MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: JURE DUNDOVIĆ

BROJ INDEKSA:

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) = \sin t, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

2. Neka je S gornja polusfera radijusa $r = 3$ sa centrom u ishodištu, $z = \sqrt{3^2 - x^2 - y^2}$. Kako preko definicije izračunati $\iint_S \mathrm{d}S$? 20

3. Izračunaj volumen prostora omeđenog plohama $y = z^2$, $y = 4$, $x = 0$ i $z = x - 8$. 20

4. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0,2)$. Izračunati $\int_{\partial K} (1 - 3x)\,\mathrm{d}x$. 20

5. Provjeri da li je $g(x, y, z) = (x - y, y - x, z)$ potencijalno polje? Zadaj neku krivulju i izračunaj krivuljni integral zadane funkcije. 20

Ukupno: 60

4. $\oint_{\partial K} (1-3x)\,dx \qquad \frac{\partial Q(x,y)}{\partial x} - \frac{\partial P(x,y)}{\partial y} = -\frac{\partial (1-3x)}{\partial y} = 0$

$P(x,y) = (1-3x)\,dx$

$Q(x,y) = 0\,dy$

$x = r\cos\varphi + 0$

$y = r\sin\varphi + 4$

$r \in [0,4]$

$\varphi \in [0, 2\pi]$

$dx\,dy = r\,dr\,d\varphi$

$$\oint_{\partial K} (1-3x)\,dx = \int_0^{2\pi}\int_0^4 0 \cdot r\,dr\,d\varphi = 0$$

3) $y = z^2$

$y = 4$

$x = 0$

$z = x - 8 \Rightarrow x = z + 8$

$y \in [4, z^2]$

$x \in [0, z+8]$

$z \in [-2, 2]$

$$\int_{-2}^{2}\int_{z^2}^{4}\int_{0}^{z+8} 1 \, dx\,dy\,dz = \int_{-2}^{2}\int_{z^2}^{4} [x]_0^{z+8} =$$

$$= \int_{-2}^{2}\int_{z^2}^{4} z+8 \, dy\, dz =$$

$$= \int_{-2}^{2} (zy + 8y)\Big|_{z^2}^{4} = \int_{-2}^{2} 4z + 32 - z^3 - 8z^2 \, dz$$

$$= 4\frac{z^2}{2} + 32z - \frac{z^4}{4} - 8\frac{z^3}{3}\Big|_{-2}^{2} =$$

$$= \frac{256}{3} = 85.33$$

1) $y'''(t) + y''(t) = \sin t \qquad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1$

$$s^3 y(s) - s^2 y(0) - s y'(0) - y''(0) + s^2 y(s) - s y(0) - y'(0) = \frac{1}{s^2+1^2}$$

$$s^3 y(s) - s^2 - 0 - 1 + s^2 y(s) - s - 0 = \frac{1}{s^2+1^2}$$

$$s^3 + s^2 = \frac{1}{s^2+1} + s^2 + s + 1$$

$$y(s) = \frac{s^2+s+1}{s^3+s^2} + \frac{1}{s^2+1}$$

$$y(s) = \frac{s^2+s+1}{s^3+s^2} + \frac{1}{s^2(s+1)(s^2+1)} =$$

$$y(s) = \frac{2}{s^2} - \frac{1}{s} + \frac{3}{2}\cdot\frac{1}{s+1} + \frac{-\frac{1}{2} + \frac{1}{2}s}{1+s^2}$$

$$y(s) = 2t - 1 + \frac{3}{2}e^{-t} + \frac{1}{2}(\cos(t) - \sin(t))$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Duje Mitrović

BROJ INDEKSA: 17-2-0205-2012

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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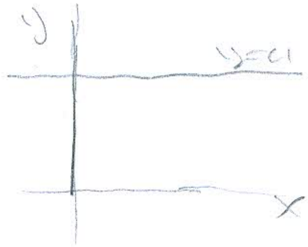
Ukupno:

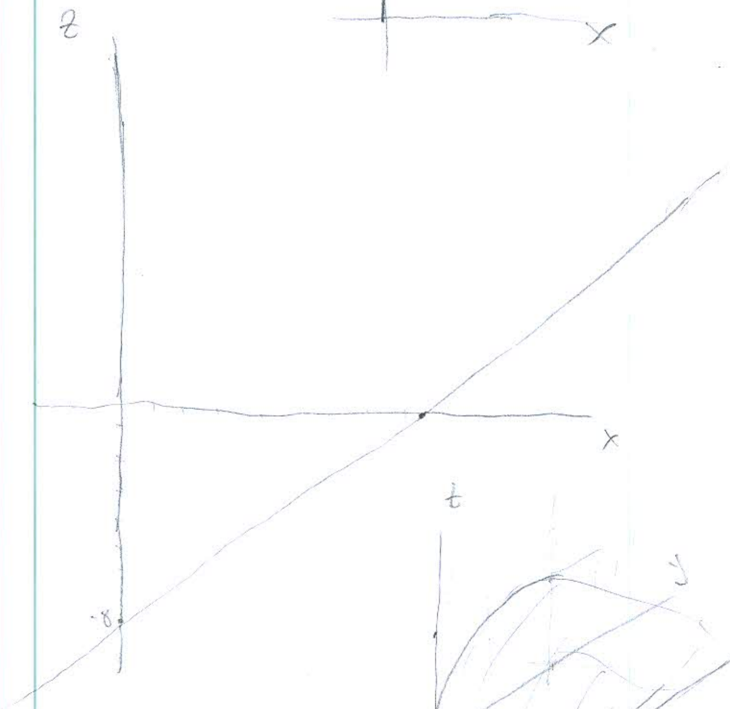
~~65~~

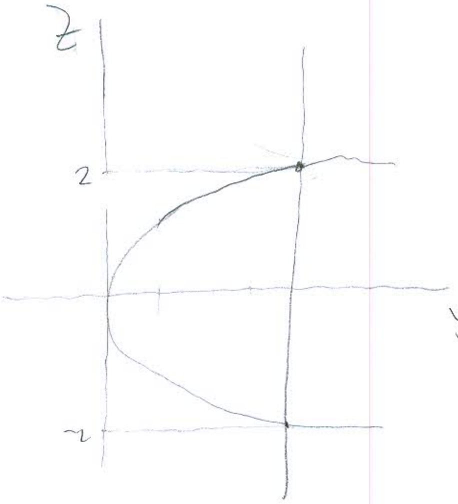
55 Kosor

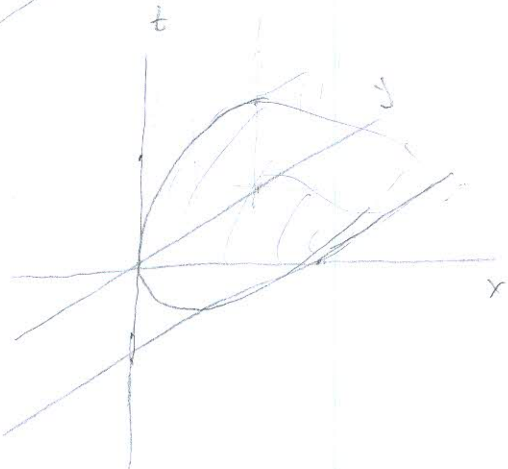
(3) $y = z^2 \quad x = 0 \quad z = x - 8$

$y = 4 \qquad\qquad x = z + 8$









$$\int_{-2}^{2} dz \int_{z^2}^{4} dy \int_{0}^{z+8} dx = \int_{-2}^{2}\int_{z^2}^{4} z+8 \, dy\, dz =$$

$$= \int_{-2}^{2} \left[zy+8y\right]_{z^2}^{4} = \int_{-2}^{2} 4z+32-(z^3+8z^2) =$$

$$= \int_{-2}^{2} 4z+32-z^3-8z^2 \, dz = \left[4\frac{z^2}{2}+32z-\frac{z^4}{4}-8\frac{z^3}{3}\right]_{-2}^{2} =$$

$$= 8+64-4-\frac{64}{3}-\left(8-64-4+\frac{64}{3}\right) =$$

$$= \cancel{8}+64\cancel{-4}-\frac{64}{3}\cancel{-8}+64\cancel{+4}-\frac{64}{3} = 128-\frac{128}{3} = \frac{256}{3}$$

Duje Mitrović

① $y'''(t) + y''(t) = \sin t \qquad y(0)=1 \quad y'(0)=0 \quad y''(0)=1$

$$s^3 Y(s) - s^2 \underbrace{y(0)}_{1} - s\underbrace{y'(0)}_{0} - \underbrace{y''(0)}_{1} + s^2 Y(s) - s\underbrace{y(0)}_{1} - \underbrace{y'(0)}_{0} = \frac{1}{s^2+1}$$

$$s^3 Y(s) - s^2 - 1 + s^2 Y(s) - s = \frac{1}{s^2+1}$$

$$s^3 Y(s) + s^2 Y(s) = \frac{1}{s^2+1} + s^2 + 1 + s$$

$$Y(s) = \frac{\frac{1}{s^2+1} + s^2 + 1 + s}{s^3+s^2} = \frac{\frac{1 + s^2(s^2+1) + s^2+1 + s(s^2+1)}{s^2+1}}{s^3+s^2} =$$

$$= \frac{1 + s^4 + s^2 + s^2 + 1 + s^3 + s}{(s^2+1)(s^3+s^2)} = \frac{s^4+s^3+2s^2+s+2}{s^2(s^2+1)(s+1)}$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} = s^4+s^3+2s^2+s+2$$

$$A[s(s^2+1)(s+1)] + B[(s^2+1)(s+1)] + C[s^2(s^2+1)] + [Ds+E][s^2(s+1)] = s^4+s^3+2s^2+s+2$$

$$A[s^4+s^3+s^2+s] + B[s^3+s^2+s+1] + C[s^4+s^2] + [Ds+E][s^3+s^2] = s^4+s^3+2s^2+s+2$$

$$As^4 + As^3 + As^2 + As + Bs^3 + Bs^2 + Bs + B + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2 = s^4+s^3+2s^2+s+2$$

$C + D = 2$
$C = 2 - D$
$C = \frac{3}{2}$

$A + C + D = 1$
$B + A + D + E = 1$
$A + B + C + E = 2$
$A + B = 1 \Rightarrow A = -1$
$B = 2$

$1 + D + E = 1$
$D = -E$
$D = 1 - D$
$2D = 1 \Rightarrow \frac{1}{2}$

$1 + C + E = 2$
$E = 1 - C \quad E = -\frac{1}{2}$

$$Y(s) = -\frac{1}{s} + 2\cdot\frac{1}{s^2} + \frac{3}{2}\cdot\frac{1}{s+1} + \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{1}{s^2+1}$$

$$\mathcal{L}^{-1}(Y(s)) = -1 + 2t + \frac{3}{2}e^{-t} + \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t)$$

PROVJERA:

$y(0)=1$

$-1 + 0 + \frac{3}{2} + \frac{1}{2} - 0 = 1$

$-1 + 2 = 1$

$1 = 1$

$y'(0)=0$

$0 + 2 - \frac{3}{2} - 0 - \frac{1}{2} = 0$

$2 - \frac{4}{2} = 0$

$0 = 0$

$y''(0)=1$

$0 + 0 + \frac{3}{2} - \frac{1}{2} + 0 = 1$

$\frac{3}{2} - \frac{1}{2} = 1$

$\frac{2}{2} = 1$

④ $r=4$ $T(0,2)$ $\oint_{\partial K}(1-3x)dx$

$x=\cos t$
$y=\sin t+2$

$W=\begin{bmatrix}1-3\cos t\\0\end{bmatrix}$ $r=\begin{bmatrix}\cos t\\ \sin t+2\end{bmatrix}$ $r'=\begin{bmatrix}-\sin t\\ \cos t\end{bmatrix}$

$W\cdot r'=-\sin t+3\cos t\sin t$ ✓

1.5

$\int_0^{2\pi}-\sin t+3\cos t\sin t\,dt=\int_0^{2\pi}-\sin t\,dt+\int_0^{2\pi}3\cos t\sin t=$ ✓

$=[\cos t]_0^{2\pi}+\int_0^{2\pi}\frac{3}{4}-\frac{3}{4}\cos(2t)\,dt=0+\left[\frac{3}{4}t-\frac{3}{8}\sin(2t)\right]_0^{2\pi}=$ ✗

$=\frac{3}{2}\pi$

$3\cos t\sin t=3\cdot\frac{1}{2}\sin 2t=3\cdot\frac{1}{2}\cdot\frac{1}{2}(1-\cos(2t))=\frac{3}{4}-\frac{3}{4}\cos(2t)=$

$\int\cos(2t)$

$u=2t$
$du=2dt$
$dt=\frac{1}{2}du$

$\frac{1}{2}\sin(2t)$

Duje Mitrović

① $s^3Y(s) - s^2\underbrace{y(0)}_{1} - s\underbrace{y'(0)}_{0} - \underbrace{y''(0)}_{1} + s^2Y(s) - s\underbrace{y(0)}_{1} - \underbrace{y'(0)}_{0} = \frac{1}{s^2+1}$

$s^3Y(s) - s^2 - 1 + s^2Y(s) - \cancel{s} = \frac{1}{s^2+1}$

$s^3Y(s) + s^2Y_s = \frac{1}{s^2+1} + s^2 + 1 + s = \frac{1 + s^4 + s^2 + s^2 + 1 + s^3 + s}{s^2+1}$

$= \frac{2 + s^4 + 2s^2 + s^3 + s}{(s^2+1)(s^3+s^2)} = \frac{s^4 + s^3 + 2s^2 + s + 2}{s^2(s^2+1)(s+1)}$

$\frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+1} + \frac{E}{s+1}$

$A[s(s^2+1)(s+1)] + B[(s^2+1)(s+1)] + [Cs+D][s^2(s+1)] + E[s^2(s^2+1)]$

$As^4 + As^3 + As^2 + As + Bs^3 + Bs^2 + Bs + B + Cs^4 + Cs^3 + Ds^3 + Ds^2 + Es^4 + Es^2$

$A + C + E$

(5)

$$\begin{bmatrix} x-y \\ y-x \\ z \end{bmatrix} = -\operatorname{grad} f = \begin{bmatrix} -\frac{\partial f}{\partial x} \\ -\frac{\partial f}{\partial y} \\ -\frac{\partial f}{\partial z} \end{bmatrix}$$

$$-x+y = \frac{\partial f}{\partial x} \quad / \int dx$$

$$-\frac{x^2}{2} + yx + C(y)$$

$$C'(y) \cancel{+x} = -y + x$$

$$C'(y) = -y \quad / \int dy$$

$$C(y) = -\frac{y^2}{2}$$

✗ ~~10~~

$$-\frac{x^2}{2} + yx - \frac{y^2}{2} - z \;\checkmark$$

$f(x,y,z)$ je potencijalno polje ✓

$$\int_{1,0,1}^{2,3,4} -\frac{x^2}{2} + yx - \frac{y^2}{2} - z \, ds$$

$$f(1,0,1) - f(2,3,4$$

$$-\frac{1}{2} + 0 - 0 - 1 - \left(-2 + 6 - \frac{9}{2} - 4\right) =$$

$$= -\frac{1}{2} - 1 + 2 - 6 + \frac{9}{2} + 4 = \frac{1}{2} - 6 + \frac{9}{2} + 4 = \underline{3}$$

NISTE ZADALI NIKAKVU KRIVULJU !

Duje Mitrović

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Alen Bura **BROJ INDEKSA:** 17-2-0095-2011

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$y'''(t) + y''(t) = \sin t, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

2. Neka je S gornja polusfera radijusa $r = 3$ sa centrom u ishodištu, $z = \sqrt{3^2 - x^2 - y^2}$. Kako preko definicije izračunati $\iint_S \mathrm{d}S$? 20

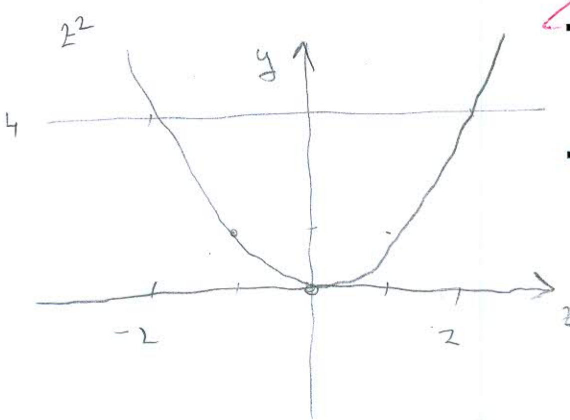
3. Izračunaj volumen prostora omeđenog plohama $y = z^2$, $y = 4$, $x = 0$ i $z = x - 8$. 20

4. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0, 2)$. Izračunati $\int_{\partial K} (1 - 3x)\, \mathrm{d}x$. 20

5. Provjeri da li je $g(x, y, z) = (x - y, y - x, z)$ potencijalno polje? Zadaj neku krivulju i izračunaj krivuljni integral zadane funkcije. ~~20~~ 10

Ukupno: 50

3. $y=z^2$, $y=4$, $x=0$, $z=x-8$



$4=z^2$ $\quad y\in[z^2,4]$

$\boxed{z=\pm 2}$ $\quad x\in[0,z+8]$

$y\in[z^2,4]$ $\quad z\in[-2,2]$

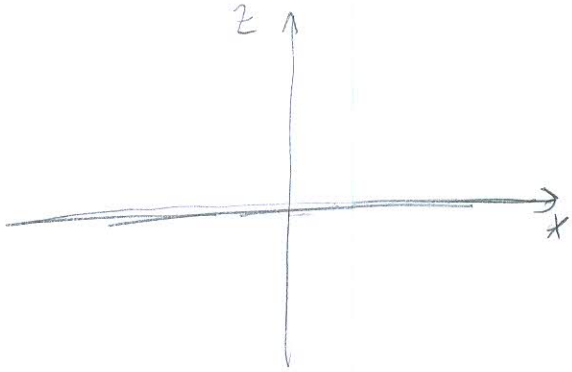
z	0	1	-1	2	-2
$y=z^2$	0	1	1	4	4

$x=0$

$z=x-8$

$x=z+8$

$$\int_{-2}^{2}\int_{z^2}^{4}\int_{0}^{z+8} dx\,dy\,dz = \int_{-2}^{2}\int_{z^2}^{4}\Big[x\Big]_{0}^{z+8} dy\,dz$$



$$=\int_{-2}^{2}\int_{z^2}^{4}(z+8)\,dy\,dz = \int_{-2}^{2}\Big[(zy+8y)\Big]_{z^2}^{4} dz$$

$x=$	0	1	2	3
$z=x-8$	-8	-7	-6	-5

$$=\int_{-2}^{2}\Big[(4\cdot z+8\cdot 4)-(z\cdot z^2+8\cdot z^2)\Big]dz$$

$$=\int_{-2}^{2}(4z+32)-(z^3+8z^2)\,dz = \int_{-2}^{2}(4z+32-z^3-8z^2)\,dz = \Big[\Big(2z^2+32z-\frac{z^4}{4}-8\cdot\frac{z^3}{3}\Big)\Big]_{-2}^{2}$$

$$\left[\left(2\cdot 2^2+32\cdot 2-\frac{2^4}{4}-8\cdot\frac{2^3}{3}\right)-\left(2\cdot(-2)^2+32\cdot(-2)-\frac{(-2)^4}{4}-8\cdot\frac{(-2)^3}{3}\right)\right]$$

$$=\frac{140}{3}-\left(-\frac{116}{3}\right)=\frac{256}{3}$$ ✓

$\frac{166}{3}$

4. $r=4$ $T(0,2)$ $\int_{\partial K}(1-3x)\,dx$

$P(x,y)=1-3x$ $Q(x,y)=0$

$r\in[0,4]$

$\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=\frac{0}{\partial x}-\frac{1-3x}{\partial y}=0-0=0$

$\varphi\in[0,2\pi]$

$$\int_0^{2\pi}\int_0^4 o\,r\,dr\,d\varphi=0$$ ✓

5. $g(x,y,z)=(x-y,y-x,z)$ $g(x)=\begin{pmatrix}x-y\\y-x\\z\end{pmatrix}$ $-\mathrm{grad}f=\begin{pmatrix}-x+y\\x-y\\-z\end{pmatrix}$

$f(x,y)=\int -x+y=-\frac{x^2}{2}+yx+C(y,z)$

integral je potencijalno polje!

$$\boxed{f(x,y,z)=-\frac{x^2}{2}+yx-\frac{y^2}{2}-\frac{z^2}{2}}$$ ✓

$\frac{\partial f}{\partial y}=\cancel{x}+C'(y,z)=\cancel{x}-y$ $C(y,z)=\int -y=\boxed{-\frac{y^2}{2}}$

10

$C'(z)=0+C'(z)=-z$ $/\int$

$C'(z)=-\frac{z^2}{2}$

NISTE ZADALI NIKAKVU KRIVULJU!

$$\int_{(1,3,5)}^{(1,1,8)}\left(\frac{-1^2}{2}+3\cdot 1-\frac{(3)^2}{2}-\frac{5^2}{2}\right)-\left(\frac{-(2)^2}{2}+1\cdot 2-\frac{1^2}{2}-\frac{8^2}{2}\right)$$

$$-\frac{29}{2}-\left(-\frac{65}{2}\right)$$

$$=-\frac{29}{2}+\frac{65}{2}=18$$

1. $y'''(t)+y''(t)=\sin t$ $\quad y(0)=1,\ y'(0)=0,\ y''(0)=1$ $\quad$ ALEN BUDA 17-2-0095-2011

$$s^3F(s)-s^2\underset{1}{f(0)}-s\underset{0}{\cancel{f'(0)}}-\underset{1}{f''(0)}+s^2F(s)-sf(0)-\underset{0}{\cancel{f'(0)}}=\frac{1}{s^2+1}$$

$$F(s)(s^3+s^2)-s^2-1-s=\frac{1}{s^2+1}$$

$$(s^3+s^2)=s^2(s+1)$$

$$F(s)(s^3+s^2)=s^2+1+s+\frac{1}{s^2+1}$$

$$F(s)(s^3+s^2)=\frac{s^4+1+s^3+1+1}{s^2+1}$$

$$F(s)(s^3+s^2)=\frac{s^4+s^3+3}{s^2+1}\ \Big/\cdot\frac{1}{s^2(s+1)}$$

$$\frac{s^4+s^3+3}{s^2(s+1)(s^2+1)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+1}+\frac{Ds+E}{s^2+1}\ \Big/\cdot s^2(s+1)(s^2+1)$$

$$As(s+1)(s^2+1)+B(s+1)(s^2+1)+Cs^2(s^2+1)+(Ds+E)\cdot(s^3+s^2)$$

$$As(s^3+s+s^2+1)+B(s^3+s+s^2+1)+Cs^4+Cs^2+Ds^4+Ds^3+Es^3+Es^2$$

$$s^4+s^3+3=As^4+As^2+As^3+As+Bs^3+Bs+Bs^2+B+Cs^4+Cs^2+Ds^4+Ds^3+Es^3+Es^2$$

$A+C+D=1$ $\quad -3+C+D=1$ $\quad -3+C+D=1$

$A+B+D+E=1$ $\quad C=1+3-D$ $\quad \boxed{D=4-C}$ $\quad \boxed{D=\frac{5}{2}}$ ✓

$A+B+C+E=0$ ✓ $\quad \boxed{C=4-D}$

$-3+3+C-\frac{3}{2}=0$

$A+B=0$

$\boxed{B=3}$ ✓ $\quad A+B+D+E=1$ $\quad \boxed{C=\frac{3}{2}}$ ✓

$A+B+4-D+E=0$

$A+3=0$

$\boxed{A=-3}$ ✓

$-3+3+D+E=1$

$-3+3-D+E=-4$ $\quad +$

$2E=-3\ /:2$

$\boxed{E=\frac{-3}{2}}$ ✓

$$\frac{-3}{s}+\frac{3}{s^2}+\frac{\frac{3}{2}}{s+1}+\frac{\frac{5}{2}s-\frac{3}{2}}{s^2+1}=$$

$$=3\cdot\frac{1}{s}+3\cdot\frac{1}{s^2}+\frac{3}{2}\cdot\frac{1}{s+1}+\frac{5}{2}\cdot\frac{s}{s^2+1}-\frac{3}{2}\cdot\frac{1}{s^2+1}$$

$$f(t)=3+3t+\frac{3}{2}\cdot e^{-t}+\frac{5}{2}\cdot\cos t-\frac{3}{2}\cdot\sin t$$ X

PROVERA ?

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: GABRIJELA JORDAN

BROJ INDEKSA: 17-2-0118-2011

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno: 20

1. $y'''(t) + y''(t) = \sin(t)$ $\quad y(0)=1 \quad y'(0)$

$s^3 y(s) - s^2 y(0) - \cancel{s y'(0)} - y''(0) + s^2 y(s) - s y(0) - \cancel{y'(0)} = \frac{1}{s^2+1}$

$y(s)(s^3+s^2) - s^2 - 1 - s = \frac{1}{s^2+1}$

$y(s)(s^3+s^2) = \frac{s^4 + s^2 + s^3 + s + s^2 + 1 + 1}{s^2+1} = \frac{s^4 + s^3 + 2s^2 + s + 2}{s^2+1}$

$\frac{s^4 + s^3 + 2s^2 + s + 2}{(s^2+1)s^2(s+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs+D}{s^2+1} + \frac{E}{s+1}$

$A(s^2+1)(s+1) + B(s^2+1)(s^2+s) + (Cs+D)(s+1)\cdot s^2 + E\cdot s^2(s^2+1)$

$A(s^3+s^2+s+1) + B(s^4+s^3+s^2+s) + C(s^4+s^3) + D(s^3+s^2) + E(s^4+s^2)$

$B + C + E = 1$ $\quad C + E = 2$ $\quad -D + E = 2$

$A + B + C + D = 1$ $\quad C + D = 0$ $\quad D + E = 1$

$A + B + D + E = 2$ $\quad D + E = 1$ $\quad E = \frac{1}{2}$ $\quad D = \frac{-3}{2}$

$A + B = 1 \to B = -1$

$A = 2$ $\quad C = \frac{3}{2}$

$= \frac{2}{s^2} + \frac{-1}{s} + \frac{3}{2}\,\frac{s+1}{s^2+1} + \frac{1}{2}\cdot\frac{1}{s+1}$

$y(t) = 2t - 1 + \frac{3}{2}\cdot(\cos t - \sin t) + \frac{1}{2}e^{-t}$

$= 2t - 1 + \frac{3}{2}\cos t - \frac{3}{2}\sin t + \frac{1}{2}e^{-t}$

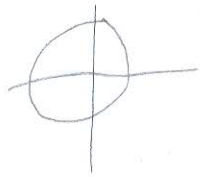
PROVJERA?

GABRIJELA JORDAN

17-2-0118-2011

4. $x = r\cos\varphi$

$y = r\sin\varphi + 2$



$$= \int_0^{2\pi} d\varphi \int_0^4 r(1-3r\cos\varphi)\,dr$$

$$= \int_0^{2\pi}\left(\frac{r^2}{2}-3\cos\varphi\,\frac{r^3}{3}\right)\Big|_0^4 d\varphi$$

$$= \int_0^{2\pi}(8-64\cos\varphi)\,d\varphi = 16\pi$$

3. $y = z^2$

$y = 4$

$x = 0$

$z = x - 8$

$$\int_{-2}^{2} dz \int_{z^2}^{4} dy \int_0^{z+8} dx\; 1$$

$$= \int_{-2}^{2}\int_{z^2}^{4} z+8 \; dz\,dy$$

$$= \int_{-2}^{2}\left((z+8)(4-z^2)\right)dz$$

$$= \int_{-2}^{2}(4z - z^3 + 32 - 8z^2)\,dz$$

$$= \cancel{4}^{2}\,\frac{z^2}{\cancel{2}}\Big|_{-2}^{2} - \frac{z^4}{4}\Big|_{-2}^{2} + 32z\Big|_{-2}^{2} - 8\cdot\frac{z^3}{3}\Big|_{-2}^{2}$$

$$= 2\cdot(4-4) - \frac{1}{4}(16-16) + 32(2+2) - \frac{8}{3}\cdot(8+8)$$

$$= 32\cdot 4 - \frac{8\cdot 16}{3} = 128 - \frac{128}{3} = \frac{3\cdot 128 - 128}{3} = \frac{256}{3}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: VESNA ŠARIĆ **BROJ INDEKSA:**	Broj ↓ bodova
Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

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4. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0, 2)$. Izračunati $\int\limits_{\widehat{\partial K}} (1 - 3x)\,\mathrm{d}x$. 20

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Ukupno: 10

(5) $g(x,y,z) = (x-y,\ y-x,\ z)$

(1) $g = -\operatorname{grad} f$

$f = ?$

$$g = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix}$$

$$x-y = -\frac{\partial f}{\partial x} \quad \Big/ \int dx$$

$$\int \frac{\partial f}{\partial x} = \int (-x+y)\,dx$$

$$f = -\int x\,dx + y\int dx$$

$$f = \frac{-x^2}{2} + yx + dy$$

(2) $$\frac{\partial f}{\partial y} = -y+x$$

$$\frac{\partial}{\partial y}\left(\frac{-x^2}{2} + yx + c(y)\right) = -y+x$$

$$x + c'(y) = -y+x$$

$$c'(y) = -y$$

$$\frac{d(c'y)}{dy} = -y \quad \Big/ \int dy$$

$$c(y) = -\frac{y^2}{2} + c(z)$$

(3) $$\frac{\partial f}{\partial z} = -z \qquad c'(z) = -z$$

$$c(z) = \int -z\,dz$$

$$\frac{\partial f}{\partial z} = c'(z) \qquad c(z) = -z^2 + C$$

$$\boxed{f = \frac{-x^2}{2} + yx - \frac{x^2}{2} - \frac{z^2}{2} + C}$$

g je potencijalno polje

✓ 10

OSTATAK ZADATKA?

(1)

$y'''(t)+y''(t)=\sin t \qquad y(0)=1,\ y'(0)=0,\ y''(0)=1$

$\mathcal{L}[y'''(t)+y''(t)]=\mathcal{L}[\sin t]$

$\mathcal{L}[y'''(t)]+\mathcal{L}[y''(t)]=\mathcal{L}[\sin t]$

$$(s^3Y(s)-s^2\underbrace{y(0)}_{1}-s\cdot\underbrace{y'(0)}_{0}-\underbrace{y''(0)}_{1}+s^2Y(s)-s\cdot\underbrace{y(0)}_{1}-\underbrace{y'(0)}_{0}=\frac{1}{s^2+1}$$

$$(s^3Y(s)-s^2-1+s^2Y(s)-1)=\frac{1}{s^2+1}$$

$$Y(s)(s^3+s^2)=\frac{1}{s^2+1}+2$$

$$Y(s)=\frac{\frac{1}{s^2+1}+2}{s^3+s^2}$$

$$(s^3+s^2)=s(s^2+s)$$

$$Y(s)=\frac{\frac{1}{s^2+1}}{s(s^2+s)}+\frac{\frac{2}{s^2+1}}{s^3+s^2}$$

VESNA ŠARIĆ MATEMATIKA (3) (2) DIO

(1) $\dfrac{\frac{1}{s^2+1}}{s^3+s^2} = \dfrac{1}{(s^2+1)s(s^2+s)} = \dfrac{A}{s} + \dfrac{Bs+C}{s(s^2+s)} + \dfrac{Ds+F}{(s^2+1)}$

$1 = A((s^2+1)(s^2+s)) + (Bs+C)(s^2+1) + (Ds+F)(s^2+s)$

$1 = A(s^4+s^3+s^2+s) + Bs^3 + Bs + Cs^2 + C + Ds^3 + Ds^2 + Fs^2 + Fs$

$1 = As^4 + As^3 + \cancel{As^2} + As + Bs^3 + Bs + \cancel{Cs^2} + C + Ds^3 + \cancel{Ds^2} + \cancel{Fs^2} + Fs$

s^4 $\boxed{0 = A}$

s^3 $0 = A + B + D$

s^2 $0 = A + C + D + F$

s $0 = A + B + F$

$\boxed{1 = C}$

VESNA ŠARIĆ MATEMATIKA (3) 03/09/2015

(1)

$$3 = A\left((s^2+1)(s^2+s)\right) + (Bs+C)\left(s(s^2+s)\right) + (Ds+F)\left(s(s^2+1)\right)$$

$$3 = A(s^4+s^3+s^2+s) + (Bs+C)(s^3+s^2) + (Ds+F)(s^3+s)$$

$$3 = As^4 + As^3 + \cancel{As^2} + As + Bs^4 + Bs^3 + Cs^3 + \cancel{Cs^2} + Ds^4 + Ds^3 + Fs^3 + Fs$$

s^4 $0 = A+B+D$

s^3 $0 = A+B+C+F$

s^2 $0 = A+C$

s $0 = A+F$

NIJE DOVEDENO DO KRAJA.

0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARIN MATEK **BROJ INDEKSA:** 17-1-0111-12

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) = \sin t, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

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3. Izračunaj volumen prostora omeđenog plohama $y = z^2$, $y = 4$, $x = 0$ i $z = x - 8$. 20

4. Neka je K krug radijusa $r = 4$ sa centrom u točki $T(0,2)$. Izračunati $\int\limits_{\partial K} (1 - 3x)\,\mathrm{d}x$. 20

5. Provjeri da li je $g(x,y,z) = (x - y, y - x, z)$ potencijalno polje? Zadaj neku krivulju i izračunaj krivuljni integral zadane funkcije. 20

Ukupno: $\cancel{0}$

1\. $y'''(t) + y''(t) = \sin t \qquad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1$

$$s^3 Y(t) - s^2 \overbrace{y(0)}^{1} - s\overbrace{y'(0)}^{0} - \overbrace{y''(0)}^{1} + s^2 Y(t) - s\overbrace{y(0)}^{1} - \overbrace{y'(0)}^{0} = \frac{1}{s^2+1}$$

$$s^3 Y(t) - s^2 - 1 + s^2 Y(t) - s = \frac{1}{s^2+1}$$

$$s^3 Y(t) + s^2 Y(t) = \frac{1}{s^2+1} + s^2 + s + 1$$

$$Y(t)(s^3 + s^2) = \frac{1}{s^2+1} + s^2 + s + 1 = \frac{1 + s^2 + s + 1}{s^2+1}$$

$$Y(t) = \frac{\frac{s^2+s+2}{s^2+1}}{s^3+s^2} = \frac{s^2+s+2}{(s^3+s^2)(s^2+1)} = \frac{s^2+s+2}{s^2(s+1)(s^2+1)}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} \quad \Big/ \cdot (s^2(s+1)(s^2+1))$$

$$= \frac{A}{s(s+1)(s^2+1)} + \frac{B}{(s+1)(s^2+1)} + \frac{C}{s^2(s^2+1)} + \frac{Ds+E}{s^2(s+1)} =$$

$$= A(s(s+1)(s^2+1)) + B(s+1)(s^2+1) + C(s^2(s^2+1)) + (Ds+E)(s^2(s+1))$$

$$= A((s^2+s)(s^2+1)) + B(s^3+s+s^2+1) + C(s^4+s^2) + (Ds+E)(s^3+s^2)$$

$$= A(s^4+s^2+s^3+s) + B(s^3+s^2+s+1) + C(s^4+s^2) + Ds^4 + Ds^3 + Es^3 + Es^2$$

$$= As^4 + As^3 + As^2 + As + Bs^3 + Bs^2 + Bs + B + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$$

$$= s^4(A+C+D) + s^3(A+B+D+E) + s^2(A+B+C+E) + s(A+B) + B$$

$A+C+D=0 \Rightarrow -1+C+C=0 \Rightarrow 2C=1$ $\quad C=\frac{1}{2}$ //

$A+B+D+E=0 \Rightarrow -1+2+D-C=0 \Rightarrow D-C=-1$

$A+B+C+E=1 \Rightarrow -1+2+C+E=1$ $\quad D=C$ // $\Rightarrow D=\frac{1}{2}$

$A+B=1 \Rightarrow A+2=1$ $\quad C+E=0$

$B=2$ // $\quad A=-1$ //

$E=-C$ // $\Rightarrow E=-\frac{1}{2}$ //

$$= -\frac{1}{s} + 2\cdot\frac{1}{s^2} + \frac{1}{2}\frac{1}{s+1} + \frac{\frac{1}{2}s-\frac{1}{2}}{s^2+1}$$

$$= -\frac{1}{s} + 2\cdot\frac{1}{s^2} + \frac{1}{2}\frac{1}{s+1} + \frac{1}{2}\frac{s-\frac{1}{2}}{s^2+1}$$ $\quad \frac{1}{2}\left(1-\frac{1}{2}+\frac{s}{s^2+1}\right)$

$$= -\frac{1}{s} + 2\frac{1}{s^2} + \frac{1}{2}\cdot\frac{1}{s+1} - \frac{1}{4}\left(\frac{s}{s^2+1}\right)$$

$$= -1 + 2t + \frac{1}{2}e^{-t} - \frac{1}{4}\cos t \text{ //}$$ ✗ PROVJERA

MARIN MATEK

2. $r = 3$ $\quad z = \sqrt{3^2 - x^2 - y^2}$

$$\iint_S ds$$

$z^2 = 3^2 - r^2$

$z = \sqrt{9-9}$

$z = 0$

$$\int_0^{2\pi}\int_0^3 dr\,d\varphi = \int_0^{2\pi} r\Big|_0^3 d\varphi = \int_0^{2\pi} 3\,d\varphi = 3\cdot\varphi\Big|_0^{2\pi} = 3\cdot 2\pi = 6\pi$$

3. $y = z^2,\ y = 4,\ x = 0,\ z = x - 8$

$y = 4$

$y = (x-8)^2$

$x = 0$

$x = z + 8 \Rightarrow$

$z = x - 8$

0

4. $r = 4$ $\quad T(0,2)$ $\quad \int_{\partial k} (1-3x)\,dx$

$x = r\cos\varphi + 0$ $\quad \varphi \in [0, 2\pi]$

$y = r\sin\varphi + 2$ $\quad r\,[0, 4]$

$$\int_0^{2\pi}\int_0^{4} 1 - 3r\cos\varphi \, d\, r\cos\varphi \, d\varphi =$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: TOMISLAV GLAVAN BROJ INDEKSA: 17-0115-2011

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

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3. Izračunaj volumen prostora omeđenog plohama $y = z^2$, $y = 4$, $x = 0$ i $z = x - 8$. 20

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Ukupno: Ø

1) $y'''(t) + y''(t) = \sin t \qquad y(0) = 1,\ y'(0) = 0 \qquad y''(0) = 1$

$$s^3 y(s) - s^2 y(0) - s \cdot y'(0) - y''(0) + s^2 y(s) - s\, y(0) - y'(0) = \frac{1}{s^2+1}$$

$$s^3 y(s) - s^2 \cdot 1 - s \cdot 0 - 1 + s^2 y(s) - s \cdot 1 - 0 = \frac{1}{s^2+1}$$

$$s^3 y(s) - s^2 - 1 + s^2 y(s) - s = \frac{1}{s^2+1}$$

$$s^3 y(s) + s^2 y(s) = \frac{1}{s^2+1} + \frac{s^2}{1} + \frac{1}{1}$$

$$s^3 y(s) + s^2 y(s) = \frac{1 + s^2(s^2+1) + (s^2+1)}{s^2+1}$$

$$s^3 y(s) + s^2 y(s) = \frac{1 + s^4 + s^2 + s^2 + 1}{s^2+1}$$

$$s^3 y(s) + s^2 y(s) = \frac{2 + 2s^2 + s^4}{s^2+1}$$

$$y(s)(s^3 + s^2) = \frac{2 + 2s^2 + s^4}{s^2+1} \Big/ (s^3 + s^2)$$

$$y(s) = \frac{\frac{2 + 2s^2 + s^4}{s^2+1}}{\frac{s^3 + s^2}{1}} = \frac{2 + 2s^2 + s^4}{s^5 + s^3 + s^4 + s^2} = \frac{2 + 2s^2 + s^4}{s^2(s^3 + s^2 + s + 1)}$$

$s^2(s+1)(s+1)$

$(s+1)(s^2+1)$

$$\frac{2+2s^2+s^4}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+1)} + \frac{Ds+E}{(s^2+1)}$$

$$\frac{2+2s^2+s^4}{s^2(s+1)(s^2+1)} = \frac{A\underbrace{s(s+1)(s^2+1)}_{}^{(s^4+s+s^3+1)} + \overbrace{B(s+1)(s^2+1)}^{(s^3+s+s^2+1)} + \overbrace{C\cdot s^2(s^2+1)}^{(s^4+s^2)} + (Ds+E)\cdot \overbrace{s^2\cdot(s+1)}^{(s^3+s^2)}}{s^2(s+1)(s^2+1)}$$

$$\cancel{2+2s^2+s^4=As^4+As^3+As^2+As+Bs^3+Bs^2+Bs+B}+Cs^4+Cs^2+Ds^4+Ds^3+Es^3+Es^2$$

$A+C+D=1 \Rightarrow -2+C+D=1 \Rightarrow C+D=1+2 \Rightarrow C+D=3$

$A+B+D+E=0 \Rightarrow -2+2+D+E=0 \Rightarrow D+E=+2-2 \Rightarrow D+E=0$

$A+B+C+E=2 \Rightarrow -2+2+C+E=2 \Rightarrow C+E=+2-2+2 \Rightarrow C+E=2$

$A+B=0 \Rightarrow A+2=0$

$\boxed{B=2}$ $\boxed{A=-2}$

PR:

$C+D=3$

$D+E=0 \Rightarrow D=-E$

$C+E=2 \Rightarrow C=2-E$

$D+E=0$

$D-\frac{5}{2}=0$

$\boxed{D=\frac{5}{2}} \Rightarrow \boxed{D=\frac{5}{2}}$

$-2+2+D-\frac{5}{2}=0$

$0+D-\frac{5}{2}=0$

$2-E-E=3$

$-2E=3+2$

$-2E=5\,/(-2)$

$\boxed{E=-\frac{5}{2}}$

$C+E=2$

$C-\frac{5}{2}=2$

$C=2+\frac{5}{2}$

$\boxed{C=\frac{9}{2}} \Rightarrow \boxed{C=\frac{9}{2}}$

PR:

$-2+2+C-\frac{5}{2}=2$

$0+C-\frac{5}{2}=2$

$C=2+\frac{5}{2}$

$$y(s)=\frac{-2}{s}+\frac{2}{s^2}+\frac{\frac{9}{2}}{s+1}+\frac{\frac{5}{2}s-\frac{5}{2}}{s^2+1} \Big/ \mathcal{L}^{-1}$$

$$y(t)=-2+2t+\frac{9}{2}e^{-t}+\frac{5}{2}\cos t-\frac{5}{2}\sin t$$ ✗

PROVJERI!

TOMISLAV GLAVAN

17-0115-2011

5) $g(x,y,z) = (x-y, y-x, z)$

$$\frac{x-y}{\partial y} = -1 \rightarrow \frac{-1}{\partial z} = 0$$

$$\frac{y-x}{\partial x} = 1 \rightarrow \frac{1}{\partial z} = 0$$

$$\frac{z}{\partial x} = 0 \rightarrow \frac{0}{\partial y} = 0$$

Nije potencijalno polje jer ne ovisi o putu integracije.

4) $r(t) = 4\cos t\vec{i} + (4\sin t + 2)\vec{j}$ $\quad t \in [0, 2\pi]$

$$\int_{dk} (1-3x)\,dx = ?$$

$$r'(t) = -4\sin t\vec{i} + 4\cos t\vec{j}$$

$$\|r'(t)\| = \sqrt{(-4\sin t)^2 + (4\cos t)^2} = \sqrt{\underbrace{16\sin^2 t + 16\cos^2 t}_{16}}$$

$$\|r'(t)\| = \sqrt{16} = 4$$

$$f(r) = 1 - 3 \cdot 4\cos t = 1 - 12\cos t$$

$$\int_{K} (1-3x) = \int_{0}^{2\pi} (1-12\cos t)\cdot 4\,dt = 4t\Big|_{0}^{2\pi} - 48\sin\Big|_{0}^{2\pi}$$

$$= 4\cdot(2\pi-0) - 48\cdot(0-0) = 8\pi$$

$$= 4\cdot(2\pi-0) - 48\cdot(0-0) = 8\pi - 0 = 8\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: FILIP MEŠTROVIĆ BROJ INDEKSA: 0171256000

Zaokružiti nastavnika za ustmeni: prof. Uglešić asistent Kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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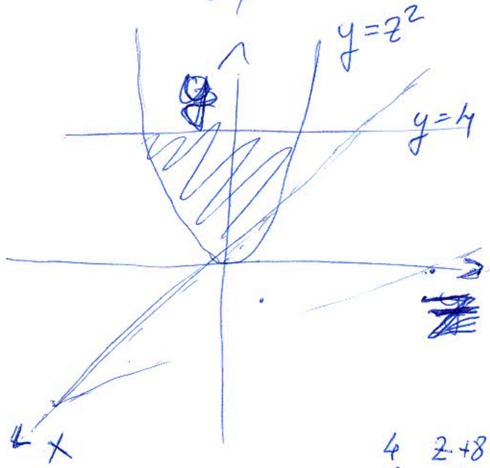
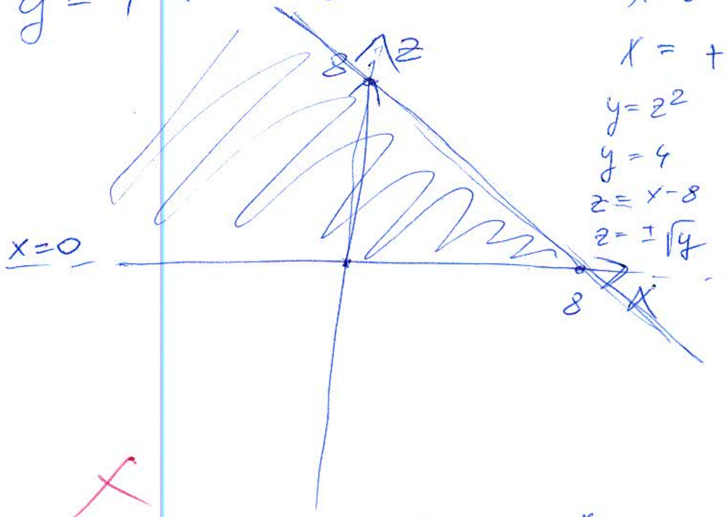
Ukupno: 0

3. $y = z^2 \quad y = 4 \quad x = 0 \quad z = x - 8$





$x = 0$
$x = +z + 8$
$y = z^2$
$y = 4$
$z = x - 8$
$z = \pm\sqrt{y}$

$$\frac{y^{\frac12 + 1}}{-\frac12 + \frac22}$$

$$\cancel{\int_0^4\int_0^{z+8}\int_0^{\sqrt y} dz\,dx\,dy = \iint_{0\,0}^{4\ z+8} (z)\Big|_0^{\sqrt y} dx\,dy} =$$

$$= \cancel{\int_0^4\int_0^{z+8} \sqrt{y}\,dx\,dy = \int_0^4 \left(\frac{2}{3} y^{\frac32}\right)\Big|_0^{z+8} dy} =$$