MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 23

IME I PREZIME: KREŠIMIR ANTOLOVIĆ **VRIJEME POČETKA:** 17:15

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0402-2014

Želim ustmeni kod (zaokružiti): (prof. Uglešića) asistenta Kosora

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Za funkciju $f(x) = \sqrt{x^2 + 8x + 15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf

2. Odrediti tok funkcije $f(x) = \dfrac{x^2 - 2}{x^2 + 3}$ i skicirati graf. 20 graf

3. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = 2^{x^2-3}$. Posebno komentirati (ne)ograničenost. 7+7+6

4. Gaussovom metodom riješiti matrični sustav: 12+3

$$\begin{aligned} x + 2y - z + u &= 2 \\ 2x + 5y - z + 2u &= 3 \\ 3x - y - 2z + u &= 2 \\ x - y + 3z - 5u &= 2 \end{aligned}$$

Provjeri uvrštavanjem!

5. Ispitati domenu funkcije $g(x) = \dfrac{x}{\ln x}$. 15

6. Riješiti jednadžbu u kompleksnim brojevima: $z^3 = \dfrac{2 + 2i}{2 - 2i}$. 15

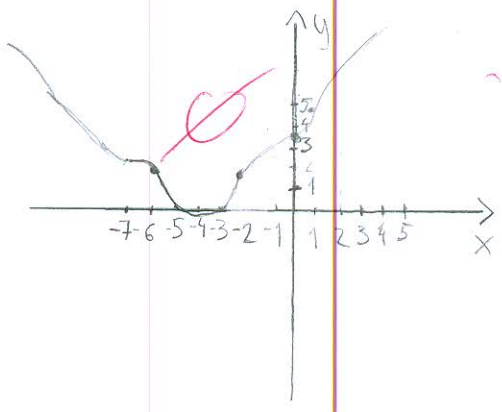
Ukupno: 30

① $f(x) = \sqrt{x^2+8x+15}$ 1.) $x^2+8x+15 \geq 0$ $x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-8 \pm \sqrt{64-60}}{2} = \frac{-8 \pm 2}{2}$

$x_1 = -3$ $x_2 = -5$

$D(f) = \langle -\infty, -5] \cup [-3, +\infty \rangle$

$f(x)$	x
$\sqrt{3}$	-6
0	-5
0	-3
$2\sqrt{6}$	1
$\sqrt{35}$	2
$\sqrt{3}$	-2
$2\sqrt{2}$	-7
$\sqrt{15}$	0



④ $x + 2y - z + u = 2$
$2x + 5y - z + 2u = 3$
$3x - y - 2z + u = 2$
$x - y + 3z - 5u = 2$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 2 \\ 1 & -1 & 3 & -5 & 2 \end{bmatrix} \begin{matrix} \\ +1R\cdot(-2) \\ +1R\cdot(-3) \\ +1R\cdot(-1) \end{matrix} \approx \begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & -7 & 1 & -2 & -4 \\ 0 & -3 & 4 & -6 & 0 \end{bmatrix} \begin{matrix} +2R\cdot(-2) \\ \\ +2R\cdot 7 \\ +2R\cdot 3 \end{matrix} \approx \begin{bmatrix} 1 & 0 & -3 & 1 & 4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 8 & -2 & -11 \\ 0 & 0 & 7 & -6 & -3 \end{bmatrix} /:8 \approx$$

$$\begin{bmatrix} 1 & 0 & -3 & 1 & | & 4 \\ 0 & 1 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & -\frac{1}{4} & | & -\frac{11}{8} \\ 0 & 0 & 7 & -6 & | & -3 \end{bmatrix} \begin{matrix} \\ \\ \\ +3R\cdot(-7) \end{matrix} \begin{matrix} +3R\cdot 3 \\ +3R\cdot(-1) \\ \\ \end{matrix} \approx \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} & | & -\frac{1}{8} \\ 0 & 1 & 0 & \frac{1}{4} & | & \frac{3}{8} \\ 0 & 0 & 1 & -\frac{1}{4} & | & -\frac{11}{8} \\ 0 & 0 & 0 & -\frac{17}{4} & | & \frac{53}{8} \end{bmatrix} /:\left(-\frac{17}{4}\right) \approx$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} & | & -\frac{1}{8} \\ 0 & 1 & 0 & \frac{1}{4} & | & \frac{3}{8} \\ 0 & 0 & 1 & -\frac{1}{4} & | & -\frac{11}{8} \\ 0 & 0 & 0 & 1 & | & -\frac{53}{34} \end{bmatrix} \begin{matrix} +4R\cdot\left(-\frac{1}{4}\right) \\ +4R\cdot\left(-\frac{1}{4}\right) \\ +4R\cdot\left(\frac{1}{4}\right) \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & | & \frac{9}{34} \\ 0 & 1 & 0 & 0 & | & \frac{13}{17} \\ 0 & 0 & 1 & 0 & | & -\frac{30}{17} \\ 0 & 0 & 0 & 1 & | & -\frac{53}{34} \end{bmatrix}$$

$x = \frac{9}{34}$

$y = \frac{13}{17}$

$z = -\frac{30}{17}$

$u = -\frac{53}{34}$ ✓

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 5 & -1 & 2 \\ 3 & -1 & -2 & 1 \\ 1 & -1 & 3 & -5 \end{bmatrix} \cdot \begin{bmatrix} \frac{9}{34} \\ \frac{13}{17} \\ -\frac{30}{17} \\ -\frac{53}{34} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

$\frac{9}{34} + 2\cdot\frac{13}{17} - 1\cdot\left(-\frac{30}{17}\right) + \left(-\frac{53}{34}\right) = 2$ ✓

$2\cdot\frac{9}{34} + 5\cdot\frac{13}{17} - 1\cdot\left(-\frac{30}{17}\right) + 2\cdot\left(-\frac{53}{34}\right) = 3$ ✓

$3\cdot\frac{9}{34} - 1\cdot\frac{13}{17} - 2\cdot\left(-\frac{30}{17}\right) + \left(-\frac{53}{34}\right) = 2$ ✓

$\frac{9}{34} - \frac{13}{17} + 3\cdot\left(-\frac{30}{17}\right) - 5\cdot\left(-\frac{53}{34}\right) = 2$ ✓

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5. $g(x) = \frac{x}{\ln x}$ $\quad D(g) = \langle 1, +\infty \rangle$ ✗

1) $\ln x \neq 0$

2) $\ln x > 0$

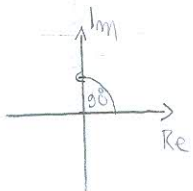
$\ln x \neq 0 \;/\; e^{\wedge}$

$x \neq e^0$

$x \neq 1$

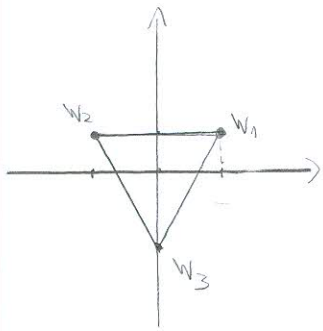
$\frac{2+2i}{2-2i} \cdot \frac{2+2i}{2+2i} = \frac{4+8i-4}{4+4} = \frac{8i}{8} = i$ ✓

6. $z^3 = \frac{2+2i}{2-2i} = i$ ✓ $\quad z = \sqrt[3]{i}$



$r = |w| = \sqrt{x^2+y^2} = \sqrt{0^2+1^2} = \sqrt{1} = 1$

$\varphi = 90^\circ = \frac{\pi}{2}$



$k=0$

$w_1 = \sqrt[3]{1}\cdot\left(\cos\frac{\frac{\pi}{2}+0\cdot 2\pi}{3} + i\sin\frac{\frac{\pi}{2}+0\cdot 2\pi}{3}\right) = \sqrt[3]{1}\cdot\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$k=1$

$w_2 = \sqrt[3]{1}\cdot\left(\cos\frac{\frac{\pi}{2}+1\cdot 2\pi}{3} + i\sin\frac{\frac{\pi}{2}+1\cdot 2\pi}{3}\right) = \sqrt[3]{1}\cdot\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

$k=2$

$w_3 = \sqrt[3]{1}\cdot\left(\cos\frac{\frac{\pi}{2}+2\cdot 2\pi}{3} + i\sin\frac{\frac{\pi}{2}+2\cdot 2\pi}{3}\right) = \sqrt[3]{1}\cdot\left(\cos\frac{9\pi}{6} + i\sin\frac{9\pi}{6}\right) = -i$ ✓

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POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: Jure Perić **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0269085660

Želim ustmeni kod (zaokružiti): prof. Uglešića asistenta Kosora

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Ukupno: 40

$$l = \lim_{x \to +\infty} \left[\left(\sqrt{x^2+8x+15} \right) - x \right] = \lim_{x \to +\infty} \frac{\sqrt{x^2+8x+15} - x}{1} \cdot \frac{\sqrt{x^2+8x+15} + x}{\sqrt{x^2+8x+15} + x} = \frac{x^2+8x+15-x^2}{\sqrt{x^2+8x+15}+x}$$

$f(x) = \sqrt{x^2+8x+15}$ $\quad f'(x) = \frac{1}{2\sqrt{x^2+8x+15}} \cdot (2x+8)$

K.A., D NAJ. ALA K $\quad l = 4$ $\quad y = kx + l$

$$\lim_{x\to+\infty} \frac{8x+15}{\sqrt{x^2+8x+15}+x} \;\begin{matrix}/:x\\/:x\end{matrix} = \lim_{x\to+\infty} \frac{8}{2} = 4$$

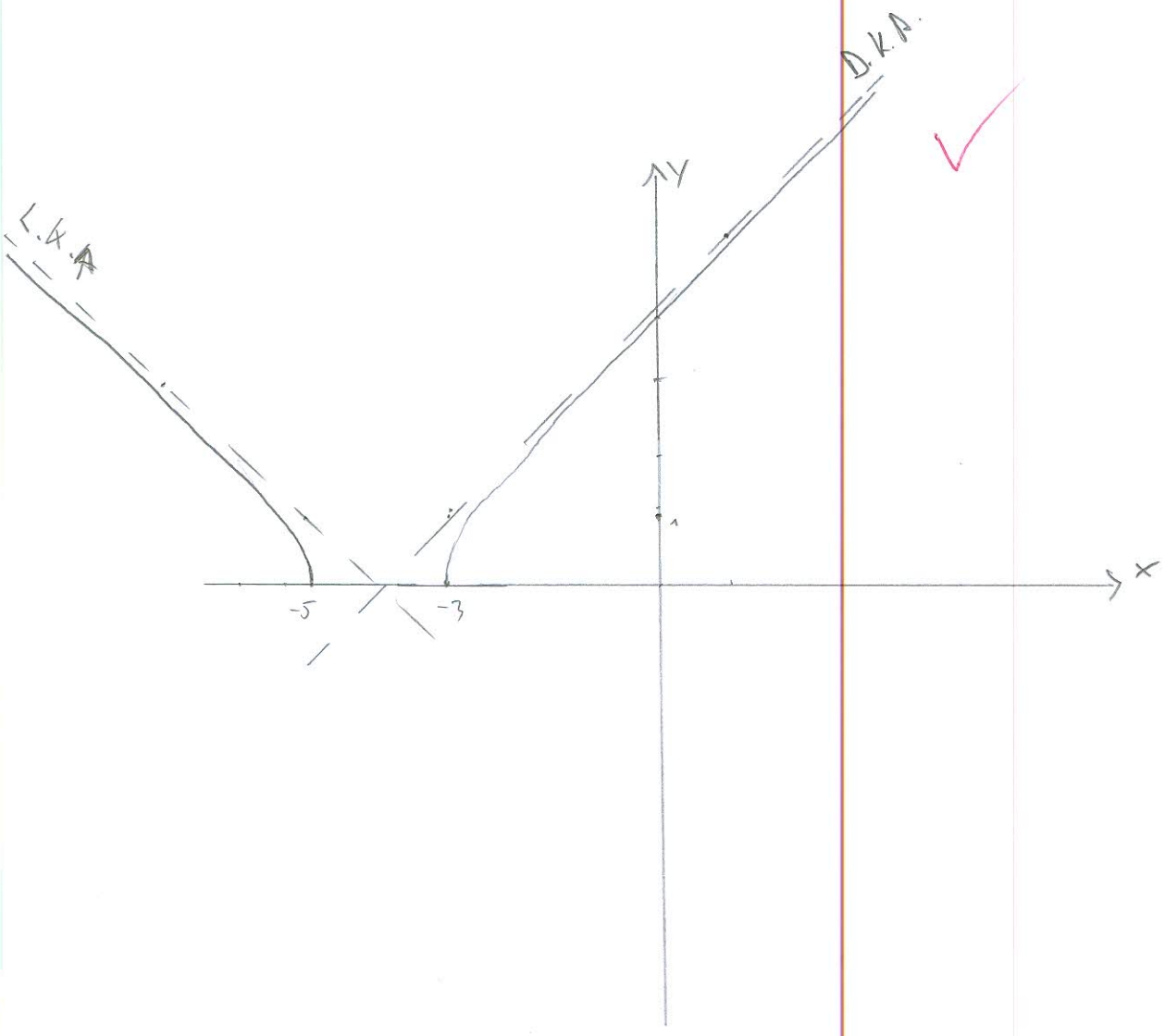
x	-3	1
y	1	5

L.K.A. $\quad k = \lim_{x\to-\infty} \frac{\sqrt{x^2+8x+15}\;/:x^-}{x\;/:x^-} = \frac{1}{-1} = -1$ $\quad k = -1$

$$l = \lim_{x\to-\infty} \left[\left(\sqrt{x^2+8x+15}\right) - (-1\cdot x)\right] = \lim_{x\to-\infty} \frac{\sqrt{x^2+8x+15}+x}{1} \cdot \frac{\sqrt{x^2+8x+15}-x}{\sqrt{x^2+8x+15}-x} = \frac{x^2+8x+15-x^2}{\sqrt{x^2+8x+15}-x} \;/:x^-$$

$$\lim_{x\to-\infty} \frac{-8}{-2} = -4 \qquad l = -4$$

x	-5	-7
y	1	3



JURE PERIĆ 0269083660

(2.) $f(x) = \frac{x^2-2}{x^2+3}$

$x^2+3 \neq 0$

$x^2+3=0$

$x^2=-3$

$x = \pm\sqrt{-3}$ $\quad D_f : \mathbb{R}$

H. A. D

$\lim\limits_{x\to+\infty} \frac{x^2-2 \;/:x^2}{x^2+3 \;/:x^2} = \lim\limits_{x\to+\infty} \frac{1}{1} = 1 \rightarrow$ DHA $y=1$ / OBOSTRANA

H. A. L

$\lim\limits_{x\to-\infty} \frac{x^2-2}{x^2+3} = \lim$

NEMA V.

K. A.

$k = \lim\limits_{x\to+\infty} \left[\frac{\frac{x^2-2}{x^2+3}}{\frac{x}{1}}\right] = \lim\limits_{x\to+\infty} \frac{x^2-2 \;/:x^3}{x^3+3x \;/:x^3} = \frac{0}{1}$ NEMA K. A.

$f(x)=0 \qquad \frac{x^2-2}{x^2+3} = 0 \;/\cdot x^2+3$

$x^2-2=0$

$x^2=2$

$x_1=\sqrt{2} \quad x_2=-\sqrt{2}$

NULTOČKE

$N_1(-\sqrt{2},0)$

$N_2(\sqrt{2},0)$

$f(0) = \frac{0^2-2}{0^2+3} = -\frac{2}{3}$

TOČKA SJECIŠTA $(-\frac{2}{3}, 0)$