

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: *Petne, Deladović*

VRIJEME POČETKA: *17:15*

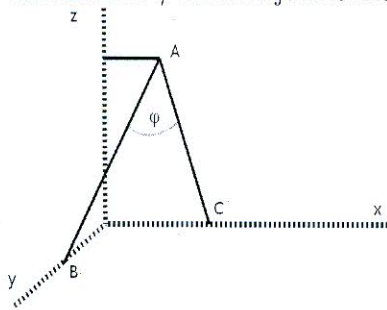
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

Želim ustmeni kod (zaokružiti):

prof. Uglešića

asistenta Kosora

1. Odrediti tangentu na funkciju  $f(x) = \log_2 x$  tamo gdje je  $x = 2$ . Nacrtati graf funkcije i nacrtati izračunatu tangentu. 15
2. Odrediti domenu funkcije  $h(x) = \arccos \ln(x^2 - 4)$ . ~~15~~
3. Odrediti tok funkcije  $f(x) = \sqrt{x^2 + 2x} - x$  i skicirati graf. ~~20 graf~~
4. Odrediti tok funkcije  $f(x) = \frac{x+2}{x^2-x-2}$  i skicirati graf. ~~20 graf~~ *13*
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije  $f(x) = (\ln x)^2$ . Komentirati (ne)omeđenost. ~~6+6+3~~
6. Zadana je konfiguracija nosača kao na slici ispod. Točke su  $A(2,2,4)$ ,  $B(1,3,1)$  i  $C(3,1,1)$ . Potrebno je odrediti kut  $\varphi$  korištenjem formule za kut između vektora. 15



Ukupno:

28

②  $h(x) = \arccos \ln(x^2 - 4)$

$\ln \circ > 0$

$D_f \dots x \in \langle -2, 2 \rangle$  ~~X~~

$x^2 - 4 > 0$   
 $x^2 > 4/\sqrt{\phantom{x}}$   
 $x \in \pm 2$

$-1 \leq \arccos \geq 1$

$-1 \leq \ln(x^2 - 4) \geq 1 \quad \ln(x^2 - 4) \geq 1$

$\ln(x^2 - 4) \geq -1$

$e^{x^2 - 4} \geq 1$

$e^{x^2 - 4} \geq -1$

$x^2 - 4 \geq 0$

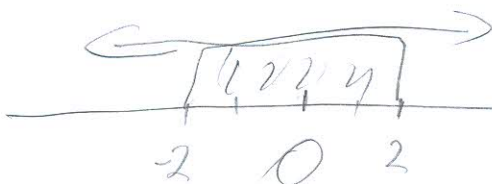
$x^2 \geq 4/\sqrt{\phantom{x}}$

$x \in \pm 2$

$x^2 - 4 \geq 0$

$x^2 \geq 4/\sqrt{\phantom{x}}$

$x \in \pm 2$



⑥

$$A(2, 2, 4)$$

$$B(1, 3, 1)$$

$$C(3, 1, 1)$$

$$\cos \varphi = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{(-i+j-3k) \cdot (i-j-3k)}{\sqrt{11} \cdot \sqrt{11}}$$

$$\vec{AB} = -1i + 1j - 3k = \frac{-1-1+9}{11} = \frac{-2+9}{11}$$

$$\vec{AC} = 1i - 1j - 3k$$

$$|\vec{AB}| = \sqrt{(-1)^2 + 1^2 + (-3)^2} = \sqrt{11}$$

$$|\vec{AC}| = \sqrt{1^2 + (-1)^2 + (-3)^2} = \sqrt{11}$$

$$= \frac{7}{11}$$

$$= 50^\circ 28' \checkmark$$

⑦  $f(x) = (\ln x)^2$

$$f'(x) = 2 \ln x \cdot \frac{1}{x}$$

$$f'(x) = \frac{2 \ln x}{x}$$

1) domain

$$x > 0 \quad \text{Df} \dots x \in (0, +\infty)$$

$f(x)$  nemá ni lokálních ni globálních extrémů ~~o~~

$$2 \ln x = 0 \quad | :2$$

$$\ln x = 0$$

et

$$x = 1$$

$$x \neq 0$$

3)  $f(x) = \sqrt{x^2+2x} - x$

2) N.T.

$$\sqrt{x^2+2x} - x = 0$$

$$\sqrt{x^2+2x} = x \quad \text{N.T. (0,0)}$$

$$x^2+2x = x^2$$

$$2x = 0$$

$$x = 0$$

3) ASIMPT.

V.A.

$$\lim_{x \rightarrow 0^+} \sqrt{x^2+2x} - x = \sqrt{0^2+2 \cdot 0} - 0 = 0 = \text{nilai V.A.}$$

H.A.

$$\lim_{x \rightarrow \pm\infty} \sqrt{x^2+2x} - x = \sqrt{x^2+2x} \sim x \quad \text{L.H.} = (\sqrt{x^2+2x})' - x' = \frac{2x+2}{2\sqrt{x^2+2x}} - 1$$

$$\begin{aligned} &= \frac{f(x+1)}{f(x)} - 1 = \frac{x+1 - \sqrt{x^2+2x}}{\sqrt{x^2+2x}} - 1 = \frac{\frac{1}{x} + \frac{1}{x} - \sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2}}}{\sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2}}} - 1 = \frac{1-\sqrt{1}}{1} \\ &= \frac{1-1}{1} = 0 \quad \boxed{g=0} \end{aligned}$$

4) I DERIVA

$$f'(x) = (\sqrt{x^2+2x})' - x' = \frac{1}{2\sqrt{x^2+2x}} \cdot (2x+2) - 1 = \frac{2x+2}{2\sqrt{x^2+2x}} - 1 = \frac{f(x+1)}{f(x)} - 1$$

$$\frac{x+1}{\sqrt{x^2+2x}} - 1 = \frac{(x+1) - (\sqrt{x^2+2x})}{\sqrt{x^2+2x}}$$

$$(x+1) - (\sqrt{x^2+2x}) = 0$$

menca m d m

$$\sqrt{x^2+2x} = x+1$$

$$x^2+2x = x^2+2x+1$$

DA

i) DOMEWA

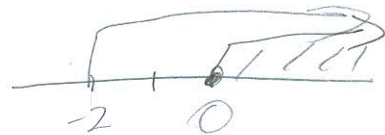
$$\sqrt{0} \geq 0 \quad x^2+2x \geq 0$$

$$x(x+2) \geq 0$$

$$x \geq 0$$

$$\begin{aligned} x+2 &\geq 0 \\ x &\geq -2 \end{aligned}$$

$$x \in [0, +\infty)$$



3) II DERIVACIJA

PETRA DOZARIC

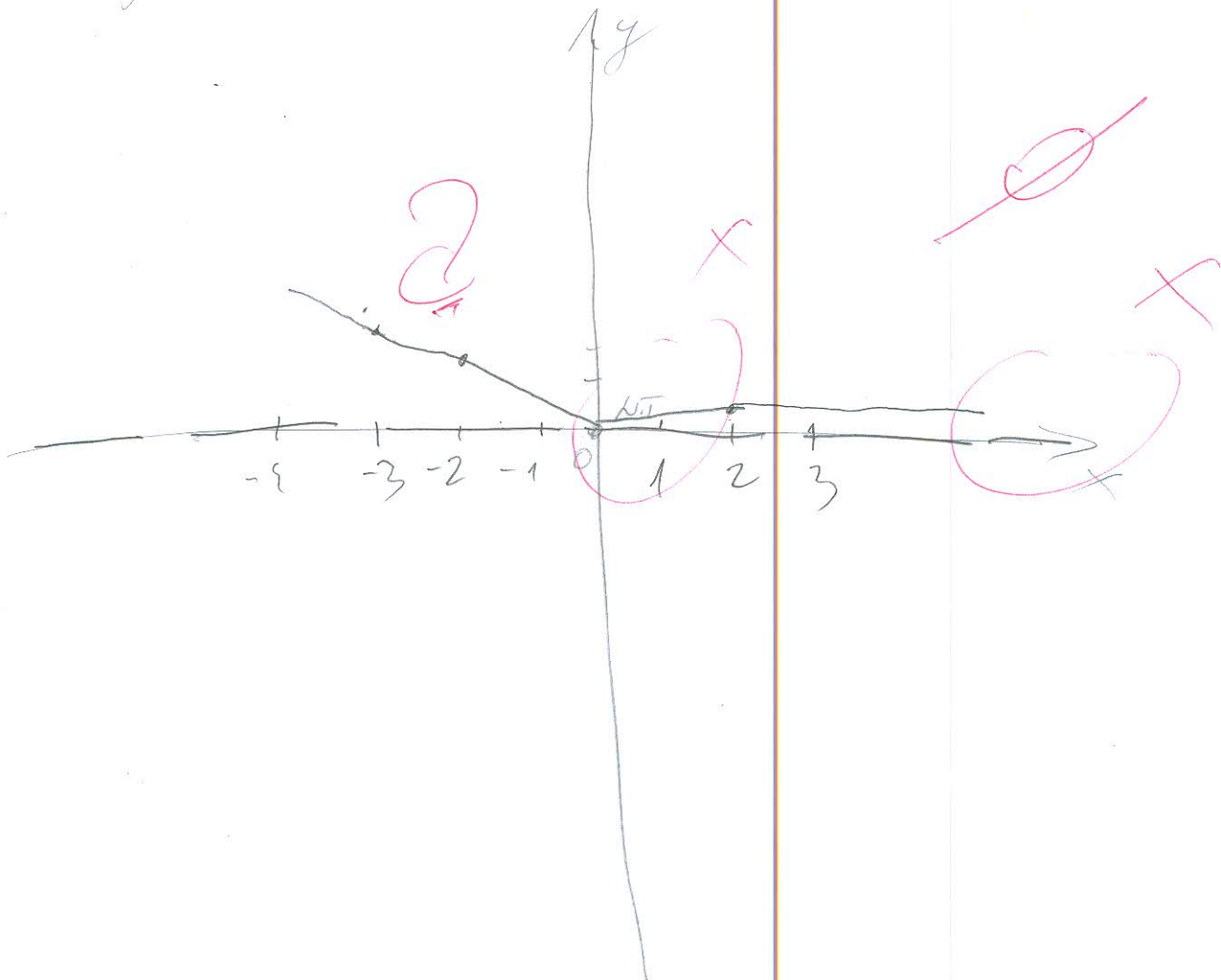
$$\left( \frac{x+1}{\sqrt{x^2+2x}} - 1 \right)' = \frac{(x+1)' \cdot (\sqrt{x^2+2x}) - (x+1) (\sqrt{x^2+2x})'}{(\sqrt{x^2+2x})^2} = \frac{\sqrt{x^2+2x} - (x+1) \cdot \frac{2x+2}{2\sqrt{x^2+2x}}}{x^2+2x}$$

$$= \frac{\sqrt{x^2+2x} - (x+1) \frac{2(x+1)}{2\sqrt{x^2+2x}}}{x^2+2x} = \frac{\sqrt{x^2+2x} - \frac{(x+1)(x+1)}{\sqrt{x^2+2x}}}{x^2+2x} = \frac{\sqrt{x^2+2x} - \frac{(x+1)^2}{\sqrt{x^2+2x}}}{x^2+2x}$$

$$= \frac{\sqrt{x^2+2x} \cdot \frac{(\sqrt{x^2+2x})^2 - (x+1)^2}{\sqrt{x^2+2x}}}{x^2+2x} = \frac{x^2+2x - (x^2+2x+1)}{\sqrt{x^2+2x}} = \frac{x^2+2x - x^2 - 2x - 1}{\sqrt{x^2+2x}} = \frac{-1}{\sqrt{x^2+2x}}$$

$$= \frac{1}{\sqrt{x^2+2x} \cdot (x^2+2x+1)} = \frac{1}{\sqrt{x^2+2x} (x^2+2x)}$$

~~no mora I~~





$$f(x) = \frac{x+2}{x^2-x-2}$$

Donera  $N \neq 0$   
 $x^2-x-2 \neq 0$

PE-10A DEALIK

2) N.T.

$x+2=0$  NT.  $(-2, 0)$   
 $x=-2$

$a=1$   $b=-1$   $c=-2$

$$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{2}$$

$$x_1 = \frac{1-3}{2} = \frac{-2}{2} = -1$$

$$x_{1,2} = \frac{1 \pm 3}{2}$$

$$x_2 = \frac{1+3}{2} = \frac{4}{2} = 2$$

3) ASIMP.T

V.A

$$\lim_{x \rightarrow -1^-} \frac{x+2}{x^2-x-2} = \frac{-1+2}{(-1)^2-(-1)-2} = \frac{1}{0} = -\infty$$

H.A.  $\lim_{x \rightarrow -1^+} \frac{x+2}{x^2-x-2} = \frac{1}{0} = +\infty$

$$\lim_{x \rightarrow \pm\infty} \frac{x+2}{x^2-x-2} = \frac{\frac{x}{x} + \frac{2}{x}}{\frac{x^2}{x^2} - \frac{x}{x} - \frac{2}{x}} = \frac{0}{1} = 0$$

$$D \dots x \in (-\infty, -1) \cup (-1, 2) \cup (2, +\infty)$$

$x=-1$  doista  
 $x=2$  doista

$$\lim_{x \rightarrow 2^-} \frac{x+2}{x^2-x-2} = \frac{4}{0} = -\infty$$

$$\lim_{x \rightarrow 2^+} \frac{x+2}{x^2-x-2} = \frac{4}{0} = +\infty$$

$y=0$  doista ona

Kose nema

4) I DERIV (KOLBEKŠOS, KONKAVOS)

$$\left( \frac{x+2}{x^2-x-2} \right)' = \frac{(x+2)' \cdot (x^2-x-2) - (x+2) \cdot (x^2-x-2)'}{(x^2-x-2)^2} = \frac{x^2-x-2 - (x+2)(2x-1)}{(x^2-x-2)^2}$$

$$= \frac{x^2-x-2 - (2x^2+x-4x+2)}{(x^2-x-2)^2} = \frac{-x^2-4x}{(x^2-x-2)^2}$$

	$x < -1$	$-1 < x < 2$	$2 < x < +\infty$
$f'$	-	+	-
$f$	$\searrow$	$\nearrow$	$\searrow$

$m(-4, -0.1)$   
 $M(0, -1)$

$$-x^2-4x=0 \quad | \cdot (-1)$$

$$x^2+4x=0$$

$$x(x+4)=0$$

$$x=0 \quad x+4=0$$

$$x=-4$$

