

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

JURE DUNDOVIĆ

BROJ INDEKSA:

prof. Uglešić

Zaokružiti nastavnika za ustmeni:

asistent Kosor

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu:

$$y'''(t) + y''(t) = \sin t, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 1.$$

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2. Neka je S gornja polusfera radijusa $r = 3$ sa centrom u ishodištu, $z = \sqrt{3^2 - x^2 - y^2}$. Kako preko definicije izračunati $\iint_S dS$?

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3. Izračunaj volumen prostora omeđenog plohama $y = z^2$, $y = 4$, $x = 0$ i $z = x - 8$.

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4. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0, 2)$. Izračunati $\int_{\partial K} (1 - 3x) dx$.

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5. Provjeri da li je $g(x, y, z) = (x - y, y - x, z)$ potencijalno polje? Zadađ neku krivulju i izračunaj krivuljni integral zadane funkcije.

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Ukupno:

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$$4. \int_{\partial K} (1 - 3x) dx$$

$$\frac{\partial Q(x, y)}{\partial x} - \frac{\partial P(x, y)}{\partial y} = -\frac{1 - 3x}{dy} = 0$$

$$P(x, y) = (1 - 3x) dx$$

$$dx dy = r dr d\theta$$

$$Q(x, y) = 0 dy$$

$$x = r \cos \theta + 0$$

$$y = r \sin \theta + 4$$

$$r \in [0, 4]$$

$$\theta \in [0, 2\pi]$$

$$\int_{\partial K} (1 - 3x) dx = \int_0^{2\pi} \int_0^4 0 \cdot r dr d\theta = 0 \quad \checkmark$$

$$3) y = z^2$$

$$y = 4$$

$$x = 0$$

$$z = x - 8 \Rightarrow x = z + 8$$

$$y \in [4, z^2]$$

$$x \in [0, z + 8]$$

$$z \in [-2, 2]$$

$$\int_{-2}^2 \int_{z^2}^4 \int_0^{z+8} 1 \, dx \, dy \, dz = \int_{-2}^2 \int_{z^2}^4 [x]_0^{z+8} \, dy \, dz =$$

$$= \int_{-2}^2 \int_{z^2}^4 (z+8) \, dy \, dz =$$

$$= \int_{-2}^2 (2y + 8y) \Big|_{z^2}^4 \, dz = \int_{-2}^2 (4z + 32 - z^3 - 8z^2) \, dz$$

$$= 4 \frac{z^2}{2} + 32z - \frac{z^4}{4} - 8 \frac{z^3}{3} \Big|_{-2}^2 =$$

$$= \frac{256}{3} = 85.33 \quad \checkmark$$

$$1) y'''(t) + y''(t) = \sin t \quad y(0) = 1, y'(0) = 0, y''(0) = 1$$

$$s^3 y(s) - s^2 y(0) - s y'(0) - y''(0) + s^2 y(s) - s y(0) - y'(0) = \frac{1}{s^2 + 1^2}$$

$$s^3 y(s) - s^2 - 0 - 1 + s^2 y(s) - s - 0 = \frac{1}{s^2 + 1^2}$$

$$s^3 + s^2 = \frac{1}{s^2 + 1} + s^2 + s + 1$$

$$y(s) = \frac{s^2 + s + 1}{s^3 + s^2} + \frac{1}{s^2 + 1}$$

$$y(s) = \frac{s^2 + s + 1}{s^2(s + 1)} + \frac{1}{s^2(s + 1)(s^2 + 1)} =$$

$$y(s) = \frac{2}{s^2} - \frac{1}{s} + \frac{3}{2} \cdot \frac{1}{s + 1} + \frac{-\frac{1}{2} + \frac{0}{2}s}{1 + s^2}$$

$$y(s) = 2t - 1 + \frac{3}{2} e^{-t} + \frac{1}{2} (\cos(t) - \sin(t)) \quad \checkmark$$

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IME I PREZIME: Duje Mitrović

BROJ INDEKSA: 17-2-0205-2012

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Zaokružiti nastavnika za ustmeni:

prof. Uglešić

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Ukupno:

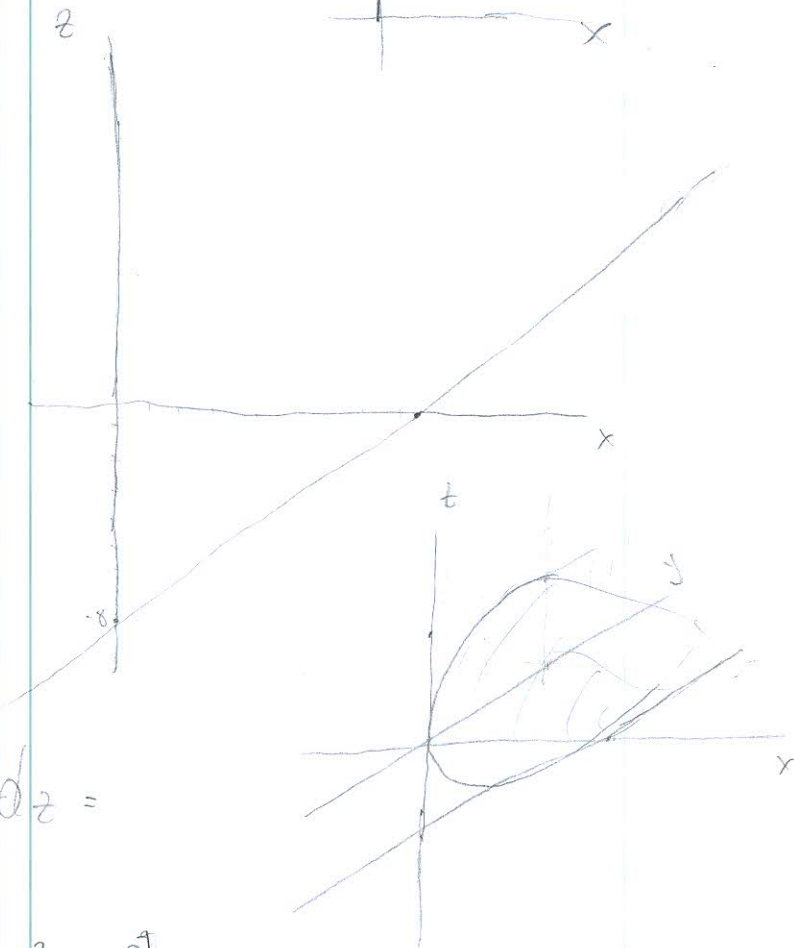
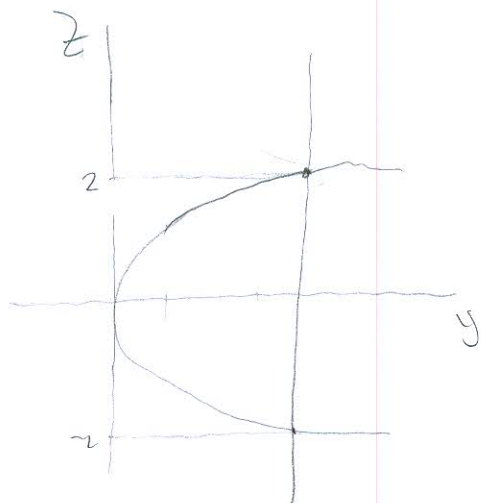
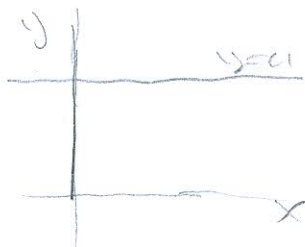
~~65~~
55 Kosor

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$$y = z^2 \quad x = 0 \quad z = x - 8$$

$$y = 4$$

$$x = z + 8$$



$$\int_{-2}^2 dz \int_{z^2}^4 dy \int_0^{z+8} dx = \int_{-2}^2 \int_{z^2}^4 (z+8) dy dz =$$

$$= \int_{-2}^2 [2y+8y]_{z^2}^4 dz = \int_{-2}^2 (4z+32 - (z^3+8z^2)) dz =$$

$$= \int_{-2}^2 (4z+32-z^3-8z^2) dz = \left[4 \frac{z^2}{2} + 32z - \frac{z^4}{4} - 8 \frac{z^3}{3} \right]_{-2}^2 =$$

$$= 8 + 64 - 4 - \frac{64}{3} - \left(8 - 64 - 4 + \frac{64}{3} \right) =$$

$$= 8 + 64 - 4 - \frac{64}{3} - 8 + 64 + 4 - \frac{64}{3} = 128 - \frac{128}{3} = \frac{256}{3} //$$



Duje Mitrović

$$\textcircled{1} y'''(t) + y'(t) = \sin t \quad y(0) = 1 \quad y'(0) = 0 \quad y''(0) = 1$$

$$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + s^2 Y(s) - s y'(0) - y''(0) = \frac{1}{s^2 + 1}$$

$$s^3 Y(s) - s^2 - 1 + s^2 Y(s) - s = \frac{1}{s^2 + 1}$$

$$s^3 Y(s) + s^2 Y(s) = \frac{1}{s^2 + 1} + s^2 + 1 + s$$

$$Y(s) = \frac{\frac{1}{s^2 + 1} + s^2 + 1 + s}{s^3 + s^2} = \frac{\frac{1 + s^2(s^2 + 1) + s^2 + 1 + s(s^2 + 1)}{s^2 + 1}}{s^3 + s^2} =$$

$$= \frac{1 + s^4 + s^2 + s^2 + 1 + s^3 + s}{(s^2 + 1)(s^3 + s^2)} = \frac{s^4 + s^3 + 2s^2 + s + 2}{s^2(s^2 + 1)(s + 1)}$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s + 1} + \frac{Ds + E}{s^2 + 1} = s^4 + s^3 + 2s^2 + s + 2$$

$$A[s(s^2 + 1)(s + 1)] + B[(s^2 + 1)(s + 1)] + C[s^2(s^2 + 1)] + [Ds + E][s^2(s + 1)] = s^4 + s^3 + 2s^2 + s + 2$$

$$A[s^4 + s^3 + s^2 + s] + B[s^3 + s^2 + s + 1] + C[s^4 + s^2] + [Ds + E][s^3 + s^2] = s^4 + s^3 + 2s^2 + s + 2$$

$$As^4 + As^3 + As^2 + As + Bs^3 + Bs^2 + Bs + B + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2 = s^4 + s^3 + 2s^2 + s + 2$$

$$C + D = 2$$

$$C = 2 - D$$

$$C = \frac{3}{2}$$

$$A + C + D = 1$$

$$B + A + D + E = 1$$

$$A + B + C + E = 2$$

$$A + B = 1 \Rightarrow A = -1$$

$$B = 2$$

$$1 + D + E = 1 \quad D = 1 - D$$

$$D = -E \quad 2D = 1 \Rightarrow \frac{1}{2}$$

$$1 + C + E = 2 \quad C = 1 - C \quad E = -\frac{1}{2}$$

$$Y(s) = -\frac{1}{s} + 2 \cdot \frac{1}{s^2} + \frac{3}{2} \cdot \frac{1}{s + 1} + \frac{1}{2} \frac{s}{s^2 + 1} - \frac{1}{2} \frac{1}{s^2 + 1}$$

$$\mathcal{L}^{-1}(Y(s)) = -1 + 2t + \frac{3}{2} e^{-t} + \frac{1}{2} \cos(t) - \frac{1}{2} \sin(t) \quad \checkmark$$

PROVVERA:

$$y(0) = 1$$

$$-1 + 0 + \frac{3}{2} + \frac{1}{2} - 0 = 1$$

$$-1 + 2 = 1$$

$$1 = 1 \quad \checkmark$$

$$y'(0) = 0$$

$$0 + 2 - \frac{3}{2} - 0 - \frac{1}{2} = 0$$

$$2 - \frac{4}{2} = 0$$

$$0 = 0 \quad \checkmark$$

$$y''(0) = 1$$

$$0 + 0 + \frac{3}{2} - \frac{1}{2} + 0 = 1$$

$$\frac{3}{2} - \frac{1}{2} = 1$$

$$\frac{2}{2} = 1 \quad \checkmark$$

