

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

B1

IME I PREZIME: LUKA BAČIĆ

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0354-2014

- Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^3$ i $g(x) = 4x$. 20
- Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = \ln\left(\frac{x}{y}\right)$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme? 20
- Pronaći:
 - koliko iznosi $\int_0^{\sqrt{3}} \sqrt{1 + \frac{4r^2}{9}} r dr$,
 - partikularno rješenje koje zadovoljava ODJ $y'' - 4y' = 0$, uz uvjete $y(1) = 0$ i $y'(1) = 0$. Na kraju provjeri rješenje. ~~10~~
- Izračunati $\int \frac{dx}{1+\sqrt{x}}$. Ova j zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. 20
- Nadi partikularno rješenje jednadžbe $y' = \frac{y}{x} - 1$ koje zadovoljava uvjet $y(1) = 1$. 20

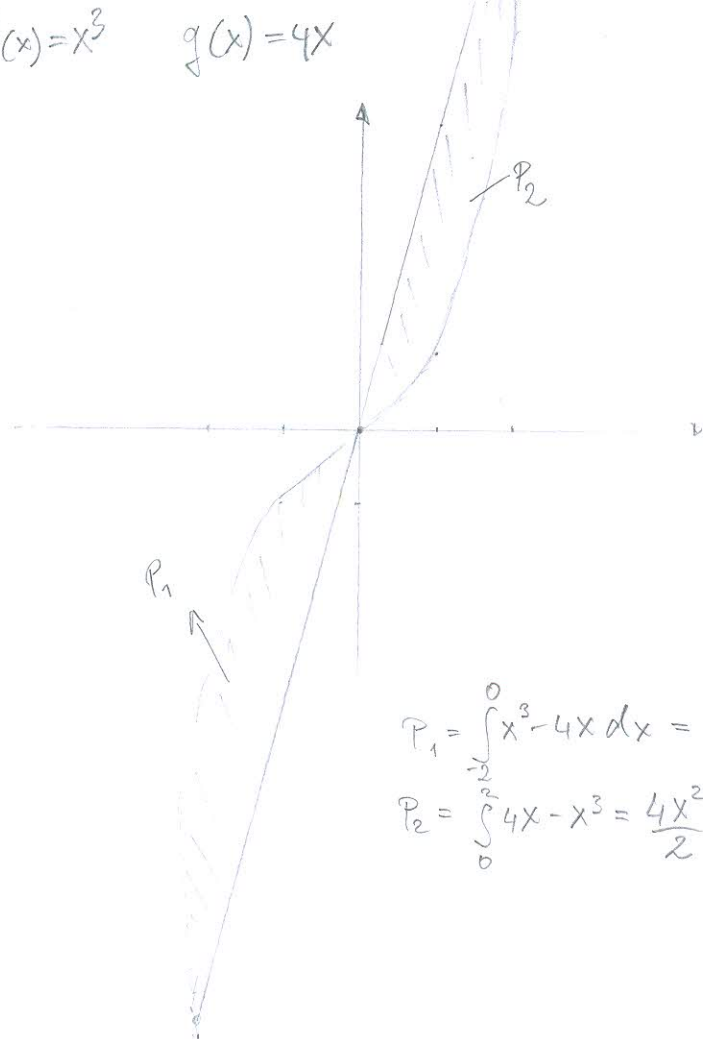
Ukupno:

90

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

① $f(x) = x^3$ $g(x) = 4x$



x	0	1	2	3
y	0	4	8	

$$x^3 = 4x$$

$$x^3 - 4x = 0$$

$$x(x^2 - 4) = 0$$

$$x = 0 \quad x^2 = 4$$

$$x = \pm 2$$

$$P_1 = \int_{-2}^0 (x^3 - 4x) dx = \left[\frac{x^4}{4} - \frac{4x^2}{2} \right]_{-2}^0 = 0 - (-4) = 4$$

$$P_2 = \int_0^2 (4x - x^3) dx = \left[\frac{4x^2}{2} - \frac{x^4}{4} \right]_0^2 = 4 - 0 = 4$$

$$P_{UKUPNO} = P_1 + P_2 = 4 + 4 = 8 \quad \checkmark$$

③ b.) $y'' - 4y' = 0$ $y(1) = 0$ $y'(1) = 0$

$$r^2 - 4r = 0 \rightarrow \frac{4 \pm \sqrt{16}}{2} = \frac{4 \pm 4}{2} \rightarrow r_1 = 0$$

$$\rightarrow r_2 = 4$$

$$y_H = C_1 + C_2 e^{4x} \rightarrow C_1 + C_2 e^4 = 0 \quad C_1 = -C_2 e^4 \quad C_1 = 0$$

$$y' = 4C_2 e^{4x} \rightarrow 4C_2 e^4 = 0 \quad C_2 = 0$$

$$y = 0 \quad \checkmark$$

PROVJERA $\rightarrow 0 - 4 \cdot 0 = 0$

$$y' = 0 \quad y'' = 0$$

$$⑤ y' = \frac{y}{x} - 1$$

$$y(x) = 1$$

$$y' - \frac{y}{x} = -1$$

$$y' - \frac{1}{x} \cdot y = -1$$

$$A = \int -\frac{1}{x} dx = -\ln|x|$$

$$y = e^{-A} \left[\int e^A \cdot g(x) dx + c \right]$$

$$y = e^{\ln|x|} \left[\int e^{-\ln|x|} \cdot (-1) dx + c \right]$$

$$y = x \left[\int x^{-1} \cdot (-1) dx + c \right]$$

$$y = x \left[\int -\frac{1}{x} dx + c \right]$$

$$y = x \left[-\ln|x| + c \right]$$

$$y = -x \ln|x| + cx$$

$$-\ln 1 + c = 1$$

$$c = 1 + \ln 1$$

$$c = 1$$

$$y = x(-\ln|x| + 1)$$

$$y = -x \ln|x| + x$$

PROVERA

$$y(1) = -1 \cdot 0 + 1 = 1 \checkmark$$

$$y' = -\ln|x| - 1 + 1 = -\ln|x|$$

ODJ:

$$-\ln|x| = \frac{-x \ln|x| + x}{x} \checkmark$$

$$④ \int_0^4 \frac{dx}{1+\sqrt{x}} = \left| \frac{x-t^2}{dx=2t dt} \right| = \int \frac{2t dt}{1+t} = 2 \int \frac{t+1-1}{1+t} dt = 2 \left(\int \frac{t+1}{t+1} dt - \int \frac{1}{t+1} dt \right)$$

$$= 2 \left(t - \ln|t+1| \right) = 2t - 2 \ln|t+1| = 2\sqrt{x} - 2 \ln|\sqrt{x}+1| \Big|_0^4 = 1,802 - 0 = 1,802 \checkmark$$

PROVERA SIMPSON

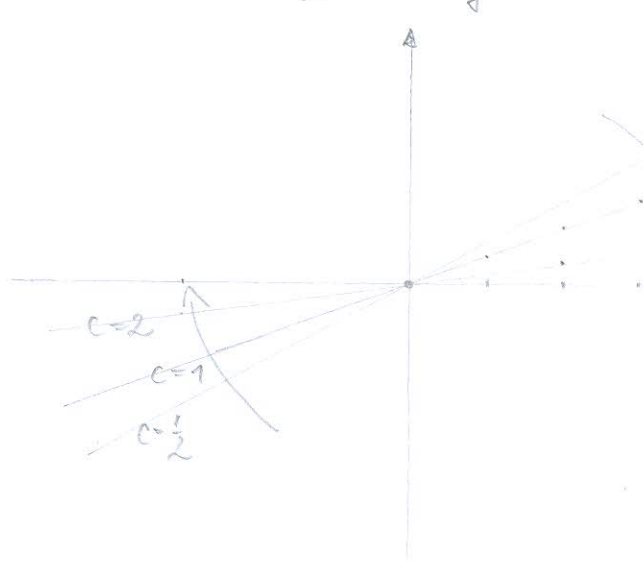
0	2	4
1	0,4142	$\frac{1}{3}$

$$S = \frac{d}{6} (f_0 + 4f_1 + f_2) = \frac{4}{6} \left(1 + 1,6568 + \frac{1}{3} \right) = 1,99$$

$$② f(x,y) = \ln\left(\frac{x}{y}\right) \quad D(f) \left\{ \frac{x^2}{y} > 0 \right\} \quad KOD \rightarrow \langle 0, +\infty \rangle$$

$$c=1 \rightarrow \ln\left(\frac{x}{y}\right) = 1 \rightarrow \frac{x}{y} = e^1 \rightarrow y = \frac{x}{e}$$

$$c=2 \rightarrow \ln\left(\frac{x}{y}\right) = 2 \rightarrow \frac{x}{y} = e^2 \rightarrow y = \frac{x}{e^2}$$



x	0	1	2	3	-1
y	0	0,13	0,27		

$$c = \frac{1}{2}$$

$$c = 1$$

$$c = 2$$

LIMES NE POSTOJE JER SE VIŠE
RAZ. KRIVULJA SIJEĆE U JEDNOJ TOČKI.

NASTAVAK



$$f(x,y) = \ln\left(\frac{x}{y}\right)$$

$$\frac{\partial f}{\partial x} = \frac{1}{\frac{x}{y}} \cdot \frac{1}{y} = \frac{1}{x} \quad \frac{\partial f}{\partial y} = \frac{1}{\frac{x}{y}} \cdot \frac{-x}{y^2} = -\frac{1}{y}$$

$$\frac{1}{x} = 0$$

$$-\frac{1}{y} = 0 \quad x=0 \quad y=0 \quad T(0,0)$$

$$A) \frac{\partial^2 f}{\partial x^2} = \frac{x^{-2}}{-2}$$

$$C) \frac{\partial^2 f}{\partial y^2} = \frac{y^{-2}}{2}$$

EKSTREMI NE POSTOJE ✓

$$3. a) \int_0^{\sqrt{3}} \sqrt{1 + \frac{4t^2}{9}} \cdot \frac{1}{2} dt = \left| \begin{array}{l} 4t^2 = t \\ 8t dt = dt \\ \frac{1}{2} dt = \frac{dt}{4} \end{array} \right|_0^{12} = \frac{1}{8} \int \sqrt{1 + \frac{t}{9}} dt = \frac{1}{8} \int \left(1 + \frac{t}{9}\right)^{\frac{1}{2}} dt$$

$$= \frac{1}{8} \left(\frac{\left(1 + \frac{t}{9}\right)^{\frac{3}{2}}}{\frac{3}{2}} \right) \Big|_0^{12} = \frac{0,2970 - 0,0833}{2} = 0,10685 \quad \times$$

PROJEKTA SIMPSON.

