

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o

stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

D6

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: KRISTIAN DOŽVIĆ

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 026958122

1. Pronađi koliko iznosi:

(a) površina između krivulja $x = y^2$ i $y = 2x - 4$,

(b) određeni integral $\int_0^\pi x \sin x \, dx$?

2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = 0$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

3. Izračunati $\int_0^4 \frac{dx}{\sqrt{x^2 - 4x + 5}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje.

4. Grafički prikazati funkciju $f(x, y) = \ln(x + y)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije.

5. Odrediti $\int_0^\pi \left(-\frac{1}{4} \sin^2 t + \frac{\sqrt{3}}{4} \cos t \right) dt$.

~~0~~
10

~~0~~

15

20

Ukupno:
45

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x \ (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

b)

$$\int_0^{\pi} x \sin x dx = \left. \begin{matrix} u=x & dv=\sin x \\ du=dx & v=-\cos x \end{matrix} \right\}$$

$$\left[-x \cdot \cos x \right]_0^{\pi} - \int_0^{\pi} -\cos x dx$$

$$\left[-x \cdot \cos x + \sin x \right]_0^{\pi}$$

$$P = \pi$$

$$3. \int_0^4 \frac{dx}{\sqrt{x^2-4x+5}} = \int_0^4 \frac{dx}{\sqrt{x^2-4x+4+1}} = \int_0^4 \frac{dx}{\sqrt{(x-2)^2+1}} \left\{ \begin{matrix} x-2 = t \\ dx = dt \end{matrix} \right\}$$

$$\int_0^4 \frac{dx}{\sqrt{t^2+1}} = \left[\frac{1}{2} \left(t \sqrt{t^2+1} + \ln(t + \sqrt{t^2+1}) \right) \right]_0^4$$

$$= \left[\frac{1}{2} \left((x-2) \sqrt{(x-2)^2+1} + \ln(x-2 + \sqrt{(x-2)^2+1}) \right) \right]_0^4$$

$$P = \left(\frac{1}{2} \left(2 \sqrt{4+1} + \ln(2 + \sqrt{4+1}) \right) - \left(\frac{1}{2} \left(-2 \sqrt{4+1} + \ln(-2 + \sqrt{4+1}) \right) \right) \right)$$

$$P = \frac{2\sqrt{5}}{2} + \frac{1.99}{2} - \left(\frac{-2\sqrt{5}}{2} - \frac{1.44}{2} \right)$$

$$P = 5.91 \quad \times$$

1. a)

$$x = y^2 \Rightarrow y = \sqrt{x}$$

$$D: [-0, +\infty)$$

$$y = 2x - 4$$

$$\sqrt{x} = 2x - 4$$

$$x = 4x^2 - 16x + 16$$

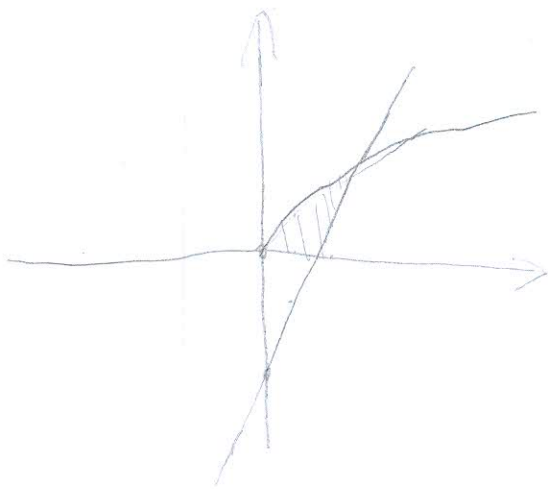
$$4x^2 - 17x + 16 = 0$$

$$x_{1,2} = \frac{17 \pm \sqrt{289 - 256}}{8}$$

$$x_{1,2} = \frac{17 \pm 5,74}{8}$$

$$x_1 = \frac{17 + 5,74}{8}$$

$$x_2 = 2,24$$



2,50

$$\int_0^{2,50} \sqrt{x} - 2x + 4 \, dx$$

X

$$= \frac{2,50^2}{2} (4 + 1 - 2 - 2,5) + 1 - 5,625$$

$$\int_0^{2,50} \sqrt{x} \, dx - \int_0^{2,50} 2x - 4 \, dx$$

2,84

$$\int_0^{2,84} \sqrt{x} \, dx - \int_0^{2,84} 2x - 4 \, dx$$

$$\int_0^{2,84} x^{\frac{1}{2}} \, dx - \left[\frac{2x^2}{2} - 4x \right]_0^{2,84}$$

$$\left[\frac{2x^{\frac{3}{2}}}{\frac{3}{2}} - x^2 + 4x \right]_0^{2,84}$$

$$= \left[\frac{2}{3} x^{\frac{3}{2}} - x^2 + 4x \right]_0^{2,84}$$

$$P = 5,18$$

5.

$$\int_0^{\pi} -\frac{1}{4} \sin^2 t + \frac{\sqrt{3}}{4} \cos t \, dt$$

$$-\frac{1}{4} \int_0^{\pi} \sin^2 t \, dt + \frac{\sqrt{3}}{4} \int_0^{\pi} \cos t \, dt$$

$$-\frac{1}{4} \int_0^{\pi} \frac{1 - \cos(2t)}{2} \, dt + \frac{\sqrt{3}}{4} \int_0^{\pi} \cos t \, dt$$

$$-\frac{1}{8} \left(\int_0^{\pi} 1 \, dt - \int_0^{\pi} \cos(2t) \, dt \right) + \frac{\sqrt{3}}{4} \int_0^{\pi} \cos t \, dt$$

$$-\frac{1}{8} \left(\int_0^{\pi} 1 \, dt - \frac{1}{2} \int_0^{\pi} \cos u \, du \right) + \frac{\sqrt{3}}{4} \int_0^{\pi} \cos t \, dt$$

$$\left[-\frac{x}{8} + \frac{1}{16} \sin(2t) + \frac{\sqrt{3}}{4} \sin t \right]_0^{\pi}$$

$$P = \pi - 0.39 = 0$$

$$P = -\frac{\pi}{8} \approx -0.39 \checkmark$$

4.

$$f(x,y) = \ln|x+y|$$

$$x+y > 0$$

$$y > -x$$

$$D_f \{ (x,y) \in \mathbb{R}^2 : y > -x \}$$

KODOMENNA

$c=1$

$$\ln|x+y| = 1$$

$$x+y = 2.71$$

$$y = 2.71 - x$$

15

$c=2$

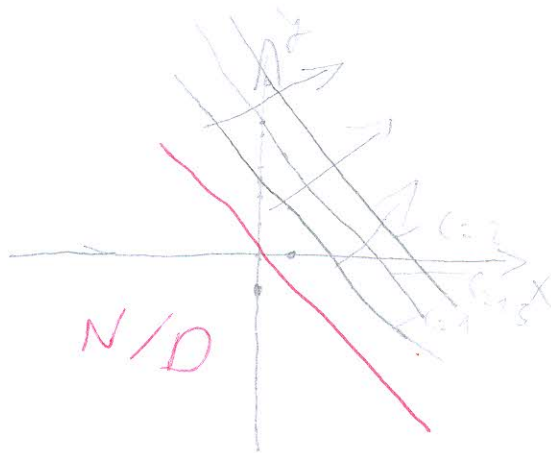
$$x+y = 7.38$$

$$y = 7.38 - x$$

$c = \frac{3}{2}$

$$x+y = 4.48$$

$$y = 4.48 - x$$



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IME I PREZIME: DORA BUŽONJA

VRIJEME POČETKA:

17:15

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

0269081190

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