

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

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POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **TONI GRBIĆ**

VRIJEME POČETKA: **17:13**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **17-1-0288-2014**

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

1. Odrediti kada je $\frac{15 + 8x + x^2}{9 - x^2} = \frac{1}{3}$.

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2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije.

20 graf

3. Odrediti tok funkcije $f(x) = \frac{x^2 + 9}{x - 4}$ i skicirati graf.

20 graf

4. Zadana je funkcija $f(x) = 2x + \sqrt{x^2 + x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf.

~~4+4+4+6~~

5. Gaussovom metodom riješi sustav linearnih jednažbi:

15+3

$$\begin{aligned} 2x_1 - x_2 + x_3 - x_4 &= -1 \\ 2x_1 - x_2 &= 1 \\ 3x_1 - x_3 + x_4 &= -1 \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 &= -1 \end{aligned}$$

Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2} - x}{x-1}$.

10+2

Ukupno:

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① $\frac{15 + 8x + x^2}{9 - x^2} = \frac{1}{3}$

⑥ $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2} - x}{x-1} = \frac{0}{0}$

$$\begin{array}{c}
 \textcircled{5} \\
 \left[\begin{array}{cccc|c}
 2 & -1 & 1 & -1 & -1 \\
 2 & -1 & 0 & -3 & 1 \\
 3 & 0 & -1 & 1 & -1 \\
 2 & 2 & -2 & 5 & -1
 \end{array} \right] \sim \left[\begin{array}{cccc|c}
 1 & -1 & 2 & -1 & -1 \\
 0 & -1 & 2 & -3 & 1 \\
 -1 & 0 & 3 & 1 & -1 \\
 -2 & 2 & 2 & 5 & -1
 \end{array} \right] \begin{array}{l} \\ \\ \text{III} + \text{I} \\ \text{IV} + 2\text{I} \end{array} \sim
 \end{array}$$

PAZI!

$$\sim \left[\begin{array}{cccc|c}
 1 & -1 & 2 & -1 & -1 \\
 0 & -1 & 2 & -3 & 1 \\
 0 & -1 & 5 & 0 & -2 \\
 0 & 0 & 6 & 3 & -3
 \end{array} \right] \begin{array}{l} \\ \\ \text{III} - \text{II} \\ \\ \end{array} \sim \left[\begin{array}{cccc|c}
 1 & -1 & 2 & -1 & -1 \\
 0 & -1 & 2 & -3 & 1 \\
 0 & 0 & 3 & 3 & -3 \\
 0 & 0 & 6 & 3 & -3
 \end{array} \right] \begin{array}{l} \\ \\ \\ \text{IV} - 2\text{III} \end{array}$$

$$\left[\begin{array}{cccc|c}
 1 & -1 & 2 & -1 & -1 \\
 0 & -1 & 2 & -3 & 1 \\
 0 & 0 & 3 & 3 & -3 \\
 0 & 0 & 0 & -3 & 3
 \end{array} \right]$$

Provera uvrstavanjem

$$\begin{aligned}
 2 \cdot 0 - 2 + 0 - (-1) &= -1 \\
 -1 &= -1 \checkmark
 \end{aligned}$$

$$2 \cdot 0 + 2 \cdot 2 - 2 \cdot 0 + 5 \cdot (-1) = 4 - 5 = -1 \checkmark$$

$$-3x_4 = 3$$

$$x_4 = -1$$

$$3x_1 + 3x_4 = -3$$

$$3x_1 = -3x_4 - 3$$

$$x_1 = 0$$

$$-x_2 - 3x_3 = 1$$

$$-x_2 = 1 - 3x_3$$

$$x_2 = 2$$

$$x_3 - x_2 - x_4 = -1$$

$$x_3 - 2 + 1 = -1$$

$$x_3 = -1 + 2 - 1$$

$$x_3 = 0$$

$$3) f(x) = \frac{x^2 + 9}{x - 4}$$

1. DOMENA

$$x - 4 \neq 0 \\ x \neq 4$$

$$Df: \mathbb{R} / \{4\}, \langle -\infty, 4 \rangle \cup \langle 4, +\infty \rangle$$

N.T.

$$x^2 + 9 = 0 \\ x^2 = -9 \quad | \sqrt{\quad}$$

NEMA N.T.

V.A. $\lim_{x \rightarrow 4} \frac{x^2 + 9}{x - 4} = +\infty$

H.A. $\lim_{x \rightarrow \pm\infty} \frac{x^2 + 9}{x - 4} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \pm\infty} \frac{2x + 9}{1} = \frac{\infty}{1} = \infty$ NEMA H.A. ∇

K.A. $\lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = \lim_{x \rightarrow \pm\infty} \frac{\frac{x^2 + 9}{x - 4}}{\frac{x}{9}} = \frac{x^2 + 9 \cdot 9}{x^2 - 4x} = \lim_{x \rightarrow \pm\infty} \frac{\frac{x^2}{x^2} + \frac{9}{x^2}}{\frac{x^2}{x^2} - \frac{4x}{x^2}} = \frac{1 + 0}{1 - 0} = 1$

$$y = kx + l$$

$$\lim_{x \rightarrow \pm\infty} f(x) - kx$$

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 + 9}{x - 4} - x \quad \lim$$

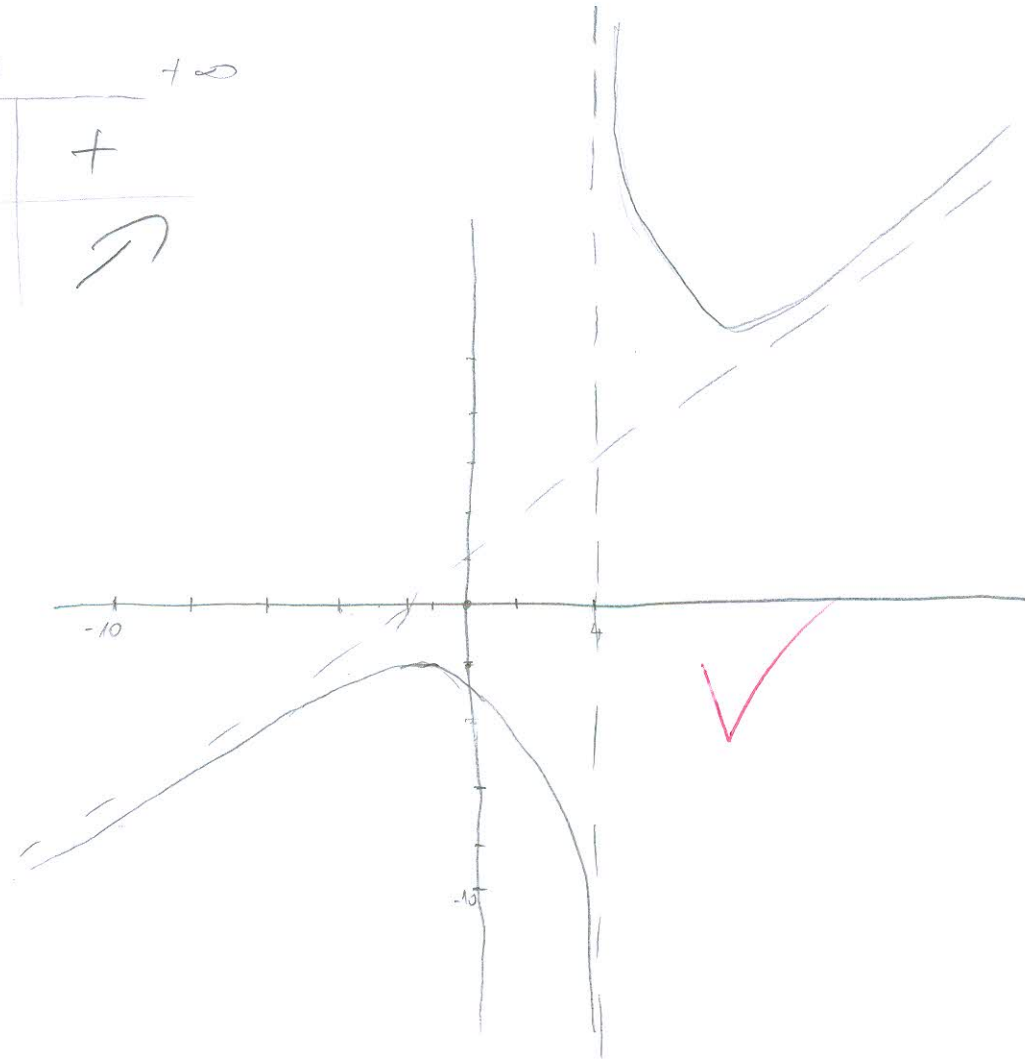
$$f'(x) = \frac{2x \cdot (x-4) - (x^2+9) \cdot 1}{(x-4)^2} = \frac{2x^2 - 8x - x^2 - 9}{(x-4)^2} = \frac{x^2 - 8x - 9}{(x-4)^2}$$

N.T.

$$x^2 - 8x - 9 = 0$$

$$x_{1,2} = \frac{8 \pm \sqrt{8^2 - 4 \cdot 1 \cdot (-9)}}{2} \quad \begin{matrix} x_1 = -1 \\ x_2 = 9 \end{matrix}$$

∞	-1	4	9	$+\infty$
$f'(x)$	+	-	-	+
$f(x)$	\nearrow	\searrow	\searrow	\nearrow



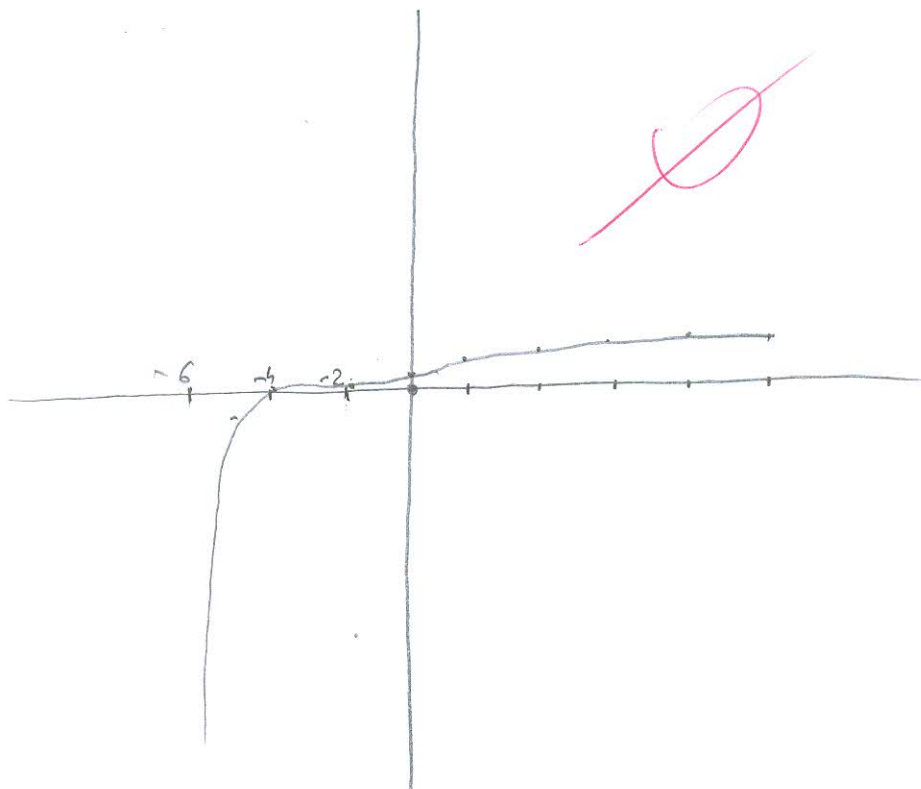
$$2) g(x) = \arctan(e^x)$$

$$Df: \mathbb{R}$$

N.T. NEMA

V.A \rightarrow NEMA jer je $Df: \mathbb{R}$

$$g(x)' = \frac{1}{1+(e^x)^2} \cdot e^x \cdot 1 = \frac{e^x}{1+(e^x)^2} = \frac{1}{1+e^x}$$



$$④ f(x) = 2x + \sqrt{x^2 + x}$$

$$x^2 + x \geq 0$$

$$Df: \mathbb{R}$$

$$f'(x) = 2 + \frac{1}{2\sqrt{x^2+x}} \cdot (2x+1) = 2 + \frac{2x+1}{2\sqrt{x^2+x}} = 2 + \frac{x+1}{\sqrt{x^2+x}}$$

$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$x = 2$$

$$y - 4 + \sqrt{6} = \frac{4 + \sqrt{6}}{2}(x - 2)$$

$$f(2) = 4 + \sqrt{6}$$

$$y = 3,22x$$

$$f'(x_0) = \frac{4 + \sqrt{6}}{2}$$

NEMA LOKALNIH EKSTREMA
NEMA GLOBALNIH EKSTREMA

