MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{B}1$	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: LUKA BAČIĆ **Vrijeme početka:**	
Matični broj studenta (iznad slike u indeksu): 17-2-0354-2014	

1. Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^3$ i $g(x) = 4x$. 20

2. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = \ln(\frac{x}{y})$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme? 20

3. Pronaći:

   (a) koliko iznosi $\int_0^{\sqrt{3}} \sqrt{1 + \frac{4r^2}{9}}\, r\, \mathrm{d}r$,

   (b) partikularno rješenje koje zadovoljava ODJ $y'' - 4y' = 0$, uz uvjete $y(1) = 0$ i $y'(1) = 0$. Na kraju provjeri rješenje.

   ~~0~~ 10

4. Izračunati $\int\limits_0^4 \frac{\mathrm{d}x}{1+\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. 20

5. Nađi partikularno rješenje jednadžbe $y' = \frac{y}{x} - 1$ koje zadovoljava uvjet $y(1) = 1$. 20

Ukupno: 90

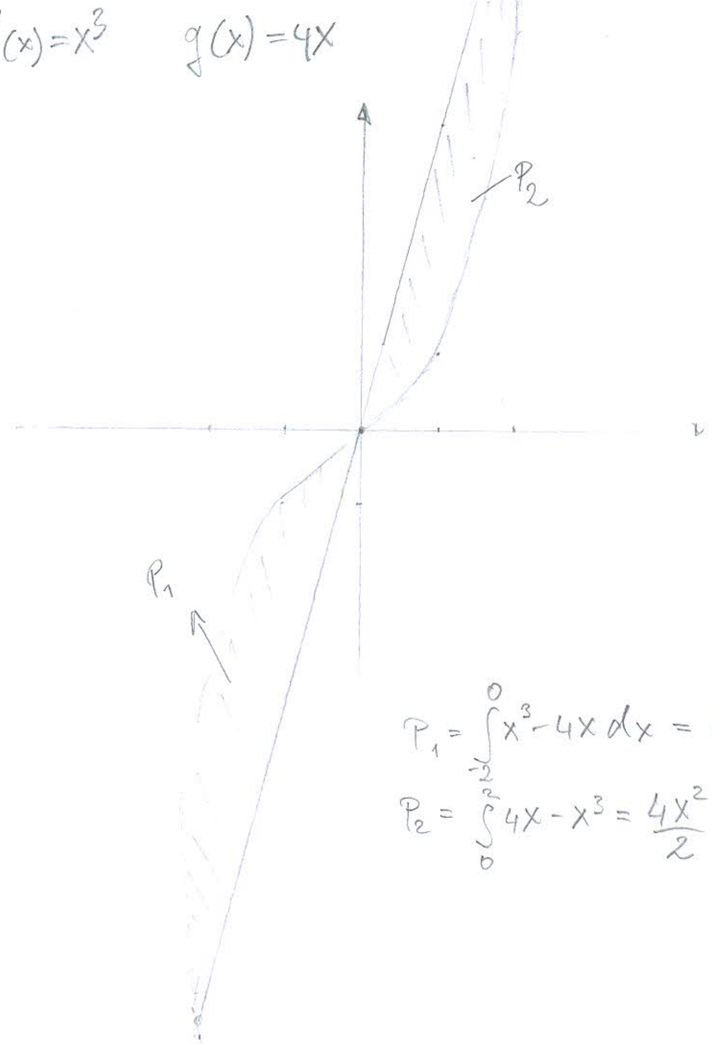
f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

LUKA BACIC

① $f(x)=x^3 \quad g(x)=4x$



x	0	1	2	3
y	0	4	8	

$x^3=4x$

$x^3-4x=0$

$x(x^2-4)=0$

$x=0 \qquad x^2=4$

$x=\pm 2$

$P_1=\int_{-2}^{0} x^3-4x\,dx=\frac{x^4}{4}-\frac{4x^2}{2}\Big|_{-2}^{0}=0-(-4)=4$

$P_2=\int_{0}^{2} 4x-x^3=\frac{4x^2}{2}-\frac{x^4}{4}\Big|_{0}^{2}=4-0=4$

$P_{UKUPNO}=P_1+P_2=4+4=8$

③ b.) $y''-4y'=0 \qquad \overset{x}{y}(1)=\overset{y}{0} \qquad y'(\overset{x}{1})=\overset{y}{0}$

$r^2-4r=0 \rightarrow \frac{4\pm\sqrt{16}}{2}=\frac{4\pm4}{2} \rightarrow r_1=0,\ r_2=4$

$y_H=C_1+C_2e^{4x} \rightarrow C_1+C_2e^4=0 \qquad C_1=-C_2e^4 \qquad C_1=0$

$y'=4C_2e^{4x} \rightarrow 4C_2e^4=0 \qquad C_2=0$

$y=0$

PROVJERA $\rightarrow \quad 0-4\cdot0=0$

$y'=0 \quad y''=0$

LUKA LACIC

⑤ $y' = \frac{y}{x} - 1$ $\qquad y(\underset{x}{1}) = \underset{y}{1}$

$y' - \frac{y}{x} = -1$

$y' - \frac{1}{x} \cdot y = -1$

$A = \int -\frac{1}{x} dx = -\ln|x|$

$y = e^{-A}\left[\int e^{A} \cdot g(x)\,dx + c\right]$

$y = e^{\ln|x|}\left[\int e^{-\ln|x|} \cdot (-1)\,dx + c\right]$

$y = x\left[\int x^{-1} \cdot (-1)\,dx + c\right]$

$y = x\left[\int -\frac{1}{x}\,dx + c\right]$

$y = x\left[-\ln|x| + c\right]$

$y = -x\ln|x| + cx$ //

$-\ln 1 + c = 1$

$c = 1 + \ln 1$

$c = 1$

$y = x(-\ln|x| + 1)$ //

$y = -x\ln|x| + x$ ✓

PROVJERA

$y(1) = -1 \cdot 0 + 1 = 1$ ✓

$y' = -\ln|x| - 1 + 1 = -\ln|x|$

ODJ:

$-\ln|x| = \frac{-x\ln|x| + x}{x} - 1$ ✓

④ $\int_0^4 \frac{dx}{1+\sqrt{x}} = \left| \begin{matrix} x = t^2 \\ dx = 2t\,dt \end{matrix} \right| = \int \frac{2t\,dt}{1+t} = 2\int \frac{t+1-1}{1+t}\,dt = 2\left(\int \frac{t+1}{t+1}\,dt - \int \frac{1\,dt}{t+1}\right)$ ✓

$= 2(t - \ln|t+1|) = 2t - 2\ln|t+1| = 2\sqrt{x} - 2\ln|\sqrt{x}+1|\Big|_0^4 = 1{,}802 - 0 = 1{,}802$ //

PROVJERA SIMPSON →

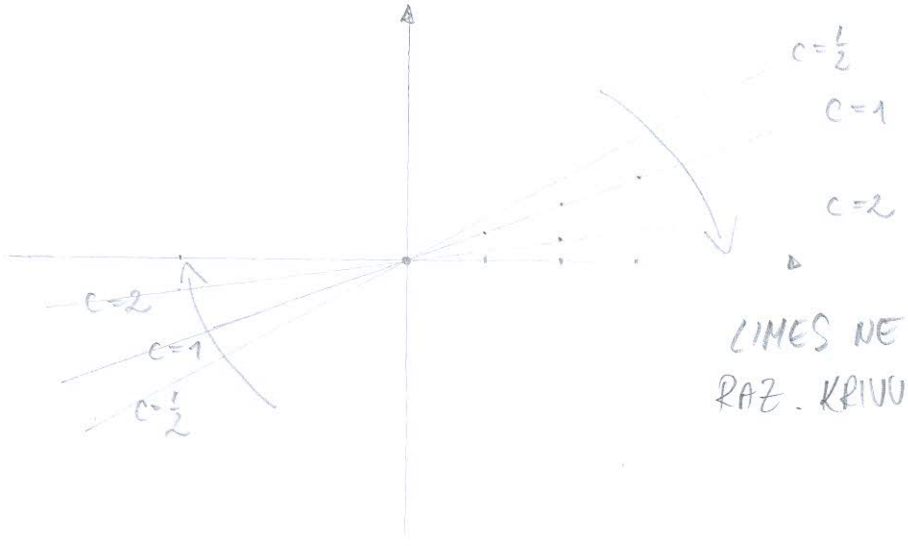
0	2	4
1	0,4142	$\frac{1}{3}$

$S = \frac{d}{6}(f_0 + 4f_1 + f_2) = \frac{4}{6}\left(1 + 1{,}6568 + \frac{1}{3}\right) = 1{,}99$

② $f(x,y) = \ln\left(\frac{x}{y}\right)$ $\qquad D(f)\left\{\mathbb{R}^2 / \frac{x}{y} > 0\right\}$ $\qquad$ KOD → $\langle 0, +\infty \rangle$

$c=1 \rightarrow \ln\left(\frac{x}{y}\right) = 1 \rightarrow \frac{x}{y} = e^1 \rightarrow y = \frac{x}{e}$

$c=2 \rightarrow \ln\left(\frac{x}{y}\right) = 2 \rightarrow \frac{x}{y} = e^2 \rightarrow y = \frac{x}{e^2}$



x	0	1	2	3	-1
y	0	0,13	0,27		

✓

LIMES NE POSTOJI JER SE VIŠE RAZ. KRIVULJA SIJEČE U JEDNOJ TOČKI.

NASTAVAK

⇓

LUKA BAČIĆ

$f(x,y) = \ln\left(\frac{x}{y}\right)$

$\frac{\partial f}{\partial x} = \frac{1}{\frac{x}{y}} \cdot \frac{1}{y} = \frac{1}{x}$ $\quad \frac{\partial f}{\partial y} = \frac{1}{\frac{x}{y}} \cdot \frac{-x}{y^2} = -\frac{1}{y}$

$\frac{1}{x} = 0 \qquad -\frac{1}{y} = 0 \qquad x=0 \quad y=0 \quad T(0,0)$

A.) $\frac{\partial^2 f}{\partial^2 x} = \frac{x^{-2}}{-2}$

EKSTREMI NE POSTOJE // ✓

C) $\frac{\partial^2 f}{\partial^2 y} = \frac{y^{-2}}{2}$

3. a.) $\int_0^{\sqrt{3}} \sqrt{1+\frac{4r^2}{9}}\, r\, dr = \left| \begin{array}{l} 4r^2 = t \\ 8r\,dr = dt \\ r\,dr = \frac{dt}{8} \end{array} \right|_0^{12} = \frac{1}{8}\int \sqrt{1+\frac{t}{9}}\, dt = \frac{1}{8}\int \left(1+\frac{t}{9}\right)^{\frac{1}{2}} dt$

$= \frac{1}{8}\left(\frac{\left(1+\frac{t}{9}\right)^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_0^{12} = \underline{0{,}2970 - 0{,}0833} = \underline{0{,}2137} \ // \ \times$

PROVJERA SIMPSON?

<u>MATEMATIKA 2</u>: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{B}1$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Lovre Štdić **VRIJEME POČETKA:** 17.00h	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0281-2014/0269092224	

1. Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^3$ i $g(x) = 4x$. 19

2. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x,y) = \ln(\frac{x}{y})$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme? 13

3. Pronaći:

   (a) koliko iznosi $\int_0^{\sqrt{3}} \sqrt{1 + \frac{4r^2}{9}}\, r \, dr$, 10

   (b) partikularno rješenje koje zadovoljava ODJ $y'' - 4y' = 0$, uz uvjete $y(1) = 0$ i $y'(1) = 0$. Na kraju provjeri rješenje. $\cancel{0}$

4. Izračunati $\int\limits_0^4 \frac{dx}{1+\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. 15

5. Nađi partikularno rješenje jednadžbe $y' = \frac{y}{x} - 1$ koje zadovoljava uvjet $y(1) = 1$. $\cancel{0}$

Ukupno: 57

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

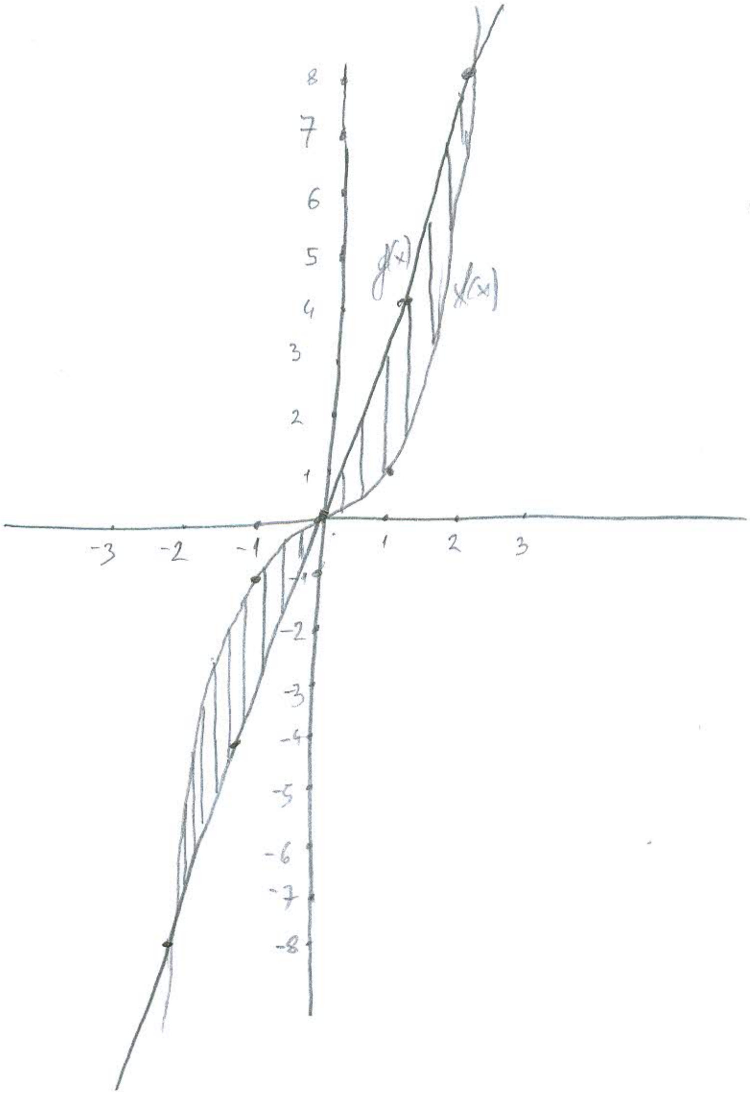
Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x\rvert + C$	$\int \cot x \, dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

Lovre Stokić 01.07.2015.g.

(1.) $f(x) = x^3$, $g(x) = 4x$

x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

x	$g(x)$
-2	-8
-1	-4
0	0
1	4
2	8



Površina lika se sastoji od dva jednaka dijela tako da je potrebno izračunati samo jedan dio te ga zbrojit jedan put sa samim sobom. ✓

$$\int_0^2 [g(x) - f(x)]\,dx$$

$$= \int_0^2 (4x - x^3)\,dx \Rightarrow 4\int x\,dx - \int x^3\,dx = 4^2\frac{x^2}{\cancel{2}} - \frac{x^4}{4} + C$$

$$\Rightarrow \left[2x^2 - \frac{1}{4}x^4\right]_0^2 = 2\cdot 2^2 - \frac{1}{4}2^4 - \cancel{\left(2\cdot 0^2 - \frac{1}{4}0^4\right)}$$

$$= 8 - 4 = 4 \,//$$ ✓

KONAČAN REZULTAT = 8 ✓

19

2. $f(x,y) = \ln\left(\frac{x}{y}\right)$ $D = \{(x,y) \in \mathbb{R}\times\mathbb{R} : x$ i y imaju isti predznak$\}$

$x \neq 0,\ y \neq 0$

$\frac{x}{y} > 0 \Rightarrow D\ x,y \in \mathbb{R} \,/\, \{x$ i y moraju imati isti predznak$\}$

$\Rightarrow K\ x,y \in \mathbb{R}$

$C_1 = 0$

$f(x,y) = C_1$

$\ln\left(\frac{x}{y}\right) = 0$

$x = y$

$C_2 = 1$

$\ln\left(\frac{x}{y}\right) = 1$

$\frac{x}{y} = e \approx 2.71$

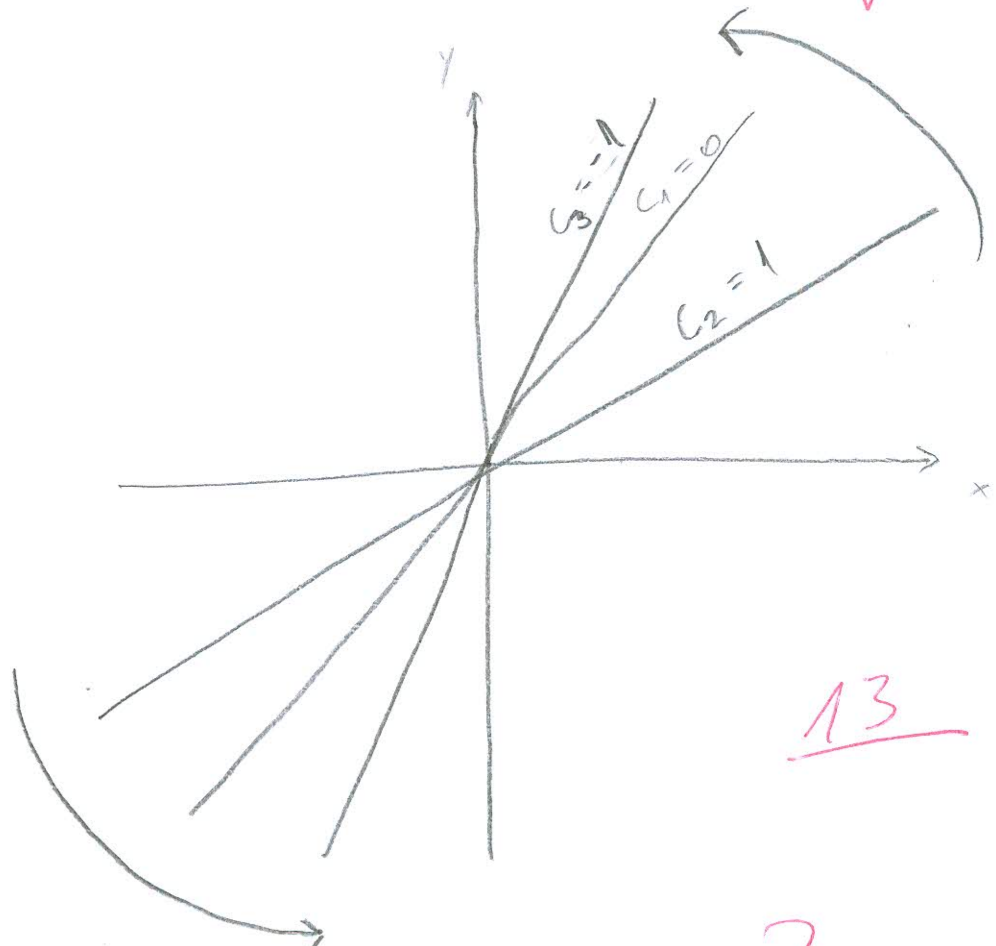
$y = \frac{x}{e}$

$C_3 = -1$

$\ln\left(\frac{x}{y}\right) = -1$

$\frac{x}{y} = e^{-1} \approx 0.37$

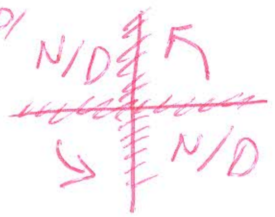
$y = x \cdot e$



13

GLOBALNI I LOKALNI EKSTREMI?

EKSTREMI NE POSTOJE ŠTO SE VIDI SA SKICE



Lovre Stokić 01.07.2015.g.

3\. a) $\int_0^{\sqrt{3}} \sqrt{1+\frac{4r^2}{9}} \cdot r\,dr \Rightarrow \left\{ \begin{array}{l} \frac{4}{9}r^2+1=t / ' \\ \frac{4}{9}\cdot 2r\,dr = dt \\ \frac{8}{9}r\,dr = dt \end{array} \right\}$

$= \frac{9}{8}\int \sqrt{t}\,dt = \frac{9}{8}\int t^{\frac{1}{2}}\,dt = \frac{9}{8}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{\cancel{9}^{3}}{\cancel{8}_{4}}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot t^{\frac{3}{2}} + C = \frac{3}{4}t^{\frac{3}{2}} + C = \frac{3}{4}\cdot\left(\frac{4}{9}r^2+1\right)^{\frac{3}{2}}$

$\Rightarrow \left[\frac{3}{4}\cdot\left(\frac{4}{9}r^2+1\right)^{\frac{3}{2}}\right]_0^{\sqrt{3}}$

$= \frac{3}{4}\cdot\left(\frac{4}{9}\cdot(\sqrt{3})^2+1\right)^{\frac{3}{2}} - \frac{3}{4}\left(\cancel{\frac{4}{9}\cdot 0^2}^{\,0}+1\right)^{\frac{3}{2}}$

$= 2.67 - 0.75 \approx 1.923$ //

✓ 10

b) $y'' - 4y' = 0$, $y(1) = 0$, $y'(1) = 0$

$r^2 - 4r = 0$ $\qquad r_1 \neq r_2 \in \mathbb{R}$

$r(r-4) = 0$ $\qquad y(x) = e^{r_1 \cdot x} C_1 + e^{r_2 x} C_2$

$r_1 = 0$, $r_2 = 4$ $\qquad y(x) = C_1 + C_2 e^{4x}$ ✓

$y'(x) = C_2 e^{4x} \cdot 4 \, /'$

$y''(x) = 4C_2 e^{4x} \cdot 4 = 16 C_2 e^{4x}$

$16C_2 e^{4x} - 4 \cdot 4 C_2 e^{4x} = 0$

$\cancel{16C_2 e^{4x}} - \cancel{16C_2 e^{4x}} = 0$ ✓

TRAŽI SE PARTIKULARNO RJEŠENJE KOJE ZADOVOLJAVA $y(1)=0$ i $y'(1)=0$. KOJE JE TO?

$\emptyset$

Lovre Stokić 01.07. 2015. g.

(4.) $\int_0^4 \frac{dx}{1+\sqrt{x}} \Rightarrow f(x) = \frac{1}{1+\sqrt{x}}$

Prvi dio : [0,1]

k	0	1	2
xk	0	0.5	1
fk	1	0.5858	0.5

$S_1 = \frac{d}{6}(f_1 + 4f_2 + f_3)$

$S_1 = \frac{1}{6}(1 + 4 \cdot 0.5858 + 0.5)$

$= 0.6405$

Drugi dio : [1,2]

k	0	1	2
xk	1	1.5	2
fk	0.5	0.4495	0.4142

$S_2 = \frac{1}{6}(0.5 + 4 \cdot 0.4495 + 0.4142)$

$S_2 = 0.452$

Treći dio : [2,3]

k	0	1	2
xk	2	2,5	3
fk	0.4142	0.3874	0.366

$S_3 = \frac{1}{6}(0.4142 + 4 \cdot 0.3874 + 0.366)$

$S_3 = 0,3883$

Četvrti dio : [3,4]

k	0	1	2
xk	3	3,5	4
fk	0.366	0.3483	0.3333

$S_4 = \frac{1}{6}(0.366 + 4 \cdot 0.3483 + 0.3333)$

$S_4 = 0.3488$

$S = \sum_{n=1}^{i} S_i = 0.6405 + 0.452 + 0.3883 + 0.3488$

$S = 1.8296$ // ✓ 15

(5.) $y' = \frac{y}{x} - 1 \quad , \quad y(1) = 1$

$$y' = \frac{1}{x} \cdot y - 1$$

$$y' = \frac{1}{x} \cdot y$$

$$\frac{dy}{dx} = \frac{1}{x} \cdot y \quad \Big/ \cdot \frac{1}{y} \Big/ \cdot dx$$

$$\frac{dy}{y} = \frac{dx}{x} \quad \Big/ \int$$

$$\ln|y| = \ln|x + c| \quad / e$$

$$y = x \cdot C$$

KOJE PARTIKULARNO RJEŠENJE ZADOVOLJAVA $y(1) = 1$?

0

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B1

IME I PREZIME: IVAN MARDETKO **VRIJEME POČETKA:** 17:15

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0216-2013

POPUNJAVA NASTAVNIK

Broj ↓ bodova

20

1. Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^3$ i $g(x) = 4x$.
2. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = \ln(\frac{x}{y})$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme?
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4. Izračunati $\int\limits_0^4 \frac{dx}{1+\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje.
5. Nađi partikularno rješenje jednadžbe $y' = \frac{y}{x} - 1$ koje zadovoljava uvjet $y(1) = 1$.

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2) $f(x,y) = \ln\left(\frac{x}{y}\right)$

$\frac{x}{y} > 0$

$Hf: \langle 0, +\infty \rangle$

$Df: \langle 0, +\infty \rangle$

IVAN MARĐETKO

4) $\int_0^4 \frac{dx}{1+\sqrt{x}}$

$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$

k	0	1	2
x_k	0	2	4
f_k	1	2.41	3

$f(0) = 1 + \sqrt{0} = 1$

$f(2) = 1 + \sqrt{2} = 2.41$

$f(4) = 1 + \sqrt{4} = 3$

$S = \frac{4}{6}(1 + 4 \cdot 2.41 + 3)$

$S = 9.09$

IVAN MARDETKO

1) $f(x) = x^3$

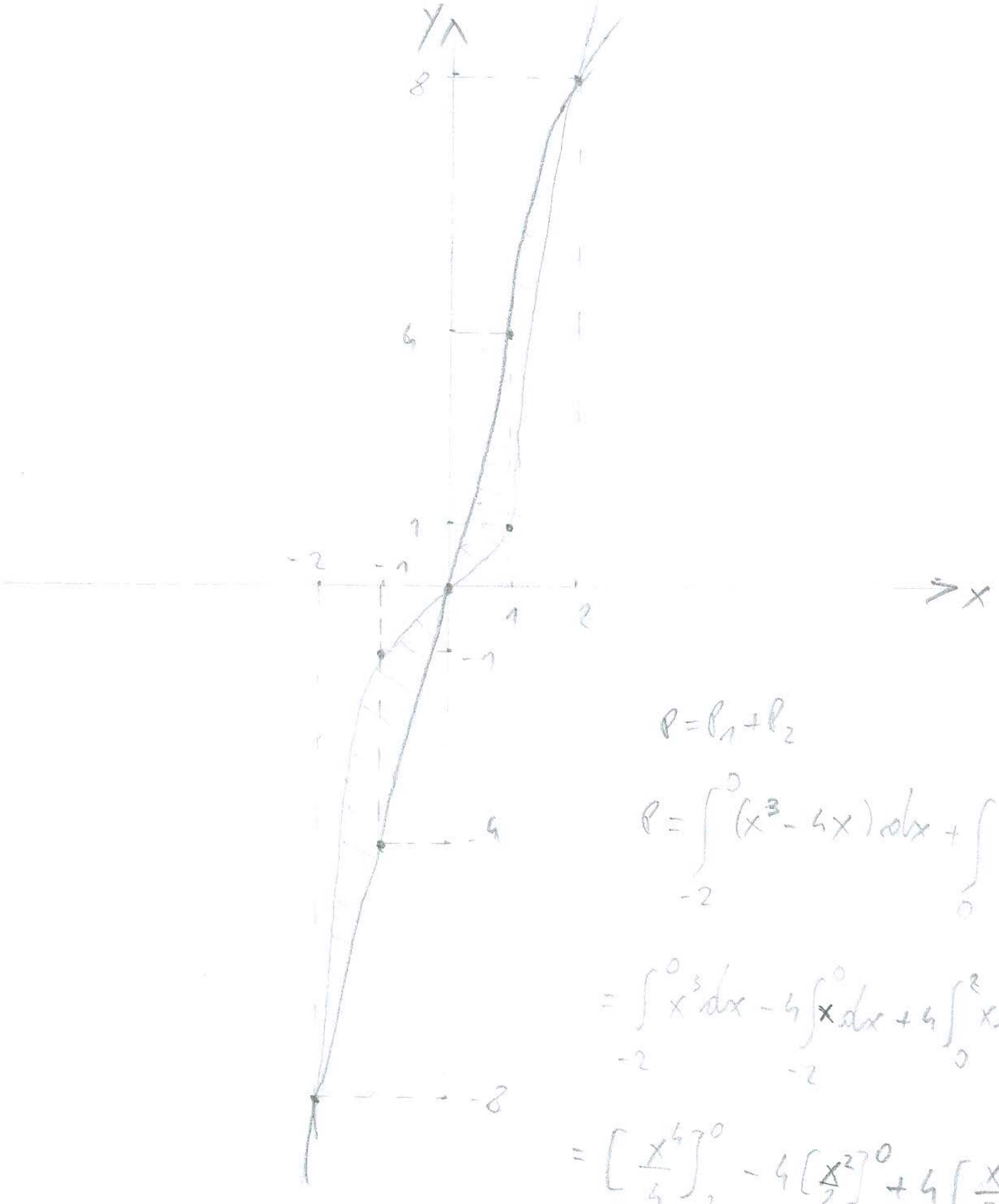
$g(x) = 4x$

$y = x^3$

X	-2	-1	0	1	2
Y	-8	-1	0	1	8

$y = 4x$

X	-2	-1	0	1	2
Y	-8	-4	0	4	8



$P = P_1 + P_2$

$$P = \int_{-2}^{0} (x^3 - 4x)\,dx + \int_{0}^{2} (4x - x^3)\,dx$$

$$= \int_{-2}^{0} x^3\,dx - 4\int_{-2}^{0} x\,dx + 4\int_{0}^{2} x\,dx - \int_{0}^{2} x^3\,dx$$

$$= \left[\frac{x^4}{4}\right]_{-2}^{0} - 4\left[\frac{x^2}{2}\right]_{-2}^{0} + 4\left[\frac{x^2}{2}\right]_{0}^{2} - \left[\frac{x^4}{4}\right]_{0}^{2}$$

$$= \left[\frac{0^4}{4} - \frac{(-2)^4}{4}\right] - 4\cdot\left[\frac{0^2}{2} - \frac{(-2)^2}{2}\right] + 4\cdot\left[\frac{2^2}{2} - \frac{0^2}{2}\right] - \left[\frac{2^4}{4} - \frac{0^4}{4}\right] = 8$$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A1	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MARKO MARASOVIĆ **VRIJEME POČETKA:** 17:20	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0242-2019	

1. Pronaći:

   (a) koliko iznosi $\int_0^{\sqrt{3}} \sqrt{1+\frac{4r^2}{9}}\, r\,\mathrm{d}r$,

   (b) partikularno rješenje koje zadovoljava ODJ $y''-4y'=0$, uz uvjete $y(1)=0$ i $y'(1)=0$. Na kraju provjeri rješenje. $\emptyset$

2. Izračunajte površinu lika omeđenog grafovima funkcija $f(x)=x^3$ i $g(x)=4x$. 13

3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x,y)=\ln(\frac{x}{y})$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme?

4. Nađi partikularno rješenje jednadžbe $y'=\frac{y}{x}-1$ koje zadovoljava uvjet $y(1)=1$.

5. Izračunati $\int\limits_0^4 \frac{\mathrm{d}x}{1+\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. $\emptyset$

Ukupno: 13

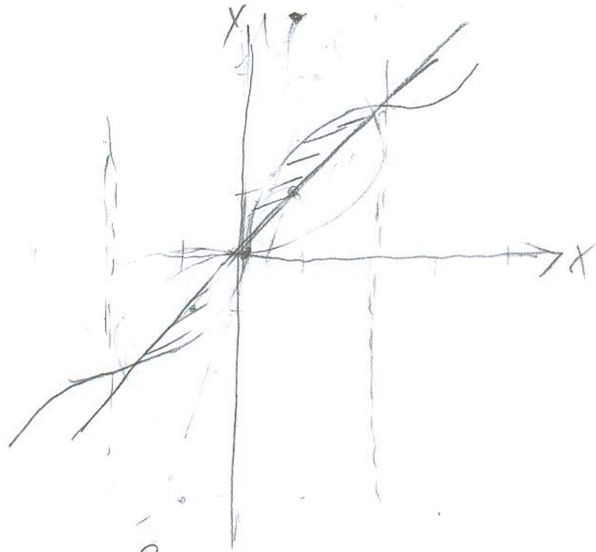
f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
α^x $(\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq-1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|+C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\|+C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\|+C$
$\int e^x\,\mathrm{d}x = e^x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x+C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a}+C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x+C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right)+C$
$\int \sin x\,\mathrm{d}x = -\cos x+C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int \cos x\,\mathrm{d}x = \sin x+C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

2)

$f(x) = x^3$

$g(x) = 4x$



x	0	1	2	-1	-2
$f(x)$	0	1	8	-1	-8

x	0	1	2	-1	-2
$y(x)$	0	4	8	-4	-8

$x^3 = 4x$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x_1 = 0$

$x^2 - 4 = 0$

$x^2 = 4 \quad \sqrt{}$

$\boxed{x_2 = -2}$

$\boxed{x_3 = 2}$

$\int_0^2 (g(x) - f(x)$

$= \int_0^2 (4x - (x^3)) dx$

$= \int_0^2 (4x - x^3) dx$

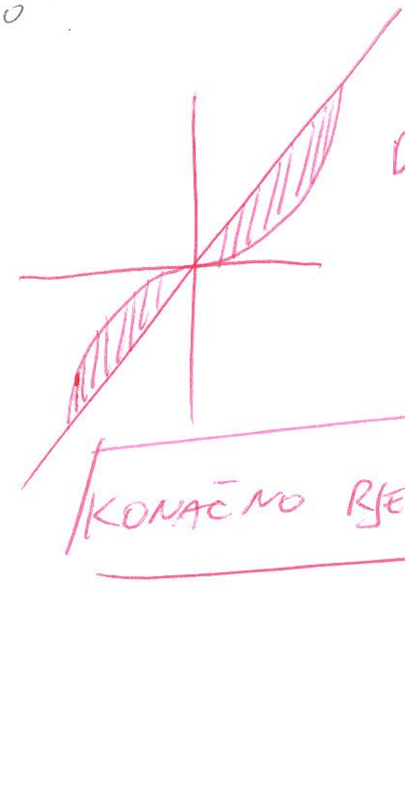
$= \int_0^2 4x\, dx - \int_0^2 x^3 dx$

$= \frac{4}{2} \cdot x^2 \Big|_0^2 - \left(\frac{2^3}{2}\right) \Big|_0^2$

$= 8 - 4$

$\boxed{= 4}$

13



DVA DIJELA

$\boxed{\text{KONAČNO RJEŠENJE } 8}$

(5) MARKO MARASOVIĆ

$$\int_0^4 \frac{dx}{1+\sqrt{x}} = \ln|1+\sqrt{4}|+C \Big|_0^4$$

$$= \ln|3| + C \Big|_0^4 = 1{,}09861$$

(1) a)

$$\int_0^{\sqrt{3}} \sqrt{1+\frac{4r^2}{9}}\, r\, dr$$

$$= \int_0^{\sqrt{3}} \sqrt{1}\, dr + \int_0^{\sqrt{3}} \sqrt{\frac{4r^2}{9}}\, dr$$

$$= 1.732 + 1 = \underline{2.732}$$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}1$	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: Ivan-Maximilian Šimović **Vrijeme početka:** 14:10	
Matični broj studenta (iznad slike u indeksu): 14-2-0107-2011	

1. Pronaći:

   (a) koliko iznosi $\int_0^{\sqrt{3}} \sqrt{1+\frac{4r^2}{9}}\, r\, dr$,

   (b) partikularno rješenje koje zadovoljava ODJ $y''-4y'=0$, uz uvjete $y(1)=0$ i $y'(1)=0$. Na kraju provjeri rješenje. 9

2. Izračunajte površinu lika omeđenog grafovima funkcija $f(x)=x^3$ i $g(x)=4x$.

3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x,y)=\ln(\frac{x}{y})$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme?

4. Nađi partikularno rješenje jednadžbe $y'=\frac{y}{x}-1$ koje zadovoljava uvjet $y(1)=1$.

5. Izračunati $\int\limits_0^4 \frac{dx}{1+\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. $\varnothing$

Ukupno: 9

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x+C$	$\int \frac{dx}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int \frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+C$
$\int x^\alpha dx=\frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq-1$	$\int \tan x\,dx=-\ln\|\cos x\|+C$	$\int \frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+C$
$\int \frac{dx}{x}=\ln\|x\|+C$	$\int \cot x\,dx=\ln\|\sin x\|+C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}}=\ln\left\|x+\sqrt{x^2\pm a^2}\right\|+C$
$\int e^x dx=e^x+C$	$\int \frac{dx}{\cos^2 x}=\tan x+C$	$\int \frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x dx=\frac{a^x}{\ln a}+C$	$\int \frac{dx}{\sin^2 x}=-\cot x+C$	$\int \frac{dx}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int \sin x\,dx=-\cos x+C$	$\int \sqrt{x^2\pm a^2}\,dx=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int \cos x\,dx=\sin x+C$	$\int \sqrt{a^2-x^2}\,dx=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

1)

a) $\int_0^{\sqrt{3}} \sqrt{1-\frac{4r^2}{9}}\, r\, dr = \int_0^{\sqrt{3}} \left(1-\frac{4r^2}{9}\right)^{\frac{1}{2}} \cdot r\, dr =$

$1-\frac{4r^2}{9} \geq 0$

$\frac{4r^2}{9} \geq -1 \,/\cdot 9$

$4r^2 \leq -\frac{1}{9} \,/\cdot 4$

$r^2 \leq -\frac{4}{9} \,/\sqrt{\ }$

$r \leq -\frac{2}{3}$

$D = \mathbb{R} / \left\{-\frac{2}{3}\right\}$

$\varnothing$

1)
b) $y'' - 4y' = 0$ $\qquad y(1)=0 \quad y'(1)=0$

$r^2 - 4r = 0$ $\qquad y_H = C_1 e^{4x} + C_2 \quad y_H' = 4C_1 e^{4x} \quad y_H'' = 16C_1 e^{4x}$

$r_{1,2} = \frac{4 \pm \sqrt{16}}{2}$

$r_1 = 4$ $\qquad y = C_1 e^{4x} + C_2$

$r_2 = 0$ $\qquad y' = 4C_1 e^{4x}$

NIGDJE NIJE NAVEDENO RJEŠENJE

$\boxed{y = 0}$

$y(1) = C_1 e^{4\cdot 1} + C_2 =$ uz $e^4 \to \boxed{C_1 = 0}$ $\quad$ provjera:

$= C_1 e^4 + C_2 = 0$ $\quad$ bez $e^4 \to \boxed{C_2 = 0}$ $\quad 16C_1 e^{4x} - 4\cdot(4C_1 e^{4x}) = 0$

$= 0 \cdot e^4 + 0 = 0$ ✓✓ $\qquad 16C_1 e^{4x} - 16C_1 e^{4x} = 0$

9

$y'(1) = 4C_1 e^{4x}$

$= 4 \cdot 0 \cdot e^{4\cdot 1} = 0$ ✓✓ $\qquad 0 = 0$ ✓✓

---

4) $y' = \frac{y}{x} - 1$

$\frac{dy}{dx} = \frac{y}{x} - 1 \quad / \cdot dx$

$dy = \frac{y}{x} - 1 dx \quad / \cdot x \quad / : y$

Ø

$x dy$

Ivan-Maximilian Šimović

$$= \frac{0 - 8r \cdot 9 - 4r^2 \cdot 0}{9^2} = 72r$$

~~$y = 7H$~~

~~$\frac{a}{b} = \frac{a'b - a \cdot b'}{b^2}$~~

~~$y = c_1 e^{4x} + c_2 = 0$~~

5) $\int_0^4 \frac{dx}{1+\sqrt{x}} = \int_0^4 1 + \int_0^4 \frac{dx}{\sqrt{x}} = x \Big|_0^4 + \int_0^4 \frac{dx}{x^{\frac{1}{2}}} = \left(x + \frac{1}{2}\ln|x|\right)\Big|_0^4$

$$\lim_{a \to 0^+} \int_a^4 \frac{dx}{1+\sqrt{x}} = \lim_{a \to 0^+} \left[\left(4 + \frac{1}{2}\ln 4\right) - \left(a + \frac{1}{2}\ln a\right)\right]\Big|_a^4 = +\infty$$

divergira

Numerička

$d = 4 - 0 = 4 \quad x_0 = 0 \quad x_1 = 2 \quad x_2 = 4$

x	0	2	4
$\frac{1}{1+\sqrt{x}}$	1	0,4142135	$\frac{1}{3}$

TOČNO RJEŠENJE ≈ 1.8028

INTERVAL $\pm 10\%$

OD 1.6225 DO 1.983

$$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$$

$$S = \frac{2}{3}\left(1 + 4 \cdot 0{,}4142135 + \frac{1}{3}\right)$$

$S = 1{,}9934582$

← PROCJENA JE S GREŠKOM IZNAD 10%

0

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B1	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: KARLO ŠTURA **VRIJEME POČETKA:** 17:15	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0373-2014 0269087719	

1. Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^3$ i $g(x) = 4x$.

2. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = \ln(\frac{x}{y})$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme?

3. Pronaći:

   (a) koliko iznosi $\int_0^{\sqrt{3}} \sqrt{1 + \frac{4r^2}{9}}\, r\, \mathrm{d}r$, Ø

   (b) partikularno rješenje koje zadovoljava ODJ $y'' - 4y' = 0$, uz uvjete $y(1) = 0$ i $y'(1) = 0$. Na kraju provjeri rješenje.

4. Izračunati $\int_0^4 \frac{\mathrm{d}x}{1+\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. Ø

5. Nađi partikularno rješenje jednadžbe $y' = \frac{y}{x} - 1$ koje zadovoljava uvjet $y(1) = 1$.

Ukupno: Ø

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\vert\frac{a+x}{a-x}\right\vert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\vert\cos x\vert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\vert\frac{x-a}{x+a}\right\vert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\vert x\vert + C$	$\int \cot x\,\mathrm{d}x = \ln\vert\sin x\vert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\vert x + \sqrt{x^2 \pm a^2}\right\vert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

LOZINO STURA

3a)

$$\int_0^{\sqrt{3}} \sqrt{1+\frac{4r^2}{9}}\, r\,dr =$$

$$\int \sqrt{1+\frac{4x^2}{9}}\, x\,dx = \left[\left(1+\frac{4x^2}{9}\right) = t^2 \right.$$

$\emptyset$

4) $$\int_0^4 \frac{dx}{1+\sqrt{x}} = \left(\operatorname{arctg} 4 - \operatorname{arctg} 0\right) = 1,33$$

$\emptyset$

$$\int \frac{dx}{1+\sqrt{x}} = \frac{1}{1}\operatorname{arctg}\frac{x}{1} + C$$

NUM. METODA

$$S = \frac{d}{6}\left(f_0 + 4f_1 + f_2\right)$$

k	0	2	4
$\frac{1}{1+\sqrt{x}}$	1	0,41421	$\frac{1}{3}$

$$S = \frac{4}{6}\left(1 + 4\cdot 0,41421 + \frac{1}{3}\right) = 1,99345$$

VIŠE OD 10% GREŠKE.

$\emptyset$

4) $\int_0^4 \frac{dx}{1+\sqrt{x}} =$

$$\int \frac{dx}{1+\sqrt{x}} = \left[\begin{matrix} x = t^2 \\ dx = 2t\,dt \end{matrix}\right] = \int \frac{1}{1+\sqrt{t^2}} \cdot 2t\,dt = 2\int \frac{1}{1+t} \cdot t\,dt = 2\underbrace{\int \frac{t\,dt}{1+t}}_{I_1}$$

$$\boxed{\begin{aligned} \int \frac{t\,dt}{1+t} &= \int 1 - \frac{1}{1+t}\,dt \\ &= t - \ln|1+t| \end{aligned}}$$

?

$$I_1 = \int \frac{t\,dt}{1+t} = \left[\begin{matrix} t = u & \underbrace{\frac{1}{1+t}}_{I_2} dt = dv \\ dt = du & \ln|1+t| = v \end{matrix}\right] = t\ln|1+t| - \int \ln|1+t|\,dt$$

$$I_2 = \int \frac{1}{1+t}\,dt = \left[\begin{matrix} 1+t = r \\ dt = dr \end{matrix}\right] = \int \frac{dr}{r} = \ln|r| = \ln|1+t|$$

$$\int \frac{dx}{1+\sqrt{x}} = \int \frac{1}{1}\,dx + \underbrace{\int \frac{1}{\sqrt{x}}\,dx}_{I_1} = x + 2x^2$$

$$I_1 = \int \frac{1}{\sqrt{x}}\,dx = \left[\begin{matrix} x = t^2 \\ dx = 2t\,dt \end{matrix}\right] = 2\int \frac{\cancel{t}\,dt}{\cancel{t}} = 2t = 2x^2$$

RAVNO STRUKA

4) $\displaystyle\int_0^4 \frac{dx}{1+\sqrt{x}} =$

$$\int \frac{dx}{1+\sqrt{x}} = \begin{bmatrix} x = t^2 \\ dx = 2t\,dt \end{bmatrix} = \int \frac{1}{1+t} \cdot 2t\,dt = 2\int \frac{t\,dt}{1+t} = 2\left(\int \frac{t}{1+t}\,dt + \int \frac{1}{1+t}\,dt \right)$$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{B}1$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Marina Bešker **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0193-2012	

1. Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^3$ i $g(x) = 4x$. $\emptyset$

2. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x,y) = \ln(\frac{x}{y})$. Ima li f globalne ekstreme? Ima li f lokalne ekstreme?

3. Pronaći:

   (a) koliko iznosi $\int_0^{\sqrt{3}} \sqrt{1 + \frac{4r^2}{9}}\, r\, dr$, $\emptyset$

   (b) partikularno rješenje koje zadovoljava ODJ $y'' - 4y' = 0$, uz uvjete $y(1) = 0$ i $y'(1) = 0$. Na kraju provjeri rješenje.

4. Izračunati $\int\limits_0^4 \frac{dx}{1+\sqrt{x}}$. Ovaj zadatak vrijedi 20 bodova. U slučaju da ne znaš pronaći točno rješenje, možeš dobiti 15 bodova ukoliko numeričkom metodom izračunaš aproksimaciju s relativnom greškom 10% ili manje. $\emptyset$

5. Nađi partikularno rješenje jednadžbe $y' = \frac{y}{x} - 1$ koje zadovoljava uvjet $y(1) = 1$.

Ukupno: $\emptyset$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

Marcia Beskey

3.

$$\int_0^{\sqrt{3}} \sqrt{1+\frac{4x^2}{9}\cdot\frac{x}{1}}\,dx$$

$$\int_0^{\sqrt{3}} \sqrt{1+\frac{4x^3}{9}}\,dx$$

$$\frac{2}{3}\cdot\int_0^{\sqrt{3}} \sqrt{1+x^3}\,dx \quad \left|\begin{array}{ll} 1+x^3=t\,|' & a=1 \\ 3x^2dx=dt & b=1+3\sqrt{3} \end{array}\right|$$

$$\frac{2}{3}\cdot\int_1^{1+3\sqrt{3}} \sqrt{t}\,dx\cdot\int_1^{1+3\sqrt{3}} 3x^2dx \quad I_2$$

0.70

$$\frac{2}{3}\,\cancel{x}\cdot\left(\frac{-1+3\sqrt{3}}{6}\right)\Bigg|_1 = 2.89-0.47$$

$$=2.42$$

//

$$C\left(\overset{x_1}{0}, \overset{y_1}{-2}\right)$$

Marina Bešker

1. $f(x) = x^3$ $g(x) = 4x$

x	0	1	2
y	0	1	2

$x^3 = 4x$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0$

$x^2 - 4 = 0$

$x^2 = 4 /\sqrt{}$

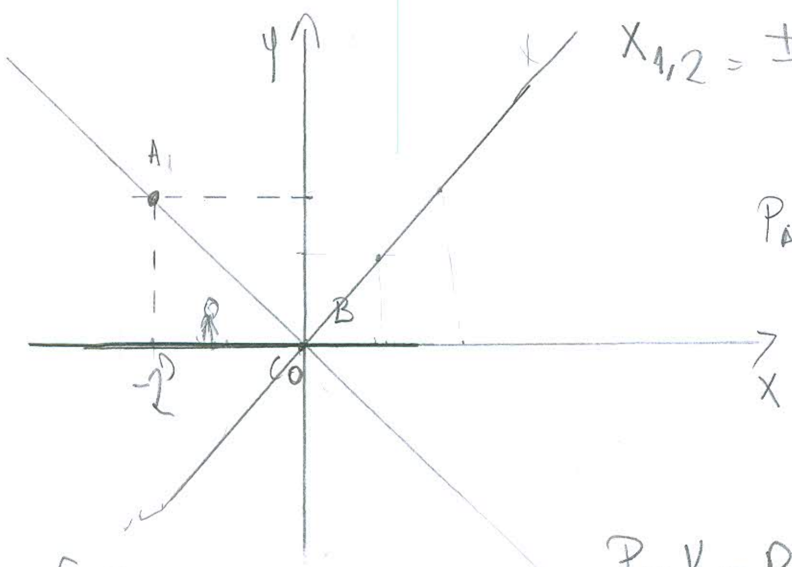
$x_{1,2} = \pm 2$

$x_1 = 2$

$x_2 = -2$

$A\left(\overset{x_1}{2}, \overset{y_1}{-2}\right)$

$B\left(\overset{x_2}{0}, \overset{y_2}{0}\right)$



P_{AB} $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y + 2 = \frac{-2 - 0}{0 - 2}(x - 2)$

$y + 2 = \frac{-2}{-2}(x - 2)$

$y + 2 = 1x - 2$

$y = 1x - 2 - 2$

$y = 1x - 4$

P_{CD} $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$P = K - P$

$P = \left[\int_{-2}^{0} \left[(1x - 4) - x\right] dx\right]$

0

$P = \int_{-2}^{0} (1x - 4 - x)\, dx$

$P = \int_{-2}^{0} -4\, dx$

$P = -4 \cdot x \Big|_{-2}^{0} = 0 - 8 = 8$

a) $\int_0^{\sqrt{3}} \sqrt{1+\frac{4x^2}{9}}\, x\, dx$

$$\int_0^{\sqrt{3}} \sqrt{1+\frac{4x^2}{9}}\, x\, dx$$

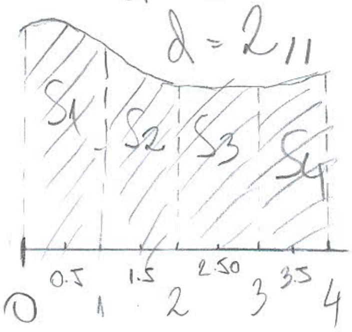
$S_1 = \frac{d}{6} \cdot \left[f_0 + 4 \cdot f_1 + f_2\right]$

4) $\int_0^4 \frac{1}{1+\sqrt{x}}\, dx =$

$d = b - a$

$d = 2 - 0$

$d = 2$



$f_0 = f(0) = \frac{1}{1+\sqrt{0}} = 1$

$f_1 = f(0.5) = \frac{1}{1+\sqrt{0.5}} = 0.5859$

$f_2 = f(1) = \frac{1}{1+\sqrt{1}} = 0.5$

$S_1 = \frac{d}{6} \cdot \left[f_0 + 4 \cdot f_1 + f_2\right] = 1.2853$

$\boxed{S_1 = 1.2853}$

$f_0 = f(1) = 0.5$

$f_1 = f(1.5) = \frac{1}{1+\sqrt{1.5}} = 0.4495$

$f_2 = f(2) = \frac{1}{1+\sqrt{2}} = 0.4142$

$S_2 = \frac{\cancel{2}^{1}}{\cancel{6}_{3}} \cdot \left[0.5 + 4 \cdot 0.4495 + 0.4142\right]$

$\boxed{S_2 = 0.9041}$

Marina Beško

$f_0 = f(2) = 0.4142$

$f_1 = f(2.50) = \frac{1}{1+\sqrt{2.50}} = 0.3874$

$f_2 = f(3) = \frac{1}{1+\sqrt{3}} = 0.3660$

$S_3 = \frac{\cancel{2}\,\frac{1}{3}}{\cancel{6}} \cdot \left[0.4142 + 4 \cdot 0.3874 + 0.3660\right]$

$\boxed{S_3 = 0.7766}$

$f_0 = f(3) = 0.3660$

$f_1 = f(3.5) = \frac{1}{1+\sqrt{3.5}} = 0.3483$

$f_2 = f(4) = \frac{1}{1+\sqrt{4}} = 0.3333$

$S_4 = \frac{1}{3} \cdot \left[0.3660 + 4 \cdot 0.3483 + 0.3333\right]$

$\boxed{S_4 = 0.6975}$ $\qquad S_1 + S_2 + S_3 + S_4 = 3.6635$ // $\emptyset$

VIDI BAČIĆ, ŠTOKIĆ

3.

a) $\cancel{\int_0^{\sqrt{3}} \sqrt{1+\frac{4x^2}{9}\cdot\frac{x}{1}}\,dx}$

$\frac{4x^2}{9}\cdot\frac{x}{1} = \frac{4x^3}{9}$

$\cancel{\frac{4}{9}\cdot\int_0^{\sqrt{3}} \sqrt{1+x^3}\,dx}$ $\left|\begin{array}{l} 1+x^3=t \,|' \\ 3x^2dx=dt \end{array}\right|$

$\cancel{\frac{4}{9}\cdot\underbrace{\int_0^{\sqrt{3}} \sqrt{t}\,dt}_{I_1}\cdot\underbrace{\int_0^{\sqrt{3}} 3x^2\,dx}_{I_2}}$

$\cancel{\frac{4}{9}\cdot 3\sqrt{3} =}$

$I_2 = \int_{1}^{1+3\sqrt{3}} 3x^2\,dx$

~~82~~ $82+90\sqrt{3}$

$\int_1^{1+3\sqrt{3}} 3x^2\,dx = \left|\begin{array}{l} 3x^2=t \,|' \\ 6x\,dx=dt \,|\,6 \\ dx=\frac{dt}{6} \end{array}\right|$

$\int_1^{3\sqrt{3}} t\cdot\int_1^{3\sqrt{3}}\frac{1}{6}\,dt$

$X\cdot\frac{1}{6}\Big|_1^{3\sqrt{3}}$

$F(b)-F(a)$

$\cancel{3\sqrt{3}-0}$

$\frac{\sqrt{3}}{2}-\frac{1}{6}$

$=\frac{-1+3\sqrt{3}}{6}$