y^2}{4} - \frac{y^2}{6} \Big|_{-1}^{1} = \left(\frac{1}{4} - \frac{1}{6}\right) - \left(\frac{(-1^2)}{4} - \frac{(-1)^2}{6}\right)$$

$$= \frac{6-4}{24} - \left(\frac{1}{4} - \frac{1}{6}\right) = \frac{2}{24} - \left(\frac{2}{24}\right) = 0$$

2) $x^2+y^2=5z \qquad z=4$

$r^2=5\cdot 4 \qquad\qquad r^2=5z$

$r^2=20 \qquad\qquad 5z=r^2$

$r=\sqrt{20} \qquad\qquad z=\frac{r^2}{5}$

$r\in[0,\sqrt{20}] \qquad\qquad z\in\left[\frac{r^2}{5},4\right]$

$\varphi\in[0,2\pi]$

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$$\int_0^{2\pi}\int_0^{\sqrt{20}}\int_{\frac{r^2}{5}}^{4} r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{20}} r\cdot z\Big|_{\frac{r^2}{5}}^{4} dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{20}} 4r-\frac{r^2}{5}\,dr\,d\varphi$$

$$\int_0^{2\pi} \frac{\cancel{4}^{2}r^2}{\cancel{2}_{1}}-\frac{1}{5}\frac{r^3}{3}\Big|_0^{\sqrt{20}} d\varphi = \int_0^{2\pi} 2r^2-\frac{1}{5}\frac{r^3}{3}\Big|_0^{\sqrt{20}} d\varphi =$$

$$\int_0^{2\pi} 2\cdot(\sqrt{20})^{\cancel{2}}-\frac{1}{5}\cdot\frac{(\sqrt{20})^3}{3}\,d\varphi = \int_0^{2\pi} 40-\frac{(\sqrt{20})^3}{3} = 40\varphi-\frac{(\sqrt{20})^3}{3}\varphi\Big|_0^{2\pi}$$

$$= 40\varphi-\frac{40\sqrt{5}}{3}\varphi\Big|_0^{2\pi} = 40\cdot 2\pi-\frac{40\sqrt{5}}{3}\cdot 2\pi = 80\pi-62{,}44279761$$

$= 188{,}88$

MARIO IVANAC

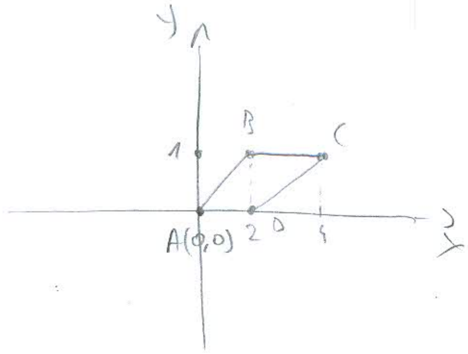
3) $\iint_S x+y\,dx\,dy$

A(0,0)

B(2,1)

C(4,1)

D(0,2)



OVO JE PARALELOGRAM, A NE TRAPEZ

$$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$$

$\overline{AD}$ $\overset{x_1\ y_1}{(0,0)}$ $\overset{x_2\ y_2}{(2,1)}$

$$(2-0)(y-0)=(1-0)(x-0)$$

$$2y=x$$

$$\boxed{x=2y}$$

$\overline{BC}$ $\overset{x_1\ y_1}{(2,1)}$ $\overset{x_2\ y_2}{(4,1)}$

$$(4-2)(y-1)=(1-1)(x-2)$$

$$2(y-1)=0(x-2)$$

$$2y-2=0$$

$$2y=2$$

$$\boxed{y=1}$$

$\overline{CD}$ $\overset{x_1\ y_1}{(4,1)}$ $\overset{x_2\ y_2}{(0,2)}$

$$(0-4)(y-1)=(2-1)(x-4)$$

$$-4(y-1)=-1(x-4)$$

$$-4y+4=-x+4$$

$$x=\cancel{4}+4y-\cancel{4}$$

$$\boxed{x=4y}$$

$\overline{AD}$ $\overset{x_1\ y_1}{(0,0)}$ $\overset{x_2\ y_2}{(0,2)}$

$$(0-0)(y-0)=(2-0)(x-0)$$

$$0=2x$$

$$x=0$$

$$\int_0^2\int_0^1 x+y\,dx\,dy+\int_2^4\int_{4y}^1 x+y\,dx\,dy=\int_0^2 \frac{x^2}{2}+yx\Big|_0^1 dy+\int_2^4 \frac{x^2}{2}+yx\Big|_{4y}^1 dy$$

$$\int_0^2 \frac12+y\,dy+\int_0^2\left(\frac12+y\right)-\left(\frac{(4y)^2}{2}+4y^2\right)=\int_0^2\frac12+y\,dy+\int_0^2\frac12 16y^2-\left(\frac12(16y^2)\right)+4y^2\,dy$$

$$=\left(\frac12 y+\frac{y^2}{2}\Big|_0^2\right)+\left[\frac12\cdot\frac{16y^3}{3}-\left(\frac12\cdot\frac{16y^3}{3}\right)+\frac{4y^3}{3}\Big|_0^2\right]$$

$$=\left(\frac12\cdot2+\frac{2^2}{2}\right)+\left[\cancel{\frac12\cdot\frac{16\cdot2^3}{3}}-\left(\cancel{\frac12\cdot\frac{16\cdot2^3}{3}}\right)+\frac{4\cdot2^3}{3}\right]$$

$$=4+\frac{32}{3}=\frac{12+32}{3}=\frac{44}{3}\ //$$

1) $y'''(t) + y''(t) = \cos t \quad , y(0) = 0, y'(0) = 0 \quad , y''(0) = 0$

$$s^2 y(t) - s^2 y(0) - s y'(0) - y''(0) + s^2 y(t) - s y(0) - y'(0)) = \frac{s}{s^2+1}$$

$$s^2 y(t) - (s^2 \cdot 0) - (s \cdot 0) - 0 + s^2 y(t) - (s \cdot 0) - (y \cdot 0) = \frac{s}{s^2+1}$$

$$y(t)(s^2 + s^2) = \frac{s}{s^2+1}$$

$$y(t)\, 2s^2 = \frac{s}{s^2+1} \quad / : 2s^2$$

$$y(t) = \frac{s}{2s^2(s^2+1)} = \frac{s}{2s^4 + 2s^2} = \frac{s}{s^2(2s^2+2)}$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{2s^2+2} \quad / \cdot (s^2(2s^2+2))$$

$$s = A(s(2s^2+2)) + B(2s^2+2) + C(s^2)$$

$$s = A(2s^3 + 2s) + 2Bs^2 + 2B + Cs^2$$

$$s = 2As^3 + 2As + 2Bs^2 + 2B + Cs^2$$

$2A = 1$

$\boxed{A = 2}$

$\boxed{B = 0}$

$\boxed{C = 0}$

$$\frac{2}{s} + \frac{0}{s^2} + \frac{0}{2s^2+2} =$$

$$\frac{2}{s} = \boxed{2}$$

0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MAURO MIŠLOV **BROJ INDEKSA:** 17-2-0170-2012.

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

2. Zadan je P paraboloid $x^2 + y^2 = 5z$, $z \leq 4$. Izračunati $\iint_P \mathrm{d}S$? 20

3. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_S x + y \,\mathrm{d}x\mathrm{d}y$. 20

4. Izaberi bilo koji trapez S u ravnini i na njemu odredi integral $\iint_{\partial S} x + y \,\mathrm{d}x$. 20

5. Izračunati integral funkcije $f(x, y, z) = y$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$. 20

Ukupno: Ø

1. $f'''(t) + f''(t) = \cos t, \quad f(0)=0,\ f'(0)=0,\ f''(0)=0$

$$s^3F(s) - \underbrace{s^2 f(0)}_{=0} - \underbrace{s f'(0)}_{=0} - \underbrace{f''(0)}_{=0} + s^2F(s) - \underbrace{s f(0)}_{=0} - \underbrace{f'(0)}_{=0} = \frac{s}{s^2+a^2}$$

$$s^3F(s) + s^2(F(s) = \frac{s}{s^2+a^2}$$

$$F(s)\left(s^3 \cancel{+} s^2\right) = \frac{s}{s^2+a^2} \quad /: (s^3+s^2)$$

Ø

$$F_s = \frac{\frac{s}{s^2+a^2}}{s(s^2+s^2}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: TONI PERKOVIĆ BROJ INDEKSA: 17201342011	Broj ↓ bodova
Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor	

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Ukupno: 0

1. $y'''(t) + y''(t) = \cos t \quad y(0)=0,\ y'(0)=0,\ y''(0)=0$

$s^3Y(s) - s^2\cancel{y(0)} - s\cancel{y'(0)} - \cancel{y''(0)} + s^2Y(s) - s\cancel{y(0)} - \cancel{y'(0)} = \frac{s}{s^2+1}$

$s^3Y(s) + s^2Y(s) = \frac{s}{s^2+1}$

$Y_s(s^3+s^2) = \frac{s}{s^2+1}$

$Y_s = \frac{\frac{s}{s^2+1}}{\frac{s^3+s^2}{1}}$

$Y_s = \frac{s}{s^2+1(s^3+s^2)}$

$Y_s = \frac{s}{s^2+1 \cdot s(s^2+s)}$

$Y_s = \frac{s}{s^3+s(s^2+s)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+s} \quad / : s^3+s(s^2+s)$

$s = A\ s^2+s(s^2+s) + B\ s+s(s^2+s) + C\ s(s^2+s) + (Ds+E)(s^2+s)$

$s = As^4 + As^3 + s^3 + s^2 + Bs^3 + Bs^2 + s^3 + s^2 + Cs^3 + Cs^2 + Ds^3 + Ds^2 + Es^2 + Es$

$s = As^4 + (A+B+C+D)s^3 + (B+C+D+E)s^2 + Es$

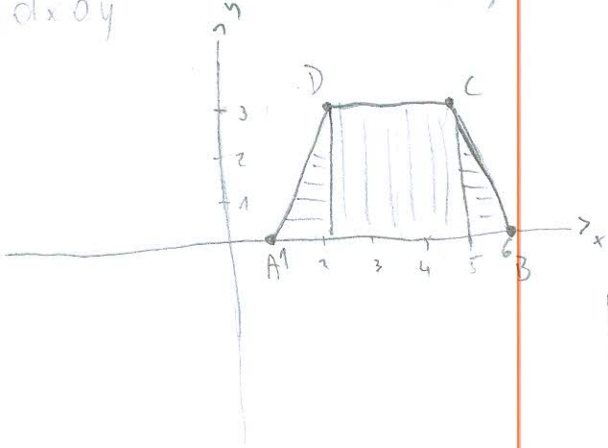
$A=0$
$B=0$
$C=0$
$D=0$
$E=1$

$Y_s = \cancel{\frac{0}{s}} + \cancel{\frac{0}{s^2}} + \cancel{\frac{0}{s^3}} + \cancel{\frac{0 \cdot s}{s^2+s}} + \frac{1}{s^2+s}$

$Y_s = \sin(t)$

③ $\iint_S x+y \, dx\, dy$

$A=(1,0) \quad B(6,0) \quad C(5,3) \quad D(0,3)$



$A(1,0)\ B(6,0)$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(6-1)(y-0)=(0-0)(x-1)$

$5y=0$

$\boxed{\overline{AB} \ldots y=0}$

$B(6,0)\ C(5,3)$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(5-6)(y-0)=(3-0)(x-6)$

$-y=3x-18 \quad /\cdot(-1)$

$\boxed{\overline{BC} \ldots y=-3x+18}$

$C(5,3)\ D(0,3)$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(0-5)(y-3)=(3-3)(x-5)$

$-5y+15=0$

$-5y=-15 \quad /:-5$

$\boxed{\overline{CD} \ldots y=3}$

$D(0,3)\ A(1,0)$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(1-0)(y-3)=(0-3)(x-0)$

$y-3=-3x$

$\boxed{\overline{DA} \ldots y=-3x+3}$

$$\iint_S x+y\,dx\,dy = \int_1^2\int_0^{-3x+3} x+y\,dy\,dx + \int_2^5\int_0^3 x+y\,dy\,dx + \int_5^6\int_0^{-3x+18} x+y\,dy\,dx$$

$$= \int_1^2 x+\frac{y^2}{2}\Big|_0^{-3x+3} dx + \int_2^5 x+\frac{y^2}{2}\Big|_0^3 dx + \int_5^6 x+\frac{y^2}{2}\Big|_0^{-3x+18} dx$$

$$= \int_1^2 x+\frac{(-3x+3)^2}{2}-\frac{0^2}{2}\,dx + \int_2^5 x+\frac{(3)^2}{2}-\frac{0^2}{2}\,dx + \int_5^6 x+\frac{(-3x+18)^2}{2}\,dx$$

$$= \int_1^2 x-\frac{9x^2+\cancel{18x}+9}{\cancel{2}}\,dx + \int_2^5 x+\frac{9}{2}\,dx + \int_5^6 x-\frac{-9x^2+108x+324}{2}\,dx$$

$$= \int_1^2 x-9x^2+9\,dx + \int_2^5 x+\frac{9}{2}\,dx + \int_5^6 x-9x^2+54x+324\,dx$$

$$= \frac{x^2}{2}-9\cdot\frac{x^3}{3}+9\cdot x\Big|_1^2 + \frac{x^2}{2}+\frac{9}{2}\cdot x\Big|_2^5 + \frac{x^2}{2}-9\cdot\frac{x^3}{3}+54\cdot\frac{x^2}{2}+324\cdot x\Big|_5^6$$

$$= \left(\frac{2^2}{2}-9\cdot\frac{2^3}{3}+9\cdot 3\right)-\left(\frac{1^2}{2}-9\cdot\frac{1^3}{3}+9\cdot 1\right)+\left(\frac{5^2}{2}+\frac{9}{2}\cdot 5\right)-\left(\frac{2^2}{2}+\frac{9}{2}\cdot 2\right)+\left(\frac{6^2}{2}-9\cdot\frac{6^3}{3}+54\cdot\frac{6^2}{2}+324\cdot 6\right)$$
$$-\left(\frac{5^2}{2}-9\cdot\frac{5^3}{3}+54\cdot\frac{5^2}{2}+324\cdot 5\right)$$

$= 376$

5. $f(x,y,z) = y$ $x = z^2,\ z = x,\ y = -1,\ y = 1$

$x = z^2$
$x = z$

$z^2 = z$
$z^2 - z = 0$
$z(z-1) = 0$

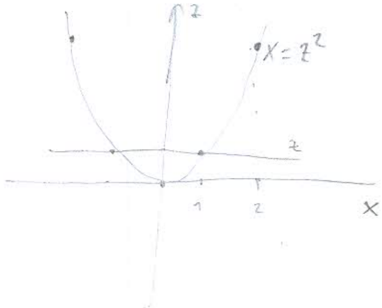
$z_1 = 0$
$z_2 = (z-1)$

$x = z^2$

z	0	1	-1	2	-2
z^2	0	1	1	4	4

$y \in [-1, 1]$

$z \in [0, z-1]$



TONI PERKOVIĆ

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