

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

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B3

IME I PREZIME: STIPE BREUJ

VRIJEME POČETKA: 17:15

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0192-2013

1. Izračunati:

(a) određeni integral $\int_0^{\pi} \sin^5 x \, dx$.

(b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$.

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^1 \cos(x^2) \, dx$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\lesssim 3\%$, 8 za rel. grešku $\lesssim 6\%$)

3. Riješiti diferencijalnu jednadžbu: $y'' - y = e^x + 1$.

4. Izračunaj $\int_0^2 \frac{x+2}{3x^2-2x-5} \, dx$

5. Odredi ekstreme funkcije $f(x, y) = 2xy - 3x^2 - 2y^2 + 10$.

~~23~~

20
20

Ukupno:

~~43~~

43

Kor

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2-x^2} dx = \frac{1}{2} \left[x\sqrt{a^2-x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

2. $\int_{-1}^1 \cos(x^2) \, dx$

k	0	1	2
x_k	-1	0	1
f_k	0.54	1	0.54

$S = \frac{d}{6} (f_0 + 4f_1 + f_2)$

$d = 1 - (-1) = 2$

$S = \frac{2}{6} (0.55 + 4 \cdot 1 + 0.55) = 1.7$

$S = 1.8$

GREŠKE

~~3~~

$f_0 = \cos(-1^2) = \cos(1) = 0.55$

$f_1 = \cos(0^2) = \cos(0) = 1$

$f_2 = \cos(1^2) = \cos(1) = 0.55$

$\cos(1) = 0.54030$

3

$$5. f(x,y) = 2xy - 3x^2 - 2y^2 + 10$$

y -KONSTANTA

$$\frac{\partial f}{\partial x} = 2x' \cdot y + 2x \cdot y' - 6x$$

$$\frac{\partial^2 f}{\partial x^2} = -6$$

$$\frac{\partial^2 f}{\partial x \partial y} = 2$$

$$\frac{\partial f}{\partial x} = 2y - 6x$$

$$\frac{\partial^2 f}{\partial y^2} = -4$$

$$\frac{\partial^2 f}{\partial y \partial x} = 2$$

x -KONSTANTA

$$\frac{\partial f}{\partial y} = 2x' \cdot y + 2x \cdot y' - 4y$$

$$\frac{\partial f}{\partial y} = 2x - 4y$$

$$2y - 6x = 0$$

$$2y = 6x / :2$$

$$y = 3x$$

$$y = 0$$

$$2x - 4y = 0$$

$$2x = 4y / :2$$

$$x = 2y$$

$$x = 2 \cdot 3x$$

$$x = 6x$$

$$5x = 0 / :5$$

$$x = 0$$

$T(0,0)$

$$\Delta = \begin{vmatrix} -6 & 2 \\ 2 & -4 \end{vmatrix} = 24 - 4 = 20$$

$$\Delta > 0$$

$$\frac{\partial^2 f}{\partial x^2} = -6 < 0$$

Točka $T(0,0)$ je maksimum funkcije.



$$4. \int_0^2 \frac{x+2}{3x^2-2x-5} dx$$

$x_1 = \frac{5}{3} \Rightarrow$ u granicama $(0, 2)$ se nalazi pa je integral nepravil.

$$3x^2 - 2x - 5 = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 + 60}}{6}$$

$$x_{1,2} = \frac{2 \pm 8}{6}$$

$$x_{1,2} = \frac{5}{3}, -1$$

$$3x^2 - 2x - 5 = 3\left(x - \frac{5}{3}\right)(x+1) \checkmark$$

$$\frac{x+2}{3x^2-2x-5} = \frac{A}{\left(x - \frac{5}{3}\right)} + \frac{B}{(x+1)} \quad | \cdot 3\left(x - \frac{5}{3}\right)(x+1)$$

$$x+2 = 3A(x+1) + 3B\left(x - \frac{5}{3}\right)$$

$$x+2 = 3Ax + 3A + 3Bx - 5B$$

$$1 = 3A + 3B \Rightarrow 3A = 1 - 3B$$

$$2 = 3A - 5B \quad A = \frac{1-3B}{3}$$

$$3 \cdot \left(\frac{1-3B}{3}\right) - 5B = 2 \quad A = \frac{1+3 \cdot \frac{1}{8}}{3}$$

$$1 - 3B - 5B = 2 \quad A = \frac{\frac{8+B}{8}}{3}$$

$$-8B = 1 \quad A = \frac{\frac{11}{8}}{3}$$

$$B = -\frac{1}{8} \quad A = \frac{11}{24}$$

$$\int_0^2 \frac{\frac{11}{24}}{x - \frac{5}{3}} dx + \int_0^2 \frac{-\frac{1}{8}}{x+1} dx = \frac{11}{24} \int_0^2 \frac{dx}{x - \frac{5}{3}} - \frac{1}{8} \int_0^2 \frac{dx}{x+1} =$$

$$= \left[\begin{array}{l} x - \frac{5}{3} = t \\ dx = dt \end{array} \right] \left[\begin{array}{l} x+1 = u \\ dx = du \end{array} \right] = \frac{11}{24} \int \frac{dt}{t} - \frac{1}{8} \int \frac{du}{u} =$$

$$= \frac{11}{24} \left[\ln \left| x - \frac{5}{3} \right| \right]_0^2 - \frac{1}{8} \left[\ln |x+1| \right]_0^2$$

$$= \frac{11}{24} \left(\ln \left| 2 - \frac{5}{3} \right| - \ln \left| 0 - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln |2+1| - \ln |0+1| \right) =$$

$$= \frac{11}{24} (-1.1 - 0.51) - \frac{1}{8} (1.1 - 0) = (-0.8754)$$

NJE RIŠENJE!

$$= \left[\frac{11}{24} \left(\ln \left| x - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln |x+1| \right) \right]_0^2$$

$$= \lim_{a \rightarrow \frac{5}{3}} \left[\frac{11}{24} \left(\ln \left| x - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln |x+1| \right) \right]_0^a + \lim_{b \rightarrow \frac{5}{3}} \left[\frac{11}{24} \left(\ln \left| x - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln |x+1| \right) \right]_b^2$$

$$= \lim_{a \rightarrow \frac{5}{3}} \left[\left(\frac{11}{24} \left(\ln \left| a - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln |a+1| \right) \right) - \left(\frac{11}{24} \left(\ln \left| 0 - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln |0+1| \right) \right) \right] + \lim_{b \rightarrow \frac{5}{3}} \left[\left(\frac{11}{24} \left(\ln \left| 2 - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln |2+1| \right) \right) - \left(\frac{11}{24} \left(\ln \left| b - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln |b+1| \right) \right) \right] =$$

$$= \left(\frac{11}{24} \left(\ln \left| \frac{5}{3} - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln \left| \frac{5}{3} + 1 \right| \right) \right) - \left(\frac{11}{24} \left(\ln \left| 0 - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln |0+1| \right) \right) + \left(\frac{11}{24} \left(\ln \left| 2 - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln 3 \right) \right) - \left(\frac{11}{24} \left(\ln \left| \frac{5}{3} - \frac{5}{3} \right| \right) - \frac{1}{8} \left(\ln \left| \frac{5}{3} + 1 \right| \right) \right)$$

$$= -\infty - 0.12 - 0.23 - 0.5 - 0.14 + \infty - 0.12 =$$

$$= -\infty + \infty = N/P$$



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IME I PREZIME: **IVAN GAČINA**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0075-2011

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$\int_{-1}^1 \cos(x^2) \, dx$

SIMPSON

$d = 1 - (-1) = 2$
 $S = \frac{2}{6} (1 + 4 \cdot 0,9998 + 0,99756)$

$= 1,9916$

POGREŠNO!

k	0	1	2
x_k	0	1	2
f_k	1	0,9998	0,99756