MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3

IME I PREZIME: STIPE BREULJ **VRIJEME POČETKA:** 17:15

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0192-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati:
   (a) određeni integral $\int_0^{\pi} \sin^5 x \, dx$.
   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$.
2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos(x^2)\, dx$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) 3
3. Riješiti diferencijalnu jednadžbu: $y'' - y = e^x + 1$.
4. Izračunaj $\int_0^2 \frac{x+2}{3x^2-2x-5} dx$ 20
5. Odredi ekstreme funkcije $f(x, y) = 2xy - 3x^2 - 2y^2 + 10$. 20

Ukupno: 43

Kos

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln|\cos x| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$
$\int \frac{dx}{x} = \ln|x| + C$	$\int \cot x \, dx = \ln|\sin x| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2. $\int_{-1}^{1} \cos(x^2) dx$

$d = 1-(-1) = 2$

$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$

$S = \frac{2}{6}(0{,}55 + 4 \cdot 1 + 0{,}55) = 1{,}7$

$S = 1.8$?

k	0	1	2
x_k	-1	0	1
f_k	0.54	1	0.54

GREŠKE

↓

$\cos(1) = 0{,}54030$

3

$f_0 = \cos(-1^2) = \cos(1) = 0{,}55$?

$f_1 = \cos(0^2) = \cos(0) = 1$

$f_2 = \cos(1^2) = \cos(1) = 0.55$?

5. $f(x,y) = 2xy - 3x^2 - 2y^2 + 10$

Y- KONSTANTA

$$\frac{\partial f}{\partial x} = 2x' \cdot y + 2x \cdot y' - 6x$$

$$\frac{\partial f}{\partial x} = 2y - 6x$$

$$\frac{\partial^2 f}{\partial x^2} = -6 \qquad \frac{\partial^2 f}{\partial x \partial y} = 2$$

$$\frac{\partial^2 f}{\partial y^2} = -4 \qquad \frac{\partial^2 f}{\partial y \partial x} = 2$$

X-KONSTANTA

$$\frac{\partial f}{\partial y} = 2x' \cdot y + 2x \cdot y' - 4y$$

$$\frac{\partial f}{\partial y} = 2x - 4y$$

$2y - 6x = 0$

$2y = 6x \ /:2$

$y = 3x$

$y = 0$

$2x - 4y = 0$

$2x = 4y \ /:2$

$x = 2y$

$x = 2 \cdot 3x$

$x = 6x$

$5x = 0 \ /:5$

$x = 0$

$T(0,0)$

$$\Delta = \begin{vmatrix} -6 & 2 \\ 2 & -4 \end{vmatrix} = 24 - 4 = 20$$

$\Delta > 0$

$$\frac{\partial^2 f}{\partial x^2} = -6 < 0$$

Točka $T(0,0)$ je maksimum funkcije.

STIPE BREULJ

4. $$\int_0^2 \frac{x+2}{3x^2-2x-5}\,dx$$

$x_1 = \frac{5}{3} \Rightarrow$ u granicama $(0,2)$ se nalazi pa je integral nepravi.

$$3x^2-2x-5=0$$

$$x_{1,2}=\frac{2\pm\sqrt{4+60}}{6}$$

$$x_{1,2}=\frac{2\pm 8}{6}$$

$$x_{1,2}=\frac{5}{3}, -1$$

$$3x^2-2x-5=3\left(x-\frac{5}{3}\right)(x+1) \checkmark$$

$$\frac{x+2}{3x^2-2x-5}=\frac{A}{\left(x-\frac{5}{3}\right)}+\frac{B}{(x+1)} \Big/ \cdot 3\left(x-\frac{5}{3}\right)(x+1)$$

$$x+2=3A(x+1)+3B\left(x-\frac{5}{3}\right)$$

$$x+2=3Ax+3A+3Bx-5B$$

$$1=3A+3B \Rightarrow 3A=1-3B$$

$$2=3A-5B \qquad A=\frac{1-3B}{3}$$

$$3\cdot\left(\frac{1-3B}{3}\right)-5B=2 \qquad A=\frac{1+3\cdot\frac{1}{8}}{3}$$

$$1-3B-5B=2 \qquad A=\frac{\frac{8+3}{8}}{3}$$

$$-8B=1 \qquad A=\frac{\frac{11}{8}}{3}$$

$$B=-\frac{1}{8} \qquad A=\frac{11}{24}$$

$$\int_0^2 \frac{\frac{11}{24}}{x-\frac{5}{3}}\,dx+\int_0^2\frac{-\frac{1}{8}}{x+1}\,dx=\frac{11}{24}\int_0^2\frac{dx}{x-\frac{5}{3}}-\frac{1}{8}\int_0^2\frac{dx}{x+1}=$$

$$=\begin{bmatrix} x-\frac{5}{3}=t/' \\ dx=dt \end{bmatrix}\begin{bmatrix} x+1=u/' \\ dx=du \end{bmatrix}=\frac{11}{24}\int\frac{dt}{t}-\frac{1}{8}\int\frac{du}{u}=$$

$$=\frac{11}{24}\Big[\ln\left|x-\frac{5}{3}\right|\Big]_0^2-\frac{1}{8}\Big[\ln|x+1|\Big]_0^2$$

$$= \frac{11}{24}\left(\ln\left|2-\frac{5}{3}\right| - \ln\left|0-\frac{5}{3}\right|\right) - \frac{1}{8}\left(\ln|2+1| - \ln|0+1|\right) =$$

$$= \frac{11}{24}\left(-1.1 - 0.51\right) - \frac{1}{8}\left(1.1 - 0\right) = \left(-0.8754\right)$$

NIJE RJEŠENJE!

$$= \left[\frac{11}{24}\left(\ln\left|x-\frac{5}{3}\right|\right) - \frac{1}{8}\left(\ln|x+1|\right)\right]_0^2$$

$$= \lim_{a\to\frac{5}{3}}\left[\frac{11}{24}\left(\ln\left|x-\frac{5}{3}\right|\right) - \frac{1}{8}\left(\ln|x+1|\right)\right]_0^a + \lim_{b\to\frac{5}{3}}\left[\frac{11}{24}\left(\ln\left|x-\frac{5}{3}\right|\right) -\right.$$

$$\left. - \frac{1}{8}\left(\ln|x+1|\right)\right]_b^2 = \lim_{a\to\frac{5}{3}}\left[\left(\frac{11}{24}\left(\ln\left|a-\frac{5}{3}\right|\right) - \frac{1}{8}\left(\ln|a+1|\right)\right) -\right.$$

$$\left. - \left(\frac{11}{24}\left(\ln\left|0-\frac{5}{3}\right|\right) - \frac{1}{8}\left(\ln|0+1|\right)\right)\right] + \lim_{b\to\frac{5}{3}}\left[\left(\frac{11}{24}\left(\ln\left|2-\frac{5}{3}\right|\right) -\right.\right.$$

$$\left.\left.\frac{1}{8}\left(\ln|2+1|\right)\right) - \left(\frac{11}{24}\left(\ln\left|b-\frac{5}{3}\right|\right) - \frac{1}{8}\left(\ln|b+1|\right)\right)\right] =$$

$$= \left(\frac{11}{24}\left(\ln\left|\frac{5}{3}-\frac{5}{3}\right|\right) - \frac{1}{8}\left(\ln\left|\frac{5}{3}+1\right|\right)\right) - \left(\frac{11}{24}\left(\ln\left|0-\frac{5}{3}\right|\right) \cancel{-\frac{1}{8}\left(\ln|0+1|\right)}\right)$$

$$+ \left(\frac{11}{24}\left(\ln\left|2-\frac{5}{3}\right|\right) - \frac{1}{8}\left(\ln 3\right)\right) - \left(\frac{11}{24}\left(\ln\left|\frac{5}{3}-\frac{5}{3}\right|\right) - \frac{1}{8}\left(\ln\left|\frac{5}{3}+1\right|\right)\right)$$

$$= -\infty - 0.12 - 0.23 - 0.5 - 0.14 + \infty - 0.12 =$$

$$= -\infty + \infty = N/P$$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3

IME I PREZIME: IVAN GAĆINA **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0075-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati:

   (a) određeni integral $\int_0^\pi \sin^5 x \, dx$. Ø

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. ~~Ø~~

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos(x^2)\, dx$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) 20

3. Riješiti diferencijalnu jednadžbu: $y'' - y = e^x + 1$. ~~Ø~~

4. Izračunaj $\int_0^2 \frac{x+2}{3x^2-2x-5} dx$ 20

5. Odredi ekstreme funkcije $f(x, y) = 2xy - 3x^2 - 2y^2 + 10$.

Ukupno: 40

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

$\boxed{2}$ $\int_{-1}^{1} \cos(x^2)\, dx$

SIMPSON

$d = 1-(-1) = 2$

$S = \frac{2}{6}\left(1 + 4 \cdot 0{,}9998 + 0{,}99756\right)$

$= 1{,}9316$

k	0	1	2
x_k	0	1	2
f_k	1	0,9998	0,99756

POGREŠNO!

Ø

$f(x,y) = 2xy - 3x^2 - 2y^2 + 10$ 5/

$$\frac{\partial x}{\partial y} = 2y - 6x$$

$$\frac{\partial y}{\partial x} = 2x - 4y$$

$$\frac{\partial^2 x}{y} = -6 \qquad \begin{vmatrix} -6 & 0 \\ 0 & -4 \end{vmatrix} = 24 \qquad 24 > 0$$

$$\frac{\partial^2 y}{x} = -4$$

FUNKCIJA IMA EXSTREM [MAXIMUM]

IVAN GAĆINA

3| $y'' - y = e^x + 1$

$\cancel{\frac{-b \pm \sqrt{b^2 - 4ac}}{2}}$ $\cancel{\frac{0 \pm 2}{2}}$ $\cancel{\pm 1}$

$r^2 - 1 = 0$

$r = \pm 1$ $\quad r_1 = -1$, $r_2 = 1$

$$\boxed{y_H = C_1 e^{-x} + C_2 e^x}$$

$y_{P_1} \Rightarrow e^x = e^{\alpha x}\left(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x)\right)$ $\quad \cos(\beta x) \to 1, \ \sin(\beta x) \to 0$

$\alpha = 1 \quad P_m(x) = 1 \quad m = 0$
$\beta = 0 \Rightarrow Q_n(x) = 0 \quad n = N/P$ $\Big\} N = 0$

$k = \alpha + \beta i = 1 + 0i$ $\quad k = 1$

$y_{P_1} = x \cdot e^x \cdot A \Rightarrow Ax \cdot e^x$

$y_{P_1}' = A \cdot e^x + Ax \cdot e^x$
$y_{P_1}'' = A \cdot e^x + A \cdot e^x + Ax \cdot e^x$

$A \cdot e^x + A \cdot e^x + Ax \cdot e^x - Ax \cdot e^x = e^x \ / e^x$

$A + A + \cancel{Ax} - \cancel{Ax} = 1 \Rightarrow 2A = 1$
$A = \frac{1}{2}$

$$\boxed{y_{P_1} = \frac{x}{2} \cdot e^x}$$

$y_{P_2} \Rightarrow 1 = e^{\alpha x}\left(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x)\right)$ $\quad \cos(\beta x) \to 1$

$\alpha = 0 \quad P_m(x) = 1 \quad m = 0$
$\beta = 0 \quad Q_n(x) = 0 \quad n = N/P$ $\Big\} N = 0$

$k = 0 + 0i$
$k = 0$

$y_{P_2} = B \qquad y_{P_2}' = 0 \quad y_{P_2}'' = 0 \Rightarrow$

$0 - B = 1$
$B = -1$ $\Big\}$ $\boxed{y_{P_2} = -1}$

$$\boxed{\begin{aligned} y(x) &= y_H + y_{P_1} + y_{P_2} \\ &= C_1 \cdot e^{-x} + C_2 e^x + \frac{x}{2} \cdot e^x - 1 \end{aligned}}$$

PROVJERA

$$y = c_1 e^{-x} + c_2 e^{x} + \frac{1}{2} x \cdot e^{x} - 1$$

$$y' = -c_1 \cdot e^{-x} + c_2 \cdot e^{x} + \frac{1}{2} e^{x} + \frac{1}{2} x \cdot e^{x}$$

$$y'' = c_1 \cdot e^{-x} + c_2 \cdot e^{x} + \frac{1}{2} e^{x} + \frac{1}{2} \cdot e^{x} + \frac{1}{2} x \cdot e^{x}$$

$$\left(c_1 \cdot e^{-x} + c_2 \cdot e^{x} + \frac{1}{2} e^{x} + \frac{1}{2} \cdot e^{x} + \frac{1}{2} x \cdot e^{x}\right)$$

$$-\left(c_1 e^{-x} + c_2 e^{x} + \frac{1}{2} x \cdot e^{x} - 1\right) = e^{x} + 1$$

$$\cancel{c_1 \cdot e^{-x}} + \cancel{c_2 \cdot e^{x}} + \frac{1}{2} e^{x} + \frac{1}{2} e^{x} + \cancel{\frac{1}{2} x \cdot e^{x}} \cancel{- c_1 e^{-x}} \cancel{- c_2 e^{x}} \cancel{- \frac{1}{2} x e^{x}} + 1 = e^{x} + 1$$

$$\boxed{e^{x} + 1 = e^{x} + 1}$$

IVAN GAĆINA

4.

$$\int_0^2 \frac{x+2}{3x^2-2x-5}\,dx$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{2 \pm \sqrt{4+60}}{6} = \frac{2 \pm 8}{6} \qquad x_1 = \frac{5}{3} \qquad x_2 = -1$$

$$\frac{x+2}{3x^2-2x-5} = \frac{A}{(x-\frac{5}{3})} + \frac{B}{(x+1)} \quad / \cdot 3x^2-2x-5 \quad (3(x-\tfrac{5}{3})(x+1)$$

$$x+2 = A(3x+3) + B(3x-5) \quad // \quad x+2 = 3xA + 3A + 3xB - 5B$$

uz x^0: $2 = 3A - 5B$ $\qquad 5B = 3A - 2 \Rightarrow B = \frac{3}{5}A - \frac{2}{5}$

uz x^1: $1 = 3A + 3B$

$$1 = 3A + (\tfrac{3}{5}A - \tfrac{2}{5}) \qquad 1 = \tfrac{18}{5}A - \tfrac{2}{5}$$

$$\tfrac{18}{5}A = \tfrac{7}{5}$$

$$A = \tfrac{7}{6}$$

$$B = \tfrac{3}{10}$$

$$\int \frac{x+2}{3x^2-2x-5}dx \Rightarrow \int \frac{\frac{7}{6}}{(x-\frac{5}{3})}dx + \int \frac{\frac{3}{10}}{(x+1)}dx$$

$$\int \frac{7}{6}\frac{dx}{x-\frac{5}{3}} \quad \left\{ \begin{array}{l} x - \frac{5}{3} = t \\ dx = dt \end{array} \right\} = \frac{7}{6}\int \frac{dt}{t} = \frac{7}{6}\ln(t)$$

$$= \frac{7}{6}\ln|x-\tfrac{5}{3}|$$

$$+\frac{3}{10}\int \frac{dx}{x+1} \quad \left\{ \begin{array}{l} x+1 = t \\ dx = dt \end{array} \right\} + \frac{3}{10}\int \frac{dt}{t} = +\frac{3}{10}\ln|t|$$

$$= +\frac{3}{10}\ln|x+1|$$

NEPRAVI INTEGRAL

$$\left[\frac{7}{6}\ln|x-\tfrac{5}{3}| + \frac{3}{10}\ln|x+1|\right]_0^2$$

$$-\tfrac{5}{3} = \tfrac{5}{3}$$

$$\left(\frac{7}{6}\ln|2-\tfrac{5}{3}| + \frac{3}{10}\ln|2+1|\right) - \left(\frac{7}{6}\ln|0-\tfrac{5}{3}| + \frac{3}{10}\ln|0+1|\right)$$

$-0{,}95$ $\qquad$ $0{,}59$

$$= -\frac{77}{50} = -1{,}54$$

IVAN GACINA

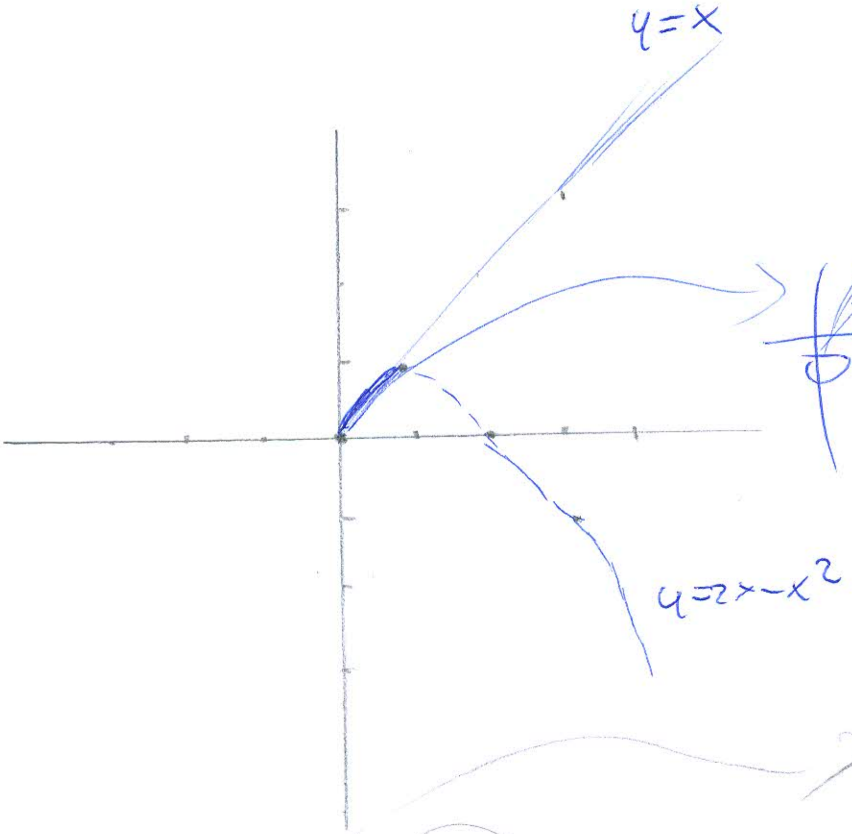
1.

$y = 2x - x^2$

x	0	1	2	3	4
y	0	1	0	-3	-8

$y = x$

x	0	1	2	3
y	0	1	2	3



$$\int 2x - x^2 \, dx \quad \begin{cases} -x^2 = t \\ -2x \, dx = dt \end{cases}$$

$$\int -t \, dt \quad -\frac{t^2}{2} \quad \boxed{-\frac{x^4}{2}}$$

$$P = \int_0^1 2x - x^2 \, dx - \int_0^1 x \, dx$$

$$\int x \, dx \rightarrow -\frac{x^2}{2}$$

$$\left[-\frac{x^4}{2}\right]_0^1 - \left[\frac{x^2}{2}\right]_0^1$$

$$-\frac{1}{2} - \cancel{0} \quad -\frac{1}{2} + \cancel{0}$$

$$\boxed{-1}$$

×

0

$\boxed{1}$ $\int_0^{\pi} \sin^5 x \, dx$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C4

IME I PREZIME: MATIJA SEGARIĆ **VRIJEME POČETKA:** 17:15

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0283-2014

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Odredi ekstreme funkcije $f(x,y) = 2xy - 3x^2 - 2y^2 + 10$.
2. Riješiti diferencijalnu jednadžbu: $y'' - y = e^x + 1$.
3. Izračunati:
   (a) određeni integral $\displaystyle\int_0^{\pi} \sin^5 x \, dx$.
   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$.
4. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos(x^2)\, dx$. (bodovanje: 20 za rel. grešku$\leq$ 1%, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$)
5. Izračunaj $\int_0^2 \frac{x+2}{3x^2-2x-5} dx$

$\cancel{0}$

20

$\cancel{0}$

Ukupno: 20

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

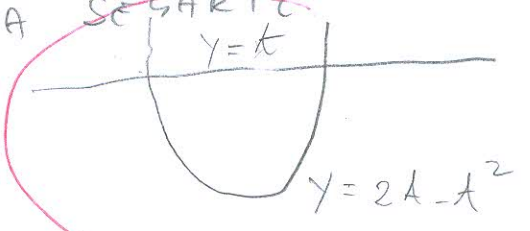
Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3 b) MATIJA

$y = -x^2 + 2x$

$y = x$



$1. \; a, b = ?$

$-x^2 + 2x = x$

$-x^2 + 2x - x = 0$

$-x^2 + x = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1 + 0}}{-2}$

$x_1 = \frac{-1 + 1}{-2} = -1$

$x_2 = \frac{-1 - 1}{-2} = 0$

$$\int_{-1}^{0} \left[(x) - (-x^2 + 2x) \right] dx$$

$$= \int_{-1}^{0} (x + x^2 - 2x)\, dx$$

$$= \int_{-1}^{0} x^2\, dx - \int_{0}^{1} x\, dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} \Big|_{-1}^{0}$$

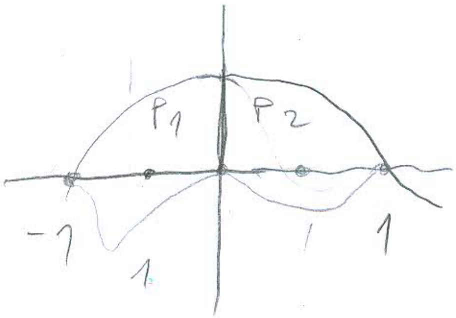
$$\left(\frac{0^3}{3} - \frac{0^2}{2} \right) - \left(\frac{-1^3}{3} - \frac{-1^2}{2} \right)$$

$P = \frac{5}{6}$

0

MATIJA SEGARIC

4. $\int_{-1}^{1} \cos(x^2)\, dx$



SIMPSON

$\frac{d}{6}(f_0 + 4f_1 + f_2)$

$P_1 = ?$

$d = 1$

$f_0 = \cos((-1)^2) = 0.540$

$f_1 = \cos((-0.5)^2) = 0.969$

$f_2 = \cos(0^2) = 1$

$P_1 = \frac{1}{6}(0.54 + 4 \cdot 0.969 + 1)$

$P_1 = 0.903$

$P_2 = ?$

$d = 1$

$f_0 = 1$

$f_1 = \cos(0.5^2) = 0.969$

$f_2 = \cos(1^2) = 0.540$

$P_2 = 0.903$

$P_{UKUPNO} = 2 \cdot 0.903$
$= 1.806$

20

MATIJA ŠEGARIĆ

5. $\int_0^2 \frac{x+2}{3x^2-2x-5}\,dx =$

$3x^2-2x-5=0$

$x_{1,2}=\frac{2\pm\sqrt{9+60}}{6}$

$x_1=1.718$

$x_2=-1.05$

$\frac{x+2}{3x^2-2x-5}=\frac{A}{x-1.718}+\frac{B}{x+1.05} \Big/ 3x^2-2x-5$

$x+2=A(x+1.05)+B(x-1.718)$

$x+2=Ax+1.05A+Bx-1.718B$

$x+2=(A+B)x+(1.05A-1.718B)$

$A+B=1 \Rightarrow B=1-A$

$1.05A-1.718B=2$

$1.05A-1.718(1-A)=2$

$1.05A-1.718+1.718A=2$

$2.768A=2+1.718$

$A=\frac{2+1.718}{2.768}$

$A=1.343$

$B=1-1.343$

$B=-0.343$

$$\int_0^2 \frac{x+2}{3x^2-2x-5} = \int_0^2 \left(\frac{1.343}{x-1.718} + \frac{-0.343}{x+1.05}\right) dx$$

$$1.343 \underbrace{\int_0^2 \frac{dx}{x-1.718}}_{I_1} - 0.343 \underbrace{\int_0^2 \frac{dx}{x+1.05}}_{I_2}$$

$$I_1 \int_0^2 \frac{dx}{x-1.718} \quad \left| \begin{array}{l} t = x - 1.718 \\ dt = dx \end{array} \right.$$

$$\ln|x-1.718| \Big|_0^2$$

$$I_2 \int_0^2 \frac{dx}{x+1.05}$$

$$= \ln|x+1.05| \Big|_0^2$$

$$I_U = 1.343 \cdot \ln|x-1.718| - 0.343 \cdot \ln|x+1.05|$$

Rješenje

$$\left(1.343 \cdot \ln|2-1.718| - 0.343 \cdot \ln|2+1.05|\right) -$$
$$1.343 \cdot \ln|0-1.718| - 0.343 \cdot \ln|0+1.05|)$$

$$= -2.08 - 0.710 = -2.79$$

OVO JE NEPRAVI INTEGRAL!

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: LUKA ĆALUŠIĆ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-270193-2012	

1. Izračunati:

   (a) određeni integral $\int_0^{\pi} \sin^5 x \, dx$.

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. 10

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos(x^2)\, dx$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) 4

3. Riješiti diferencijalnu jednadžbu: $y'' - y = e^x + 1$.

4. Izračunaj $\int_0^2 \frac{x+2}{3x^2-2x-5} dx$ $\emptyset$

5. Odredi ekstreme funkcije $f(x, y) = 2xy - 3x^2 - 2y^2 + 10$. $\emptyset$

Ukupno: 14

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

(2) $\int_{-1}^{1} \cos(x^2)\, dx$

	-1	0	1
	f_1	f_0	f_2
$\cos(x^2)$	0,540	1	0,540

$S = \frac{d}{6}(f_1 + 4f_0 + f_2)$

$d = 2$

$S = \frac{1}{3}(0,540 + 4 \cdot 1 + 0,540) = \frac{5,08}{3} = 1,69333$ 4

4

TOČNO RJEŠENJE: 1.80905

APS. GREŠKA: 1.80905 - 1.69333 = 0.063967

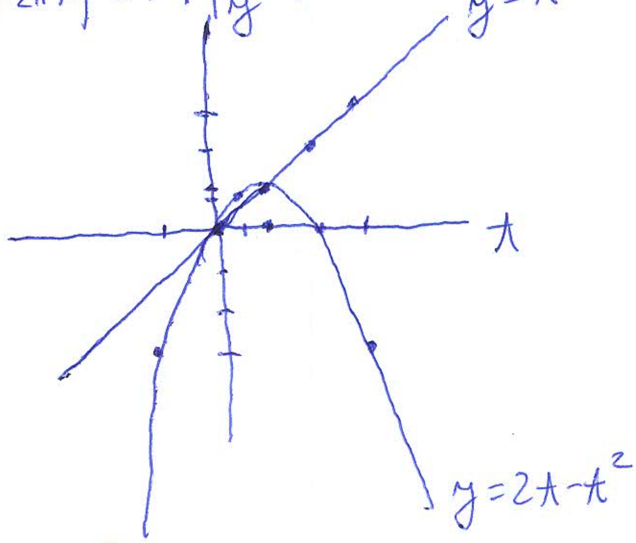
REL. GREŠKA: 6.4 %

① b) GORNJA $y = 2x - x^2$ DONJA $y = x$

x	1	2	3
$y=x$	1	2	3

x	0	1	2	3	−1	$\frac{1}{2}$
$y=2x-x^2$	0	1	0	−3	−3	$\frac{3}{4}$

$2x - x^2 = x$



$x - x^2 = 0$

$x(1-x) = 0$

$x_1 = 0$

$1 - x = 0$

$x_2 = 1$

$\cancel{\left(\int_0^1 x\,dx - \int_0^1 (2x - x^2)\,dx = \frac{x^2}{2}\Big|_0^1 - 2\frac{x^2}{2}\Big|_0^1 - \frac{x^3}{3}\Big|_0^1\right)}$

$\cancel{\frac{1}{2} - 1 - \frac{1}{3} =}$

$P = \int_0^1 2x - x^2\,dx - \int_0^1 x\,dx = 2\int_0^1 x\,dx - \int_0^1 x^2\,dx - \int_0^1 x\,dx = 2\frac{x^2}{2}\Big|_0^1 - \frac{x^3}{3}\Big|_0^1 - \frac{x^2}{2}\Big|_0^1 =$

GORNJA, DONJA

$= 1 - \frac{1}{3} - \frac{1}{2} = \frac{1}{6}$

LUKA ČALUŠIĆ

⑤

LUKA ČALUŠIĆ

$f(x,y)=2xy-3x^2-2y^2+10$

$\frac{\delta f}{\delta x}=2y-6x=0$

$\frac{\delta f}{\delta y}=2x-4y=0 \Rightarrow 2x=4y \Rightarrow x=2y$

$2y-12y=0$

$-10y=0$

$\boxed{y=0}$

$x=2y$

$\boxed{x=0}$

$S(0,0)$

$\frac{\delta^2 f}{\delta x^2}=-6$ $\frac{\delta^2 f}{\delta x \delta y}=2$

$\frac{\delta^2 f}{\delta y \delta x}=2$ $\frac{\delta^2 f}{\delta y^2}=-4$

$\Delta=\begin{bmatrix} \overset{0}{\overset{\vee}{-6}} & 2 \\ 2 & -4 \end{bmatrix}=24-4=20$

$\Delta>0$

NEMA
EKSTREMA
NE MOŽEMO ODREDITI
STACIONARNU TOČKU

④ $\int_0^2 \frac{x+2}{3x^2-2x-5}$

$x_{1,2} = \frac{\cancel{4}\,2 \pm \sqrt{4+60}}{6}$

$\cancel{x_1} \ldots \quad x_2 = -1$

$x_1 = \frac{10}{6} = \frac{5}{3}$

$3\cdot\left(\frac{5}{3}\right)^2 - 2\cdot\frac{5}{3} - 5 = 0$

$\boxed{x_1 = \frac{5}{3}} \to$ PREKID FUNKCIJE = NEPRAVI INTEGRAL

$\frac{x+2}{(x+\frac{5}{3})(x-1)} = \frac{A}{(x+\frac{5}{3})} + \frac{B}{(x-1)} \Big/ \cdot$ NAZ

POGREŠNA FAKTORIZACIJA:

$3x^2-2x-5 \neq 3(x+\frac{5}{3})(x-1)$

$3x^2-2x-5 = 3(x-\frac{5}{3})(x+1)$

$x+2 = Ax - A + \cancel{Bx} + \frac{5}{3}B$

$A+B = 1 \to A = 1-B$

$\frac{5}{3}B - A = 2$

$\frac{5}{3}B - (1-B) = 2$

$\frac{5}{3}B + B = 3$

$\frac{8}{3}B = 3$

$\boxed{B = \frac{8}{9}}$

$A = 1 - \frac{8}{9}$

$\boxed{A = \frac{1}{9}}$

$\begin{cases} x+\frac{5}{3} = t \\ dx = dt \end{cases}$

$= \int \frac{\frac{1}{9}}{x+\frac{5}{3}}dx + \int \frac{\frac{8}{9}}{x-1}dx = \frac{1}{9}\ln|x+\frac{5}{3}| + \frac{8}{9}\ln|x-1|$

$\lim_{a\to\frac{5}{3}^-}\left(\frac{1}{9}\ln|x+\frac{5}{3}| + \frac{8}{9}\ln|x-1|\right)\Big|_0^a = 0{,}1338 + (-0{,}3604)$

$+$

$\lim_{b\to\frac{5}{3}^+}\left(\frac{1}{9}\ln|x+\frac{5}{3}| + \frac{8}{9}\ln|x-1|\right)\Big|_b^2 = 0{,}1444 + 0 - 0{,}1338 + (-0{,}3604)$

$= -0{,}5764$ //

X

LUKA ČALUŠIĆ

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ C4	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: MISLAV ROGOZNICA **Vrijeme početka:** **Matični broj studenta (iznad slike u indeksu):** 17-2-0345-2013	

1. Odredi ekstreme funkcije $f(x, y) = 2xy - 3x^2 - 2y^2 + 10$. 10
2. Riješiti diferencijalnu jednadžbu: $y'' - y = e^x + 1$.
3. Izračunati:

   (a) određeni integral $\int\limits_0^\pi \sin^5 x\, dx$.

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. ~~0~~
4. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos(x^2)\, dx$. (bodovanje: 20 za rel. grešku$\leq$ 1%, 15 za rel. grešku $\lessapprox$ 3%, 8 za rel. grešku $\lessapprox$ 6%) 4
5. Izračunaj $\int_0^2 \frac{x+2}{3x^2-2x-5} dx$ ∅

Ukupno: 14

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
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$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4. $\int\limits_{-1}^{1} \cos(x^2)\, dx =$

$f_0(-1) = \cos(-1^2) = \cos(1) = 0{,}540302$

$f_1(0) = \cos(0) = 1$

$f_2(1) = \cos(1) = 0{,}540302$

$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$

$S = \frac{\cancel{2}^1}{\cancel{6}_3}(0{,}540302 + 4 \cdot 1 + 0{,}540302)$

$S = \frac{1}{3} \cdot 5{,}080604$

$\boxed{S = 1{,}693535}$ 4

TOČNO RJEŠENJE: 1.80905

REL. GREŠKA: 6.4%

3. b) POVRŠINA LIKA OMEĐENOG KRIVULJAMA $y=2t-t^2$ i $y=t$.

1. SJECIŠTE:

$y=t$

$y=2t-t^2$

t	0	1	-1
$y=t$	0	1	-1

t	0	1	-1	2
$y=2t-t^2$	0	1	-3	0

$2t-t^2=t$

$2t-t^2-t=0$

$-t^2+t=0 \,/\cdot(-1)$

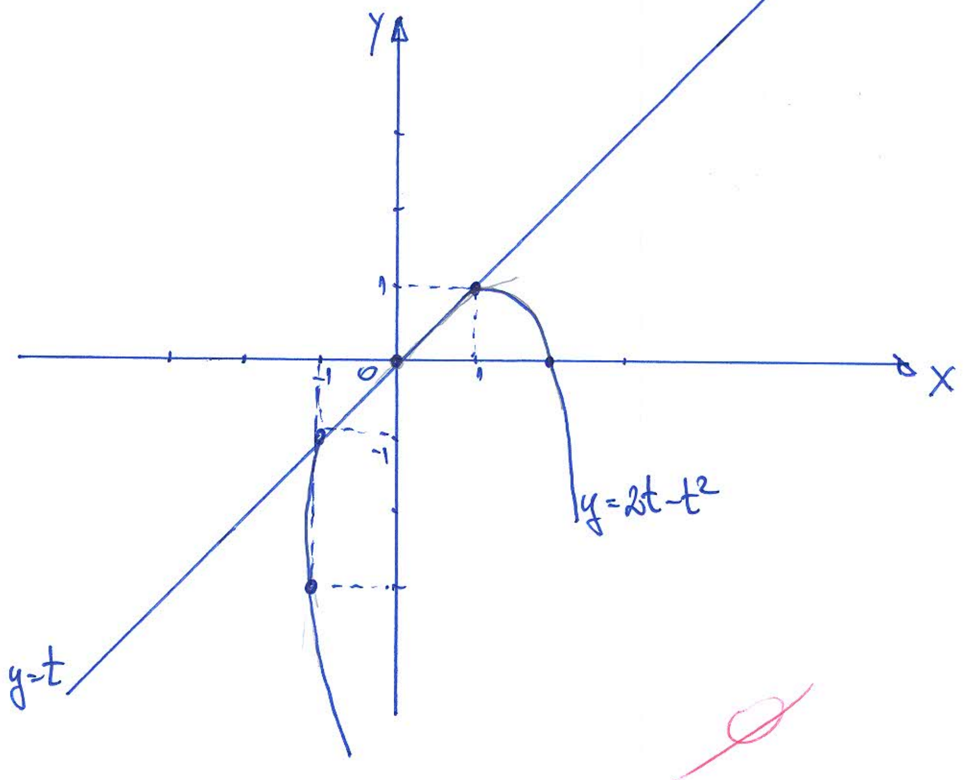
$t^2-t=0$

$t_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$t_{1,2}=\frac{1\pm\sqrt{1-4}}{2}$ ✗

$t_{1,2}=\frac{1\pm\sqrt{-3}}{2}$

$\Downarrow$



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- NE POSTOJI SJECIŠTE OVIH DVIJU KRIVULJA PA TAKO NE POSTOJI NI OMEĐENA POVRŠINA P_0

3. a) $\int_0^{\pi} \sin^5 x\, dx = \int_0^{\pi} (\sin^2 x)^2 \cdot \sin x \cdot dx = \int_0^{\pi} \left(\frac{1-\cos(2x)}{2}\right)^2 \cdot \sin x \cdot dx =$

① ODREDI EKSTREME FUNKCIJE $f(x,y) = 2xy - 3x^2 - 2y^2 + 10$

1. EKSTREMI:

$\partial_x f = 2y - 6x$

$\partial_{xx} f = -6$

$\partial_{xy} f = 1$

$\partial_y f = 2x - 4y$

$\partial_{yy} f = -4$

2. STACIONARNE TOČKE

$\partial_x f = 2y - 6x$

$\partial_y f = 2x - 4y$

$2y - 6x = 0$

$2y = 6x \; / :2$

$y = 3x \quad \Rightarrow \quad y = 3 \cdot 0$

$2x - 4 \cdot 3x = 0 \quad\quad y = 0$

$2x - 12x = 0$

$-10x = 0 \; / :(-10)$

$\underline{\underline{x = 0}}$

$T(0,0)$

$A = \partial_{xx} f(T) = -6$

$$\Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} -6 & 1 \\ 1 & -4 \end{vmatrix} = +24 - 1 = 23$$

$\left.\begin{matrix} A > 0 \\ \Delta > 0 \end{matrix}\right\}$ FUNKCIJA IMA LOKALNI ~~MINIMUM~~ MAXIMUM

<u>1 0</u>

$f(0,0) = 2 \cdot 0 \cdot 0 - 3 \cdot 0^2 - 2 \cdot 0^2 + 10 = 10$

FUNKCIJA IMA LOKALNI MINIMUM U TOČKI $T(0,0)$ I IZNOSI $f_{MIN} = 10$!

MISLAV BOGOZNICA
17-2-0345-2013

(5) $\int_0^2 \frac{x+2}{3x^2-2x-5}\,dx$

$$\int \frac{x+2}{3x^2-2x-5}\,dx = \begin{bmatrix} t = 3x^2-2x-5 \\ dt = 6x-2\,dx \\ dx = \frac{dt}{6x-2} \end{bmatrix} = \int \frac{x+2}{t} \cdot \frac{dt}{6x-2} =$$

$$= \int \frac{x+2}{t} \cdot \frac{dt}{\quad}$$

DOMENA:

$3x^2-2x-5 \neq 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{2 \pm \sqrt{4+60}}{2}$

$x_{1,2} = \frac{2 \pm \sqrt{64}}{2}$

$x_{1,2} = \frac{2 \pm 8}{2}$

$\boxed{x_1 = 5}$ $\boxed{x_2 = -3}$

$D(f) = \mathbb{R}$ $[0,2] \in \mathbb{R}$

INTEG. NIJE NEPRAVI

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C4

IME I PREZIME: FILIP FCORAXI **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0308-2013

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Odredi ekstreme funkcije $f(x,y) = 2xy - 3x^2 - 2y^2 + 10$. 10
2. Riješiti diferencijalnu jednadžbu: $y'' - y = e^x + 1$.
3. Izračunati:

   (a) određeni integral $\displaystyle\int_0^\pi \sin^5 x \, dx$.

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$.
4. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos\left(x^2\right) dx$. (bodovanje: 20 za rel. grešku$\leq$ 1%, 15 za rel. grešku $\lessapprox$ 3%, 8 za rel. grešku $\lessapprox$ 6%) Ø
5. Izračunaj $\int_0^2 \frac{x+2}{3x^2-2x-5} dx$ Ø

Ukupno: 10

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x\rvert + C$	$\int \cot x \, dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4)

$$\int_{-1}^{1} \cos(x^2)\, dx$$

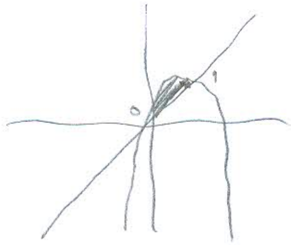
$$\boxed{P = 2}$$

0	1	2
-1	0	1
0,9995	1	0,9998

$$\left(\frac{d}{6} \left(f_0 + 4 f_1 + f_2 \right) \right)$$

3) b)

$$y = 2t - t^2$$
$$y = t$$



$$2t - t^2 = t \qquad \boxed{t = 0}$$
$$2t - t - t^2 = 0 \qquad \boxed{t = 1}$$
$$t - t^2 = 0$$
$$t(1 - t) = 0$$
$$1 - t = 0$$
$$-t = -1$$
$$t = 1$$

SJECIŠTA

$$\int_0^1 (-t^2 + 2t - t)\, dx$$

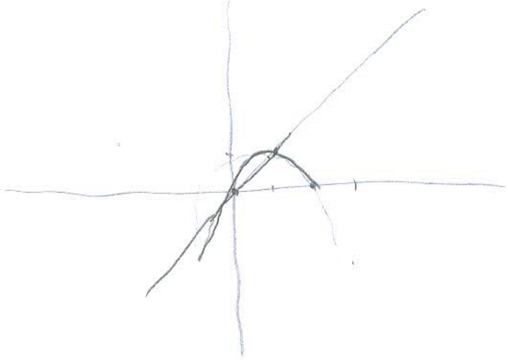
$$= \int_0^1 t - t^2\, dx = \int_0^1 t\, dt - \int_0^1 t^2\, dt$$

$$= \left[\frac{1}{2} t^2\right]_0^1 - \left[\frac{1}{3} t^3\right]_0^1 = \frac{1}{2} \cdot 0 - \frac{1}{3} = \boxed{\frac{1}{6}}$$

FILIP
FLORANI

③b $y_1 = 2t - t^2 = -t^2 + 2t$

$y_2 = t$



$\cancel{(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)}$

$y -$ $2t - t^2 = t$

$2t - t^2 - t = 0$ $\boxed{t = 0}$ SJECIŠTA

$t - t^2 = 0$ $\boxed{t = 1}$

$t(1 - t) = 0$

$1 - t = 0$

$-t = -1$

$t = 1$

$$\int_0^1 (-t^2 + 2t - t)\,dx = \int_0^1 t - t^2\,dx = \int_0^1 t\,dt - \int_0^1 t^2\,dt$$

$$= \left[\frac{t^2}{2}\right]_0^1 - \left[\frac{t^3}{3}\right]_0^1 = \left[\frac{1}{2}t^2\right]_0^1 - \left[\frac{1}{3}t^3\right]_0^1$$

$$= \frac{1}{2} - 0 - \frac{1}{3} - 0 = \boxed{\frac{1}{6}}$$

FILIP

FLORANI

$$\int_0^{\pi} \sin^5 x\,dx = \int_0^{\pi} \sin^4 x \cdot \sin x\,dx = \int_0^{\pi} (\sin^2 x)^2 \cdot \sin x\,dx$$

$$= \int_0^{\pi} (1-\cos^2 x)^2 \cdot \sin x\,dx$$

$$= \int_0^{\pi} \cancel{1 - 2\cos^2 x + \cos^4 x \cdot \sin x\,dx}$$

$$= \int_0^{\pi} (1 - t$$

$\sin^2 x = 1$

$1 - \cos x$

$\sin^2 6 = 0{,}01092613963$

$$\frac{x+2}{3x^2-2x-5}$$

$$\lim_{x \to -1^+} \frac{1}{0} = \frac{1}{3^+} -$$

FILIP

FLORANI

$$\int_0^2 \frac{x+2}{3x^2-2x-5}\,dx \qquad x_1=\frac{5}{3} \qquad x_2=-1$$

$$\frac{x+2}{3x^2-2x-5}=\frac{A}{x-\frac{5}{3}}+\frac{B}{x+1}$$

$$\frac{x+2}{3x^2-2x-5}=\frac{Ax+A+Bx-\frac{5}{3}B}{\left(x-\frac{5}{3}\right)(x+1)}$$

$$x+2=x(A+B)+A-\frac{5}{3}B$$

$$1=A+B \qquad A=\frac{11}{8}$$

$$2=A-\frac{5}{3}B \qquad B=-\frac{3}{8}$$

$$\int_0^2 \frac{\frac{11}{8}}{x-\frac{5}{3}}\,dx+\int_0^2 \frac{-\frac{3}{8}}{(x+1)}\,dx=\frac{11}{8}\int_0^{\frac{5}{3}}\frac{dt}{t}-\frac{3}{8}\int_0^{\frac{5}{3}}\frac{dt}{t}$$

$$=\left[\frac{11}{8}\ln\left|x-\frac{5}{3}\right|\right]_0^{\frac{5}{3}}-\left[\frac{3}{8}\ln|x+1|\right]_0^{\frac{5}{3}}$$

DALJE... 0

$=$

FILIP FLORANI

$$\int_0^2 \frac{x+2}{3x^2-2x-5}\,dx =$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{4-4\cdot 3\cdot(-5)}}{6} =$$

$$x_1 = \frac{5}{3} \quad x_2 = -1$$

$$\left(x-\frac{5}{3}\right)\left(x+1\right)$$

$$\frac{x+2}{3x^2-2x-5} = \frac{A}{\left(x-\frac{5}{3}\right)} + \frac{B}{(x+1)}$$

$$6 = 4+2$$

$$\boxed{3 = 7-4}$$

$$= \frac{x+2}{3x^2-2x-5} = \frac{Ax+A+Bx-\frac{5}{3}B}{}$$

$$x+2 = x(A+B)+A-\frac{5}{3}B$$

$$1 = A+B \quad \rightarrow \quad A = 1-B$$

$$2 = A-\frac{5}{3}B \quad \rightarrow \quad A = 2+\frac{5}{3}B$$

$$2+\frac{5}{3}B = 1-B$$

$$\frac{5}{3}B + B = 1-2$$

$$\frac{8}{3}B = -1 \quad / \frac{8}{3}$$

$$\boxed{B = -\frac{3}{8}}$$

$$1 = A - \frac{3}{8}$$

$$A = 1+\frac{3}{8} = \boxed{\frac{11}{8}}$$

$$\int_0^2 \frac{\frac{11}{8}}{\left(x-\frac{5}{3}\right)}dx + \int_0^2 \frac{-\frac{3}{8}}{(x+1)}dx = \frac{11}{8}\int_0^2 \frac{dx}{x-\frac{5}{3}} - \frac{3}{8}\int_0^2 \frac{dx}{x+1} =$$

$$= \left| \begin{array}{l} t = x-\frac{5}{3} \\ x = t+\frac{5}{3} \\ dt = dx \end{array} \right| = \frac{11}{8}\int_{-\frac{5}{3}}^{\frac{1}{3}} \frac{dt}{t} - \frac{3}{8}\int_1^3 \frac{dt}{t} = \left[\frac{11}{8}\ln|t|\right]_{-\frac{5}{3}}^{\frac{1}{3}} - \left[\frac{3}{8}\ln t\right]_1^3$$

$$\left| \begin{array}{l} t = x+1 \\ dt = dx \end{array} \right| =$$

PALJE...

FILIP

FLORANI

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3

IME I PREZIME: Živković Kristijan **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati:

   (a) određeni integral $\int_0^\pi \sin^5 x \, \mathrm{d}x$.

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$.

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos(x^2)\,\mathrm{d}x$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$)

3. Riješiti diferencijalnu jednadžbu: $y'' - y = \mathrm{e}^x + 1$.

4. Izračunaj $\int_0^2 \frac{x+2}{3x^2-2x-5}\mathrm{d}x$

5. Odredi ekstreme funkcije $f(x, y) = 2xy - 3x^2 - 2y^2 + 10$.

Ukupno:

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
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$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

KRISTIAN ŽIVKOVIĆ

$$\int_{-1}^{1} \cos(x^2)\, dx = \int_{-1}^{1} -\sin(x^2) + 2x + x$$

Živković Kristina

$$\int_{-1}^{1} \cos(x^2)\,dx = \int_{-1}^{1} -\sin(x^2) + 2x$$

k	0	1	2	
xk	-1	0	1	
fk	-2,84	0	1,158	

POGREŠNO!

$$s = \frac{d}{6}(f_0 + 4f_1 + f_2)$$

$$s = 1,8090$$

KRISTIAN ŽIVKOVIĆ

4. $$\int_0^2 \frac{x+2}{3x^2-2x-5}\,dx =$$

$3x^2-2x-5=0$

$x_1 = \frac{5}{3} = 1{,}666$

$x_2 = -1$

$$= \lim_{a \to 1{,}66} \int_0^{1{,}66} \frac{x+2}{3x^2-2x-5}\,dx + \lim_{b \to 1{,}66} \int_{1{,}66}^{2} \frac{x+2}{3x^2-2x-5}\,dx$$

$= -2{,}65246 +$

$$\int \frac{x+2}{3x^2-2x-1-4}$$

$(a \cdot b)^2 = a^2 - 2ab + b^2$

$-2ab = -2x$

$-2xb = -2x \quad /:(-2x)$

$\boxed{b=1}$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ B3	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: SEBASTIJAN KOŠTA **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0094-2011	

1. Izračunati:

   (a) određeni integral $\int\limits_0^{\pi} \sin^5 x\,dx$. Ø

   (b) površinu lika omeđenog krivuljama $y = 2t - t^2$ i $y = t$. Ø

2. Numeričkom integracijom odrediti vrijednost $\int_{-1}^{1} \cos(x^2)\,dx$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$)

3. Riješiti diferencijalnu jednadžbu: $y'' - y = e^x + 1$. Ø

4. Izračunaj $\int_0^2 \frac{x+2}{3x^2-2x-5}dx$

5. Odredi ekstreme funkcije $f(x,y) = 2xy - 3x^2 - 2y^2 + 10$.

Ukupno: Ø

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$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4. $$\int_0^2 \frac{x+2}{3x^2-2x-5}dx = \begin{bmatrix} t = 3x^2-2x-5 \\ dt = 6x-2\,dx \\ dx = \frac{dt}{6x-2} \end{bmatrix} = \int_0^2 \frac{\cancel{x+2}}{t}\cdot\frac{dt}{6x-2} = \int_0^2 \frac{2}{t}\cdot\frac{dt}{3}$$

$$= \int_0^2 \frac{2\,dt}{3t} = \frac{2}{3}\int_0^2 \frac{dt}{t} = \frac{2}{3}\ln t$$

$$= \frac{2}{3}\ln(3x^2-2x-5)\Big|_0^2 = \frac{2}{3}\ln(3\cdot2^2-2\cdot2-5) - \frac{2}{3}\ln(3\cdot0^2-2\cdot0-5)$$

$$= \frac{2}{3}\ln(3) - \frac{2}{3}\ln(-5) = \frac{2}{3}\ln 3 - \frac{2}{3}\ln 5$$

$$= 0{,}732 - 1{,}073 = -0{,}341$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4\cdot3\cdot(-5)}}{2\cdot3}$$

$$x = \frac{2 \pm \sqrt{4+60}}{6}$$

$$x = \frac{2 \pm \sqrt{64}}{6}$$

$$x_1 = \frac{2+8}{6} = \frac{10}{6} = \frac{5}{3} \qquad x_2 = \frac{2-8}{6} = -1$$

2.) $\int_{-1}^{1} \cos(x^2)\,dx = \begin{bmatrix} \cancel{t = x^2} \\ \cancel{dt = 2x\,dx} \\ \cancel{dx = \frac{dt}{2x}} \end{bmatrix} = \cancel{\int_{-1}^{1} \cos t\, \frac{dt}{2x}}$

1.) $\int_0^{\pi} \sin^5 x\,dx = \int_0^{\pi} (1-\cos^2 x)\cdot \sin^3 x\,dx = \int_0^{\pi} (1-\cos^2 x)\cdot(1-\cos^2 x)\cdot \sin x\,dx$

$= \begin{bmatrix} t = \cos x \\ dt = -\sin x\,dx \\ dx = \frac{dt}{-\sin x} \end{bmatrix} = \int_0^{\pi} (1-t^2)\cdot(1-t^2)\cdot \cancel{\sin x}\cdot \frac{dt}{\cancel{-\sin x}} = \int_0^{\pi} 1-t^2-t^2+t^4\,dt$

$= 1\int_0^{\pi} t^4-t^2-t^2\,dt = 1\cdot \frac{t^5}{5} - \frac{t^3}{3} - \frac{t^3}{3}\Big|_1^1$

$t_1 = \cos\pi = 1$

$t_2 = \cos 0 = 1$

$\boxed{t_1 = \cos\pi = -1}$

$= \cancel{1} \frac{1}{5} - \frac{1}{3} - \frac{1}{3} = \cancel{\frac{1}{5}}$

$= \frac{1}{5} - \frac{6}{9} = \frac{-21}{45}$ ×

SEBASTIJAN KOŠTA

1.) $y = 2t - t^2$ $\quad t = x$

$y = t$

$y = 2x - x^2$

$y = x$

$\cancel{2x - x^2 - x}$

$\cancel{2x - x^2 - x = 0 \;/\cdot(-1)}$

$\cancel{x^2 - 2x - x = 0}$

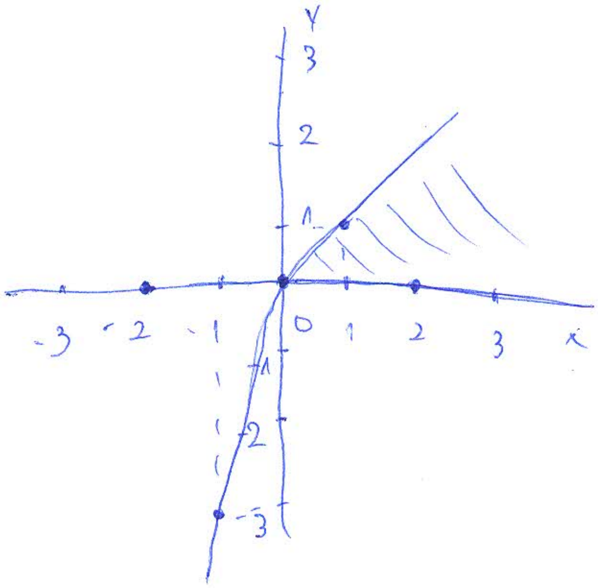
$2x - x^2 = 0$ $\leftarrow$ OVO NISU SJECIŠTA ~~NULTOČKE~~

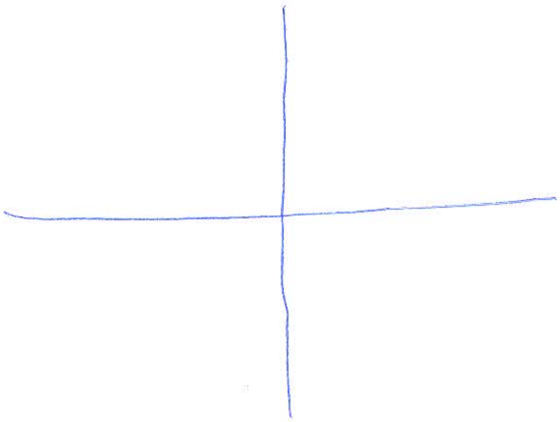
$x^2 - 2x = 0$

$$x_1 = \frac{2 \pm \sqrt{(-2)^2 - 4\cdot 1\cdot 0}}{2} = \frac{2 \pm \sqrt{4}}{2} = \frac{2-2}{2} = \frac{0}{2}$$

$x_2 = 2$

	0	1	2	-1	-2
$2x - x^2$	0	1	0	-3	0





$$P = \int_0^2 (2x - x^2) - x$$

$$= \frac{2x^2}{2} - \frac{x^3}{3} - \frac{x^2}{2} \Bigg|_0^2$$

$$= \frac{2\cdot 2^2}{2} - \frac{2^3}{3} - \frac{2^2}{2} - \frac{2\cdot 0}{2} - \frac{0^3}{3} - \frac{0^2}{2}$$

$$= 4 - \frac{8}{3} - 2$$

$$= \frac{4}{3} - 2 = \frac{-2}{3}$$