MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{D}6$	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: KRISTIAN DOŠ[illegible]VIĆ **Vrijeme početka:**	
Matični broj studenta (iznad slike u indeksu): 0269 08122 [illegible]	

1. Pronaći koliko iznosi:
   (a) površina između krivulja $x = y^2$ i $y = 2x - 4$, Ø
   (b) određeni integral $\int_0^{\pi} x \sin x \, dx$? 10

2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = 0$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

3. Izračunati $\int\limits_0^4 \frac{dx}{\sqrt{x^2 - 4x + 5}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. Ø

4. Grafički prikazati funkciju $f(x, y) = \ln(x + y)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. 15

5. Odrediti $\int\limits_0^{\pi} \left(-\frac{1}{4} \sin^2 t + \frac{\sqrt{3}}{4} \cos t \right) dt$. 20

Ukupno: 45

f	$\frac{df}{dx}$
$x^{\alpha}\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_{\alpha} x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln \|\cos x\| + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int \frac{dx}{x} = \ln \|x\| + C$	$\int \cot x \, dx = \ln \|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\| x + \sqrt{x^2 \pm a^2} \right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

b)

$$\int_0^{\pi} x \sin x \, dx = \left\{ \begin{matrix} u = x & dv = \sin x \\ du = dx & v = -\cos x \end{matrix} \right\}$$

$$\left[-x \cdot \cos x\right]_0^{\pi} - \int_0^{\pi} -\cos x \, dx$$

$$\left[-x \cdot \cos x + \sin x\right]_0^{\pi}$$

$$P = \underline{\pi}$$

3.
$$\int_0^4 \frac{dx}{\sqrt{x^2-4x+5}} = \int_0^4 \frac{dx}{\sqrt{x^2-4x+4+1}} = \int_0^4 \frac{dx}{\sqrt{(x-2)^2+1}} \left\{ \begin{matrix} x-2 = t \\ dx = dt \end{matrix} \right\}$$

$$\int_0^4 \frac{dt}{\sqrt{t^2+1}} = \left[\frac{1}{2}\left(t\sqrt{t^2+1}\right) + \ln\left(t + \sqrt{t^2+1}\right)\right]_0^4$$

$$= \left[\frac{1}{2}\left((x-2)\sqrt{(x-2)^2+1} + \ln\left((x-2) + \sqrt{(x-2)^2+1}\right)\right]_0^4$$

$$P = \left(\frac{1}{2}\left(2\sqrt{4+1} + \ln\left(2 + \sqrt{4+1}\right)\right) - \left(\frac{1}{2}\left(-2\sqrt{4+1} + \ln\left(-2 + \sqrt{4+1}\right)\right)\right)\right)$$

$$P = \frac{2\sqrt{5}}{2} + \frac{1.48}{2} - \left(\frac{-2\sqrt{5}}{2} - \frac{1.44}{2}\right)$$

$$P = 5.91$$

KRISTIAN DUŠEVIĆ

1.

a)

$x = y^2 \Rightarrow y = \sqrt{x}$ $\quad D = \langle 0, +\infty \rangle$

$y = 2x - 4$

$\sqrt{x} = 2x - 4$

$x = 4x^2 - 16x + 16$

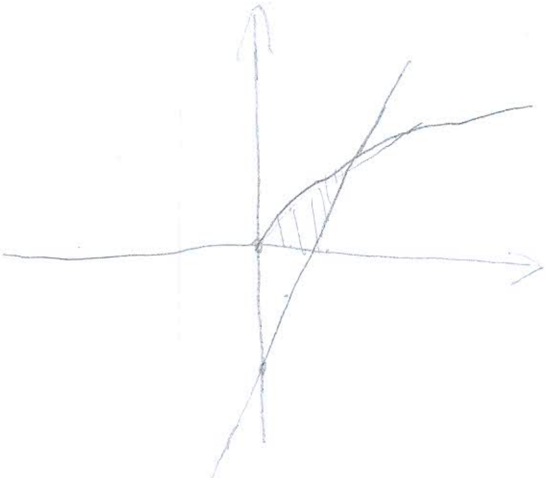
$4x^2 - 17x + 16 = 0$

$x_{1,2} = \frac{17 \pm \sqrt{289 - 256}}{8}$

$x_{1,2} = \frac{17 \pm 5.74}{8}$

$x_1 = \frac{17 + 5.74}{8}$

$x_1 = 2.84$



$\int_0^{2.84} \sqrt{x} - 2x + 4 \, dx$ X

$\int_0^{2.84} \sqrt{x}\,dx - \int_0^{2} \ldots$

$\int_0^{2.84} \sqrt{x}\,dx$

$\int_0^{2.84} t^2\,dt$

$\left[\frac{2\sqrt{x^3}}{3} \ldots \right]_0$

$P = 5.18$

Kristijan Dobović

5.

$$\int_0^\pi -\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t \, dt$$

$$-\frac{1}{4}\int_0^\pi \sin^2 t \, dt + \frac{\sqrt{3}}{4}\int_0^\pi \cos t$$

$$-\frac{1}{4}\int \frac{1-\cos(2t)}{2} dt + \frac{\sqrt{3}}{4}\int_0 \cos t$$

$$-\frac{1}{8}\left(\int_0^\pi 1 dt - \int \cos 2t \quad \left\{\begin{matrix} 2t = u \\ 2dt = du \\ dt = \frac{du}{2} \end{matrix}\right\}\right) + \frac{\sqrt{3}}{4}\int_0^\pi \cos t$$

$$-\frac{1}{8}\left(\int_0^\pi dt - \frac{1}{2}\int_0 \cos u \, du\right) + \frac{\sqrt{3}}{4}\int_0 \cos t$$

$$\left[-\frac{x}{8} + \frac{1}{16}\sin 2t + \frac{\sqrt{3}}{4}\sin t\right]_0^\pi$$

$$P = (-0.39 - 0$$

$$P = -\frac{\pi}{8} \approx -0.39$$

4.

$f(x,y) = \ln(x+y)$

$x+y>0$

$y>-x$

$D_f = \{(x,y)\in R^2 : y>-x\}$

KODOMENA?

15

$c=1$

$\ln(x+y)=1$

$x+y=2.71$

$y=2.71-x$

$c=2$

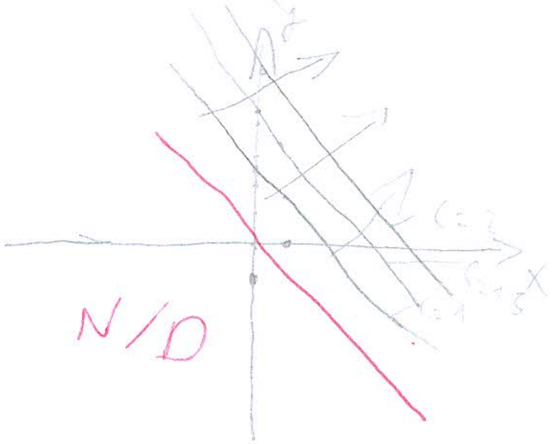
$x+y=7.38$

$y=7.38-x$

$c=\frac{3}{2}$

$x+y=4.48$

$y=4.48-x$



MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{D}6$

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: DORA BUŽONJA

VRIJEME POČETKA: 17:15

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

0269081190

1. Pronaći koliko iznosi:

   (a) površina između krivulja $x = y^2$ i $y = 2x - 4$, 10

   (b) određeni integral $\int_0^\pi x \sin x \, \mathrm{d}x$? 10

2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = 0$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

3. Izračunati $\displaystyle\int_0^4 \frac{\mathrm{d}x}{\sqrt{x^2 - 4x + 5}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. ~~0~~ 10

4. Grafički prikazati funkciju $f(x, y) = \ln(x + y)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije.

5. Odrediti $\displaystyle\int_0^\pi \left(-\frac{1}{4} \sin^2 t + \frac{\sqrt{3}}{4} \cos t \right) \mathrm{d}t.$ ~~0~~

Ukupno: 30

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
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$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln \|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln \|x\| + C$	$\int \cot x \, \mathrm{d}x = \ln \|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\| x + \sqrt{x^2 \pm a^2} \right\| + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

DORA BUŽONJA

$$\int_0^4 \frac{dx}{\sqrt{x^2-4x+5}} = \ln\left|(x-2)+\sqrt{(x-2)^2+1}\right| \Big|_0^4 = \ln\left|2+\sqrt{5}-\left(-2+\sqrt{5}\right)\right| =$$

$$= \ln\left|2+\sqrt{5}+2-\sqrt{5}\right| = \ln|4| = 1.3863$$

$$\int \frac{dx}{\sqrt{x^2-4x+5}} = \int \frac{dx}{\sqrt{(x^2-4x+4)-4+5}} = \int \frac{dx}{\sqrt{(x-2)^2+1}} = \ln\left|(x-2)+\sqrt{(x-2)^2+1}\right|$$

$a=0,\ b=4$

$$h = \frac{b-a}{n} = \frac{\cancel{4}^{1}}{\cancel{8}_{2}} = \frac{1}{2} = 0.50$$

k	0	1	2	3	4	5	6	7	8
x_k	0	0.5	1	1.5	2	2.5	3	3.5	4
y_k	0.44721	0.5547	0.7071	0.8944	1	0.8944	0.7071	0.5547	0.44721

$$y_0 = \frac{1}{\sqrt{5}} = 0.44721$$

$$y_1 = f(x_0) = \frac{1}{\sqrt{(0.5)^2-4\cdot 0.5+5}} = 0.5547$$

$$y_2 = f(x_1) = \frac{1}{\sqrt{1^2-4\cdot 1+5}} = 0.7071$$

$$y_3 = f(x_3) = \frac{1}{\sqrt{(1.5)^2-4\cdot 1.5+5}} = 0.8944$$

$$y_4 = f(x_4) = \frac{1}{\sqrt{2^2-4\cdot 2+5}} = 1$$

$$y_5 = f(x_5) = \frac{1}{\sqrt{(2.5)^2-4\cdot 2.5+5}} = 0.8944$$

$$y_6 = f(x_6) = \frac{1}{\sqrt{3^2-4\cdot 3+5}} = 0.7071$$

$$y_7 = f(x_7) = \frac{1}{\sqrt{(3.5)^2-4\cdot 3.5+5}} =$$

$$= 0.5547$$

$$y_8 = f(x_8) = \frac{1}{\sqrt{4^2-4\cdot 4+5}} =$$

$$= 0.44721$$

$$S = \frac{b-a}{3n}\left[y_0 + y_n + 2\cdot(y_2 + y_4 + y_6 + y_8 + \ldots) + 4\cdot(y_1 + y_3 + y_5 + y_7 + \ldots)\right] =$$

$$= \frac{\cancel{4}^{1}}{\cancel{24}_{6}}\left[0.44721 + 0.44721 + 2\cdot(0.7071 + 1 + 0.7071) + 4\cdot(0.5547 + 0.8944 + 0.8944 + 0.5547)\right] =$$

$$= \frac{1}{6}\cdot\left[0.89442 + 4.8284 + 11.5928\right] = 2.8859$$

PONUDILI STE 2 RJEŠENJA. SAMO JEDNO JE TOČNO.

10

5.

$$\int_0^{\pi}\left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right)dt = -\frac{1}{8} - \frac{1}{8}\cos(2t) + \frac{\sqrt{3}}{4}\sin t\Big|_0^{\pi} =$$

$$= -\frac{1}{8} - \frac{1}{8}\cos(2\pi) + \frac{\sqrt{3}}{4}\sin\pi - \left(-\frac{1}{8} - \frac{1}{8}\cos(0) + \frac{\sqrt{3}}{4}\sin 0\right) =$$

$$= 0$$

$\emptyset$

$$\int\left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right)dt = -\frac{1}{4}\int\sin^2 t\,dt + \frac{\sqrt{3}}{4}\int\cos t\,dt =$$

$$= -\frac{1}{4}\cdot\left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right) + \frac{\sqrt{3}}{4}\sin t =$$

$$= -\frac{1}{8} - \frac{1}{8}\cos(2x) + \frac{\sqrt{3}}{4}\sin t$$

DORA BUŽONJA

1. a) $x = y^2$

$y = 2x - 4$ $\quad x \longleftrightarrow y$

$y = x^2$

$x = 2y - 4$

$y = x^2$

$-2y = -x - 4$

$2y = x + 4$

$y = \frac{x+4}{2} = \frac{x}{2} + \frac{\cancel{4}^{2}}{\cancel{2}_{1}} = \frac{x}{2} + 2$

$x^2 = \frac{x}{2} + 2$

$x^2 - \frac{x}{2} - 2 = 0 \, / \cdot 2$

$2x^2 - x - 4 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1+32}}{4} = \frac{1 \pm \sqrt{33}}{4}$

$x_1 = \frac{1+\sqrt{33}}{4} = 1.7$

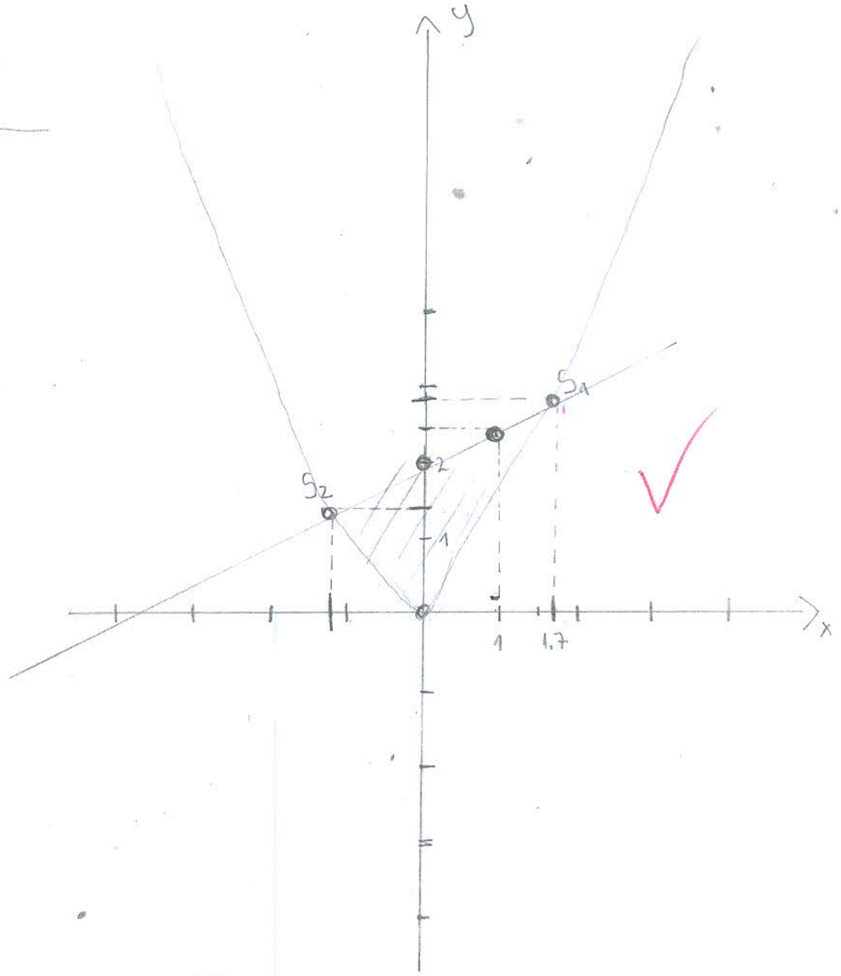
$x_2 = \frac{1-\sqrt{33}}{4} = -1.2$

$y_1 = \frac{1.7}{2} + 2 = 2.85$

$y_2 = -\frac{1.2}{2} + 2 = 1.4$

$S_1(1.7, 2.85)$

$S_2(-1.2, 1.4)$



$y = x^2, \ a > 0, \ \cup$

$x^2 = 0 \, / \sqrt{\ }$

$\boxed{x = 0}$

$y = \frac{x}{2} + 2$

x	0	1
y	2	2.5

$P = \int_{-1.2}^{1.7} \left(\frac{x}{2} + 2 - x^2\right) dx = \int_{-1.2}^{1.7} \frac{1}{2} x + 2 \int_{-1.2}^{1.7} dx - \int_{-1.2}^{1.7} x^2 dx =$

$= \frac{1}{2} \cdot \frac{x^2}{2} + 2x - \frac{x^3}{3} \Big|_{-1.2}^{1.7} =$

$= \frac{1}{2} \cdot \frac{(1.7)^2}{2} + 2 \cdot (1.7) - \frac{(1.7)^3}{3} - \left(\frac{1}{2} \cdot \frac{(-1.2)^2}{2} + 2 \cdot (-1.2) - \frac{(-1.2)^3}{3}\right) =$

$= \frac{1}{2} \cdot \frac{2.89}{2} + 3.4 - \frac{4.913}{3} - \left(\frac{1}{2} \cdot \frac{1.44}{2} - 2.4 - \frac{-1.728}{3}\right) =$

$= \frac{2.89}{4} + 3.4 - \frac{4.913}{3} - \frac{1.44}{4} + 2.4 - \frac{1.728}{3} = 3.95$ //

b) $\int x\sin x\,dx = \begin{bmatrix} x=u & dv=\sin x\,dx\ /\int \\ dx=du & v=\int\sin x\,dx \\ & v=-\cos x \end{bmatrix} = -x\cos x - \int -\cos x\,dx =$

$= -x\cos x + \int\cos x\,dx = -x\cos x + \sin x + C$

$$\int_0^{\pi} x\sin x\,dx = -x\cos x + \sin x \Big|_0^{\pi} = -\pi\cos\pi + \sin\pi - (\sin 0) = \pi$$ ✓

3.) $\int_0^4 \frac{dx}{\sqrt{x^2-4x+5}}$

$a=0$ $\quad h=\frac{b-a}{n}=\frac{4-0}{4}=\frac{4}{4}=1$

$b=4$

k	0	1	2	3	4
x_k	0	1	2	3	4
y_k	0,44721	0,70711	1	0,70711	0,44721

$y_0 = f(x_0) = \frac{1}{\sqrt{5}} = 0.44721$

$y_1 = f(x_1) = \frac{1}{\sqrt{1-4+5}} = 0.70711$

$y_2 = f(x_2) = \frac{1}{\sqrt{2^2-4\cdot2+5}} = 1$

$y_3 = f(x_3) = \frac{1}{\sqrt{3^2-4\cdot3+5}} = 0.70711$

$y_4 = f(x_4) = \frac{1}{\sqrt{4^2-4\cdot4+5}} = 0,44721$

$x^2-4x+5 \neq 0$

$x_{1,2} = \frac{4\pm\sqrt{16-20}}{2} = \frac{4\pm\sqrt{-4}}{2}$

$D_f = \mathbb{R}$

$\int_0^4 \frac{dx}{\sqrt{x^2-4x+5}} = \ln\left|x-2+\sqrt{(x-2)^2+1}\right|\Big|_0^4 =$

$= \ln\left|4-2+\sqrt{(4-2)^2+1} - \left(-2+\sqrt{(-2)^2+1}\right)\right| =$

$= \ln\left|2+\sqrt{5}+2-\sqrt{5}\right| = 1.3863.$

$\int\frac{dx}{\sqrt{x^2-4x+5}} = \int\frac{dx}{\sqrt{(x^2-4x+4)+1}} = \int\frac{dx}{\sqrt{(x-2)^2+1^2}} =$

$= \ln\left|x-2+\sqrt{(x-2)^2+1}\right|$

$S = \frac{b-a}{3n}\left[y_0+y_n+2\cdot(y_2+y_4+\dots)+4\cdot(y_1+y_3+\dots)\right] =$

$= \frac{4}{12}\left[0.44721+0.44721+2\cdot1+4\cdot(0.70711+0.70711)\right] =$

$= \frac{1}{3}\left[2,89442+5,65688\right] = \frac{1}{3}\cdot 8.5513 = 2,85043$ ✓

OCIJENJENO VEĆ PRIJE

4) $f(x,y) = \ln(x+y)$

DORA BUŽONJA

$D_f.: \quad x+y>0$

$D_f. = \{(x,y) \in \mathbb{R}^2 : x>0, y>0\}$

KODOMENA :

$C = \ln(x+y)$

$D(f) = \langle 0, +\infty \rangle$

<u>MATEMATIKA 2</u>: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}3$

Ime i prezime: AUGUSTIN PTIČAR **Vrijeme početka:**

Matični broj studenta (iznad slike u indeksu): [illegible]

Popunjava nastavnik
Broj ↓ bodova

1. Grafički prikazati funkciju $f(x,y) = \ln(x+y)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Ø

2. Odrediti $\displaystyle\int\limits_0^\pi \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) \mathrm{d}t$.

3. Pronaći koliko iznosi: Ø
   (a) površina između krivulja $x = y^2$ i $y = 2x - 4$,
   (b) određeni integral $\int_0^\pi x \sin x \, \mathrm{d}x$? 9

4. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = 0$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje. 20

5. Izračunati $\displaystyle\int\limits_0^4 \frac{\mathrm{d}x}{\sqrt{x^2 - 4x + 5}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. Ø

Ukupno: 29

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

AUGUSTIN PTIČAR

2. $\int_0^{\pi}\left(-\frac{1}{4}\sin^2 t+\frac{\sqrt{3}}{4}\cos t\right)dt=$

$(x-2)(x+2)=$
$=x^2+\cancel{2x}-\cancel{2x}+4$
$=x^2-4$

$=\int_0^{\pi}-\frac{1}{4}\sin^2 t\,dt+\int_0^{\pi}\frac{\sqrt{3}}{4}\cos t\,dt=$

$=-\frac{1}{4}\int_0^{\pi}\sin^2 t\,dt+\int_0^{\pi}\frac{\sqrt{3}}{4}\cos t\,dt=$

$=-\frac{1}{4}^{*}\left[\begin{array}{c|c}\sin t=u & d\\ \cos t\,dt=du & \end{array}\right]=\int-\underbrace{u^2}_{U}\cdot\underbrace{\frac{\cancel{1}\,du}{\cos t}}_{dV}$

$\left[\begin{array}{l|l}U=-u^2 & dv=\frac{1}{\cos t}\\ du=-2u & V=\end{array}\right.$

---

4. $y''-5y'+4y=0$

$y(0)=5\quad T(0,5)$
$y'(0)=8\quad T(0,8)$

$\lambda^2-5\lambda+4=0$

$\lambda_{1,2}=\frac{5\pm\sqrt{25-16}}{2}$

$\lambda_{1,2}=\frac{5\pm\sqrt{9}}{2}$

$\lambda_{1,2}=\frac{5\pm3}{2}$

$\lambda_1=1$

$\lambda_2=4$

$y=C_1e^x+C_2e^{4x}$

$y'=(C_1e^x)'+(C_2e^{4x})'$

$y'=C_1e^x+4C_2e^{4x}$

$y''=C_1e^x+16C_2e^{4x}$

$C_1e^x+16C_2e^{4x}-5(C_1e^x+4C_2e^{4x})+4(C_1e^x+C_2e^{4x})=0$

$\cancel{C_1e^x}+\cancel{16C_2e^{4x}}-\cancel{5C_1e^x}-\cancel{20C_2e^{4x}}+\cancel{4C_1e^x}+\cancel{4C_2e^{4x}}=0$

$0=0$, TOČNO

$y = C_1 e^x + C_2 e^{4x}$; $y(0) = 5$ $T(0,5)$

$5 = C_1 + C_2$

$\underline{C_1 = 5 - C_2}$

$y'' = C_1 e^x + 4C_2 e^{4x}$; $y(0) = 8$ $T(0,8)$

$8 = C_1 + 4C_2$

$8 = (5 - C_2) + 4C_2$

$8 = 5 - C_2 + 4C_2$

$8 = 5 + 3C_2$

$3C_2 = 8 - 5$

$3C_2 = 3$

$C_2 = 1$

$C_1 = 5 - C_2$

$C_1 = 5 - 1$

$\underline{C_1 = 4}$ //

$y = C_1 e^x + C_2 e^{4x}$

$\underline{y = 4e^x + e^{4x}}$

$y' = C_1 e^x + 4C_2 e^{4x}$

$\underline{y' = 4e^x + 4e^{4x}}$

$y'' = C_1 e^x + 16C_2 e^{4x}$

$\underline{y'' = 4e^x + 16e^{4x}}$

3.

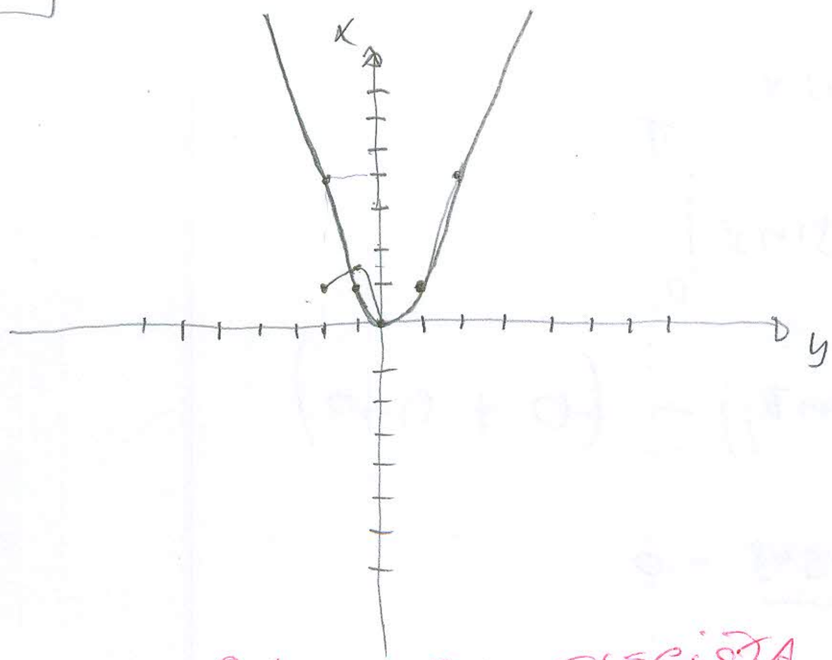
a) $x = y^2$ AUGUSTIN PTIČAR

$y = 2x - 4$ → ZAMJENA VARIJABLI

$2x = y + 4$

$\boxed{x = \dfrac{y+4}{2}}$

y	-2	-1	0	1	2
$x = y^2$	4	1	0	1	4
$x = \frac{y+4}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	4



OVO NISU SJECIŠTA

$$\int\left(\frac{y+4}{2} - y^2\right)dy = \int\left(\frac{y+4}{2}\right)dy - \int y^2 dy$$

$$= \frac{1}{2}\int_0^{\frac{3}{2}} (y+4)\,dy - \frac{y^3}{3}\Big|_0^{\frac{3}{2}}$$

$$= \frac{1}{2}\left(\frac{y^2}{2}\Big|_0^{\frac{3}{2}} + 4y\Big|_0^{\frac{3}{2}}\right) - \frac{y^3}{3}\Big|_0^{\frac{3}{2}}$$

$$= \frac{1}{2}\left(\frac{1}{2}\left(\left(\frac{3}{2}\right)^2 - 0\right) + 4\frac{3}{2}\right) - \frac{1}{3}\left(\left(\frac{3}{2}\right)^3 - 0\right)$$

$$= \frac{1}{2}\left(\frac{1}{2}\cdot\frac{9}{4} + 6\right) - \frac{1}{3}\left(\frac{9}{4}\right) = \frac{1}{2}\left(\frac{9}{8} + 6\right) - \frac{3}{4}$$

$$= \frac{1}{2}\left(\frac{9+48}{8}\right) - \frac{3}{4} = \frac{1}{2}\,\frac{57}{8} - \frac{3}{4} = \frac{57}{16} - \frac{3}{4} = \frac{57-12}{16} = \frac{45}{16} = 2{,}81$$

b) $\int_0^{\pi} x \sin x \, dx = \left[\begin{array}{ll} u = x & dv = \sin x \,/\int \\ du = dx & v = -\cos x \end{array} \right.$

$uv - \int v \, du =$

$-x\cos x + \int \cos x \, dx$

$= -x\cos x + \sin x \Big|_0^{\pi}$

$= \left(-\pi(\cos\pi) + (\sin\pi)\right) - \left(-0 + \sin 0\right)$

$= \underline{-3.1369} + \underline{0.0548} - 0$

$= \boxed{3.0821}$

$\cos\pi = -1$

$\sin\pi = 0$

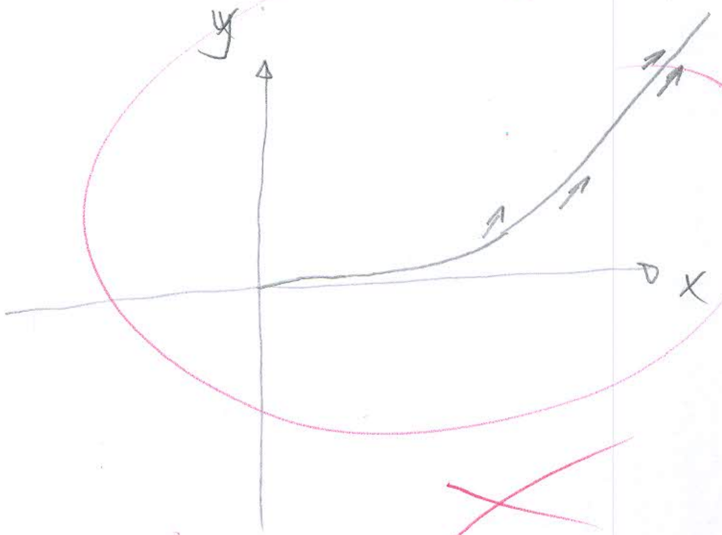
REZULTAT $= \pi = 3.14$

AUGUSTIN [illegible]

1. $f(x,y) = \ln(x+y)$

$\boxed{x + y > 0}$

$D_f : \mathbb{R} > 0$



~~x, y~~	~~1~~	~~2~~	~~3~~	~~4~~
~~ln(x+y)~~	~~2 / 0,69~~			

5. $\int_0^4 \frac{dx}{\sqrt{x^2-4x+5}}$

$\sqrt{x^2-4x+5 \geq 0} \Big/^2$

$x^2-4x+5=0$

$x_{1,2} = \frac{4 \pm \sqrt{16-20}}{2}$

$\boxed{x_{1,2} = \frac{4 \pm \sqrt{-4}}{2} \rightarrow \text{NEMA NULTOČKE}}$

0

$\cancel{\boxed{(x-2)(x+2)+5}}$

$\cancel{\int_0^4 \frac{dx}{\sqrt{(x-2)(x+2)+5}}}$

$\int_0^4 \frac{\pi}{\sqrt{x^2-4x+5}}$

$\cancel{x(x-4)+5=0}$

$\boxed{x=0 \quad x=4}$

$\sqrt{x^2-4x+5=0} \Big/^2$

$x^2-4x+5=0$

$x(x-4)=-5$

$\boxed{x=-5}$

$(x-4)=-5$

$x-4=-5$

$\boxed{x=-1}$

$$\frac{1}{\sqrt{x^2-4x+5}} = \frac{Ax^2+Bx+C}{\sqrt{x^2-4x+5}} \Big/ \cdot \sqrt{x^2-4x+5}$$

$$\frac{\sqrt{x^2-4x+5}}{\sqrt{x^2-4x+5}} = Ax^2+Bx+C$$

$$1 = Ax^2+Bx+C$$

$0 = A$

$0 = B$

$\boxed{1 = C}$

$$\frac{1}{\sqrt{x^2-4x+5}} = \frac{1}{\sqrt{}}$$

PTIČAR

$$-x\cos x\Big|_0^{\pi} + \sin x\Big|_0^{\pi}$$

$$(-\pi\cos\pi)-(0)$$

$$+(\sin\pi - \sin 0)$$

$$= -3.1369 - 0 +$$

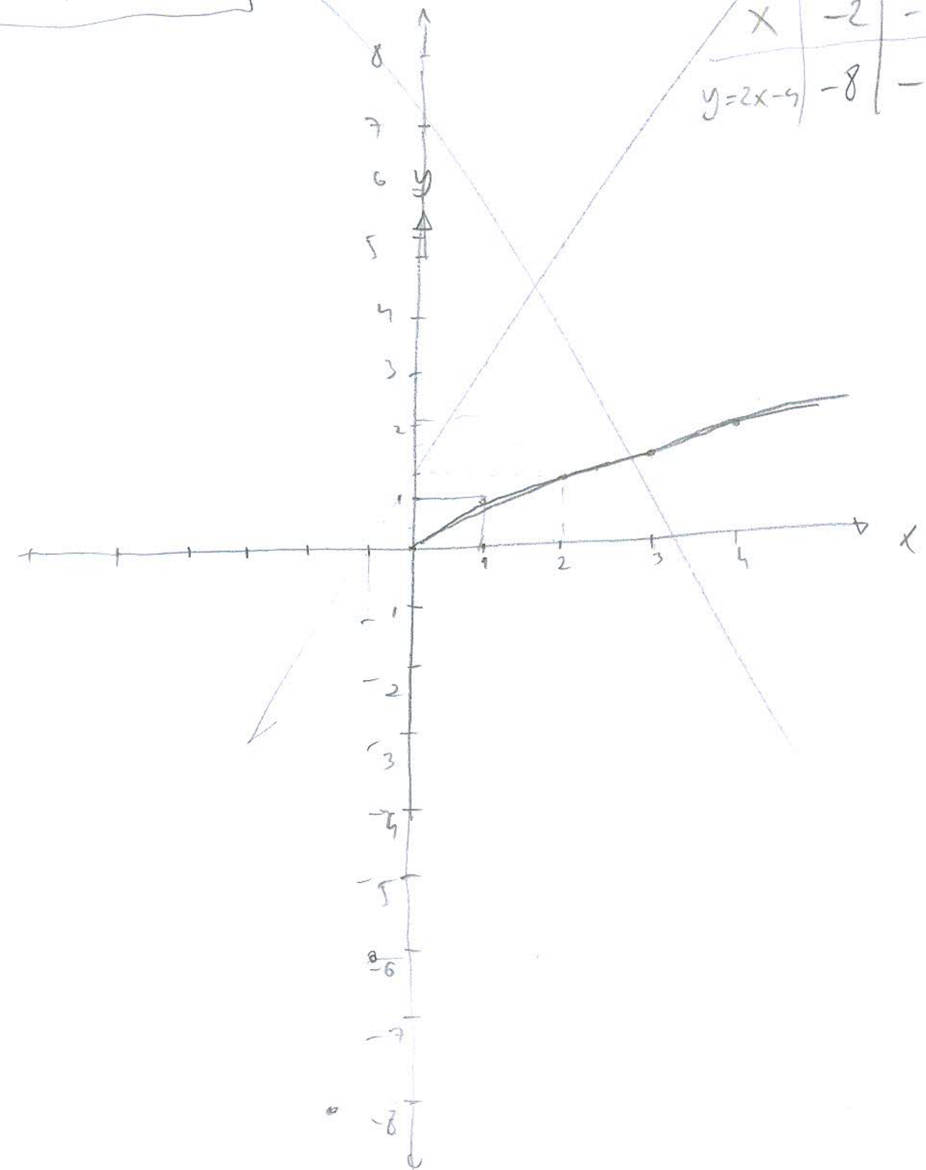
3.

$x = y^2 \Rightarrow y = \sqrt{x}$

$\boxed{y = 2x-4}$

x	~~-2~~	~~-1~~	0	1	2	3	4
$y=\sqrt{x}$	~~-1~~		0	1	$\sqrt{2}$	$\sqrt{3}$	2

x	-2	-1	0	1	2
$y=2x-4$	-8	-6	-4	-2	0



MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A3 **IME I PREZIME:** JOSIP JANKOVIĆ **VRIJEME POČETKA:** **MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Grafički prikazati funkciju $f(x,y) = \ln(x+y)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. 10

2. Odrediti $\displaystyle\int_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t \right) \mathrm{d}t.$

3. Pronaći koliko iznosi:

   (a) površina između krivulja $x = y^2$ i $y = 2x - 4$,

   (b) određeni integral $\int_0^{\pi} x \sin x \, \mathrm{d}x$? 10

4. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = 0$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

5. Izračunati $\displaystyle\int_0^4 \frac{\mathrm{d}x}{\sqrt{x^2 - 4x + 5}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje.

Ukupno: 20

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \, \mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

JOSIP JANKOVIĆ

5) $\int_{0\,a}^{4\,b} \frac{1}{\sqrt{x^2-4x+5}}\,dx$

x^2-4x+5

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{(-4)^2-4\cdot 5}}{2} = //$

NEPRAVILNI INTEGRAL ?

$h = \frac{b-a}{n} = \frac{4-0}{6} = \frac{2}{3}$

$x_0 = 0$

i	$i\cdot h$	$x_i = x_0 + ih$	$y_i\,(f\,x)$
0	0	0	$\frac{\sqrt{5}}{5}$
1	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{5}$
2	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{3\sqrt{13}}{13}$
3	2	2	1
4	$\frac{8}{3}$	$\frac{8}{3}$	$\frac{3\sqrt{13}}{13}$
5	$\frac{10}{3}$	$\frac{10}{3}$	$\frac{3}{5}$
6	4	4	$\frac{\sqrt{5}}{5}$

$-0,180196866$

$-1,10405$

$$\int_0^4 \frac{1}{\sqrt{x^2-4x+5}} \approx \frac{2}{3}\cdot\left(\frac{\frac{\sqrt{5}}{5}-\frac{\sqrt{5}}{5}}{2} + \frac{3}{5} + \frac{3\sqrt{13}}{3} + 1 + \frac{3\sqrt{13}}{13} + \frac{3}{5}\right)$$

≈ 4.4250 ✗

TOČNO RJEŠENJE: 7.8873

2. $\int_0^{\pi}\left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right)dt$

$I_1 = -\frac{1}{4}\int_0^{\pi}\sin^2 t$

$x = \sin^2 t$

$dx = \sin 2t\,dt$

$dt = \frac{dx}{\sin^2 t}$

$-\frac{1}{4}\int_0^{\pi} x + \frac{\sqrt{3}}{4}\int \cos$

JOSIP JANKOVIĆ

(1.) $f(x,y)=\ln(x+y)$

$x+y>0$

$y>-x$

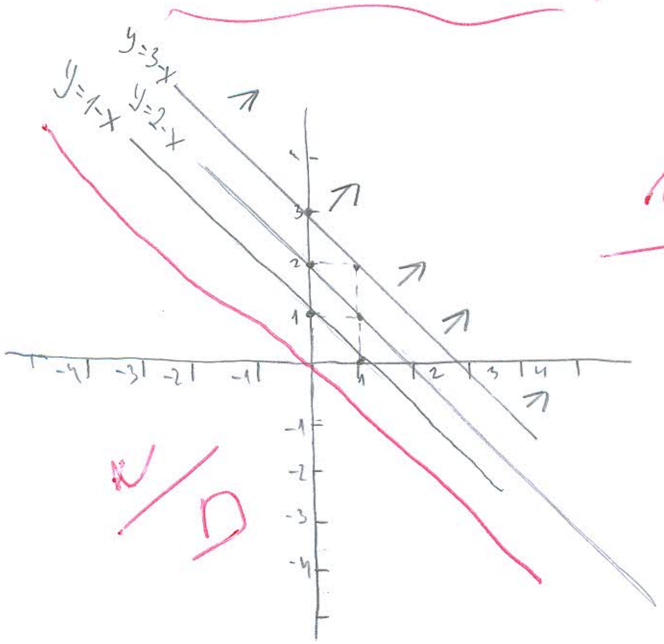
$D: \mathbb{R}\ \langle 0,+\infty\rangle$ ✗

$\ln(x+y)=c$

$\ln(x+y)=\ln e^c$

$x+y=c$

$y=c-x$



10

N/D

$C_1=1$

x	0	1
y	1	0

$C_3=3$

x	0	1
y	3	2

$C_2=2$

x	0	1
y	2	1

(3) b) $\int_0^{\pi} x\sin x\,dx \;\Big|\; \begin{matrix} u=x & dv=\int\sin x\,dx \\ du=1dx & v=-\cos x \end{matrix}$ $u\cdot v-\int v\,du$

$x\cdot(-\cos x)\Big|_0^{\pi}-\int_0^{\pi}-\cos x\,dx$

$x\cdot(-\cos x)\Big|_0^{\pi}+\int_0^{\pi}\cos x\,dx = x\cdot(-\cos x)\Big|_0^{\pi}+\sin x\Big|_0^{\pi}=\pi$ ✓

a) $x=y^2$ GORNJA

$y=2x-4$ DONJA

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}3$

IME I PREZIME: NIKOLA MILEPĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0265-2013

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Grafički prikazati funkciju $f(x,y) = \ln(x+y)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije.

2. Odrediti $\displaystyle\int_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) \mathrm{d}t.$

3. Pronaći koliko iznosi:

   (a) površina između krivulja $x = y^2$ i $y = 2x - 4$,

   (b) određeni integral $\int_0^{\pi} x \sin x \, \mathrm{d}x$?

10

4. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = 0$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

5. Izračunati $\displaystyle\int_0^4 \frac{\mathrm{d}x}{\sqrt{x^2 - 4x + 5}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje.

Ukupno: 10

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^{α} $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_{\alpha} x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^{\alpha}\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

③a) $x = y^2 \Rightarrow y = \sqrt{x}$

$y = 2x - 4$

x	0	2	4	9	16
$\sqrt{x}$	0	1.41	2	3	4

$\sqrt{0} = 0$

$\sqrt{2} \approx 1.41$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

x	0	-1	1	2	-2	3
$2x-4$	-4	-6	-2	0	-8	2

$2 \cdot 0 - 4 = -4$

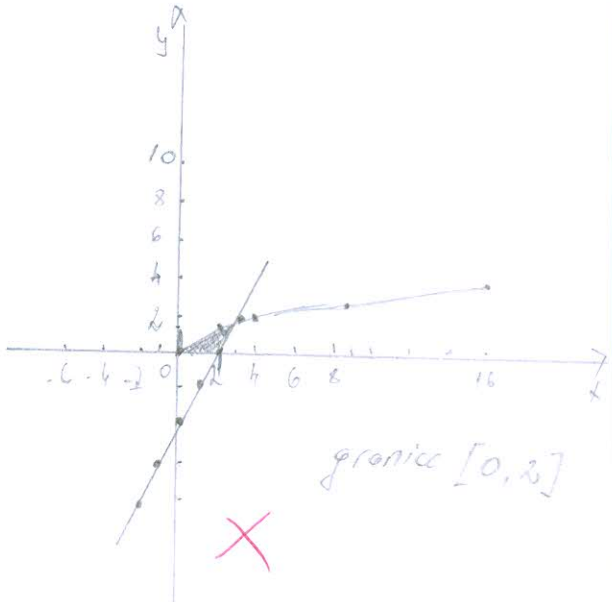
$2 \cdot (-1) - 4 = -2 - 4 = -6$

$2 - 4 = -2$

$2 \cdot (-2) - 4 = -4 - 4 = -8$

$2 \cdot 2 - 4 = 4 - 4 = 0$

$2 \cdot 3 - 4 = 6 - 4 = 2$



granice [0, 2]

$$\int_0^2 [(\sqrt{x}) - (2x-4)]\,dx = \int_0^2 (x^{\frac{1}{2}} - 2x + 4)\,dx = \left[\int x^{\frac{1}{2}}dx - 2\int x\,dx + 4\int dx\right]_0^2$$

$\sqrt{x} = x^{\frac{1}{2}}$

$$\left[\frac{2}{3}x^{\frac{3}{2}} - 2\cdot\frac{1}{2}x^2 + 4x\right]_0^2 = \left[\frac{2}{3}\cdot(2)^{\frac{3}{2}} - 2^2 + 4\cdot 2\right] - 0 = 5{,}89$$

③ b) $\displaystyle\int_0^{\pi} x\sin x\,dx = \left\{\begin{matrix} u = x & dv = \sin x\,dx \\ du = dx & \int dv = \int \sin x\,dx \\ & v = -\cos x \end{matrix}\right\} = \left[-x\cos x - \int -\cos x\,dx\right]_0^{\pi}$

$$= \left[-x\cos x + \sin x\right]_0^{\pi} = \left[-\pi\cos\pi + \sin\pi\right] - 0 = \pi$$

10

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ D6

IME I PREZIME: Franko Ba[illegible]rić

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17 - [illegible]2 - 0204 - 2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Pronaći koliko iznosi:
   (a) površina između krivulja $x = y^2$ i $y = 2x - 4$,
   (b) određeni integral $\int_0^\pi x \sin x \,\mathrm{d}x$?

10

2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = 0$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

3. Izračunati $\displaystyle\int_0^4 \frac{\mathrm{d}x}{\sqrt{x^2 - 4x + 5}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje.

∅

4. Grafički prikazati funkciju $f(x, y) = \ln(x + y)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije.

5. Odrediti $\displaystyle\int_0^\pi \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) \mathrm{d}t$.

∅

Ukupno: 10

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5. $\int_0^\pi \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) dt$

$f(x) = -\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t$

$D_f = \mathbb{R}$

$[0, \pi] \in \mathbb{R}$

Integral nije nepravi

1\. b) $\int_0^{\pi} x \sin x \, dx = -x\cos x + \sin x \Big|_0^{\pi}$

$= -\pi \cos \pi + \sin \pi - (-0 \cos 0 + \sin 0)$

$\to 0$

$= -\pi \cdot (-1) + 0$

$= \pi = 3.14159265$ ✓

$f(x) = x \sin x \, dx$

$D(f) = \mathbb{R}$

$[0, \pi] \in \mathbb{R}$

Integral nije nepravi jer je $[0, \pi]$ u domeni

$$\int x \sin x \, dx = \left[\begin{array}{ll} u = x & dv = \sin x \, dx \\ du = dx & v = -\cos x \end{array} \right]$$

$= -x\cos x - \int -\cos x \, dx$

$= -x \cos x + \sin x$

3\. $\int_0^4 \frac{dx}{\sqrt{x^2 - 4x + 5}}$

$d = 4$

$x_0 = a = 0$

$x_2 = b = 4$

$x_1 = \frac{a+b}{2} = \frac{4}{2} = 2$

k	0	1	2
x_k	0	2	4
f_k	0.4472	1	0.4472

$S = \frac{d}{6} (f_0 + 4 f_1 + f_2)$

$= \frac{4^2}{6_3} (0.4472 + 4 \cdot 1 + 0.4472)$

$= \frac{2}{3} (0.4472 + 4 + 0.4472)$

$= \frac{2}{3} \cdot 4.8944$

$= 3.2629$

TOČNO RJEŠ. = 2.88

REL. GREŠKA > 10%

Ø

5. $\int (-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t)\,dt$

$= $ [illegible]

$= \int -\frac{1}{4}\sin^2 t\,dt + \int \frac{\sqrt{3}}{4}\cos t\,dt$

$= -\frac{1}{4}\int \sin^2 t\,dt + \frac{\sqrt{3}}{4}\int \cos t\,dt$

$= \frac{1}{3}\cos^2 t + \frac{\sqrt{3}}{4}\sin t$

$\int_0^{\pi} (-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t)\,dt = \frac{1}{3}\cos^2 t + \frac{\sqrt{3}}{4}\sin t \Big|_0^{\pi}$

$= \frac{1}{3}\cos^2 \pi + \frac{\sqrt{3}}{4}\sin \pi - (\frac{1}{3}\cos^2 0 + \frac{\sqrt{3}}{4}\sin 0)$

$= \frac{1}{3} + 0 - (\frac{1}{3} + 0$

$= \frac{1}{3} - \frac{1}{3} = 0$

Franko Bakarić

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{D}6$

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: JOSIP GAUTA

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0385-2014

1. Pronaći koliko iznosi:

   (a) površina između krivulja $x = y^2$ i $y = 2x - 4$, ∅

   (b) određeni integral $\int_0^\pi x \sin x \, dx$? ∅

2. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = 0$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

3. Izračunati $\displaystyle\int_0^4 \frac{dx}{\sqrt{x^2 - 4x + 5}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje.

4. Grafički prikazati funkciju $f(x, y) = \ln(x + y)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije.

5. Odrediti $\displaystyle\int_0^\pi \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) dt$. ∅

Ukupno: ∅

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

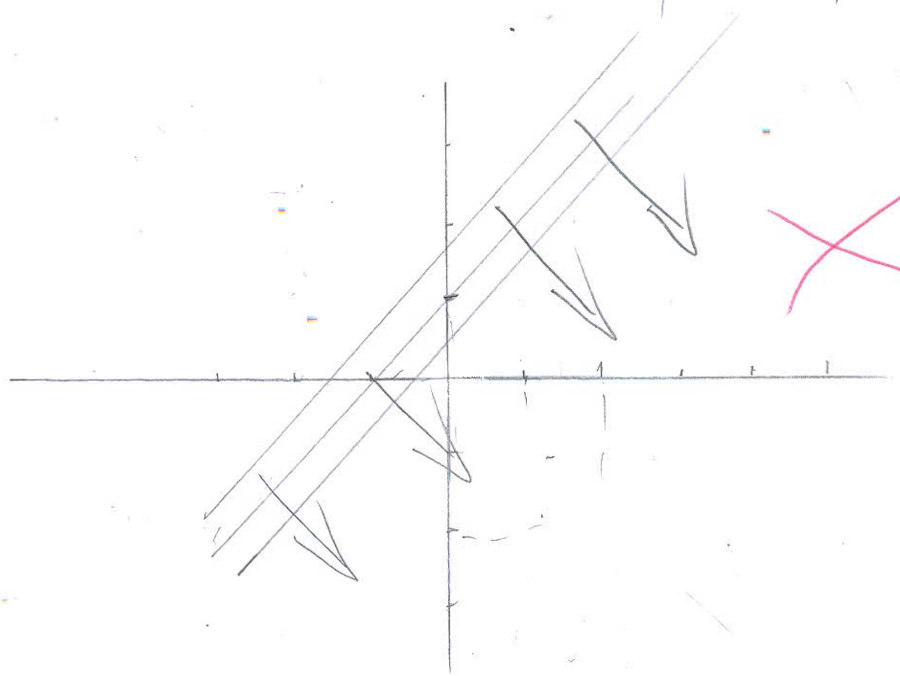
VIDI BUŽONJA

Josip Gauta

(4) $f(x,y) = \ln(x+y)$

$Df: \mathbb{R}^2$

$Kf: \langle 0, +\infty\rangle$



$1 = \ln(x+y)$

$\ln(x+y) = 1$

$2 = \ln(x+y)$

$\ln(x+y) = 2$

$3 = \ln(x+y)$

$\ln(x+y) = 3$

$\frac{\partial f}{\partial x} = \ln(x+y)$

$\frac{\partial f}{\partial y} = \frac{1}{x+y}\, x$

$\frac{1}{x+y}\, y \Rightarrow$

(5) $\int_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) dt \qquad \sin^2 t = \frac{1-\cos(2x)}{2}$

$\int_0^{\pi} \left(-\frac{1}{4}\left(\frac{1-\cos(2t)}{2}\right) + \frac{\sqrt{3}}{4}\left(\frac{1-t^2}{1+t^2}\right)\right) dt \qquad \cos t = \frac{1-t^2}{1+t^2}$

$\int_0^{\pi} \left(-\frac{1}{4}\left(\frac{1}{2} - \cos t\right) + \frac{\sqrt{3} - \sqrt{3}t^2}{4+4t^2}\right) dt$

$\int_0^{\pi} \left(-\frac{1}{8} + \frac{\cos t}{4} + \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} t^2\right)$

$-\int_0^{\pi} \frac{1}{8} + \frac{1}{4}\int_0^{\pi} \cos t + \int_0^{\pi} \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\int_0^{\pi} t^2$

$\frac{1}{8}[\pi - 0] + \frac{1}{4}[\cos\pi - \cos 0] + \frac{\sqrt{3}}{4}[\pi - 0] - \frac{\sqrt{3}}{4}\left[\frac{\pi^3}{3} - \frac{0^3}{3}\right] \Rightarrow$

$\frac{1}{8}\pi + \frac{1}{4}[-1-1] + \frac{\sqrt{3}}{4}\pi - \frac{\sqrt{3}}{4}\, 10,3354 \Rightarrow$

$= 0,3926 - \frac{1}{2} + 1,3604 - 4,4754.$

$\Rightarrow -3,2224$ //

(3) $\int_0^4 \frac{dx}{\sqrt{x^2-4x+5}}$

k	0	1	2
x_k	0	2	4
f_k	0,4472	1	0,4472

2,236067977

$4-8+5$

$16-16+5$

0,447213595

$S=\frac{d}{6}\left(f_0+4f_1+f_2\right)$

$S=\frac{4^2}{6_3}\left(0,4472+4\cdot 1+0,4472\right)$

$S=\frac{2}{3}\left(4,894427191\right)$

$S=3,262951461$ //

TOČNO RJEŠ. = 2.8873

REL. GREŠKA > 10%

∅

$\int_0^4 \frac{dx}{\sqrt{x^2-4x+5^2}} \Rightarrow \int_0^4 \frac{dx}{\sqrt{(x-3)}}$

x^2-4x+5

$x_{1/2}=\frac{+4\pm\sqrt{16-20}}{2}$

$x_{1/2}=\frac{4\pm 2}{2}$

$x_1=3 \quad x_2=1$

(5)

JOSIP GAUTA

$$\int_0^{\pi} x \sin x \, dx$$

$$= x\cos x - \int_0^{\pi} \cos x \, dt$$

$u = x \quad dv = \sin x$

$v =$

$$\Rightarrow \left[x \cos x - \ln|\sin x| + C \right]_0^{\pi}$$

$$\Rightarrow \frac{\pi^2}{2} - 1 - 0 + C$$

$$\int u \, dv = uv - \int v \, du$$

$$\Rightarrow 3{,}934802201$$

PROVJERA

k	0	1	2
x_k	0	$\frac{\pi}{2}$	π
f_k	0	$\frac{\pi}{2}$	0

$$S = \frac{\pi}{6}(f_0 + 4f_1 + f_2)$$

$$S = \frac{\pi}{6}\left(0 + 4\frac{\pi}{2} + 0\right) \Rightarrow S = \frac{\pi}{6}(2\pi) \Rightarrow S = 0{,}523598775\,(6{,}283185307)$$

$$\Rightarrow S = 3{,}28986813$$

(1)

(a) $x = y^2$ $y = 2x - 4$

x	0	1	2
y	0	±1	±1,42

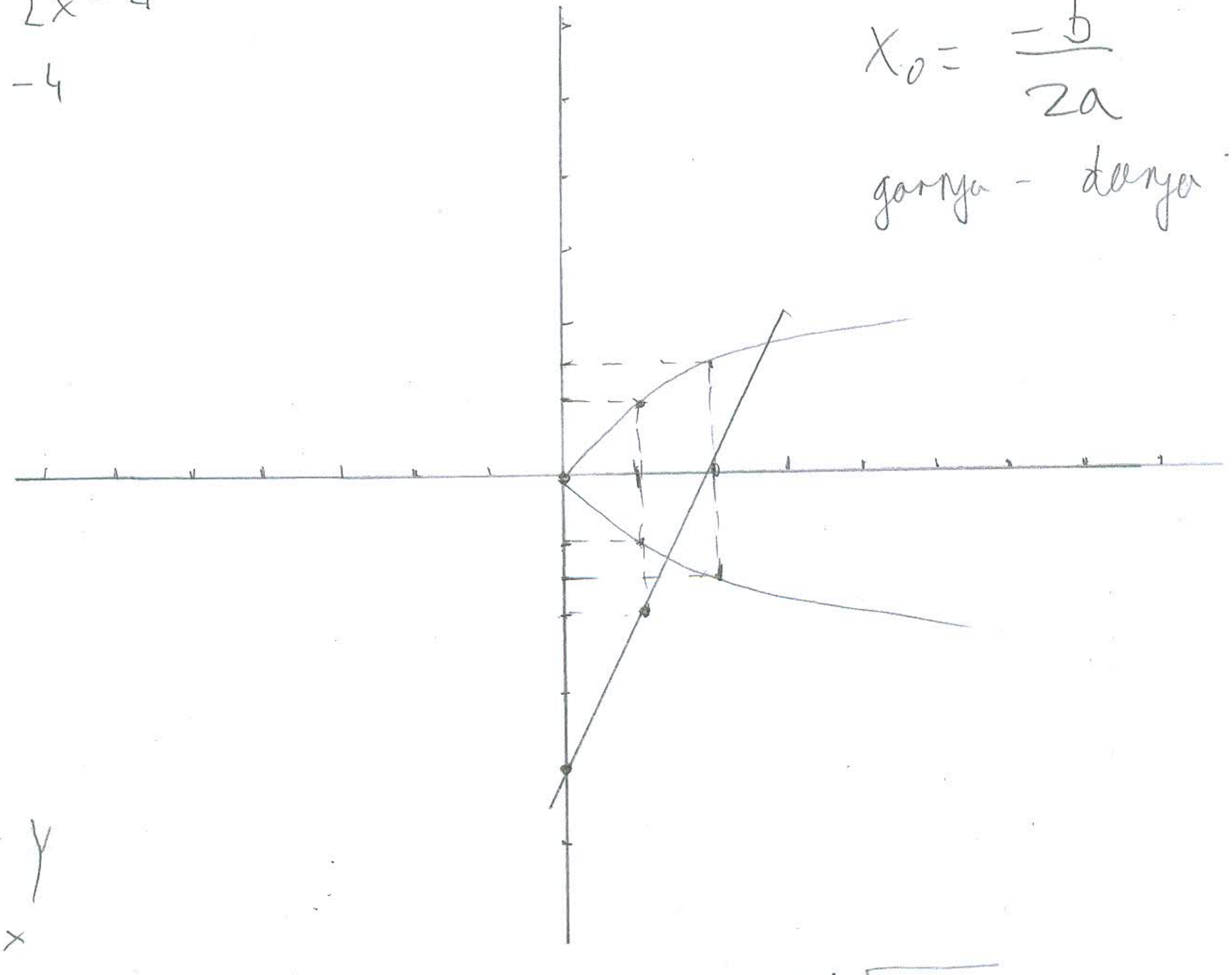
x	0	1	2
y	-4	-2	0

$x = 2x - 4$

$x = -4$

$x_0 = \frac{-b}{2a}$

gornja - donja



$x \Rightarrow y$

$y \mapsto x$

$y = x^2$

$x = 2y - 4$

$x = 2 \cdot x^2 - 4$

$-2x^2 + x + 4 = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(2) \cdot 4}}{2(-2)}$$

$$= \frac{-1 \pm \sqrt{1 + 32}}{-4}$$

$$= \frac{-1 \pm \sqrt{33}}{-4}$$

$x_1 = \frac{1 + \sqrt{33}}{-4} =$

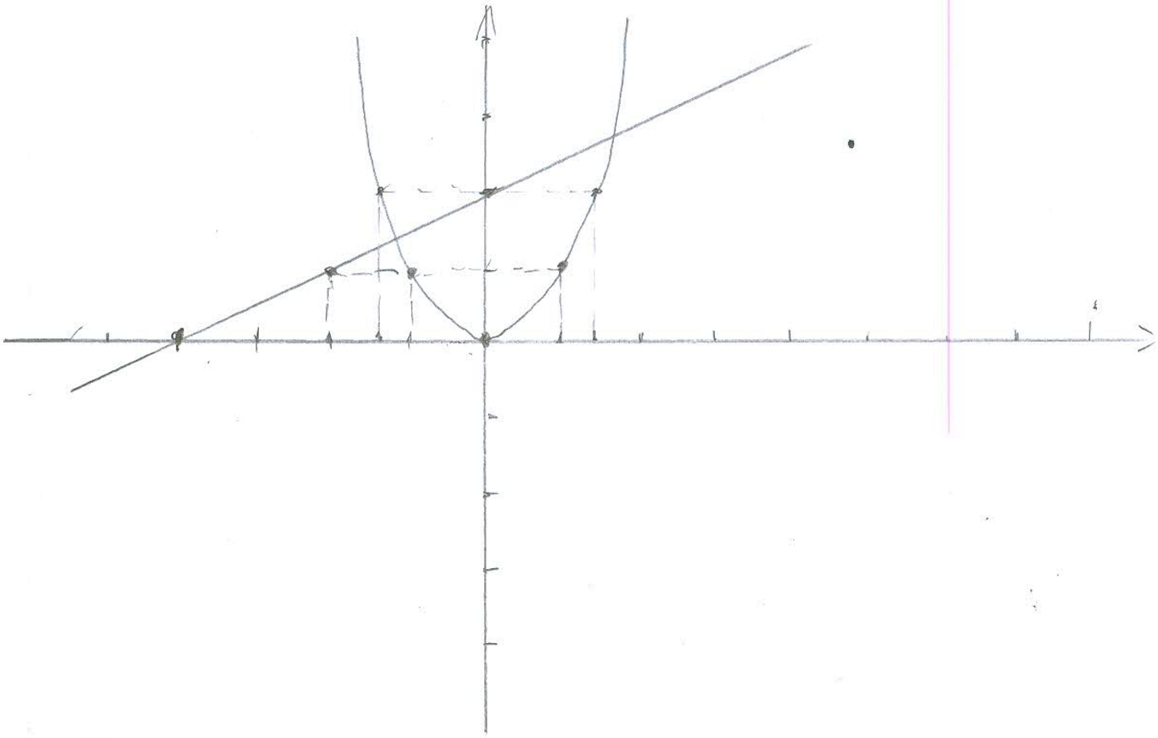
$x_1 = 1,686140662$

$x_2 = -1,186140662$

JOSIP GAUTA

y	0	1	2
x	0	±1	±1,42

y	0	1	2
x	-4	-2	0



$$\int_{-1,18}^{1,68} \left[x^2 - \left[-2x^2 + x + 4 \right] \right.$$

$$\Rightarrow \int_{-1,18}^{1,68} \left[x^2 + 2x^2 - x - 4 \right] dx \Rightarrow \int_{-1,18}^{1,68} \left[3x^2 - x - 4 \right] dx \Rightarrow 3 \int_{-1,18}^{1,68} x^2 dx - \int_{-1,18}^{1,68} x dx - 4 \int_{-1,16}^{1,68} dx$$

$$\Rightarrow 3 \left[\frac{x^3}{3} \right]_{-1,18}^{1,68} - \left[\frac{x^2}{2} \right]_{-1,18}^{1,68} - 4 \left[x \right]_{-1,18}^{1,68} \Rightarrow 3 \left[1,580544 + 1,643032 \right]$$

$$- \left[1,4112 - 0,6962 \right] - 4 \left[2,86 \right] \Rightarrow$$

$$\Rightarrow 9,670728 - 0,715 - 11,44$$

$$\Rightarrow 2,54772$$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇊ A3

IME I PREZIME: Goran Kovaček **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
17-1-0219-2013

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Grafički prikazati funkciju $f(x, y) = \ln(x + y)$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije.

2. Odrediti $\displaystyle\int_0^\pi \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right) \mathrm{d}t$.

Ø

3. Pronaći koliko iznosi:

   (a) površina između krivulja $x = y^2$ i $y = 2x - 4$,

   (b) određeni integral $\int_0^\pi x \sin x \, \mathrm{d}x$?

4. Odredi partikularno rješenje koje zadovoljava navedenu ODJ i uvjete: $y'' - 5y' + 4y = 0$, uz $y(0) = 5$ i $y'(0) = 8$. Na kraju provjeri rješenje.

5. Izračunati $\displaystyle\int_0^4 \frac{\mathrm{d}x}{\sqrt{x^2 - 4x + 5}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje.

Ø

Ukupno: Ø

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5. $\displaystyle\int_0^4 \frac{dx}{\sqrt{x^2-4x+5}}$

$d = 4 - 0 = 4$

0. $\dfrac{1}{\sqrt{x^2-4x+5}} = \dfrac{1}{\sqrt{0^2-4\cdot 0+5}} = \dfrac{\sqrt{5}}{5} = 0{,}44721$

1. $\dfrac{1}{\sqrt{x^2-4x+5}} = \dfrac{1}{\sqrt{2^2-4\cdot 2+5}} = \dfrac{1}{\sqrt{1}} = 1$

2. $\dfrac{1}{\sqrt{x^2-4x+5}} = \dfrac{1}{\sqrt{4^2-4\cdot 4+5}} = \dfrac{\sqrt{5}}{5} = 0{,}44721$

k	0	1	2
x_k	0	2	4
f_k	$\frac{\sqrt{5}}{5}$	1	$\frac{\sqrt{5}}{5}$
$S = 3{,}26295$			

$S = \dfrac{d}{6}(f_0 + 4f_1 + f_2) = \dfrac{4}{6}\left(\dfrac{\sqrt{5}}{5} + 4\cdot 1 + \dfrac{\sqrt{5}}{5}\right) = 3{,}26295$ /

TOČNO RJEŠ. = 2.8873

APS. GREŠKA = 0.38

REL. GREŠKA > 10%

3.5) $\int_0^{\pi} x \sin x \, dx =$

2. $\int_0^{\pi} \left(-\frac{1}{4}\sin^2 t + \frac{\sqrt{3}}{4}\cos t\right)dt = \underbrace{\int_0^{\pi} -\frac{1}{4}\sin^2 t \, dt}_{1.} + \underbrace{\int_0^{\pi} \frac{\sqrt{3}}{4}\cos t \, dt}_{2.}$

2. $\int \frac{\sqrt{3}}{4}\cos t \, dt = \frac{\sqrt{3}}{4}\int \cos t \, dt = \frac{\sqrt{3}}{4}\sin t$

1. $\int -\frac{1}{4}\sin^2 t \, dt = -\frac{1}{4}\int \sin^2 t \, dt$

DALJE... ?