MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇓ A2

Popunjava nastavnik Broj ↓ bodova

Ime i prezime: STANKO STANIĆ **Vrijeme početka:** 17:45

Matični broj studenta (iznad slike u indeksu): 17-1-0165-2013

1. Riješiti integrale:

   (a) $\displaystyle\int_{-2}^{0} 3x\sqrt{1-3x}\,\mathrm{d}x$, ∅

   (b) $\displaystyle\int_{0}^{\pi} \frac{dx}{\cos^2 x}$. ∅

2. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\displaystyle\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$. Bodovanje za relativnu grešku: $\leq 3\%$ 20 bodova, $\leq 7\%$ 15 bodova, $\leq 12\%$ 10 bodova, $\leq 20\%$ 5 bodova. 20

3. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.

4. Riješiti: $y' + 2y = x - 3$.

5. U koordinatnoj ravnini skicirati domenu funkcije $f(x, y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije.

Ukupno: 20

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

Stanko Stanić Brodostrojarstvo

(1.) b) $\int_0^{\pi} \frac{dx}{\cos^2 x} = [\tan x]_0^{\pi} = \tan \pi - \tan 0 =$

NEPRAVI INTEGRAL

$= 0 - 0 = 0$ ×

a) $\int_{-2}^{0} 3x\sqrt{1-3x}\,dx = \begin{bmatrix} u=(1-3x)^{\frac{1}{2}} & du=\frac{1}{2}-\frac{3}{2}x \\ dv=3x & v=2x^3 \end{bmatrix}$ ×

$= \left[(1-3x)^{\frac{1}{2}} \cdot 2x^3 - \int 2x^3\left(\frac{1}{2}-\frac{3}{2}x\right)dx\right]_{-2}^{0} = \left[\sqrt{1-3x}\cdot 2x - \int x^3 - 3x\,dx\right]_{-2}^{0} =$

$\left[\sqrt{1-3x}\cdot 2x - \frac{x^4}{4} - \frac{3x^2}{2}\right]_{-2}^{0} = \left[\sqrt{1-3\cdot 0}\cdot 2\cdot 0 - \frac{0^4}{4} - \frac{3\cdot 0^2}{2}\right] - \left[\sqrt{1-3\cdot(-2)} - 2\cdot(-2)^4 - \frac{-2^4}{4} - \frac{3(2)^2}{2}\right]$

$= 1 - \sqrt{7} + 4 - 4 - 6 = -7{,}64$

(2.)

$\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,dx$

k	0	1	2
x_k	π	$\frac{3}{2}\pi$	2π
$f(k)$	0,4019	0,2889	0,2248

$S = \frac{d}{6}(f_0 + 4f_1 + f_2) =$

$= \frac{\pi}{6}(0{,}4019 + 4\cdot 0{,}2889 + 0{,}2248)$

$= \frac{\pi}{6} \cdot 1{,}7823 = 0{,}93321$ ✓

$f_{k0} = \frac{\arctan \pi}{\pi} = 0{,}4019$

$f_{k1} = \frac{\arctan \frac{3}{2}\pi}{\frac{3}{2}\pi} = 0{,}2889$

$f_{k2} = 0{,}2248$

20

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C5

IME I PREZIME: LUKA GULAN **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.
2. Riješiti: $y' + 2y = x - 3$.
3. U koordinatnoj ravnini skicirati domenu funkcije $f(x, y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije.
4. Riješiti integrale:

   (a) $$\int_{-2}^{0} 3x\sqrt{1-3x}\,dx,$$ ~~0~~

   (b) $$\int_{0}^{\pi} \frac{dx}{\cos^2 x}.$$ ~~0~~

5. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,dx$. Bodovanje za relativnu grešku: $\leq 3\%$ 20 bodova, $\leq 7\%$ 15 bodova, $\leq 12\%$ 10 bodova, $\leq 20\%$ 5 bodova. 20

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3.)

4.) a) $\int_{-2}^{0} 3\sqrt{1-3x}dx = \begin{bmatrix} t = 1-3x \\ dt = dx \end{bmatrix} = \int_1^7 \sqrt{t}dt = \int_1^7 t^{\frac{1}{2}} = \int_1^7 \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} \cdot \sqrt{7^3} - 1 = 11.34$

POGREŠNO PREPISAN ZADATAK

5.) $\int_{\pi}^{2\pi} \frac{\arctan x}{x} dx =$

k	0	1	2
xk	π	$3\frac{\pi}{2}$	2π
fk	0.40190	0.28895	0.2009

$f_0(\pi) = \frac{\arctan \pi}{\pi} = 0.401906738$

$f_1(3\frac{\pi}{2}) = \frac{\arctan \frac{3\pi}{2}}{\frac{3\pi}{2}} = 0.2889599497$

$f_2(2\pi) = \frac{\arctan 2\pi}{2\pi} = 0.200953369$

$S = \frac{\pi}{6} = (f_0 + 4f_1 + f_2) = \frac{\pi}{6}(0.401906738 + 4 \cdot 0.2889599497 + 0.200953369)$

$= 0.9208531173$ ✓

$1 - 0.9208531173 = \frac{0.0791468827}{2} = 0.03957344135 =$ [illegible]

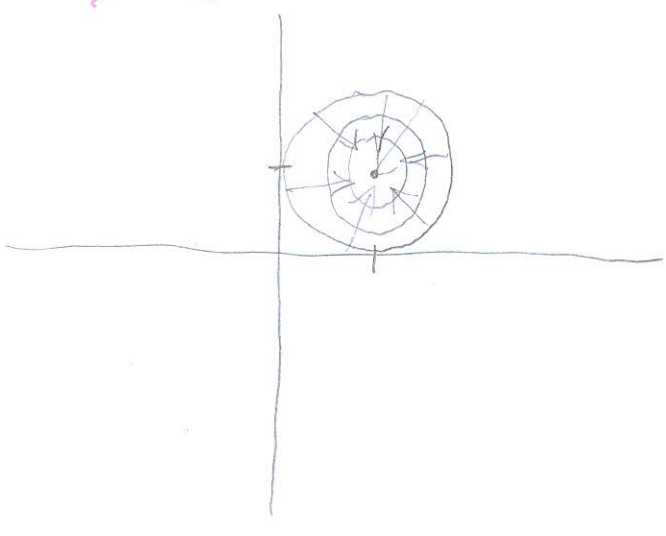
4.) b) $\int_0^{\pi} \frac{dx}{\cos^2 x} = \begin{bmatrix} t = \cos^2 x \\ dt = dx \end{bmatrix} = \int_0^{\pi} \ldots = \ln|t| \ldots$ [illegible]

$= 0.0$ [illegible]

4.) b) $\int_0^{\pi} \frac{dx}{\cos^2 x} = \begin{bmatrix} t = \cos^2 x \\ dt = dx \end{bmatrix} = \begin{bmatrix} t = 1 - \sin x \\ dt = dx \end{bmatrix} = \int_0^{\pi} \frac{dt}{t} = \ln|1 - \sin \pi| - \ln|1 - \sin 0| = 1 - 1$

$= 0$ ✗

3.)



MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}2$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MARKO MARIJANOVIĆ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):	

1. Riješiti integrale:

(a) $\int\limits_{-2}^{0} 3x\sqrt{1-3x}\mathrm{d}x,$ Ø

(b) $\int\limits_{0}^{\pi} \frac{dx}{\cos^2 x}.$ Ø

2. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\int\limits_{\pi}^{2\pi} \frac{\arctan x}{x}\mathrm{d}x$. Bodovanje za relativnu grešku: $\leq 3\%$ 20 bodova, $\leq 7\%$ 15 bodova, $\leq 12\%$ 10 bodova, $\leq 20\%$ 5 bodova. 20

3. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.

4. Riješiti: $y' + 2y = x - 3$.

5. U koordinatnoj ravnini skicirati domenu funkcije $f(x,y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije.

Ukupno: 20

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2.

$$\int\limits_{\pi}^{2\pi} \frac{\mathrm{arctg}\, x}{x} dx = \frac{1}{3} \cdot \frac{2\pi - \pi}{2}\left(f_0 + 4f_1 + f_2\right) =$$

$$= \frac{1\pi}{6}\left(0{,}402 + 1{,}156 + 0{,}225\right) = 0{,}934 \checkmark$$

K	0	1	2
x_k	π	$\frac{3\pi}{2}$	2π
f_k	0,402	0,289	0,225

$\frac{\mathrm{arctg}\,\pi}{\pi} = 0{,}402 \checkmark$ $\frac{\mathrm{arctg}\,\frac{3\pi}{2}}{\frac{3\pi}{2}} = 0{,}289 \checkmark$ $\frac{\mathrm{arctg}\,2\pi}{2\pi} = 0{,}225 \checkmark$

$4 \cdot 0{,}289 = 1{,}156 \checkmark$

MARKO MANDANOVIĆ

(1) a). $\int_{-2}^{0} 3x\sqrt{1-3x}\,dx = \int_{-2}^{0} 3x\,dx \cdot \int_{-2}^{0} \sqrt{1-3x}\,dx =$

$3\int_{-2}^{0} x\,dx \cdot \int_{-2}^{0} \sqrt{1-3x}\,dx =$

I_1 * TABLIČNI $\quad I_2$

$I_2: \int_{-2}^{0} \sqrt{1-3x}\,dx = \left(\begin{array}{l} 1-3x = t \,|' \\ -3\,dx = dt \,|:-3 \\ dx = \frac{-dt}{3} \end{array} \right)$

x	-2	0
t	7	1

$\int_{7}^{1} \sqrt{t}\left(\frac{-dt}{3}\right) = \frac{-1}{3}\int_{7}^{1} t^{\frac{1}{2}}\,dt =$

$= \frac{-1}{3}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_{7}^{1} =$

$= \frac{-1}{3}\left(\frac{1^{\frac{3}{2}}}{\frac{3}{2}} - \frac{7^{\frac{3}{2}}}{\frac{3}{2}}\right) = 3{,}893$

$I_1: 3(0-2) = 6$

KONAČNO RJEŠENJE!

$6 \cdot 3{,}893 = 23{,}358$

$\int_{-2}^{0} 3x\sqrt{1-3x}\,dx = \lim_{a \to 0^+} \int_{-2}^{a} 3x\sqrt{1-3x}\,dx = \left(\begin{array}{l} 1-3x = t \,|' \\ -3dx = dt \,|:-3 \\ dx = \frac{dt}{-3} \\ 3x = t+1 \end{array} \right)$

$\int_{-2}^{0} t+1\,\sqrt{t}\,\frac{dt}{-3} = \frac{-1}{3}\int_{-2}^{0} t + \sqrt{t}\,dt =$

$= \frac{-1}{3}\int_{-2}^{0} t\,dt + \int_{-2}^{0} t^{\frac{1}{2}}\,dt = \frac{-1}{3}\left[\left(\frac{t^2}{2}\right)\Bigg|_{-2}^{0} + \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_{-2}^{0}\right] =$

$= \frac{-1}{3}\left[\left(\frac{0^2}{2} - \frac{-2^2}{2}\right) + \left(\frac{0^{\frac{3}{2}}}{\frac{3}{2}} - \frac{-2^{\frac{3}{2}}}{\frac{3}{2}}\right)\right] = -1{,}2952$

MARKO MANDJANOVIĆ

(1.) b) $\int_0^{\pi} \frac{dx}{\cos^2 x} = \left(t = tg\frac{x}{2} \quad \cos^2 x = \frac{1-t^2}{1+t^2} \quad dx = \frac{2dt}{1+t^2} \right)$

$\lim_{a\to 0^+} \int_a^{\pi} \frac{\frac{2dt}{1+t^2}}{\frac{1-t^2}{1+t^2}} = \lim_{a\to 0^+} \int_a^{\pi} \frac{2dt}{2(1-t^2)} = \lim_{a\to 0^+} \int_a^{\pi} \frac{dt}{(1)^2 - t^2} =$ *TABL.

$\lim_{a\to 0^+} \left(\frac{1}{2} \ln \left| \frac{1+t}{1-t} \right| \right) \Big|_a^{\pi} =$

$\lim_{a\to 0^+} \left(\frac{1}{2} \ln \left| \frac{1+\pi}{1-\pi} \right| \right) - \left(\frac{1}{2} \ln \left| \frac{1+a}{1-a} \right| \right.$

$\emptyset$

(1.) b) $\int_0^{\pi} \frac{dx}{\cos^2 x} = \lim_{a\to 0^+} \left(\tan x \right) \Big|_0^{\pi}$

NEPRAVI INTEGRAL

$= \left(\tan(\pi) - \tan(0) \right) =$

$= 0$ X $\emptyset$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ *A2*

IME I PREZIME: DANIEL ŠOŠA **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17.2-0366-2019

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti integrale:

   (a) $\displaystyle\int_{-2}^{0} 3x\sqrt{1-3x}\,dx$,

   (b) $\displaystyle\int_{0}^{\pi} \frac{dx}{\cos^2 x}$.

2. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\displaystyle\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,dx$. Bodovanje za relativnu grešku: $\leq 3\%$ 20 bodova, $\leq 7\%$ 15 bodova, $\leq 12\%$ 10 bodova, $\leq 20\%$ 5 bodova. 20

3. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. ~~20~~ 0

4. Riješiti: $y' + 2y = x - 3$. 0

5. U koordinatnoj ravnini skicirati domenu funkcije $f(x,y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije. 0

Ukupno: 20

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1. $\displaystyle\int_{-2}^{0} 3x\sqrt{1-3x}\,dx =$

3. $y'' - y' = -x+1$

$r^2 - r = 0$

$r(r-1) = 0$

$r_1 = 0$

$r_2 = 1$

$y_H = C_1 e^0 + C_2 e^1$

$y_{1p} = x(Ax+B) = Ax^2 + Bx = \frac{1}{2}x^2$

$y_{2p} = Ax = -x$

$y_1' = 2Ax + B$ $y_2' = A$

$y_1'' = 2A$ $y_2'' = 0$

$2A - 2Ax + B = -x$

$-2A = -1$

$A = \frac{1}{2}$ $B = 0$

$0 - A = 1$

$-A = 1$

$A = -1$

$y_p = y_{1p} + y_{2p} = \frac{1}{2}x^2 - x$

$\boxed{y(x) = C_1 + C_2 e^{1} + \frac{1}{2}x^2 - x}$

X

POČETNI UVJETI $x=0, y=0$ $y'=0$

$0 = C_1 + C_2 e^{1}$?

$0 = C_1 + C_2\, 2.71$

$2.71 C_2 = -C_1$

$C_2 = \frac{-C_1}{2.71}$

$y'(x) = x - 1$

$x - 1 = 0$

$x = 1$

$0 = C_1 + C_2 e^1 + \frac{1}{2} - 1$

$0 = C_1 + C_2 e^1 - \frac{1}{2}$

$C_1 + 2.71 C_2 = \frac{1}{2}$

$C_1 = \frac{1}{2} - 2.71 C_2$

4. $y' + 2y = x - 3$

$\frac{dy}{dx} + 2y = 0 \,/\cdot dx$

$dy + 2y\,dx = 0 \,/ : y$

$\frac{dy}{y} + 2\,dx = 0$

$\frac{dy}{y} = -2\,dx$

$\ln y = -2x + C \,/ e$

$y = e^{-2x} + C$

VARIJACIJA KONSTANTE

$y = e^{-2x} + C(x)$ X $y' = -2e^{-2x} + C'(x)$

$-2e^{-2x} + C'(x) + 2e^{-2x} + C(x) = x - 3$

$C'(x) + C(x) = x - 3$

$C'(x) = x - 3 - C(x)$

$\emptyset$

PROVJERA ?

$y' =$

DANIEL ŠOŠA

b) $\int_0^{\pi} \frac{dx}{\cos^2 x} = (\text{TABLIČNI}) = \tan x + C \Big|_0^{\pi} = -\infty - \infty \doteq N/P$

?

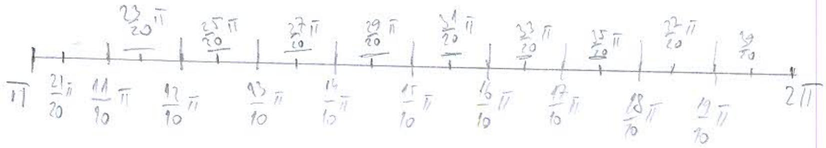
(2) $\int_{\pi}^{2\pi} \frac{\arctan x}{x}\, dx$

$\lim_{x \to \frac{\pi}{2}^-} = +\infty$

$\lim_{x \to \frac{\pi}{2}^+} \to -\infty$

SIMPSONOVA FORMULA

$P = \frac{d}{6}(f_0 + 4f_1 + f_2) \qquad d = \frac{\pi}{10}$



$P_1 = \frac{\frac{\pi}{10}}{6}(0.402 + 1.548 + 0.373) = 0.1216$

$P_2 = \frac{\frac{\pi}{10}}{6}(0.373 + 1.44 + 0.348) = 0.113$

$P_3 = \frac{\pi}{60}(0.348 + 1.346 + 0.326) = 0.1057$

$P_4 = \frac{\pi}{60}(0.326 + 1.263 + 0.306) = 0.099$

$P_5 = \frac{\pi}{60}(0.306 + 1.19 + 0.289) = 0.0935$

$P_6 = \frac{\pi}{60}(0.289 + 1.124 + 0.273) = 0.0883$

$P_7 = \frac{\pi}{60}(0.273 + 1.065 + 0.259) = 0.0836$

$P_8 = \frac{\pi}{60}(0.259 + 1.012 + 0.247) = 0.0795$

$P_9 = \frac{\pi}{60}(0.247 + 0.964 + 0.235) = 0.0757$

$P_{10} = \frac{\pi}{60}(0.235 + 0.9199 + 0.225) = 0.0723$

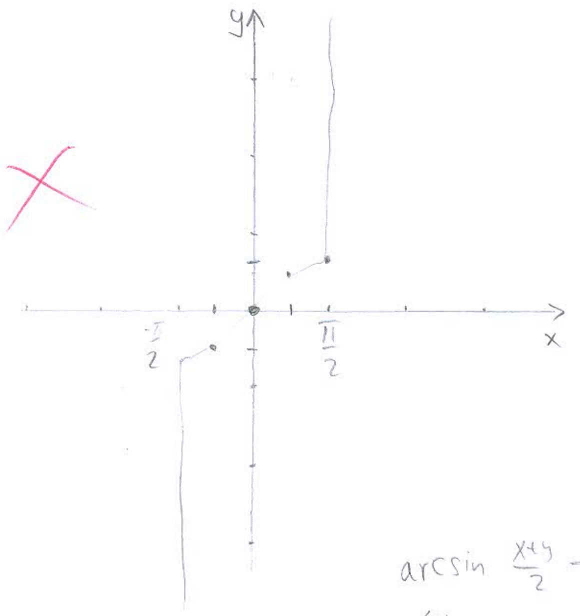
$P = P_1 + P_2 \ldots + P_{10}$

$P = 0.9322$

f(x) = arcsin x DANIEL SOJA

5. $f(x,y)=\arcsin\left(\frac{x+y}{2}\right)$

$D_f=\mathbb{R}^2;\ x\le \frac{\pi}{2},\ y\le\frac{\pi}{2},\ x\ge -\frac{\pi}{2},\ y\ge-\frac{\pi}{2}$



$\arcsin\left(\frac{x+y}{2}\right)=0$

$\frac{x+y}{2}=0$

$x+y=0$

$y=-x$

$\arcsin\left(\frac{x+y}{2}\right)=1\ /\cdot\sin$

$\frac{x+y}{2}=\sin 1$

$\frac{x+y}{2}=0.841$

$x+y=1.68$

$y=-x+1.68$

$\arcsin\frac{x+y}{2}=-1\ /\cdot\sin$

$\frac{x+y}{2}=-0.84$

$x+y=-1.68$

$y=-x-1.68$

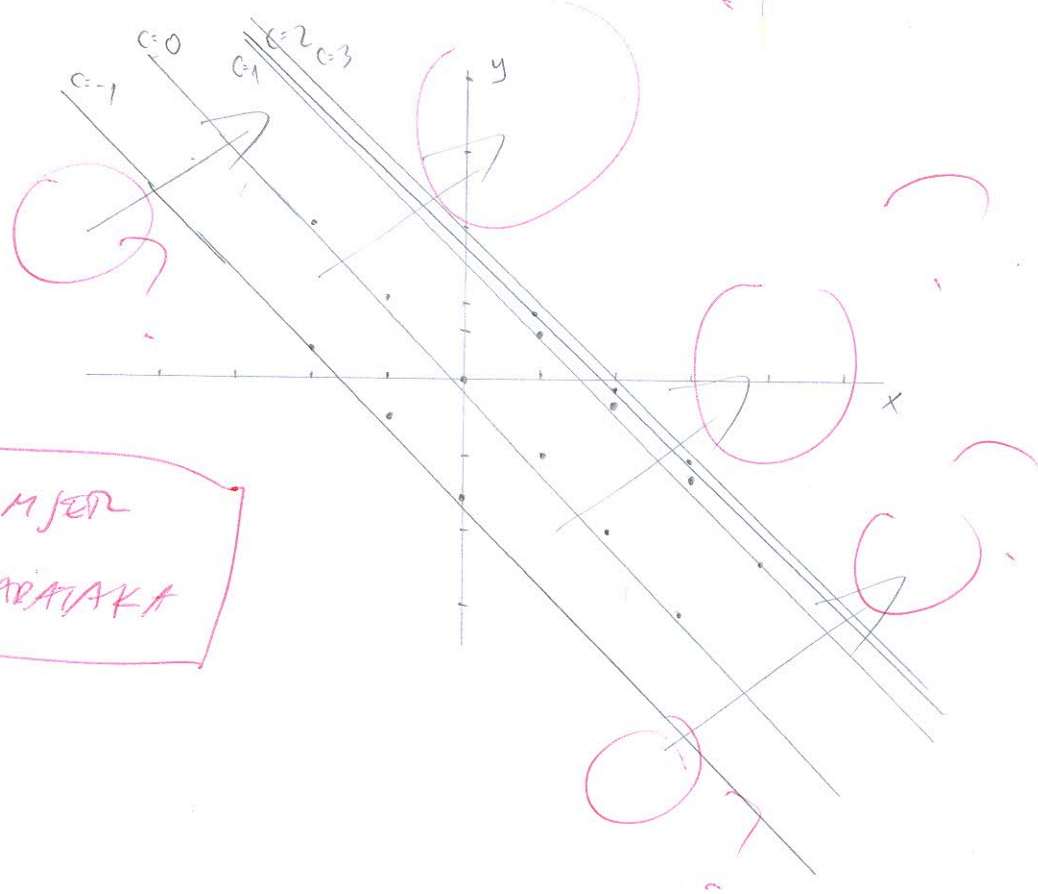
$\arcsin\left(\frac{x+y}{2}\right)=2\ /\cdot\sin$

$\frac{x+y}{2}=\sin 2$

$\frac{x+y}{2}=0.909$

$x+y=1.82$

$y=-x+1.82$



OVO JE PRIMJER
IZ ZBIRKE ZADATAKA

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ C5

Ime i prezime: Angelo Kosović

Vrijeme početka:

Matični broj studenta (iznad slike u indeksu): 17-2-0264-2013

Popunjava nastavnik
Broj ↓ bodova

1. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.
2. Riješiti: $y' + 2y = x - 3$.
3. U koordinatnoj ravnini skicirati domenu funkcije $f(x, y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije. ∅
4. Riješiti integrale:

   (a) $$\int_{-2}^{0} 3x\sqrt{1-3x}\,dx,$$ ∅

   (b) $$\int_{0}^{\pi} \frac{dx}{\cos^2 x}.$$ ∅

5. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,dx$. Bodovanje za relativnu grešku: $\leq 3\%$ 20 bodova, $\leq 7\%$ 15 bodova, $\leq 12\%$ 10 bodova, $\leq 20\%$ 5 bodova. 5

Ukupno: 5

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5. $$\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,dx$$

x	π	$\frac{3\pi}{2}$	2π
$f = \frac{\arctan x}{x}$	0.25210	0.15584	0.11264

$$S = \frac{2\pi}{6}\left(0.25210 + 4(0.15584) + 0.11264\right)$$

$$S = \frac{2\pi}{6}(0.9881)$$

$$S = \frac{1}{3}\pi\,([illegible])$$

$$S = 1.0347359$$

GREŠKA > 12%

5

Angelo Kosović

2. $y' + 2y = x - 3$

$\frac{dy}{dx} + 2y = x - 3$

$\frac{dy}{dx} = x - 3 - 2y \;/ \cdot dx$

$dy = (x - 3 - 2y)dx \;/ \int$

$y = \frac{x^2}{2} - 3x$

Ø

4. b) $\int_0^\pi \frac{dx}{\cos^2 x}$

NEPRAVI INTEGRAL

$\int \frac{dx}{\cos^2 x} = \tan x + C$

$\tan x \Big|_0^\pi = \tan \pi - \tan 0 = 0$

Ø

Angela Kosović

4. a) $\int_{-2}^{0} 3x\sqrt{1-3x}\,dx$

$1-3x \geq 0$

$3x \geq 1$

$x \leq \frac{1}{3}$

$\lim_{a \to \frac{1}{3}} -\frac{1}{2}\ln|3x^2| - \lim_{b \to \frac{1}{3}} -\frac{1}{2}\ln|3x^2|$

$\lim_{a \to \frac{1}{2}} -\frac{1}{2}\ln|3x^2| \Big|_{a}^{0} - \lim_{b \to \frac{1}{3}} -\frac{1}{2}\ln|3x^2| \Big|_{-2}^{b}$

$-\frac{1}{2}\ln|0| + \frac{1}{2}\ln|3(\frac{1}{3})^2| + \frac{1}{2}\ln|3(\frac{1}{3})^2| - \frac{1}{2}\ln|12|$

$= -\infty$ N/D

$\int 3x\sqrt{1-3x}\,dx$

$\int 3x\,(1-3x)^{\frac{1}{2}}\,dx$

$3x^2 = t$

$6x\,dx = dt$

$2(3x\,dx) = dt$

$3x\,dx = \frac{1}{2}dt$

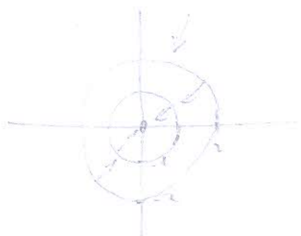
$\frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|3x^2| + C$

3. $f(x,y) = \arcsin\left(\frac{x+y}{2}\right)$

$C_1 = 0$ $f(x,y) = 0$

$C_2 = 1$ $f(x,y) = 1$

$C_3 = 2$ $f(x,y) = 2$



MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}2$

IME I PREZIME: JOSIP PREDOVAN **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0126-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti integrale:

(a) $\displaystyle\int_{-2}^{0} 3x\sqrt{1-3x}\,\mathrm{d}x$,

(b) $\displaystyle\int_{0}^{\pi} \frac{dx}{\cos^2 x}$.

Ø
Ø

2. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\displaystyle\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$. Bodovanje za relativnu grešku: $\leq 3\%$ 20 bodova, $\leq 7\%$ 15 bodova, $\leq 12\%$ 10 bodova, $\leq 20\%$ 5 bodova.

3. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.

4. Riješiti: $y' + 2y = x - 3$.

5. U koordinatnoj ravnini skicirati domenu funkcije $f(x, y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije.

Ukupno: Ø

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x + \sqrt{x^2\pm a^2}\right\| + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

b) $\int_0^{\pi} \frac{dx}{\cos^2 x} = \int_0^{\pi} \frac{1}{\cos^2 x}\, dx = \int_0^{\pi} \frac{\sin^2 x + \cos^2 x}{\cos^2 x} dx = \int_0^{\pi} \sin^2 x\, dx$ $\boxed{\sin^2 x + \cos^2 x = 1}$

$= -\cos^2 x \Big|_0^{\pi}$

ODREĐENI INTEGRAL

$= -\cos(\pi)^2 - (-\cos(0)^2)$

$= -1 + 1 = 0$

a) $\int_{-2}^{0} 3x \sqrt{1-3x}\, dx = \int_{-2}^{0} 3x \cdot (1-3x)^{\frac{1}{2}} dx = 3 \cdot \frac{x^2}{2} \cdot \frac{(1-3x)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_{-2}^{0}$

$= 3 \cdot \frac{1}{2} x^2 \cdot \frac{2}{3} \sqrt{(1-3x)^3} \Big|_{-2}^{0}$

$= \frac{3}{2} x^2 \cdot \frac{2}{3} \sqrt{(1-3x)^3} = \left(\frac{3}{2} \cdot 0^2 \cdot \frac{2}{3} \sqrt{(1-3\cdot 0)^3}\right) - \left(\frac{3}{2} \cdot (-2)^2 \cdot \frac{2}{3} \sqrt{(1-3\cdot(-2))^3}\right)$

$0 - 28\sqrt{7} = -74.08$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $A2$

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: Leila Adum **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

1. Riješiti integrale:

(a) $\int\limits_{-2}^{0} 3x\sqrt{1-3x}\mathrm{d}x$,

(b) $\int\limits_{0}^{\pi} \frac{dx}{\cos^2 x}$.

∅

2. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\int\limits_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$. Bodovanje za relativnu grešku: $\leq 3\%$ 20 bodova, $\leq 7\%$ 15 bodova, $\leq 12\%$ 10 bodova, $\leq 20\%$ 5 bodova.

3. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. ∅

4. Riješiti: $y' + 2y = x - 3$. ∅

5. U koordinatnoj ravnini skicirati domenu funkcije $f(x,y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije. ∅

Ukupno: ∅

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

Leila Adum

(1) $\int_{-2}^{0} 3x\sqrt{1-3x}\,dx = \begin{cases} x = u & dv = \sqrt{1-3x}\,dx \\ dx = du & \int dv = \int \sqrt{1-3x}\,dx \Rightarrow \int 3\sqrt{1-3x}\,dx = \end{cases}$

$= \begin{cases} 1-3x = t^2 \\ 3dx = 2t\,dt \end{cases}$

$v = \frac{1}{2}(1+3x)^2$ ✗

$= \int t^2 \cdot 2t\,dt$

$= \int 2t^3\,dt =$

$= 2\int t^3\,dt = 2\cdot\frac{t^4}{4} + c$

$= \frac{1}{2}\cdot t^4$

$= \frac{1}{2}(1+3x)^2$

$u\cdot v - \int v\,du =$

$= x\cdot\frac{1}{2}(1+3x)^2 - \int \frac{1}{2}(1+3x)^2\cdot dx =$

$= x\cdot\frac{1}{2}(1-3x)^2 - \frac{1}{2}\int (1+3x)^2\,dx =$

$= x\cdot\frac{1}{2}(1-3x)^2 - \frac{1}{2}\int (1+6x+9x^2)\,dx =$

$= x\cdot\frac{1}{2}(1-3x)^2 - \frac{1}{2}\int dx - \frac{1}{2}\int 6x\,dx - \frac{1}{2}\int 9x^2\,dx =$

$= x\cdot\frac{1}{2}(1-3x)^2 - \frac{1}{2}\cdot x - 3\cdot\frac{x^2}{2} - \frac{9}{2}\cdot\frac{x^3}{3} + c$

$0 \to 0\cdot\frac{1}{2}(1-3\cdot 0)^2 - \frac{1}{2}\cdot 0 - 3\,\frac{0^2}{2} - \frac{9}{2}\cdot\frac{0}{3} + c =$

$=$

$-2 = -2\cdot\frac{1}{2}(1-3\cdot(-2))^2 - \frac{-2}{2} - 3\cdot\frac{4}{2} - \frac{9}{2}\cdot\frac{-8}{3} =$

$= -1(1+6)^2 + 1 - 6 - \frac{9}{2} + 12 = -49 + 1 - 6 + 12 - \frac{9}{2} =$

$= -42 - 4\frac{1}{2} = -46 - \frac{1}{2} = -45\frac{1}{2}$

$\int_{-2}^{0} 3x\sqrt{1-3x}\,dx = 0 - \left(-45\frac{1}{2}\right) = 45\frac{1}{2}$ ✗

b) $\int_0^{\pi} \frac{dx}{\cos^2 x} = tg\,x + C \Rightarrow tg\,\pi - tg\,0 = 0{,}05 - 0 = 0{,}05$ $\emptyset$

NEPRAVI INTEGRAL
VIDI ISTI PRIMJER U ZBIRCI ZADATAKA

(3.) $y'' - y' = -x + 1$

$x = 0$
$y(0) = 0$
$y'(0) = 0$

$r^2 - r = 0$

$r(r-1) = 0$

$r_1 = 0 \quad r_2 = 1$

$y_H = C_1 \cdot e^{0x} + C_2 \cdot e^x$

$\boxed{y = C_1 + C_2 \cdot e^x - 1}$ X $\emptyset$

$y_p \Rightarrow -x + 1 = e^{\alpha x}(P_m(x)\cos\beta x + Q_n(x)\sin\beta x)$

$\sin 0 = 0$

$\alpha = 0 \quad \beta = 0 \quad m = 1$

$-x + 1 = e^{\alpha x} \cdot P_m(x) \cdot 1$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$y'' - y' = -x + 1$

$0 - A = -x + 1$

$-A = -x + 1$

$-x = 0 \Rightarrow x = 0$

$-A = 1$

$A = -1$

PROVJERA?

Zelma Adum

(5) $f(x,y) = \arcsin\left(\frac{x+y}{2}\right)$

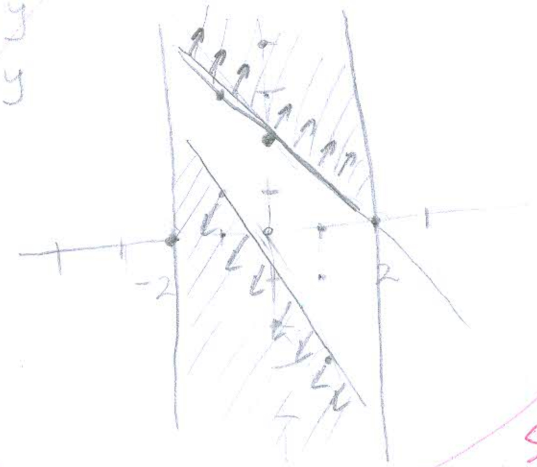
$D \to [-1, 1]$

$-1 \le \frac{x+y}{2} \le 1 \quad / \cdot 2$

$-2 \le x+y \le 2$

$-2 \le x+y \Rightarrow -2-x \le y$

$2 \ge x+y \Rightarrow 2-x \ge y$



SKICA ČEGA?

$y \ge -x-2$

$y \le 2-x$

za $x=-1$

$y = +1-2 = -1 \quad (-1,1)$

za $x=1$

$y = -1+2 = -3 \quad (1,-3)$

za $x=-1$

$y \le 2+1 = 3 \quad (-1,3)$

$x=0$

$y \le 2 \quad (0,2)$

$\emptyset$

OVO JE PRIMJER IZ ZBIRKE ZADATAKA

za $x=2$

$y \ge -2-2 = -4 \quad (2,-4$

$x=3$

$y \ge -3-2 = -5$

(4) $y' + 2y = x - 3$

$y' + 2y = 0$

$y' = -2y$

$\frac{dy}{dx} = -2y \;/\cdot dx \;/ \cdot \frac{1}{2y}$

$\int \frac{dy}{2y} = \int -dx$

$\frac{1}{2} \int \frac{dy}{y} = -x$

$\frac{1}{2} \ln|y| = -x + C \;/\cdot 2$

$\ln|y| = -2x + C \;/ e$

$y = e^{-2x} \cdot e^{C} \rightarrow D$

$y = D \cdot e^{-2x}$

RJEŠENJE ?

$y' = D' \cdot e^{-2x} + D \cdot e^{-2x} \cdot -2$

$y' = 0 \cdot e^{-2x} + -2De^{-2x} = -2De^{-2x}$

$-2D \cdot e^{-2x} + 2D \cdot e^{-2x} = x - 3$

$0 = x - 3$

$x = 3$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C5

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: VALENTINO ŠARE **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0149-2011

1. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.
2. Riješiti: $y' + 2y = x - 3$.
3. U koordinatnoj ravnini skicirati domenu funkcije $f(x, y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije.
4. Riješiti integrale:

   (a) $\displaystyle\int_{-2}^{0} 3x\sqrt{1-3x}\,\mathrm{d}x$,

   (b) $\displaystyle\int_{0}^{\pi} \frac{\mathrm{d}x}{\cos^2 x}$.

5. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\displaystyle\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$. Bodovanje za relativnu grešku: $\leq 3\%$ 20 bodova, $\leq 7\%$ 15 bodova, $\leq 12\%$ 10 bodova, $\leq 20\%$ 5 bodova.

Ukupno:

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4.

a) $\displaystyle\int_{-2}^{0} 3x\sqrt{1-3x}\,dx$ $\quad \left[\begin{array}{l} t = 1-3x \\ \cancel{dt = 3x\,dx} \\ dx = \frac{dt}{3x} \end{array}\right]$ X

$= \displaystyle\int_{-2}^{0} 3x\sqrt{t}\,\frac{dt}{3x}$

$= \displaystyle\int_{-2}^{0} \sqrt{t}\,dt$

$= \displaystyle\int_{-2}^{0} t^2\,dt = \frac{t^3}{3} = \frac{(1-3x)^3}{3}\Big|_{-2}^{0} = \left(\frac{(1-3\cdot 0)^3}{3}\right) - \left(\frac{(1-3\cdot(-2))^3}{3}\right) =$

$= \frac{1}{3} - \frac{343}{3} = -114$

b) $\int_0^{\pi} \frac{dx}{\cos^2 x}$

NEPRAVI INTEGRAL
SINGULARITET $x = \frac{\pi}{2}$

$\tan(\pi) = 0$

$= \tan x \Big|_0^{\pi} = \tan(\pi) - \tan(0)$

$= 0.05488$ ×

$= 0.049388$ GRA. tj. $= 0$ RAD.

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇊ $\mathcal{A}2$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: JURE ŠUŠIĆ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0259-2014	

1. Riješiti integrale: 0

   (a) $\displaystyle\int_{-2}^{0} 3x\sqrt{1-3x}\,\mathrm{d}x$,

   (b) $\displaystyle\int_{0}^{\pi} \frac{dx}{\cos^2 x}$.

2. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\displaystyle\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$. Bodovanje za relativnu grešku: $\le 3\%$ 20 bodova, $\le 7\%$ 15 bodova, $\le 12\%$ 10 bodova, $\le 20\%$ 5 bodova. 0

3. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.

4. Riješiti: $y' + 2y = x - 3$.

5. U koordinatnoj ravnini skicirati domenu funkcije $f(x, y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije.

Ukupno: 0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1.) a) $\int_{-2}^{0} 3x\sqrt{1-3x}\,dx$

$= \int_{-2}^{0} \left(3x\left(\frac{1}{2}\left[x\sqrt{1-3x} + 1\arcsin\left(\frac{3x}{1}\right)\right]+c\right)\right)$

$= \int_{-2}^{0} \left(3x\left(\frac{1}{2}\left[x\sqrt{1-3x} +\right.\right.\right.$

b) $\int_{0}^{\pi} \frac{dx}{\cos^2 x} = \int_0^{\pi} \tan x + c$

$= \Big|_0^{\pi} 0{,}05 + c - \Big|_0^{\pi} 0 + c$

$= 0{,}05 + c - c$

$= 0{,}05$

2.) $\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,dx$

$= \frac{df}{dx} = \frac{\frac{1}{1+x^2}}{\frac{x^2}{2}} = \frac{2}{x^2+x^4} = \frac{2}{x^2(1+x^2)}$

$= \frac{2}{1^2(1+1^2)} = \frac{2}{2} = 1$

$x^2 = 0$

$x = 0$

$1 + x^2 = 0$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm 1$

Ø

5.) $f(x,y) = \arcsin\left(\frac{x+y}{2}\right)$

$$\arcsin\left(\frac{x+y}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{x+y}{2}\right)^2}}$$

4.) $y' + 2y = x - 3$ $\quad y' = -2y + x - 3$ $\quad 2y = -y' + x - 3$

$-x = -3 - y' - 2y$

$x = y' + 2y + 3$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C5

IME I PREZIME: LUKA LUKAČIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
17-2-0365-2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti $y'' - y' = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.
2. Riješiti: $y' + 2y = x - 3$.
3. U koordinatnoj ravnini skicirati domenu funkcije $f(x, y) = \arcsin(\frac{x+y}{2})$ i nekoliko razinskih krivulja. Strelicama označiti smjer rasta funkcije.
4. Riješiti integrale:

   (a) $\int\limits_{-2}^{0} 3x\sqrt{1-3x}\mathrm{d}x$, Ø

   (b) $\int\limits_{0}^{\pi} \frac{dx}{\cos^2 x}$. Ø

5. Nekom od metoda numeričke integracije odrediti vrijednost integrala $\int\limits_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$. Bodovanje za relativnu grešku: $\leq 3\%$ 20 bodova, $\leq 7\%$ 15 bodova, $\leq 12\%$ 10 bodova, $\leq 20\%$ 5 bodova.

Ukupno: Ø

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α ($\alpha \neq 0$)	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ ($\alpha > 0$)	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x ($\alpha > 0$)	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

LUKA LUKAČIĆ

(2) $y' + 2y = x - 3$

$y' = x - 3 - 2y$

$y = \frac{x^2}{2} - 3x - 2\frac{y^2}{2}$

$y = \frac{1}{2}x^2 - 3x - y^2$

$y + y^2 = \frac{1}{2}x^2 - 3x$

$\int t^{\frac{1}{2}} dx$

$-\frac{1}{3} \cdot \left(+\frac{1}{2}\right) t^{-\frac{1}{2}}$

$t = 1 - 3x \qquad -\frac{1}{2} t^{-\frac{1}{2}}$

$dt = -3dx$

$dx = -\frac{1}{3} dt$

(4) a) $\int_{-2}^{0} 3x\sqrt{1-3x}\, dx = \begin{cases} u = 3x & dv = (1-3x)^{\frac{1}{2}} \\ du = 3dx & v = -\frac{1}{6}(1-3x)^{-\frac{1}{2}} \end{cases}$

$= uv - \int v\,du = 3x \cdot \left(-\frac{1}{6}\right)(1-3x)^{-\frac{1}{2}} - \int \left(-\frac{1}{6}\right)(1-3x)^{-\frac{1}{2}} 3\,dx$

$= \cancel{3x} \; -\frac{x}{2}(1-3x)^{-\frac{1}{2}} + \frac{1}{2}\int (1-3x)^{-\frac{1}{2}} dx$

$\int (1-3x)^{-\frac{1}{2}} dx = \begin{cases} t = 1-3x \\ dt \end{cases}$

$\emptyset$

4. b)

$$\int_0^{\pi} \frac{dx}{\cos^2 x} = \left| \tan x \right|_0^{\pi} = \tan \pi - \tan 0 = 0{,}0548$$

NEPRAVI INTEGRAL

4. a)

$$\int_{-2}^{0} 3x\sqrt{1-3x}\, dx = \begin{cases} u = (1-3x)^{\frac{1}{2}} & dv = 3x \\ du = -\frac{1}{6}(1-3x)^{-\frac{1}{2}} & v = 3 \cdot \frac{x^2}{2} \\ & v = \frac{3}{2}x^2 + C \end{cases}$$

$t = 1-3x$

$dt = -3dx$

$dx = -\frac{1}{3}dt$

$-\frac{1}{3}\int t\, dt = -\frac{1}{3} \cdot \frac{1}{2}(1-3x)^{-\frac{1}{2}}$

$= -\frac{1}{6}(1-3x)^{-\frac{1}{2}}$

$= uv - \int v\,du = \sqrt{1-3x} \cdot \frac{3}{2}x^2$

$- \int \frac{3}{2}x^2 \left(-\frac{1}{6}(1-3x)^{-\frac{1}{2}}\right)$

$(1-3x)^{\frac{1}{2}} dx$

$-\frac{1}{3}\int (1-3x)^{\frac{1}{2}} dt$

$= -\frac{1}{3} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} =$

$= -\frac{2}{9}t^{\frac{3}{2}}$

LUKA LUKAČIĆ

4) a)

$$\int_{-2}^{0} 3x\sqrt{1-3x}\,dx = \left\{\begin{array}{ll} u = 3x & dv = (1-3x)^{\frac{1}{2}} \\ du = 3\,dx & v = -\frac{2}{9}(1-3x)^{\frac{3}{2}} \end{array}\right.$$

$$= uv - \int v\,du = 3x\cdot\frac{2}{\cancel{9}_{3}}(1-3x)^{\frac{3}{2}} - \int -\frac{2}{\cancel{9}_{3}}(1-3x)^{\frac{3}{2}}\cdot 3\,dx$$

$$= \frac{2x}{3}(1-3x)^{\frac{3}{2}} + \frac{2}{3}\int(1-3x)^{\frac{3}{2}}\,dx$$

$$\int(1-3x)^{\frac{3}{2}}\,dx = \left\{\begin{array}{l} t = 1-3x \\ dt = -3\,dx \\ dx = -\frac{dt}{3} \end{array}\right\} = \frac{1}{3}\int t^{\frac{3}{2}}\,dt$$

$$= \frac{1}{3}\cdot\frac{t^{\frac{5}{2}}}{\frac{5}{2}} = \frac{1}{3}\,\frac{2t^{5/2}}{5} = \frac{2}{15}\,t^{\frac{5}{2}} = \frac{2}{15}(1-3x)^{\frac{5}{2}}$$

$$= \frac{2x}{3}(1-3x)^{\frac{3}{2}} + \frac{4}{45}(1-3x)^{\frac{5}{2}} + C$$

$$\Rightarrow \left(\frac{2\cdot 0}{3}(1-3\cdot 0)^{\frac{3}{2}} + \frac{4}{45}(1-3\cdot 0)^{\frac{5}{2}}\right) - \left(\frac{2\cdot(-2)}{3}(1-3\cdot(-2))^{\frac{3}{2}} + \frac{4}{45}(1-3(-2))^{\frac{5}{2}}\right)$$

$$= \frac{4}{45} + 13{,}169 = \underline{13{,}257} \quad \times$$

$$\int_{-2}^{0} 3x\sqrt{1-3x}\,dx = \left\{\begin{array}{l} t = 1-3x \\ dt = -3\,dx \\ x = \frac{1-t}{3} \end{array}\right\} = \int_{7}^{1} 3\cdot\frac{1-t}{\cancel{3}}\sqrt{t}\cdot\left(-\frac{1}{3}\right)dt$$

$$= -\frac{1}{3}\int_{7}^{1}\sqrt{t} - t^{3/2}\,dt = \frac{1}{3}\int_{1}^{7}\sqrt{t} - t^{\frac{3}{2}}\,dt = \frac{1}{3}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}}\right]_{1}^{7} = \frac{1}{3}\left(\frac{2}{3}\sqrt{7^3} - \frac{2}{5}\sqrt{7^5} - \frac{2}{3} + \frac{2}{5}\right)$$

$$= -13.25$$

③ $f(x,y) = \arcsin\left(\frac{x+y}{2}\right)$

D

⑤ $\int_{\pi}^{2\pi} \frac{\arctan x}{x} dx = \begin{cases} u = x^{-1} & dv = \operatorname{arctg} x \\ du = -x^{-2} & v = \frac{1}{1+x^2} \end{cases}$

$uv - \int v\,du = \frac{1}{x} \cdot \frac{1}{1+x^2} - \int \frac{1}{1+x^2} \cdot -\frac{1}{x^2} dx$

$= \frac{1}{x(1+x^2)} + \underbrace{\int \frac{1}{x^2(1+x^2)} dx}$

$\int (x^2 + x^4)^{-1} dx = \begin{cases} t = x^2 + x^4 \\ dt = (2x + 4x^3) dx \end{cases}$

$\begin{cases} u = \operatorname{arctg} x & dv = x^{-1} \\ du = \frac{1}{1+x^2} & v = \ln|x| + c \end{cases}$

$uv - \int v\,du = \operatorname{arctg} x \cdot \ln|x| - \int \frac{\ln|x|}{1+x^2} dx$

5. nastavak:

LUKA CUKAČIĆ

$$\int \frac{\ln(x)}{1+x^2}\,dx = \cancel{\int \ln|x|}$$

$\emptyset$