

**MATEMATIKA 3:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: **ANTUN ŽANETIĆ**

BROJ INDEKSA: **17-2-0169-2012**

Zaokružiti nastavnika za ustmeni:

prof. Uglešić

asistent Kosor

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t) + y''(t) = \cos t, \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 0.$$

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2. Zadan je  $P$  paraboloid  $x^2 + y^2 = 5z, z \leq 4$ . Izračunati  $\iint_P dS$ ?

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3. Izaberi bilo koji trapez  $S$  u ravnini i na njemu odredi integral  $\iint_S x + y \, dx \, dy$ .

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4. Izaberi bilo koji trapez  $S$  u ravnini i na njemu odredi integral  $\iint_{\partial S} x + y \, dx$ .

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5. Izračunati integral funkcije  $f(x, y, z) = y$  u dijelu prostora omeđenog plohama  $x = z^2, z = x, y = -1$  i  $y = 1$ .

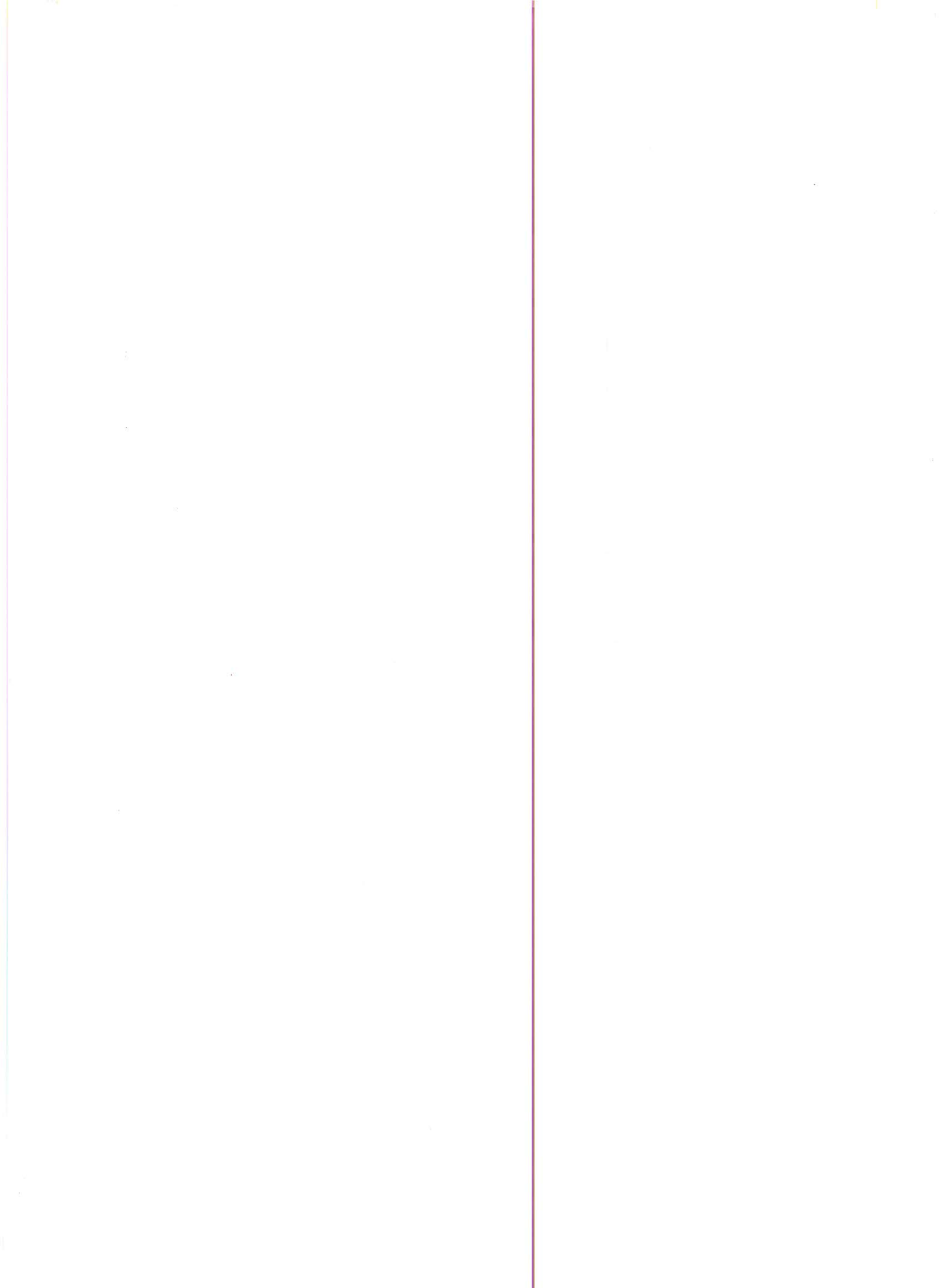
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NASTAVKE 3. ZAD

$$\int_0^1 \left[ \frac{y^2}{2} - 3y + \frac{9}{2} - y^2 + 3y - \frac{y^2}{2} - y + \frac{1}{2} + y^2 - y \right] dy =$$
$$= \int_0^1 (-2y + 10) dy = \left( -2 \cdot \frac{y^2}{2} + 10y \right) \Big|_0^1 = -1 + 10 = 9 //$$

Ukupno:

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4.  $\iint_{\partial S} x+y dx$   
 $\Downarrow$

$\int_{\partial S} (x+y) ds \Rightarrow$  KRIVOLINI INTEGRAL 1. VRSTE NA SKALARNOI  $\phi$ .

$r(t) = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

$\phi(t) = x, \quad \psi(t) = y, \quad \chi(t) = 0$

$\phi'(t) = 1, \quad \psi'(t) = 1, \quad \chi'(t) = 0$

$\|\vec{r}'\| = \sqrt{1^2 + 1^2 + 0} = \sqrt{2}$

for  $= f(x, y, \phi) = x+y$

$\iint_{\partial S} x+y dx \Rightarrow \int_{\partial S} x+y dx$

$P = x+y$

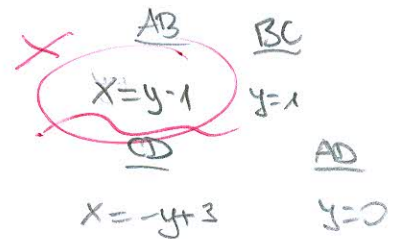
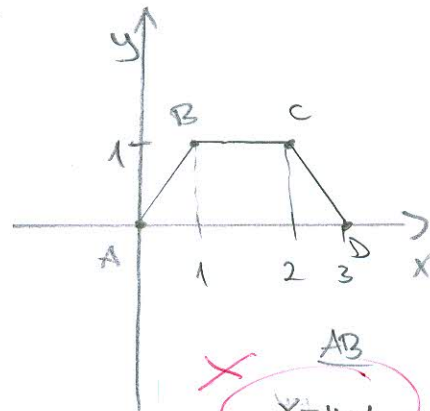
$\frac{\partial Q}{\partial x} = 0$

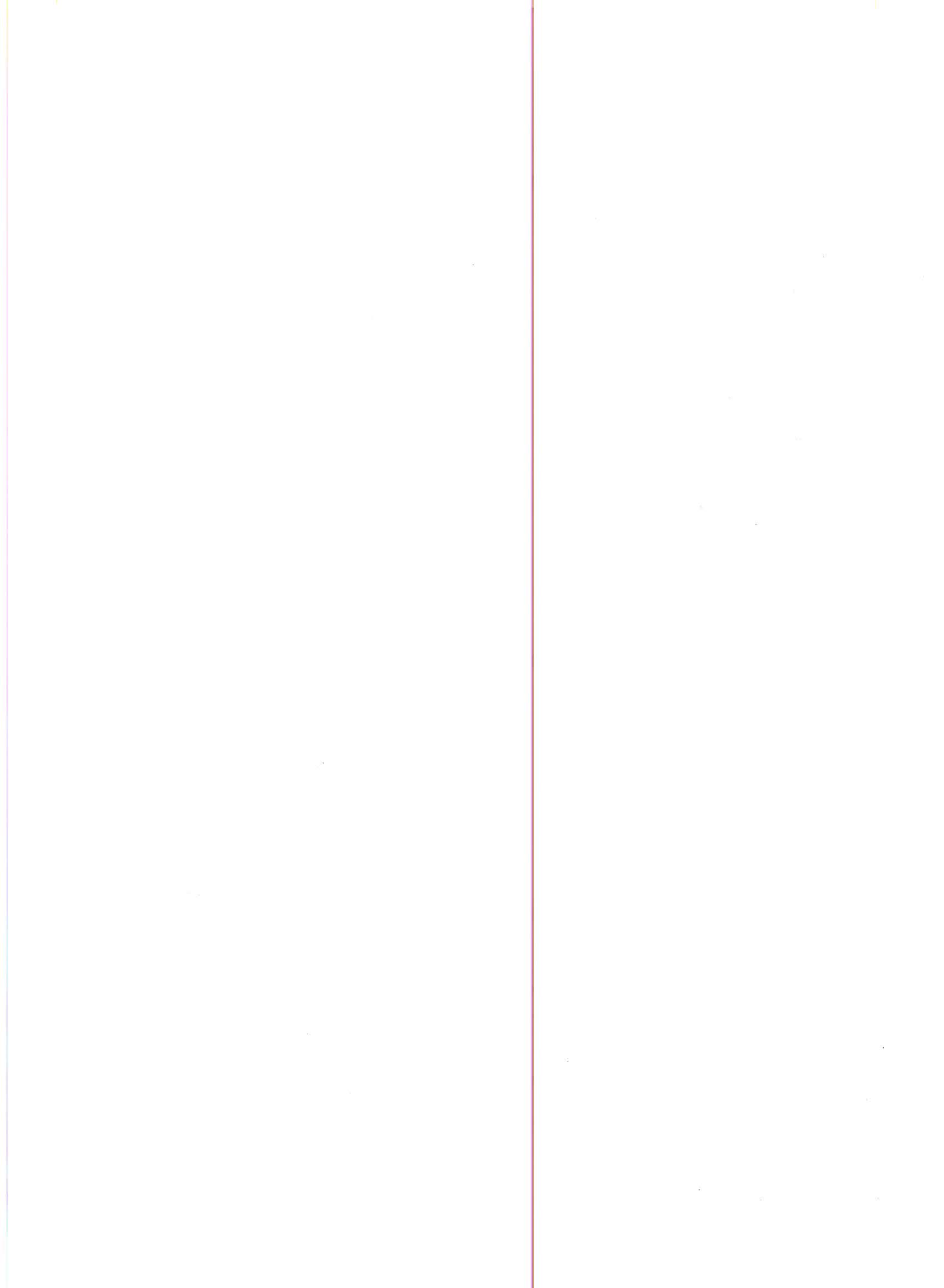
$\frac{\partial P}{\partial y} = 1$

$\iint_{\partial S} (0-1) dx dy = \iint_{\partial S} -1 dx dy$

$\int_{y-1}^{1-y+3} -1 dx dy = \int_0^1 -x \Big|_{y-1}^{1-y+3} dy = \int_0^1 -[(-y+3)-(y-1)] dy = \int_0^1 (y-3+y-1) dy =$

$= \int_0^1 2y-4 dy = 2 \cdot \frac{y^2}{2} - 4y \Big|_0^1 = 1-4 = -3 //$





1)  $y'''(t) + y''(t) = \cos t$ ,  $y(0) = 0$ ,  $y'(0) = 0$ ,  $y''(0) = 0$

$y'''(t) + y''(t) = \cos t \quad / \mathcal{L}$

$\mathcal{L}(y'''(t)) + \mathcal{L}(y''(t)) = \mathcal{L}(\cos t)$

$s^3 \cdot Y(s) - s^2 \cdot y(0) - s \cdot y'(0) - y''(0) + s^2 \cdot Y(s) - s \cdot y(0) - y'(0) = \frac{s}{s^2+1}$

$Y(s)(s^3 + s^2) - s^2 \cdot 0 - s \cdot 0 - 0 - s \cdot 0 - 0 = \frac{s}{s^2+1}$

$Y(s)(s^3 + s^2) = \frac{s}{s^2+1} \quad /: (s^3 + s^2)$

$Y(s) = \frac{s}{(s^3 + s^2)(s^2 + 1)}$

$Y(s) = \frac{s}{s^2(s+1)(s^2+1)} \Rightarrow$  RASTAV NA PARCIJALNE RAZKLOPKE

$\frac{s}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} \quad / \cdot [s^2(s+1)(s^2+1)]$

$s = A(s(s+1)(s^2+1)) + B(s+1)(s^2+1) + C(s^2(s^2+1)) + (Ds+E)(s^2(s+1))$

$s = As^4 + As^2 + As^3 + As + Bs^3 + Bs + Bs^2 + B + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$

$s = s^4(A+C+D) + s^3(A+B+D) + s^2(A+B+C+E) + s(A+B) + B$

$A+C+D=0$   
 $A+B+D+E=0$   
 $A+B+C+E=0$   
 $A+B=1 \quad \boxed{A=1}$   
 $B=0$

$1+C+D=0$   
 $1+D+E=0$   
 $1+C+E=0$

$C+D=-1 \Rightarrow C=-1-D$   
 $D+E=-1 \rightarrow D+(-\frac{1}{2})=-1$   
 $C+E=-1$   
 $D+E=-1$   
 $-1-D+E=-1$   
 $\boxed{D=-\frac{1}{2}}$

$D+E=-1$   
 $-D+E=0$   
 $2E=-1$   
 $\boxed{E=-\frac{1}{2}}$   
 $C=-1-(-\frac{1}{2})$   
 $\boxed{C=-\frac{1}{2}}$

$$V(s) = \frac{1}{s} + 0 - \frac{1}{2} \cdot \frac{1}{s+1} - \frac{1}{2} \cdot \frac{s}{s^2+1} - \frac{1}{2} \cdot \frac{1}{s^2+1} \quad / \mathcal{L}^{-1}$$

$$V(t) = 1 - \frac{1}{2} \cdot e^{-t} - \frac{1}{2} \cdot \cos t - \frac{1}{2} \sin t \quad \checkmark$$

②. paraboloid  $x^2 + y^2 = 5z$ ,  $z \leq 4$ .

$$r(x, y) = \begin{bmatrix} x \\ y \\ \frac{x^2}{5} + \frac{y^2}{5} \end{bmatrix} \quad \rightarrow \quad \begin{aligned} x^2 + y^2 &= 5z \quad /:5 \\ z &= \frac{x^2}{5} + \frac{y^2}{5} \end{aligned}$$

$$\frac{\partial r}{\partial x} = \begin{bmatrix} 1 \\ 0 \\ \frac{2}{5}x + 0 \end{bmatrix}, \quad \frac{\partial r}{\partial y} = \begin{bmatrix} 0 \\ 1 \\ 0 + \frac{2}{5}y \end{bmatrix}$$

$$\left\| \frac{\partial r}{\partial x} \times \frac{\partial r}{\partial y} \right\| = \begin{vmatrix} i & j & k \\ 1 & 0 & \frac{2}{5}x \\ 0 & 1 & \frac{2}{5}y \end{vmatrix} = i \cdot \begin{vmatrix} 0 & \frac{2}{5}x \\ 1 & \frac{2}{5}y \end{vmatrix} - j \cdot \begin{vmatrix} 1 & \frac{2}{5}x \\ 0 & \frac{2}{5}y \end{vmatrix}$$

$$+ k \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -\frac{2}{5}xi - \frac{2}{5}yj + k$$

$$\vec{n} = \begin{bmatrix} -\frac{2}{5}x \\ -\frac{2}{5}y \\ 1 \end{bmatrix}$$

$$\begin{aligned} \|\vec{n}\| &= \sqrt{\left(-\frac{2}{5}x\right)^2 + \left(-\frac{2}{5}y\right)^2 + 1} = \sqrt{\frac{4}{25}x^2 + \frac{4}{25}y^2 + 1} = \sqrt{\frac{4x^2 + 4y^2 + 25}{25}} \\ &= \frac{\sqrt{4x^2 + 4y^2 + 25}}{5} \end{aligned}$$

$$\text{for } \pm f(x, y, \frac{x^2}{5} + \frac{y^2}{5}) = 1$$

PRELAZAK U POLARNE KOORDINATE

$$\begin{aligned} x &= r \cos \varphi \\ y &= r \sin \varphi \end{aligned}$$

$$\begin{aligned} (r \cos \varphi)^2 + (r \sin \varphi)^2 &= 5 \cdot 4 \\ r^2 (\cos^2 \varphi + \sin^2 \varphi) &= 20 \\ r^2 &= 20 \\ r &= \sqrt{20} \end{aligned}$$

Granice

$$\begin{aligned} r &\in [0, \sqrt{20}] \\ \varphi &\in [0, 2\pi] \end{aligned}$$

