

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

24

IME I PREZIME: **MAJA ŠIKIĆ**

VRIJEME POČETKA: **17:13**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 4x - 2}$ i skicirati graf.

~~20 graf~~

2. Riješiti jednačinu: $8 + 4i = |z| + z$. Prikaži rješenje u kompleksnoj ravnini!

~~12+3~~

3. Odrediti sve asimptote funkcije $f(x) = \frac{x^2 - 1}{4 - x^2}$.

~~15~~ **5**

4. Odrediti tok funkcije $f(x) = \frac{x^2 + 4}{x^2 - 2}$ i skicirati graf.

~~20 graf~~

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost.

~~6+6+3~~

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 2x}$, u točki gdje je $x = 0$.

~~15~~

Ukupno:

5

4.
a) DOMENA
 $f(x) = \frac{x^2 + 4}{x^2 - 2}$

$x^2 - 2 \neq 0$
 $x^2 \neq 2 / \sqrt{\quad}$
 $x = \sqrt{2}$
 $D(f) = \{ \sqrt{2} \}$

b) NUL TOČKE

$x = 0$ $A(0, -2)$
 $\frac{0^2 + 4}{0^2 - 2} = \frac{4}{-2} = -2$

$y = 0$
 $f(x) = \frac{x^2 + 4}{x^2 - 2} = 0 / \cdot (x^2 - 2)$

$x^2 + 4 = 0$
 $x^2 = -4 / \sqrt{\quad}$

$x = \sqrt{-4} \Rightarrow$ NEMA NUL TOČKE

c) PERIODIČNOST

Funkcija nije periodična jer u sebi ne sadrži niti jedno od trigonometrijskih funkcija.

d) PARNOST I NEPARNOST

$f(x) = \frac{x^2 + 4}{x^2 - 2} = \frac{(-x)^2 + 4}{(-x)^2 - 2} = \frac{x^2 + 4}{x^2 - 2}$ Funkcija je parna!

e) ASIMPTOTE

1) HORIZONTALNA

$$\lim_{x \rightarrow \infty} \frac{x^2+4}{x^2-2} = \lim_{x \rightarrow \infty} \frac{x^2+4}{x^2-2} \cdot \frac{1}{x^2} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2} + \frac{4}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x^2}} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x^2+4}{x^2-2} = \lim_{x \rightarrow -\infty} \frac{x^2+4}{x^2-2} \cdot \frac{1}{x^2} = \frac{1 - \frac{x^2}{x^2} - \frac{2}{x^2}}{1 - \frac{x^2}{x^2} - \frac{2}{x^2}} = \frac{-1}{-1} = 1$$

$y=1$

2) VERTIKALNA

$$\lim_{x \rightarrow \sqrt{2}} \frac{x^2+4}{x^2-2} = \lim_{x \rightarrow \sqrt{2}} \frac{(\sqrt{2}+4)}{(\sqrt{2})^2-2} = \frac{6}{0} = \infty$$

3) KOSE NEMA JER IMA HORIZONTALNE ASIMPTOTE!

f) EKSTREMI

$$f(x) = \frac{x^2+4}{x^2-2} \quad f'(x) = \frac{(x^2+4)' \cdot (x^2-2) - (x^2-2)' \cdot (x^2+4)}{(x^2-2)^2}$$

$$f'(x) = \frac{2x(x^2-2) - 2x(x^2+4)}{(x^2-2)^2} = f'(x) = \frac{2x^3 - 4x - 2x^3 - 8x}{(x^2-2)^2}$$

$$f'(x) = \frac{-12x}{(x^2-2)^2}$$

	$-\infty$	0	$\sqrt{2}$	$+\infty$
$f'(x) = \frac{-12x}{(x^2-2)^2}$	-	-	-	
$f(x)$	\searrow	\searrow	\searrow	

$$x=0$$

$$f(x) = \frac{0^2+4}{0^2-2} = \frac{4}{-2} = -2$$

$E_{\min}(0|-2)$

g) KONVEKSNOST I KONKAVNOST

$$f'(x) = \frac{-12x}{(x^2-2)^2}$$

$$f''(x) = \frac{(-12x)' \cdot ((x^2-2)^2) - ((x^2-2)^2)' \cdot (-12x)}{((x^2-2)^2)^2}$$

MAJA ŠIKIĆ

Matični broj studenta:

17-2-0101-2011

03.08.2015.

NAUTIKA I
TEHNOLOGIJA
PONORSKOG PROMETA

Matematika 1

g)

$$f(x)'' = \frac{-12(x^2-2)^4 - 4x(x^2-2)^4 \cdot (-12x)}{(x^2-2)^4}$$

$$f(x)'' = \frac{-12x^6 + 192 + 48x^7 + 96x^7}{(x^2-2)^4}$$

$$f(x)'' = \frac{-12x^6 + 192 + 144x^7}{(x^2-2)^4}$$

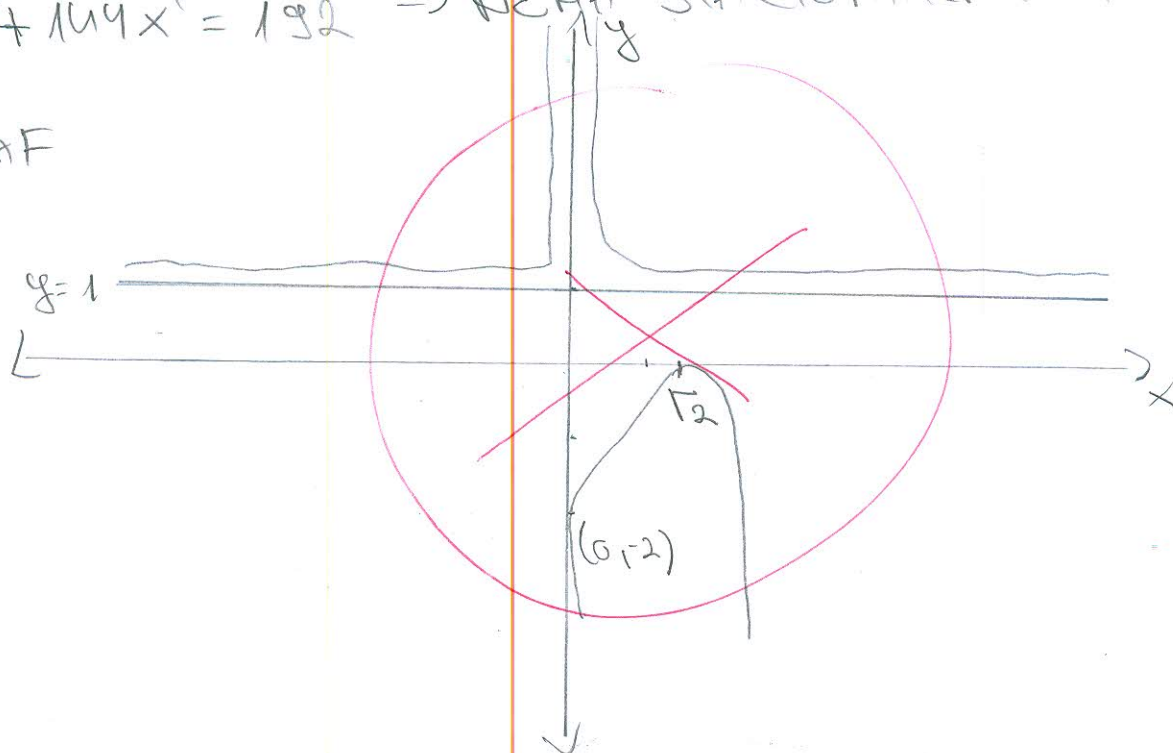
$$\frac{-12x^6 + 192 + 144x^7}{(x^2-2)^4} = 0 \quad / \cdot (x^2-2)^4$$

$$-12x^6 + 192 + 144x^7 = 0$$

$$-12x^6 + 144x^7 = 192 \Rightarrow \text{NEMA STACIONARNIH TOČAKA}$$

	$-\infty$	0	$\sqrt{2}$	$+\infty$
$f(x)''$	-	+	+	
	∩	∪	∩	

h) GRAF



3. Odredi sve asimptote funkcije:

$$f(x) = \frac{x^2 - 1}{4 - x^2}$$

a) DOMENA

$$4 - x^2 \neq 0$$

$$-x^2 \neq -4/\sqrt{\quad}$$

$$x = \sqrt{4}$$

$$D(f) \in \mathbb{R}$$

b) ASIMPTOTE

a) HORIZONTALNA

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{4 - x^2} = \lim_{x \rightarrow \infty} \frac{x^2 - 1}{4 - x^2} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \rightarrow \infty} \frac{\left(\frac{x^2}{x^2}\right) \left(\frac{1}{x^2}\right)^0}{\left(\frac{4}{x^2}\right) \left(\frac{x^2}{x^2}\right)^0} = \frac{1}{1} = 1$$

$$\boxed{y=1}$$

b) KOSE NEMA JER IMA HORIZONTALNA

c) VERTIKALNA

- Nema je jer nema domene!

5

1. $f(x) = x + \sqrt{x^2 - 4x - 2}$

$$D(f) \in \{-0.45, 4.45\}$$

a) DOMENA

$$x^2 - 4x - 2 \geq 0$$

$$x^2 - 4x - 2$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-2)}}{2}$$

$$x_{1,2} = \frac{4 \pm 2\sqrt{6}}{2} \quad \nearrow \quad x_1 = \frac{4 + 4.9}{2} = 4.45$$

$$\searrow \quad x_2 = \frac{4 - 4.9}{2} = -0.45$$

2. NUL TOČKE

$$x = 0$$

$$0 \pm \sqrt{0^2 - 4 \cdot 0 \cdot 2}$$

~~0~~

X

MAJA ŠIKIĆ

Matični broj studenta:

17-2-0101-2011

03.09.2018.

NAUTIKA I
TEHNOLOGIJA
POMORSKOG PROMETA

Matematika 1

5.

$$f(x) = \sqrt{x+2} + \sqrt{4-x}$$

$$f(x)' = (\sqrt{x+2})' + (\sqrt{4-x})'$$

$$f(x)' = \frac{1}{2\sqrt{x+2}} + \frac{1}{2\sqrt{4-x}} = \frac{2\sqrt{x+2}}{4\sqrt{x-2}}$$

$$f(x)'' = \frac{(2\sqrt{x+2})' \cdot (4\sqrt{x-2}) - (4\sqrt{x-2})' \cdot (2\sqrt{x+2})}{(4\sqrt{x-2})^2}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

IME I PREZIME: **FILIP GORŠEK**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **17-2-0369-2014**

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

24

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 4x - 2}$ i skicirati graf. *

~~20 graf~~ 18

2. Riješiti jednačbu: $8 + 4i = |z| + z$. Prikaži rješenje u kompleksnoj ravnini!

~~12+3~~

3. Odrediti sve asimptote funkcije $f(x) = \frac{x^2 - 1}{4 - x^2}$.

~~15~~

4. Odrediti tok funkcije $f(x) = \frac{x^2 + 4}{x^2 - 2}$ i skicirati graf. *

~~20 graf~~ 7

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost.

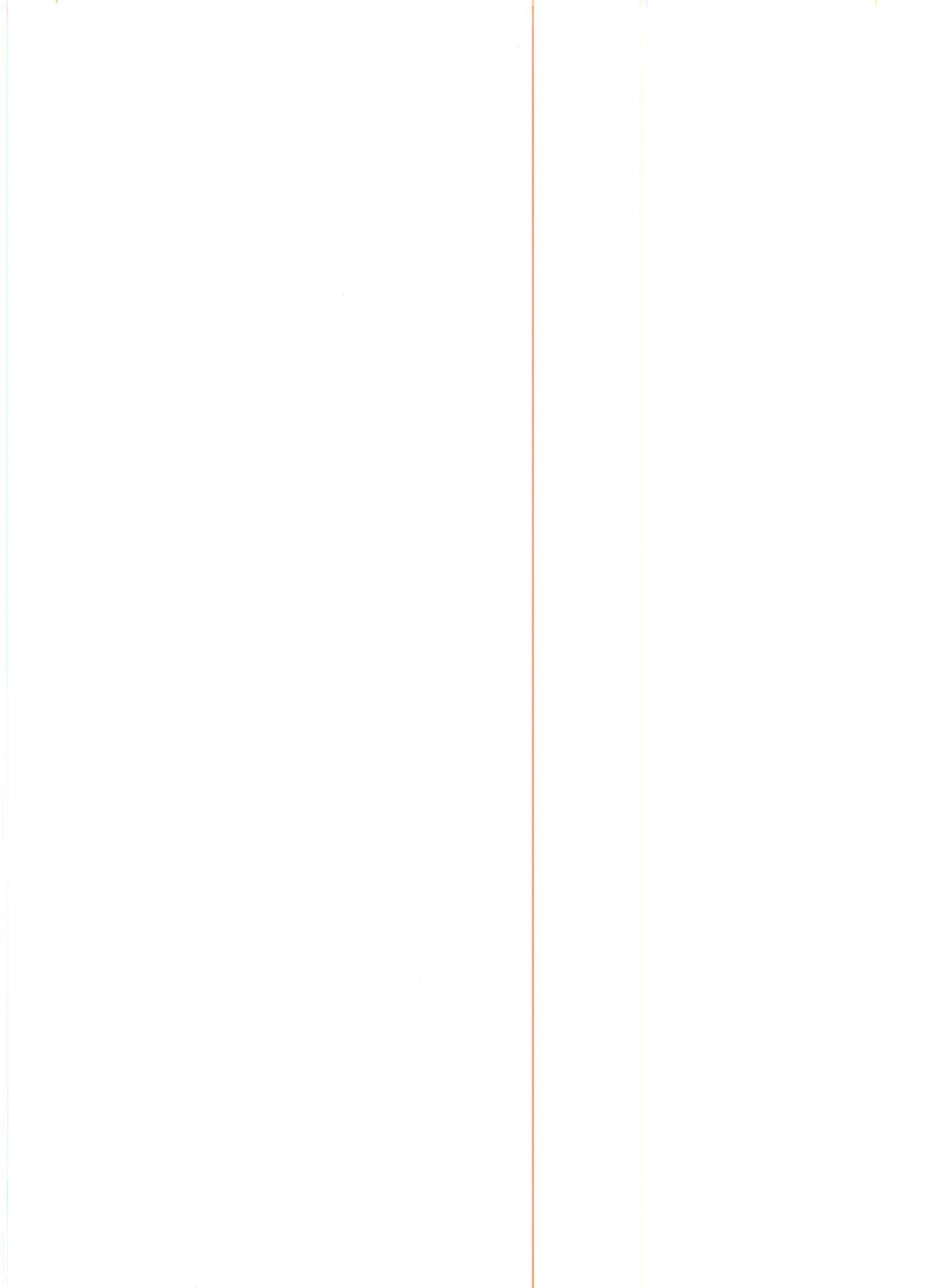
~~6+6+3~~

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-2x}$, u točki gdje je $x = 0$. *

~~15~~

Ukupno:

52



5) $f(x) = \sqrt{x+2} + \sqrt{4-x}$

$x \geq -2$ $-x+4 \geq 0$
 $-x \geq -4$
 $x \geq 4$

FILIP GORSEK

2) $\lim_{x \rightarrow -2} \sqrt{-2+2} + \sqrt{4-2} = 0 + \sqrt{2} = 1.41$ NEMA VA ZA -2

$D(f) = [-2, 4]$

$\lim_{x \rightarrow 4} \sqrt{4+2} + \sqrt{4-4} = \sqrt{6}$ NEMA VA ZA 4

4) $f(x) = 0, \sqrt{x+2} + \sqrt{4-x} = 0/2$

$f(0) = \sqrt{2} + \sqrt{4} = 3.41$ $S(0, 3.41)$

$x+2 + 4-x$ NEMA NT

5) $f'(x) = \frac{1}{2\sqrt{x+2}} + \frac{1}{2\sqrt{4-x}} \cdot (-1) = \frac{1}{2\sqrt{x+2}} - \frac{1}{2\sqrt{4-x}}$

6) $f'(x) = 0, \frac{1}{2\sqrt{x+2}} - \frac{1}{2\sqrt{4-x}} = 0 / \cdot (2\sqrt{x+2})(2\sqrt{4-x})$

$2\sqrt{4-x} - 2\sqrt{x+2} = 0 / 2$

$4(4-x) - 4(x+2) = 0$

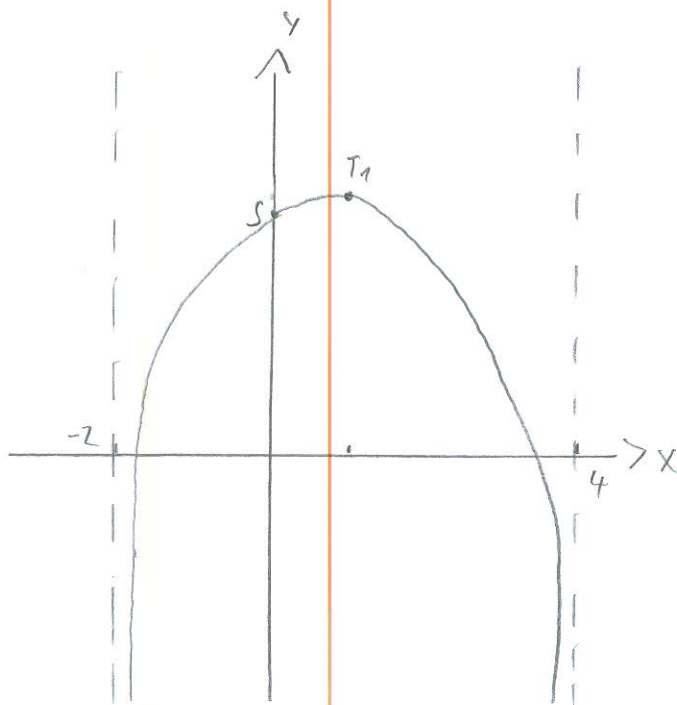
$16 - 4x - 4x - 8 = 0$

$-8x = -8 / : (-8)$

$x = 1$

$f(1) = \sqrt{3} + \sqrt{3} = 3.46$ $T_1(1, 3.46)$

	-2	0	1	2	4
$f'(x)$		+		-	
$f(x)$		↗		↘	



GLOBALNI (LOKALNI) MAKSIMUM
 JE $T_1(1, 3.46)$, MINIMUM NEMA.
 FUNKCIJA JE ODOZGO OMEĐENA.
 ODOZDO FUNKCIJA NIJE OMEĐENA.



4) $f(x) = \frac{x^2+4}{x^2-2}$

$x^2 \neq 2/\sqrt{}$
 $x \neq \pm 1.41$

FLIP GORSEK

$D(f) = \langle -\infty, -1.41 \rangle \cup \langle -1.41, 1.41 \rangle \cup \langle 1.41, +\infty \rangle$

2) $\lim_{x \rightarrow +\infty} \frac{x^2+4}{x^2-2} \stackrel{x^2}{\stackrel{x^2}{\left[\frac{\infty}{\infty} \right]}} = 1$

OHA $y=1$

$\lim_{x \rightarrow -\infty} \frac{x^2+4}{x^2-2} \stackrel{x^2}{\stackrel{x^2}{\left[\frac{\infty}{\infty} \right]}} = 1$

$\lim_{x \rightarrow -1.41} \frac{x^2+4}{x^2-2} = \frac{6}{0} = +\infty$

VA $x = -1.41$

$\lim_{x \rightarrow 1.41} \frac{x^2+4}{x^2-2} = \frac{6}{0} = \infty$

VA $x = 1.41$

4) $f(x) = 0, \quad x^2+4=0$
 ~~$x^2 = -4$ NEMA NI~~

$f(0) = \frac{4}{-2} = -2 \quad S(0, -2)$

5) $f'(x) = \frac{2x \cdot (x^2-2) - (x^2+4) \cdot 2x}{(x^2-2)^2} = \frac{2x^3 - 4x - 2x^3 - 8x}{(x^2-2)^2} = \frac{-12x}{(x^2-2)^2}$

$f''(x) = \frac{-12 \cdot (x^2-2)^2 - (-12x) \cdot 2(x^2-2) \cdot 2x}{(x^2-2)^4} = \frac{(x^2-2) [-12 \cdot (x^2-2) + 48x^2]}{(x^2-2)^4}$

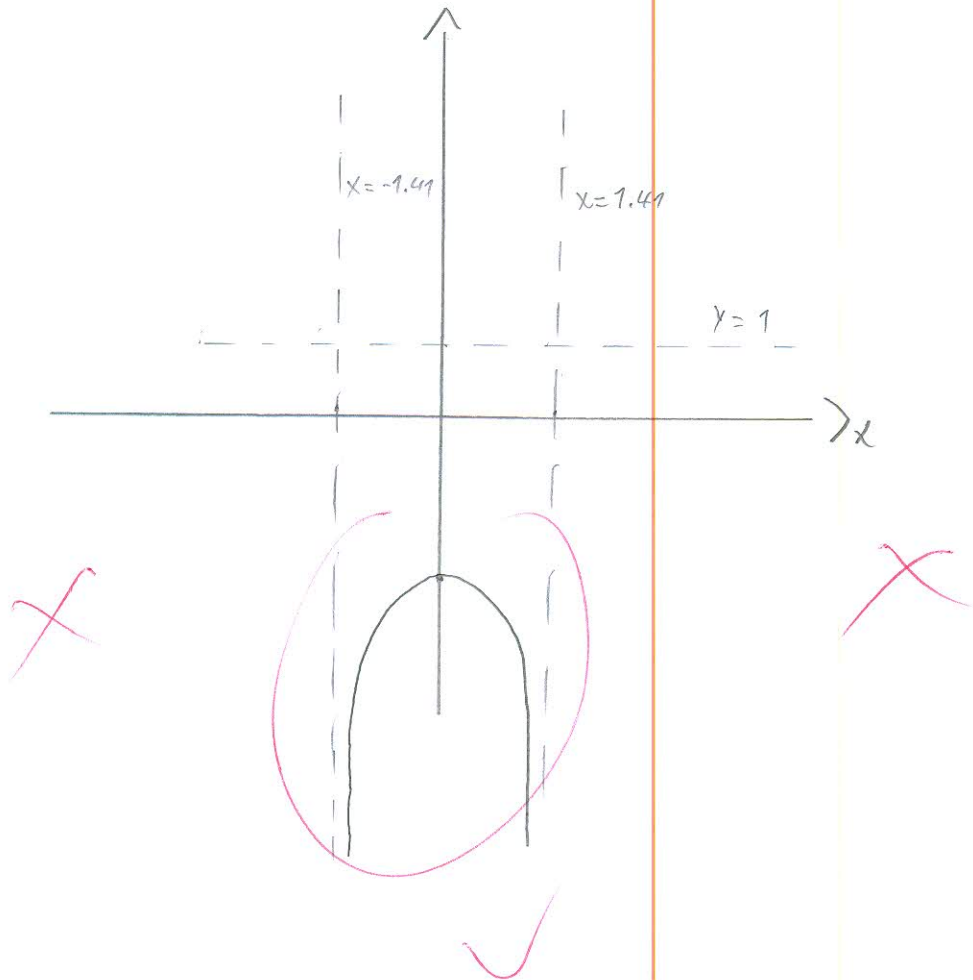
$= \frac{-12x^2 + 24 + 48x^2}{(x^2-2)^3} = \frac{-36x^2 + 24}{(x^2-2)^3}$

6) $f'(x) = 0, \quad -12x = 0 \quad T_1(0, -2)$
 $x = 0$

$f''(x) = 0, \quad -36x^2 + 24 = 0 \quad / : -6$

~~$6x^2 - 4 = 0$~~
 $x_{1/2} = \frac{0 \pm \sqrt{0}}{2}$

	-1	1
	$-\infty$	$+\infty$
$f'(x)$	+	-
$f(x)$	↗	↘



7

1) $f(x) = x + \sqrt{x^2 - 4x - 2}$

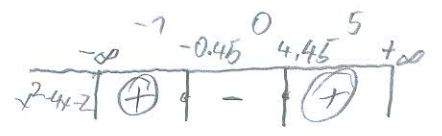
$x^2 - 4x - 2 \geq 0$

$x_{1/2} = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot (-2)}}{2}$

$x_{1/2} = \frac{4 \pm 4.90}{2}$

$x_1 = 4.45$

$x_2 = -0.45$



$D(f) = \langle -\infty; -0.45 \rangle \cup \langle 4.45; +\infty \rangle$

2) HA

$\lim_{x \rightarrow +\infty} x + \sqrt{x^2 - 4x - 2} \cdot \frac{x - \sqrt{x^2 - 4x - 2}}{x - \sqrt{x^2 - 4x - 2}} = \frac{-x^2 + 4x + 2}{x - \sqrt{x^2 - 4x - 2}} \cdot x = \frac{\infty}{\infty} = \frac{4}{0} = +\infty$
 NEMA DHA

$\lim_{x \rightarrow -\infty} x + \sqrt{x^2 - 4x - 2} = \lim_{x \rightarrow +\infty} -x + \sqrt{x^2 + 4x - 2} \cdot \frac{x + \sqrt{x^2 + 4x - 2}}{x + \sqrt{x^2 + 4x - 2}} = \lim_{x \rightarrow +\infty} \frac{-x^2 + x^2 + 4x - 2}{x + \sqrt{x^2 + 4x - 2}} \cdot x = \frac{\infty}{\infty} = \frac{4}{2} = 2$

DKA $\lim_{x \rightarrow +\infty} \frac{x + \sqrt{x^2 - 4x - 2}}{x} = \frac{\infty}{\infty} = \frac{1+1}{1} = 2$ LHA $y=2$

$\lim_{x \rightarrow +\infty} x + \sqrt{x^2 - 4x - 2} - 2x = \lim_{x \rightarrow +\infty} -x + \sqrt{x^2 - 4x - 2} \cdot \frac{x + \sqrt{x^2 - 4x - 2}}{x + \sqrt{x^2 - 4x - 2}} = \lim_{x \rightarrow +\infty} \frac{-x^2 + x^2 - 4x - 2}{x + \sqrt{x^2 - 4x - 2}} \cdot x = \frac{\infty}{\infty} = \frac{-4}{2} = -2$ DKA $y=2x-2$

VA. $\lim_{x \rightarrow -0.45}$ NEMA

$\lim_{x \rightarrow 4.45}$ NEMA

$$4) f(x) = 0 \quad x + \sqrt{x^2 - 4x - 2} = 0 \quad | \cdot 2$$

$$x^2 + x^2 - 4x - 2 = 0$$

$$2x^2 - 4x - 2 = 0$$

$$x_{1,2} = \frac{4 \pm \sqrt{16 - 4 \cdot 2 \cdot (-2)}}{4}$$

$$x_{1,2} = \frac{4 \pm 5.66}{4}$$

$$x_1 = 2.41$$

$$x_2 = -0.42$$

$$N_1(2.41, 0)$$

$$N_2(-0.42, 0)$$

$f(0) = \text{NEMA } 5$

$$5) f'(x) = 1 + \frac{1}{2\sqrt{x^2 - 4x - 2}} \cdot 2x - 4 = 1 + \frac{2x - 4}{2\sqrt{x^2 - 4x - 2}}$$

$$f'(x) = 0, \quad 2\sqrt{x^2 - 4x - 2} + 2x - 4 = 0$$

$$2\sqrt{x^2 - 4x - 2} = -2x + 4 \quad | \cdot 2$$

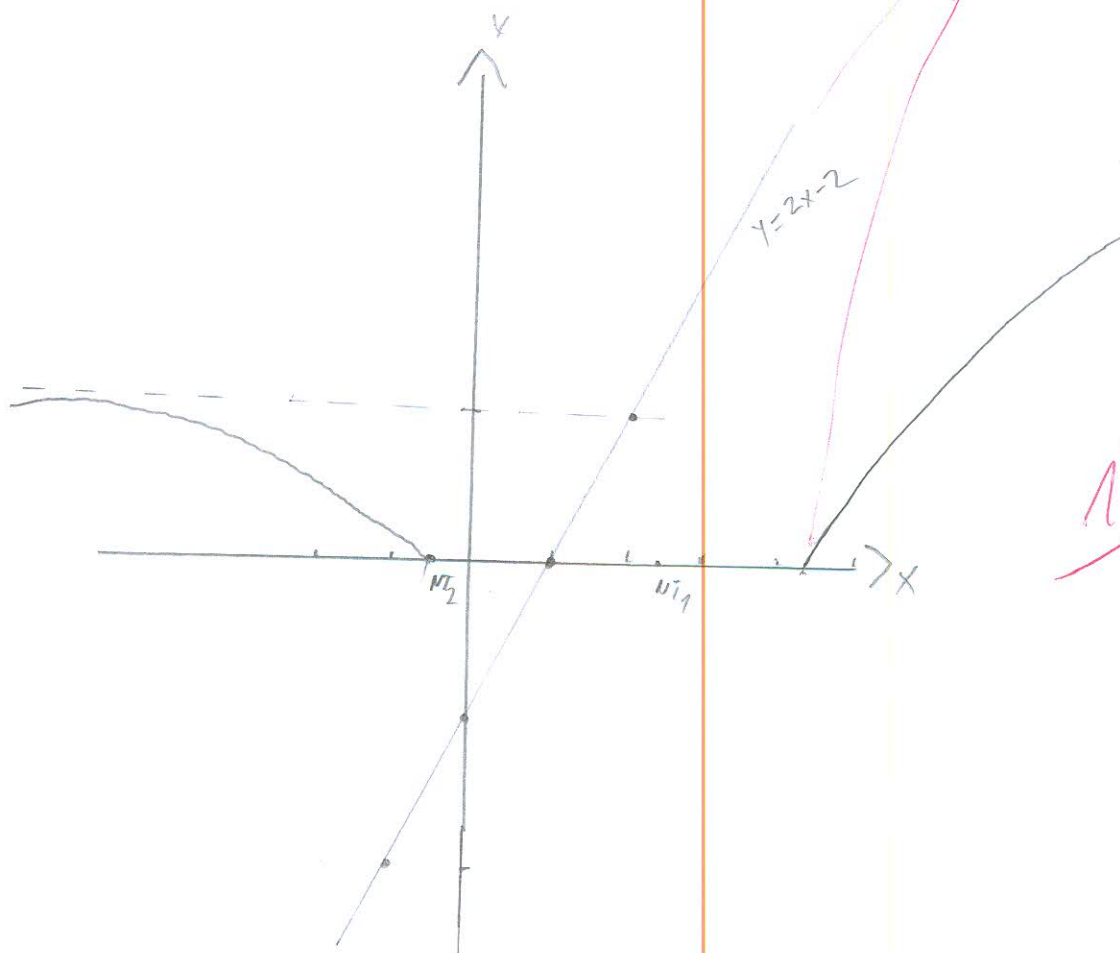
$$4(x^2 - 4x - 2) = (16 - 2 \cdot 2x \cdot 4 + (-2x)^2)$$

$$4x^2 - 16x - 8 = 16 - 16x + 4x^2$$

NEMA STACIONARNIH

DKA $y = 2x - 2$

x	0	2	-1	-2
y	-2	-2	-4	-6



$$(6.) f(x) = e^{x^2-2x}$$

$$x_0 = 0 \quad y_0 = 1 \quad f'(x) = e^{x^2-2x} \cdot (2x-2)$$

$$y - y_0 = f'(x_0) \cdot (x - x_0)$$

$$y - 1 = -2 \cdot (x - 0)$$

$$y = 1 \quad \text{X}$$

$$y = -2x + 1$$

$$(2.) 8 + 4i = |z| + iz$$

$$8 + 4i = \sqrt{x^2 + y^2} + x + yi$$

$$Im: y = 4 \quad \checkmark$$

$$Re: 8 = \sqrt{x^2 + y^2} + 1$$

$$-\sqrt{x^2 + y^2} = -8 + 1$$

$$\sqrt{x^2 + y^2} = 7 \quad |^2$$

$$x^2 + y^2 = 49$$

$$x^2 + 16 = 49$$

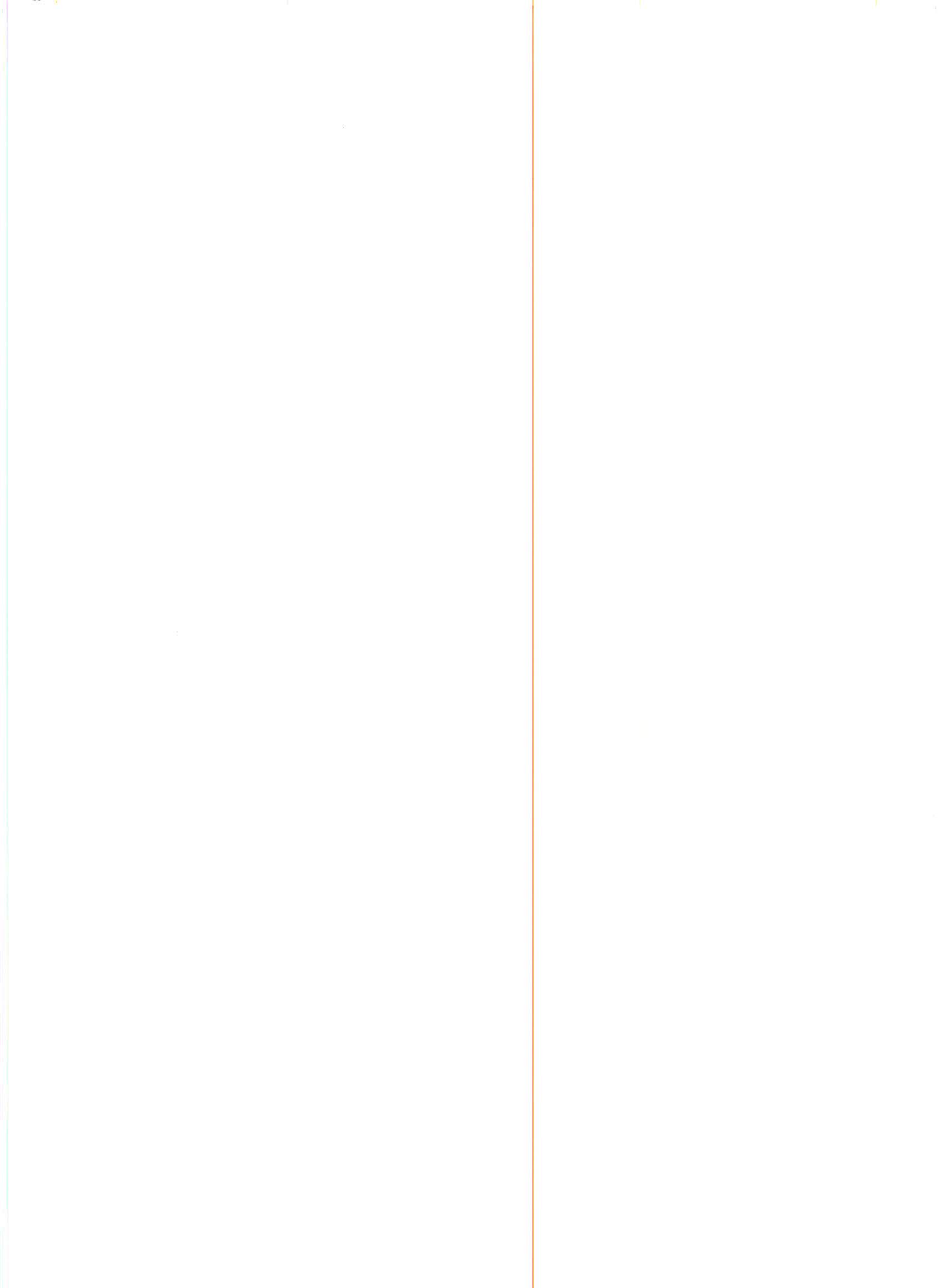
$$x^2 = 49 - 16$$

$$x^2 = 33 \quad | \sqrt{\quad}$$

$$x_1 = 5.74$$

$$x_2 = -5.74$$

$$\underline{12}$$



$$3) f(x) = \frac{x^2-1}{4-x^2}$$

$$1) 4-x^2 \neq 0$$

$$x^2 \neq 4/\sqrt{\quad}$$

$$x \neq \pm 2$$

$$D(f) = \langle -\infty, -2 \rangle \cup \langle -2, 2 \rangle \cup \langle 2, +\infty \rangle$$

$$2) \lim_{x \rightarrow +\infty} \frac{x^2-1}{4-x^2} \stackrel{\cdot x^2}{=} \left[\frac{\infty}{\infty} \right] = -1$$

$$\text{O.H.A. } y = -1$$

$$\lim_{x \rightarrow -\infty} \frac{x^2-1}{4-x^2} = \left[\begin{array}{l} x \rightarrow -x \\ -\infty \rightarrow +x \end{array} \right] = \lim_{x \rightarrow +\infty} \frac{x^2-1}{4-x^2}$$

$$\frac{(-x)^2-1}{4-(-x)^2} = \lim_{x \rightarrow +\infty} \frac{x^2-1}{4-x^2} \stackrel{\cdot x^2}{=} \left[\frac{\infty}{\infty} \right] = -1$$

$$\lim_{x \rightarrow -2_-} \frac{x^2-1}{4-x^2} = \frac{4_+-1}{4-4_+} = \frac{3}{0_+} = +\infty$$

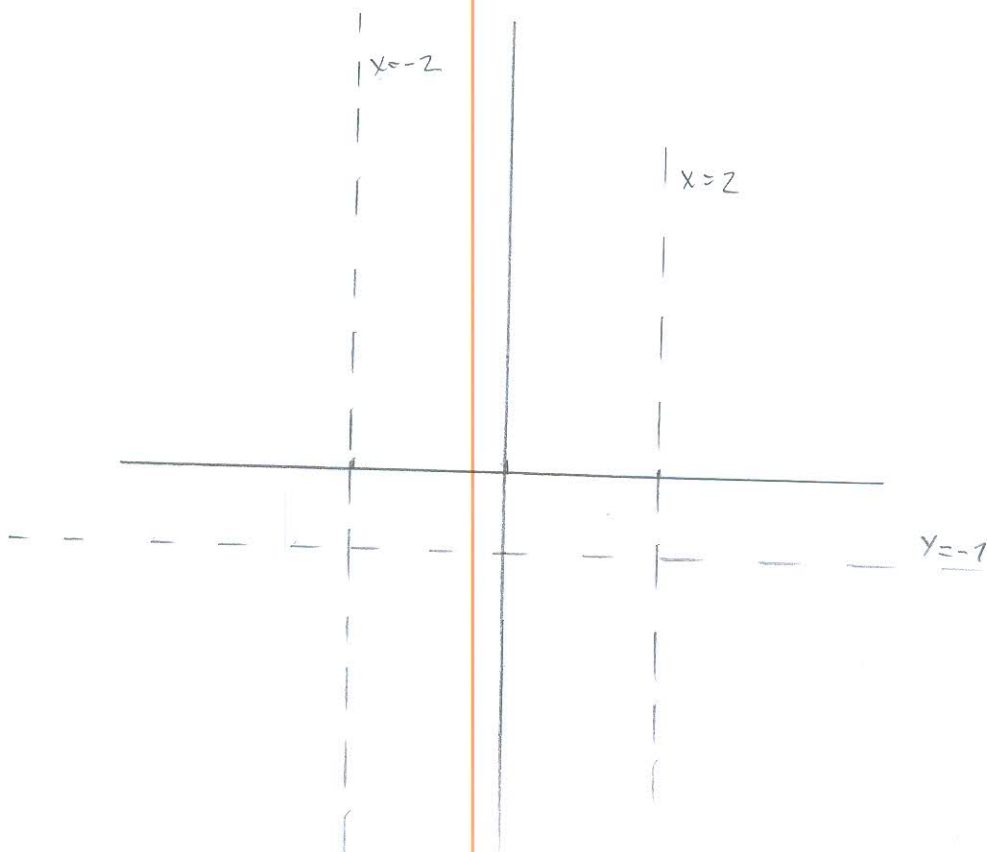
$$\text{VA } x = -2_-$$

$$\lim_{x \rightarrow -2_+} \frac{x^2-1}{4-x^2} = \frac{4_- -1}{4-4_-} = \frac{3}{0_-} = -\infty$$

$$\lim_{x \rightarrow 2_-} \frac{x^2-1}{4-x^2} = \frac{4_- -1}{4-4_-} = \frac{3}{0_-} = -\infty$$

$$\text{VA } x = 2$$

$$\lim_{x \rightarrow 2_+} \frac{x^2-1}{4-x^2} = \frac{4_+ -1}{4-4_+} = \frac{3}{0_+} = +\infty$$



odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

24

IME I PREZIME: **MARKO VUKELIĆ**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): ~~17-2-0293-13~~

17-2-0293-13

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asistenta Kosora

Želim ustmeni kod (zaokružiti):

prof. Uglešića

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 4x - 2}$ i skicirati graf. 20 graf
2. Riješiti jednačinu: $8 + 4i = |z| + z$. Prikaži rješenje u kompleksnoj ravnini! 12+3
3. Odrediti sve asimptote funkcije $f(x) = \frac{x^2 - 1}{4 - x^2}$. 15
4. Odrediti tok funkcije $f(x) = \frac{x^2 + 4}{x^2 - 2}$ i skicirati graf. 20 graf
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-2x}$, u točki gdje je $x = 0$. 15

Ukupno:

15

1.

$$x^2 - 4x - 2 \geq 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1$$

$$b = -4$$

$$c = -2$$

$$4^2 - 4 \cdot 1 \cdot (-2)$$

$$16 - 16 - 2$$

$$-2$$

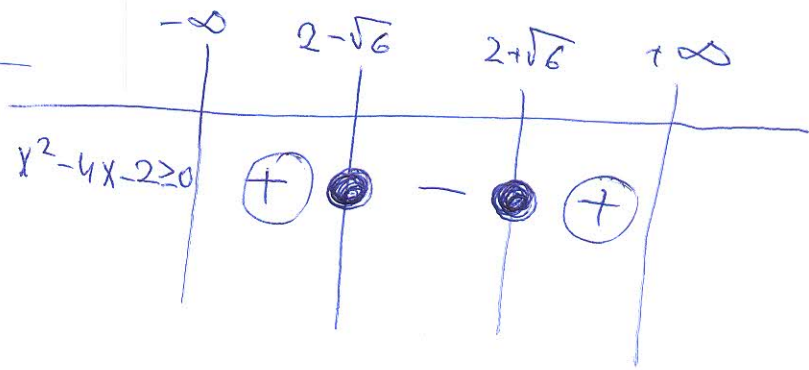
$$x_{1,2} = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot (-2)}}{2}$$

$$x_{1,2} = \frac{4 \pm \sqrt{16 + 8}}{2}$$

$$x_{1,2} = \frac{4 \pm \sqrt{24}}{2}$$

$$x_1 = 2 + \sqrt{6}$$

$$x_2 = 2 - \sqrt{6}$$



$$D_f: \langle -\infty, 2 - \sqrt{6} \rangle \cup \langle 2 + \sqrt{6}, +\infty \rangle$$

VERTIKALNIM ASIMPTOTA

NETAMO SE

SE x_1 i x_2 UKLJUČUJU

TO VIDIMO I IZ DOBROG

D.H.A

lim
 $x \rightarrow \infty$

$$\frac{x + \sqrt{x^2 - 4x - 2}}{x - \sqrt{x^2 - 4x - 2}} = \frac{x - \sqrt{x^2 - 4x - 2}}{x - \sqrt{x^2 - 4x - 2}} = \frac{x^2 - (\sqrt{x^2 - 4x - 2})^2}{x - \sqrt{x^2 - 4x - 2}}$$

$$= \frac{x^2 - (x^2 - 4x + 2)}{x - \sqrt{x^2 - 4x - 2}}$$

$$= \frac{4x+2}{x-\sqrt{x^2-4x-2}} \quad | :x \quad | :x = \frac{4 + \frac{2}{x}}{1 - \sqrt{\frac{1}{x} - \frac{4}{x} - \frac{2}{x^2}}}$$

$$4 \cdot 4 = \frac{16}{16}$$

$$= \frac{4+0}{1-\sqrt{0}} = 4$$

L.

$$\lim_{x \rightarrow -\infty} x + \sqrt{x^2 - 4x - 2} \dots \lim_{x \rightarrow -\infty} \frac{4x+2}{x-\sqrt{x^2-4x-2}} \quad | : -x \quad | : -x$$

$$= \frac{-4 + \frac{2}{x}}{-1 - \sqrt{\frac{1}{x} - \frac{4}{x} + 0}}$$

$$= \frac{-4}{-1}$$

$$= 4$$

GRAF?

$$3. \quad f(x) = \frac{x^2 - 1}{4 - x^2}$$

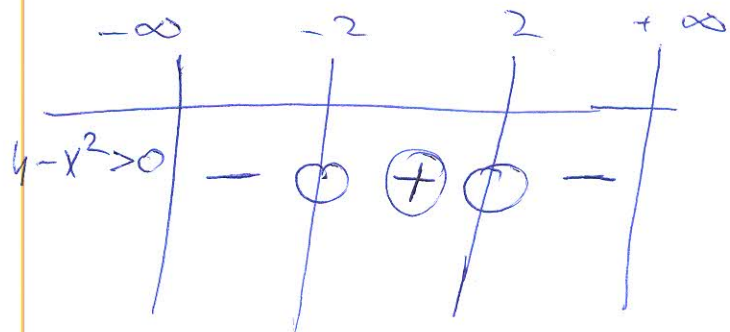
$$4 - x^2 \neq 0$$

$$x^2 = 4 \quad | \sqrt{}$$

$$x_1 = 2$$

$$x_2 = -2$$

$$P_f: [-2, 2]$$



H.A

$$\lim_{x \rightarrow -2} \frac{x^2 - 1}{4 - x^2} = \lim_{x \rightarrow -2} = \frac{3}{0} \quad + \infty$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 1}{4 - x^2} = \lim_{x \rightarrow 2} = \frac{3}{0} \quad + \infty$$

D.V.A

$$\lim_{x \rightarrow +\infty} \frac{x^2 - 1}{4 - x^2} \stackrel{1: x^2}{=} \lim_{x \rightarrow +\infty} \frac{1 - \frac{1}{x^2}}{\frac{4}{x^2} - 1} = \frac{1 - 0}{-1} = -1 \quad \checkmark$$

L. A

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 1}{4 - x^2} \stackrel{1: x^2}{=} \frac{-1 + \frac{1}{x^2}}{-\frac{4}{x^2} + 1} = \frac{-1}{1} = 0 = -1$$

POSTOJE OBA DVIJE

ASIMPTOTE

TO ZAKLJUČUJEMO PO

PREKIDU U DOLJENI

TO JEST POSTANJEM LIMESA

U -2 I 2 ✓

KOSTA

ASIMPTOTA

$y =$

~~lim~~

$$\lim_{x \rightarrow +\infty} k = \frac{k(x)}{c(x)} = \lim_{x \rightarrow +\infty} \left(\frac{x^2 - 1}{4 - x^2} \right) \cdot \frac{x}{x}$$

$$= \frac{x^2 - 1}{4x - x^3} \cdot \frac{1}{x^3} = \frac{\frac{1}{x^0} - \frac{1}{x^0}}{\infty \frac{1}{x^2} - \frac{1}{x^0}}$$

$= \frac{0}{\infty}$ NEODREĐEN
OBLIK

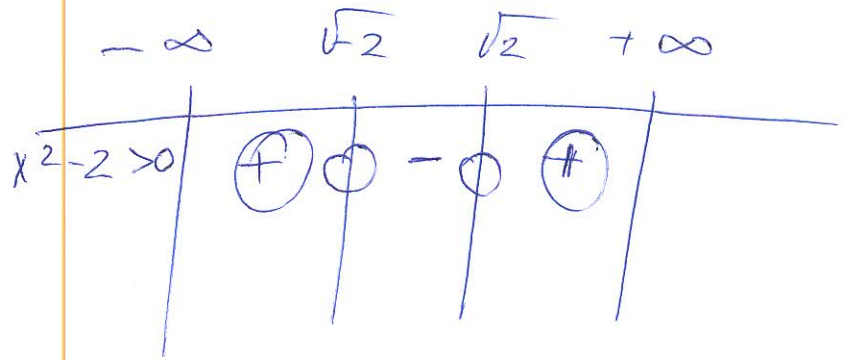
$$4. \quad f(x) = \frac{x^2 + 4}{x^2 - 2}$$

$$x^2 - 2 \neq 0$$

$$x^2 \neq 2/\sqrt{\quad}$$

$$x_1 = \sqrt{2}$$

$$x_2 = -\sqrt{2}$$



$$D_f: \langle -\infty, \sqrt{2} \rangle \cup \langle \sqrt{2}, +\infty \rangle$$

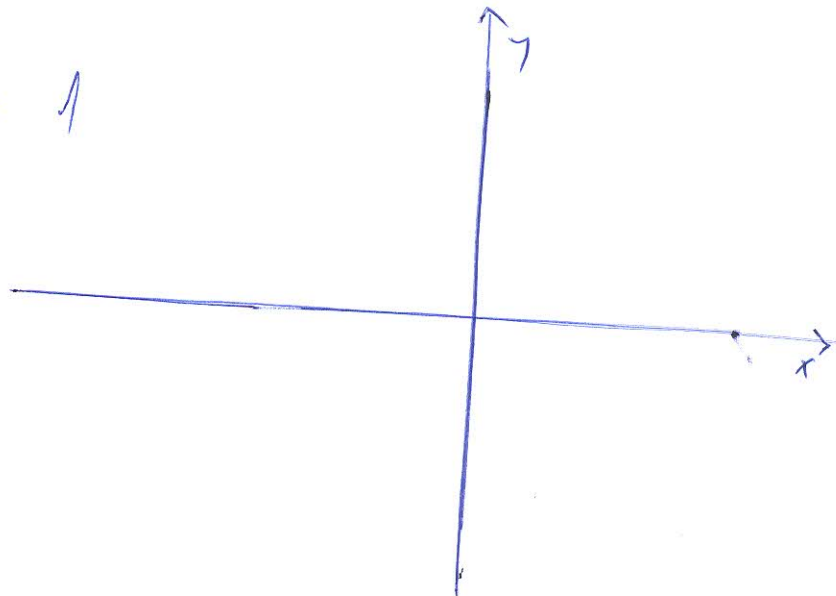
$$\lim_{x \rightarrow \sqrt{2}} \frac{x^2 + 4}{x^2 - 2} = \lim_{x \rightarrow \sqrt{2}} \frac{\cancel{x^2 + 4}}{(\sqrt{2})^2 - 2} = \lim_{x \rightarrow \sqrt{2}} \frac{4 + 4}{4 - 2} = \frac{16}{2} = 8 \quad \text{NETA}$$

$$\lim_{x \rightarrow \sqrt{2}} \frac{x^2 + 4}{x^2 - 2} = \lim_{x \rightarrow \sqrt{2}} \frac{16}{2} = 8 \quad \text{NETA}$$

$$\lim_{x \rightarrow +\infty} \frac{x^2 + 4}{x^2 - 2} = \lim_{x \rightarrow +\infty} \frac{1 + \frac{4}{x^2}}{1 - \frac{2}{x^2}} = \frac{1}{1} = 1$$

GRAF?

$$\lim_{x \rightarrow -\infty} \frac{x^2 + 4}{x^2 - 2} = \frac{-1 + 0}{-1 + 0} = 1$$



$$f'(x) = \frac{x^2 + 4}{x^2 - 2}$$

$$= \frac{(x^2 + 4)' \cdot (x^2 - 2) - (x^2 + 4) \cdot (x^2 - 2)'}{(x^2 - 2)^2}$$

$$= \frac{(2x + 0) \cdot (x^2 - 2) - (x^2 + 4) \cdot 2x - 0}{(x^2 - 2)^2}$$

$$= \frac{2x \cdot (x^2 - 2) - 2x(x^2 + 4)}{(x^2 - 2)^2}$$

$$= \cancel{2x^3} - 4x - \cancel{2x^3} - 8x$$

$$= \frac{-4x - 8x}{(x^2 - 2)^2}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

24

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **TONI BABIĆ**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **17 - 1 - 0272 - 2014**

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 4x - 2}$ i skicirati graf. ~~20 graf~~
2. Riješiti jednačinu: $8 + 4i = |z| + z$. *Prikaži rješenje u kompleksnoj ravnini!* ~~12+3~~
3. Odrediti sve asimptote funkcije $f(x) = \frac{x^2 - 1}{4 - x^2}$. 15
4. Odrediti tok funkcije $f(x) = \frac{x^2 + 4}{x^2 - 2}$ i skicirati graf. ~~20 graf~~
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. ~~6+6+3~~
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-2x}$, u točki gdje je $x = 0$. 15

Ukupno:

~~0~~

1 $f(x) = x + \sqrt{x^2 - 4x - 2}$

① $x^2 - 4x - 2 \geq 0$

$x^2 - 4x - 2 = 0$

$a = 1$

$b = -4$

$c = -2$

$x_{1,2} = \frac{4 \pm \sqrt{16 + 8}}{2} = \frac{4 \pm \sqrt{24}}{2}$

$x_1 = 4.45$

$x_2 = -0.45$

$D(f) = \langle -\infty, -0.45 \rangle \cup [4.45, +\infty \rangle$
 GP

② $x + \sqrt{x^2 - 4x - 2} = 0 \quad |^2$

$x^2 + x^2 - 4x - 2 = 0$

$2x^2 - 4x - 2 = 0$

$x_{1,2} = \frac{4 \pm \sqrt{16 + 16}}{4} = \frac{4 \pm \sqrt{32}}{4}$

$x_1 = 2.41$

$x_2 = -0.41$

$N_1(2.41, 0)$

$N_2(-0.41, 0)$

③ $\lim_{x \rightarrow 4.45} x + \sqrt{x^2 - 4x - 2} = \frac{4}{5}$

$\lim_{x \rightarrow -0.45} x + \sqrt{x^2 - 4x - 2} = 12.89$

$\lim_{x \rightarrow +\infty} x + \sqrt{x^2 - 4x - 2}$

$\lim_{x \rightarrow -\infty} x + \sqrt{x^2 - 4x - 2}$

MEMA V.A.

$$4) f(x) = x + \sqrt{x^2 - 4x - 2}$$

$$f'(x) = 1 + \frac{1}{2\sqrt{x^2 - 4x - 2}} \rightarrow$$

~~$$1 + \frac{2x - 4}{\sqrt{x^2 - 4x - 2}} = 0$$~~

~~$$\frac{2x - 4 + \sqrt{x^2 - 4x - 2}}{\sqrt{x^2 - 4x - 2}} = 0$$~~

~~$$\sqrt{x^2 - 4x - 2}$$~~

~~$$2x - 4 + \sqrt{x^2 - 4x - 2} = 0 \quad |^2$$~~

~~$$4x^2 - 16 + x^2 - 4x - 2 = 0$$~~



GRAF?

$$1 + \frac{1}{2\sqrt{x^2 - 4x - 2}} = 0$$

$$\frac{2\sqrt{x^2 - 4x - 2} + 1}{2\sqrt{x^2 - 4x - 2}} = 0$$

$$2\sqrt{x^2 - 4x - 2} + 1 = 0 \quad |^2$$

$$4(x^2 - 4x - 2) + 1 = 0$$

$$4x^2 - 16x - 8 + 1 = 0$$

$$4x^2 - 16x - 7 = 0$$

$$x_{1,2} = \frac{16 \pm \sqrt{16^2 - 4 \cdot 4 \cdot (-7)}}{8}$$

$$x_{1,2} = \frac{16 \pm \sqrt{218}}{8}$$

$$x_1 = \frac{16 + 14.58}{8} = 4.39$$

$$x_2 = \frac{16 - 14.58}{8} = -3.18$$



$$2 \quad 8+4i = |z| + 2$$

$$8+4i = \sqrt{x^2+y^2} + 2$$

$$2 = 8+4i - \sqrt{x^2+y^2}$$



$$3. \quad f(x) = \frac{x^2-1}{4-x^2}$$

$$4-x^2 \neq 0$$

$$x^2 = 4$$

$$x_1 = -2$$

$$x_2 = 2$$

$$D_f = \mathbb{R} \setminus \{-2, 2\}$$

$$\lim_{x \rightarrow -2} \frac{x^2-1}{4-x^2} = \frac{4-1}{4-4} = \frac{3}{0} = \infty$$

$$\lim_{x \rightarrow 2} \frac{x^2-1}{4-x^2} = \frac{4-1}{4-4} = \frac{3}{0} = \infty$$

$$\lim_{x \rightarrow \infty} \frac{x^2-1}{4-x^2} = \infty$$

4. $f(x) = \frac{x^2+4}{x^2-2}$

① $x^2 - 2 \neq 0$

$x^2 = 2$

$x_1 \neq \sqrt{2}$

$D(f) = \mathbb{R} \setminus \{\sqrt{2}\}$

② $x^2 + 4 = 0$

$x^2 = -4$

$x_1 = 2$

$x_2 = -2$

$N_1(2, 0)$

$N_2(-2, 0)$

③ $\lim_{x \rightarrow \sqrt{2}} \frac{x^2+4}{x^2-2} = \frac{6}{0} = \infty$

④ $f'(x) = \frac{(x^2+4)' \cdot (x^2-2) - (x^2+4) \cdot (x^2-2)'}{(x^2-2)^2} = \frac{2x(x^2-2) - 2x(x^2+4)}{(x^2-2)^2} =$
 $= \frac{2x[(x^2-2) - (x^2+4)]}{(x^2-2)^2}$

$2x[(x^2-2) - (x^2+4)] = 0$

$2x \neq 0$

$x^2 - 2 - x^2 - 4 = 0$

$-6 = 0$

$0 = 6$

NEMA RJEŠENJA!

GRAT?

$$5 \quad f(x) = \sqrt{x+2} + \sqrt{4-x}$$

$$\textcircled{1} \quad x+2 \geq 0 \quad x \geq -2$$

$$\textcircled{2} \quad 4-x \geq 0 \quad x \leq 4$$

$$f'(x) = \frac{1}{2\sqrt{x+2}} + \frac{1}{2\sqrt{4-x}}$$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

24

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: ANTE INKOVIC

VRIJEME POČETKA: 17:00

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0244-2014

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 4x - 2}$ i skicirati graf. 20 graf
2. Riješiti jednačbu: $8 + 4i = |z| + z$. Prikaži rješenje u kompleksnoj ravnini! 12+3
3. Odrediti sve asimptote funkcije $f(x) = \frac{x^2 - 1}{4 - x^2}$. 15 ~~5~~
4. Odrediti tok funkcije $f(x) = \frac{x^2 + 4}{x^2 - 2}$ i skicirati graf. 20 graf ~~20~~
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x-2x}$, u točki gdje je $x = 0$. 15

Ukupno:

5

3.

$$f(x) = \frac{x^2 - 1}{4 - x^2}$$

$$4 - x^2 \neq 0$$

$$-x^2 \neq -4 \quad | \cdot (-1)$$

$$x^2 \neq 4$$

$$x \neq \pm 2$$

$$D = \mathbb{R} \setminus \{-2, 2\}$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

H.A

$$\lim_{x \rightarrow \pm \infty} \frac{x^2 - 1}{4 - x^2} : x^2 = \lim_{x \rightarrow \pm \infty} \frac{1 - \frac{1}{x^2}}{\frac{4}{x^2} - 1} = -1$$

K.A.

$$a = \frac{f(x)}{x} = \frac{\frac{x^2 - 1}{4 - x^2}}{x} = \frac{x^2 - 1}{4x - x^3}$$

~~5~~

4.

$$f(x) = \frac{x^2 + 4}{x^2 - 2}$$

$$x^2 - 2 \neq 0$$

$$x^2 \neq 2$$

$$x \neq \sqrt{2}$$

$$D = \mathbb{R} \setminus \{\sqrt{2}\}$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

НЕМА НИЛТОЇАКА

$$VA \dots = \sqrt{2}$$

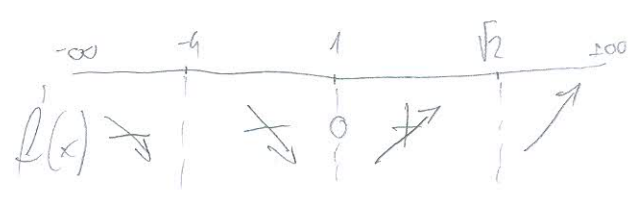
HA...

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 + 4}{x^2 - 2} = \frac{1 + \frac{4}{x^2}}{1 - \frac{2}{x^2}} = 1$$



ГРАФ ?

$$f'(x) = \frac{2x \cdot (x^2 - 2) - (x^2 + 4) \cdot 2x}{(x^2 - 2)^2} = \frac{\cancel{2x^3} - 4x - \cancel{2x^3} + 8x}{(x^2 - 2)^2} = \frac{4x}{(x^2 - 2)^2}$$



odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

24

IME I PREZIME: ĐAVID KOVAČEVIĆ

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0122-2012

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 4x - 2}$ i skicirati graf. 20 graf
2. Riješiti jednačbu: $8 + 4i = |z| + z$. Prikaži rješenje u kompleksnoj ravnini! 12+3
3. Odrediti sve asimptote funkcije $f(x) = \frac{x^2 - 1}{4 - x^2}$. 15
4. Odrediti tok funkcije $f(x) = \frac{x^2 + 4}{x^2 - 2}$ i skicirati graf. 20 graf
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 2x}$, u točki gdje je $x = 0$. 15

Ukupno:

① $f(x) = x + \sqrt{x^2 - 4x - 2}$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot (-2)}}{2}$$

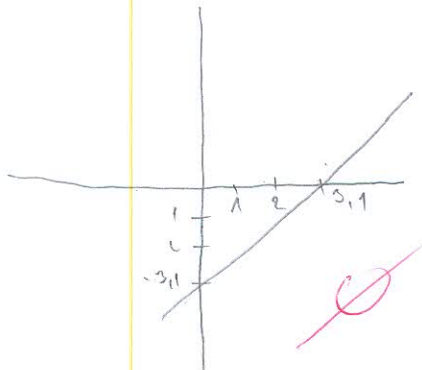
$$x_{1/2} = \frac{-4 \pm \sqrt{1 + 8}}{2}$$

$$x_{1/2} = \frac{-4 \pm 3}{2}$$

$$x_1 = -3,1$$

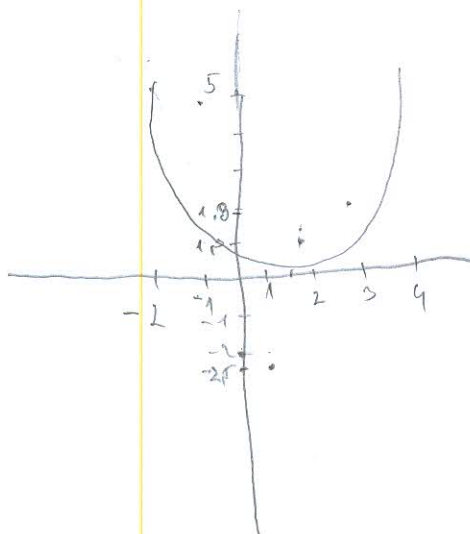
$$x_2 = -1,1$$

$$x_2 = -3,1$$



④

$$f(x) = \frac{x^2 + 4}{x^2 - 2}$$



odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

24

IME I PREZIME: JOSIP MATEŠIĆ

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0269075368

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 4x - 2}$ i skicirati graf. 20 graf
2. Riješiti jednačbu: $8 + 4i = |z| + z$. Prikaži rješenje u kompleksnoj ravnini! 12+3
3. Odrediti sve asimptote funkcije $f(x) = \frac{x^2 - 1}{4 - x^2}$. 15
4. Odrediti tok funkcije $f(x) = \frac{x^2 + 4}{x^2 - 2}$ i skicirati graf. 20 graf
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x + 2} + \sqrt{4 - x}$. Posebno komentirati (ne)ograničenost. 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 2x}$, u točki gdje je $x = 0$. 15

Ukupno:

~~6~~

3.

$$f(x) = \frac{x^2 - 1}{4 - x^2}$$

V.A.

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{4 - x^2} = \lim_{x \rightarrow 1} \frac{1^2 - 1}{4 - 1^2} = \lim_{x \rightarrow 1} \frac{0}{3} = \lim_{x \rightarrow 1} 0$$

$$x = 0$$

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{4 - x^2} = \lim_{x \rightarrow -1} \frac{-1^2 - 1}{4 - (-1)^2} = \lim_{x \rightarrow -1} \frac{0}{3} = \lim_{x \rightarrow -1} 0$$

$$H.A. \lim_{x \rightarrow \infty} \frac{x^2 - 1}{4 - x^2} = \lim_{x \rightarrow \infty} \frac{x^2 - 1 : x^2}{4 - x^2 : x^2} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{4}{x^2} - \frac{x^2}{x^2}} = \lim_{x \rightarrow \infty} \frac{0}{-1} = \lim_{x \rightarrow \infty} 0$$

$$y = 0$$

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

24

IME I PREZIME: DINO NIKOLOVSKI

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0375-2 014

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 4x - 2}$ i skicirati graf. 20 graf
2. Riješiti jednačbu: $8 + 4i = |z| + z$. Prikaži rješenje u kompleksnoj ravnini! 12+3
3. Odrediti sve asimptote funkcije $f(x) = \frac{x^2 - 1}{4 - x^2}$. 15
4. Odrediti tok funkcije $f(x) = \frac{x^2 + 4}{x^2 - 2}$ i skicirati graf. 20 graf
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 2x}$, u točki gdje je $x = 0$. 15

TAYLOROV RED?

$f(x_0) = 1$

Ukupno:

4. TOK FUNKCIJE $f(x) = \frac{x^2 + 4}{x^2 - 2}$

3) DERIVACIJA

1) DOMENA $x^2 - 2 \neq 0$

$x^2 + 2 \neq 0$

$D_f \mathbb{R} \setminus \{1, -1\}$

NULTOČKE

$x^2 + 4 = 0$

$x^2 = -4 \quad \sqrt{\quad}$

$x = \text{NEMA}$

$x_{1/2} = \frac{0 \pm \sqrt{e^2 - 4ac}}{2} = \frac{0 \pm \sqrt{-16}}{2}$

VERTIKALNE ASIMP. - NEMA

$f'(x) = \left(\frac{f}{g}\right)' = \frac{(x^2+4)' \cdot (x^2-2) - (x^2+4)(x^2-2)'}{(x^2-2)^2}$

$= \frac{(2x+0)(x^2-2) - (x^2+4)(2x-0)}{(x^2-2)^2} = \frac{2x^3 - 4x - 2x^3 + 8x}{(x^2-2)^2} = \frac{4x}{(x^2-2)^2}$

STAC. TOČKE

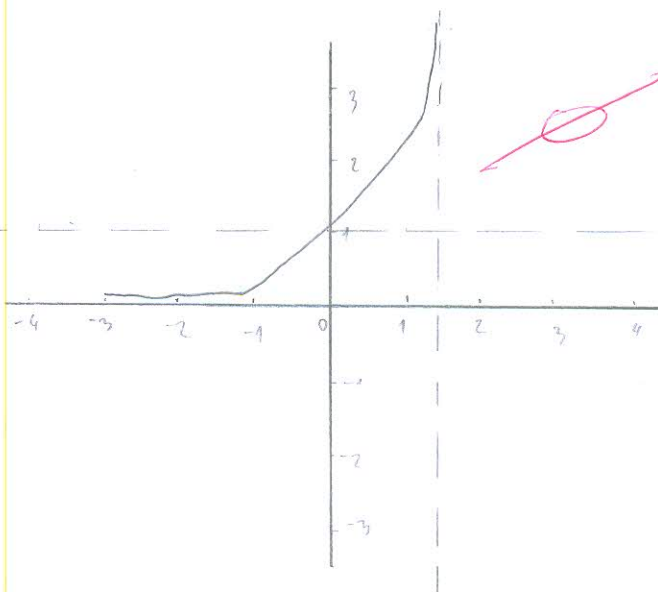
$f'(x)$	+	+	+
$f(x)$	↗	↗	↗

$f(x)$ RASTE $\leftarrow \infty, +\infty \right$

HORIZONTALNE

$\lim_{x \rightarrow \pm\infty} \frac{x^2+4}{x^2-2} = \frac{\infty}{\infty} = \frac{x^2+4 \cdot 1/x^2}{x^2-2 \cdot 1/x^2} = \frac{\frac{x^2}{x^2} + \frac{4}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x^2}} = \frac{1 + \frac{4}{x^2}}{1 - \frac{2}{x^2}} \rightarrow \frac{1}{1} = 1$

TREBA DOBITI
REŠ



1. $f(x) = x + \sqrt{x^2 - 4x - 2}$

DOMENA

$$x^2 - 4x - 2 \geq 0$$

$$a=1 \quad b=-4 \quad c=-2$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot (-2)}}{2} = \frac{4 \pm 4,9}{2}$$

$$x_1 = 4,45$$

$$x_2 = -0,45$$

$$= x + \sqrt{x^2 - 4x - 2} \cdot \frac{x - \sqrt{x^2 - 4x - 2}}{x - \sqrt{x^2 - 4x - 2}} = \frac{x^2 - (x^2 - 4x - 2)}{x - \sqrt{x^2 - 4x - 2}} = \frac{x^2 - x^2 + 4x + 2}{x - \sqrt{x^2 - 4x - 2}} = \frac{4x + 2}{x - \sqrt{x^2 - 4x - 2}}$$

$$\cdot \frac{1}{x} \cdot \frac{4x + 2}{x} = \frac{4x + 2}{x^2 - x^2 + 4x + 2} = \frac{4x + 2}{4x + 2} = 1$$

3) Asimptote $f(x) = \frac{x^2 - 1}{4 - x^2}$

VERTIKALNA ASIMPTOTA

DOMENA $4 - x^2 \neq 0$

NULOVI $x^2 - 1 = 0$

$$\lim_{x \rightarrow 1^+} \frac{x^2 - 1}{4 - x^2} = \frac{0}{0^-} = \text{NEMA}$$

$x = 1$

VERTIKALNA

$x = 2$

$$\lim_{x \rightarrow 2^-} \frac{x^2 - 1}{4 - x^2} = \frac{0}{0^-} = \text{ASIMPTOTA}$$

HORIZONTALNE ASIMPTOTE

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 - 1}{4 - x^2} = \frac{x^2 - 1 / : x^2}{4 - x^2 / : x^2} = \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{4}{x^2} - \frac{x^2}{x^2}} = \frac{1 - 0}{-1} = -1$$

KOSE ASIMPTOTE

DESNA $\lim_{x \rightarrow +\infty} \frac{x^2 - 1}{4 - x^2} = \frac{x^2 - 1 / : x^2}{4 - x^2 / : x^2} = \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{4}{x^2} - \frac{x^2}{x^2}} = \frac{1 - 0}{-1} = -1$

$\lim_{x \rightarrow +\infty} f(x) - ax = \frac{x^2 - 1}{4 - x^2} + x = \frac{(x^2 - 1) + x(4 - x^2)}{4 - x^2} = \frac{x^2 - 1 + 4x - x^3}{4 - x^2} = \frac{x^2 - 1 + 4x - x^3 / : x^3}{4 - x^2 / : x^3} = \frac{1 - \frac{1}{x^3} + \frac{4}{x^2} - x}{\frac{4}{x^3} - x} = \frac{1 - 0 + 0 - \infty}{-\infty} = 0$

LIJEVA $\lim_{x \rightarrow +\infty} \frac{x^2 - 1}{4 - x^2} = \frac{-x^2 - 1 / : x^2}{4 + x^2 / : x^2} = \frac{-1 - \frac{1}{x^2}}{\frac{4}{x^2} + \frac{x^2}{x^2}} = \frac{-1 - 0}{1} = -1$

$\lim_{x \rightarrow +\infty} f(x) - bx = \frac{x^2 - 1}{4 - x^2} + x = \frac{(x^2 - 1) + x(4 - x^2)}{4 - x^2} = \frac{x^2 - 1 + 4x - x^3}{4 - x^2} = \frac{x^2 - 1 + 4x - x^3 / : x^3}{4 - x^2 / : x^3} = \frac{1 - \frac{1}{x^3} + \frac{4}{x^2} - x}{\frac{4}{x^3} - x} = \frac{1 - 0 + 0 - \infty}{-\infty} = 0$

$x = -x$

$y = x$

$y = ax + b$
 $b = -x$

$$2) \quad 8+4i = |z| + z \quad |z| = x+yi$$

$$|z| + z = 8+4i$$

$$x+yi + z = 8+4i$$

$$x = 8$$

$$yi = 4i$$

$$|z| = \sqrt[n]{r} \cdot \left(\cos \frac{\varphi + k \cdot 2\pi}{n} + i \sin \frac{\varphi + k \cdot 2\pi}{n} \right)$$

$$e^{x^2} = e^{x^2}$$

$$6) \quad x=0$$

$$f(x) = e^{x^2-2x}$$

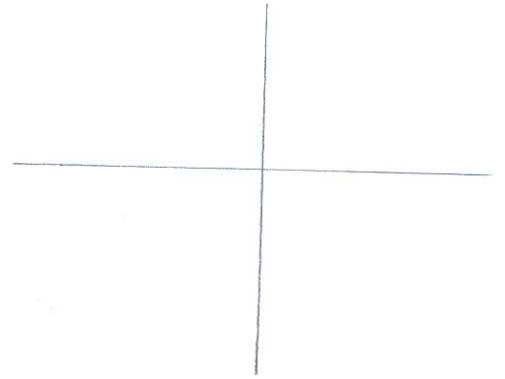
$$f(x_0) = 1$$

$$f'(x) = -2x e^{x^2-2x}$$

$$5) \quad f(x) = \sqrt{x+2} + \sqrt{4-x}$$

$$x_1 = -2$$

$$x_2 = 4$$



odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

24

IME I PREZIME: **ANTONELA KAČAN**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0391-2014

0034058493

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 4x - 2}$ i skicirati graf.

20 graf

2. Riješiti jednačbu: $8 + 4i = |z| + z$. Prikaži rješenje u kompleksnoj ravnini!

12+3

3. Odrediti sve asimptote funkcije $f(x) = \frac{x^2 - 1}{4 - x^2}$.

15

4. Odrediti tok funkcije $f(x) = \frac{x^2 + 4}{x^2 - 2}$ i skicirati graf.

20 graf

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost.

6+6+3

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 2x}$, u točki gdje je $x = 0$.

15

Ukupno:

⑥

$$f(x) = e^{x^2 - 2x}$$

$$x = 0$$

$$T(0, 1)$$

$$y = e^{x^2 - 2x} = e^{0^2 - 2 \cdot 0} = 1$$

$$y - y_1 = f(x) (x - x_0)$$

$$f(x) = e^{x^2 - 2x} \cdot (2x - 2)$$

$$f'(x_0) = f'(0) = e^{0^2 - 2 \cdot 0} \cdot (2 \cdot 0 - 2) = -0,271 \quad \times$$

$$y - 1 = -0,271 (x - 0)$$

$$y = 1 - 0,271x + 0$$

$$y = -0,271x + 1$$

④

$\sqrt{2} \approx 1.414$

⑤

$f(x) = 0$

$-12x = 0$

$x = 12$

	$-\infty$	$\sqrt{2}$	12	$+\infty$
		-2	5	15
$f(x)$	+	-	-	
		↘	↗	

$(1.414, -9.24)$



⑥

$f(x) = 0$

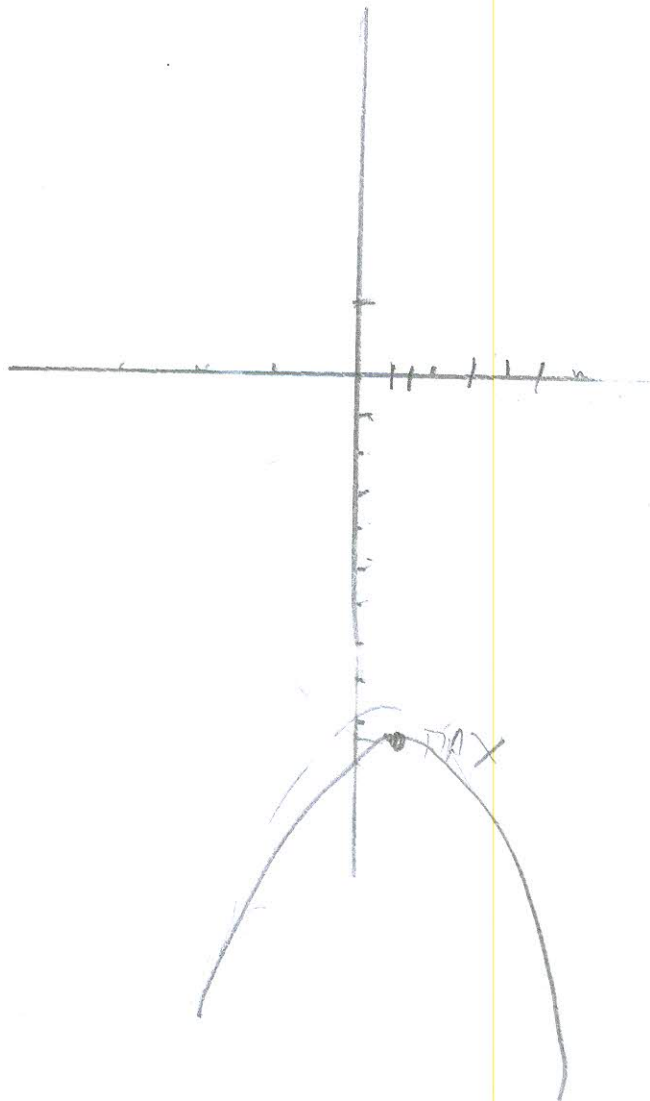
$47x^2 + 2 = 0$

$47x^2 = -2/47$

$x^2 = -\frac{2}{47} \sqrt{\quad}$

= no real

	$-\infty$	$+\infty$
		-2.2
$f(x)$	+	+
		↘



①

$$f(x) = x + \sqrt{x^2 - 4x - 2}$$

$$x^2 - 4x - 2 \geq 0$$

$$x_1 = 4,449$$

$$x_2 = -0,449$$

	$x < -0,449$	$-0,449 < x < 4,449$	$x > 4,449$	
$x^2 - 4x - 2$	+	-	+	

$$Df(x) = (-\infty, -0,449] \cup [4,449, \infty)$$

$$(4,449, 4,449)$$

$$(-0,449, -0,449)$$

V.A. -nema

H.A.

$$y = \lim_{x \rightarrow \infty} x + \sqrt{x^2 - 4x - 2} \cdot \frac{x - \sqrt{x^2 - 4x - 2}}{x - \sqrt{x^2 - 4x - 2}} =$$

$$= \frac{x^2 - x^2 - 4x - 2}{x - \sqrt{x^2 - 4x - 2}} \cdot \frac{1}{x} = \frac{-4}{1} = -4$$

$$y = \lim_{x \rightarrow -\infty} \frac{+4x - 2}{-x - \sqrt{x^2 - 4x - 2}} \cdot \frac{1}{x} = \frac{4}{-1} = -4$$

$$y = -4$$

$$f'(x) = 1 + \frac{1}{2\sqrt{x^2 - 4x - 2}} \cdot (2x - 4)$$

$$= 1 + \frac{2x - 4}{2\sqrt{x^2 - 4x - 2}}$$

$$f''(x) = \frac{(2x - 4)'}{(2\sqrt{x^2 - 4x - 2})^2}$$

$$f'(x) = 0$$

$$1 + \frac{2x - 4}{2\sqrt{x^2 - 4x - 2}} = 0$$

$$2\sqrt{x^2 - 4x - 2} = -2x + 4$$

$$4x^2 - 16x - 4 = -2x + 4$$

$$4x^2 - 16x + 2x - 4 + 4 = 0$$

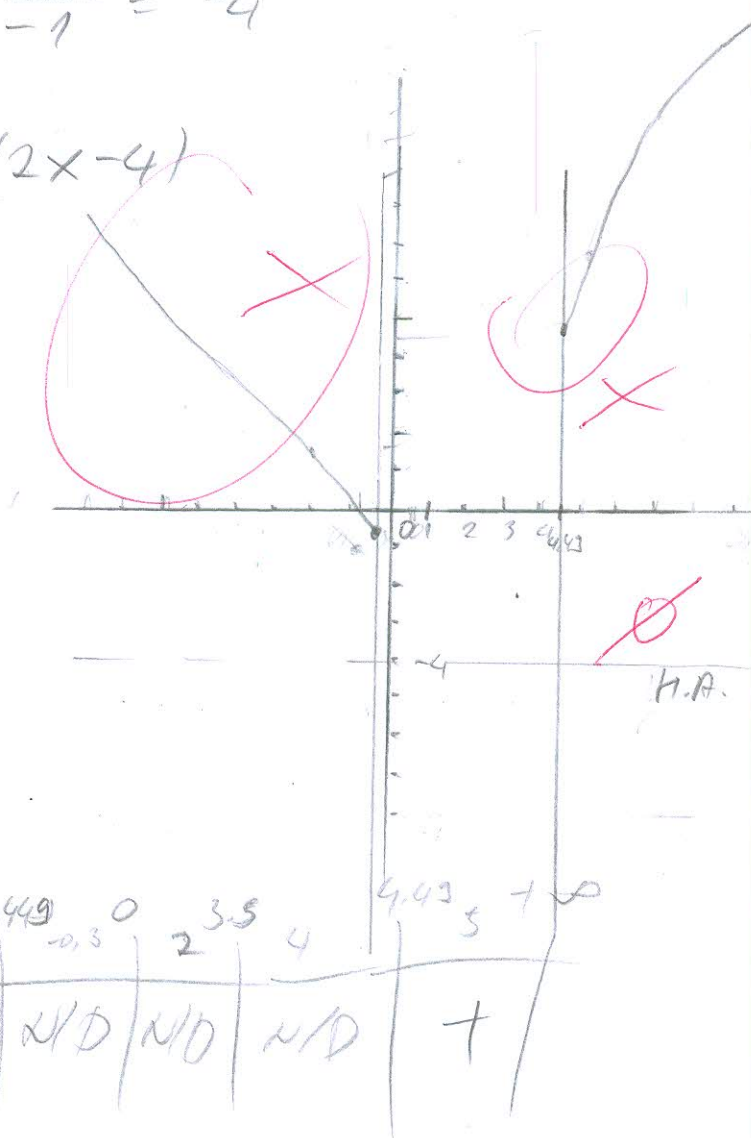
$$4x^2 - 14x = 0$$

$$x(4x - 14) = 0$$

$$x = 0 \quad 4x - 14 = 0$$

$$4x = 14 / 4$$

$$x = 3,5$$



	$x < -0,449$	$-0,449 < x < 0,3$	$0,3 < x < 2$	$2 < x < 3,5$	$3,5 < x < 4$	$4 < x < 4,449$	$x > 4,449$	
$f(x)$	-	N/D	N/D	N/D	N/D	+		

$$(3) f(x) = \frac{x^2 - 1}{4 - x^2}$$

ANTONELA KADAN

$$\begin{aligned} (1) \quad & 4 - x^2 \neq 0 \\ & -x^2 = -4 \quad | \cdot (-1) \\ & x^2 = 4 \quad | \sqrt{} \\ & x = \pm 2 \end{aligned}$$

$$Df(x) \quad x \in \mathbb{R} \setminus \{-2, 2\}$$

$$(2) \quad \text{V.A.} \quad \lim_{x \rightarrow 2} \frac{2^2 - 1}{4 - 2^2} = \frac{4 - 1}{4 - 4} = \frac{3}{0} = \infty$$

$$\lim_{x \rightarrow -2} \frac{(-2)^2 - 1}{4 - (-2)^2} = \frac{4 - 1}{4 - 4} = \frac{3}{0} = \infty$$

NETMA

H.A.

$$y = \lim_{x \rightarrow \infty} \frac{x^2 - 1}{4 - x^2} \stackrel{: x^2}{=} \frac{x^2 - 1}{-x^2} = \frac{1}{-1} = -1 \quad y = -1$$

$$y = \lim_{x \rightarrow -\infty} \frac{(-x)^2 - 1}{4 - (-x)^2} = \frac{x^2 - 1}{4 - x^2} \stackrel{: x^2}{=} \frac{1}{-1} = -1$$

$$\text{K.A.} \quad y = Kx - 1$$

kad ma H.A.

rema K.A.

$$K = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \left[\frac{x^2 - 1}{4 - x^2} \right] \stackrel{: x^3}{=} \frac{x^2 - 1}{4x - x^3} \stackrel{: x^3}{=} \frac{0}{-1} = 0$$

$$l = \lim_{x \rightarrow \infty} \frac{x^2 - 1}{4 - x^2} - \frac{x}{1} = \frac{x^2 - 1 - 4x + x^3}{4 - x^2} \stackrel{: x^3}{=} \frac{1}{0} = \infty$$

NETMA

$$K = \lim_{x \rightarrow -\infty} \frac{-x^2 - 1}{4x - x^3} \stackrel{: x^3}{=} \frac{0}{-1} = 0$$

④ $f(x) = \frac{x^2+4}{x^2-2}$

ANTONELLA KROHN

① $x^2-2 \neq 0$
 $x^2=2/\sqrt{}$
 $x=\pm\sqrt{2}$

① $Df(x) \ x \in \mathbb{R} \setminus \{\pm\sqrt{2}\}$

② VIA L'HÔPITAL

H.A $x \rightarrow \infty$
 $y = \lim_{x \rightarrow \infty} \frac{x^2+4}{x^2-2} \stackrel{:\cdot x^2}{=} \frac{1}{1} = 1 \quad \underline{y=1}$

$y = \lim_{x \rightarrow -\infty} \frac{x^2+4}{x^2-2} \stackrel{:\cdot x^2}{=} \frac{1}{1} = 1$

③ so osi y
 $x=0 \ f(0) = \frac{0^2+4}{0^2-2} = 2 \quad (0, 2)$

so osi x
 $f(x)=0 \quad x^2+4=0$
 $x^2=-4/\sqrt{}$
 $x=\pm\sqrt{-4}/\text{nono}$

④ $f'(x) = \frac{(x^2+4)' \cdot (x^2-2) - (x^2+4) \cdot (x^2-2)'}{(x^2-2)^2}$
 $= \frac{2x \cdot (x^2-2) - (x^2+4) \cdot 2x}{(x^2-2)^2} = \frac{2x^3 - 4x - 2x^3 - 8x}{(x^2-2)^2}$
 $= \frac{-12x}{(x^2-2)^2}$

$f''(x) = \frac{(-12x)' \cdot (x^2-2)^2 - (-12x) \cdot (x^2-2)^2'}{(x^2-2)^4}$
 $= \frac{-12 \cdot (x^2-2)^2 + 12x \cdot 2(x^2-2) \cdot 2x}{(x^2-2)^4}$
 $= \frac{-12 \cdot (x^2-2) + 12x \cdot 2 \cdot 2x}{(x^2-2)^3} = \frac{-x^2+2+48x^2}{(x^2-2)^3}$
 $= \frac{47x^2+2}{(x^2-2)^3}$

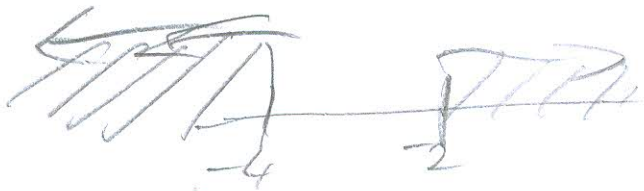
$$5. f(x) = \sqrt{x+2} + \sqrt{4-x}$$

$$x+2 \geq 0 \quad 4-x \geq 0$$

$$x \geq -2$$

$$-x \geq 4 \quad | \cdot (-1) |$$

$$x \leq -4$$



$$D_f(x) = (-\infty, -4) \cup (-2, \infty)$$

~~X~~

$$f'(x) = \frac{1}{2\sqrt{x+2}} \cdot (1) + \frac{1}{2\sqrt{4-x}} \cdot (-1)$$

$$= \frac{1}{2\sqrt{x+2}} - \frac{1}{2\sqrt{4-x}}$$

$$f'(x) = 0$$

$$\frac{1}{2\sqrt{x+2}} - \frac{1}{2\sqrt{4-x}} = 0 \quad | \cdot 2 |$$

$$\frac{2\sqrt{4-x} - 2\sqrt{x+2}}{(2\sqrt{x+2})(2\sqrt{4-x})} = 0 \quad | \cdot (2\sqrt{x+2})(2\sqrt{4-x}) |$$

$$2\sqrt{4-x} - 2\sqrt{x+2} = 0 \quad | \cdot 2 |$$

$$4\sqrt{4-x} - 4\sqrt{x+2} = 0$$

$$16 - 16x - 4x - 8 = 0$$

$$-20x + 16 = 0$$

$$-20x = -16 \quad | \cdot (-1) |$$

$$x = \frac{16}{20}$$

~~φ~~