MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $G7$ **Ime i prezime:** Dino Markov **Vrijeme početka:** 17^{35} **Matični broj studenta (iznad slike u indeksu):** 0269075721	Popunjava nastavnik Broj ↓ bodova

1. Koliko iznosi $\displaystyle\int_2^{+\infty} \frac{\mathrm{d}x}{1-x^2}$? 20

2. Pronaći:

   (a) opće rješenje diferencijalne jednadžbe $\sqrt[3]{x}\,y\,y' = 1 - x^2$. Na kraju provjeri rješenje.

   (b) partikularno rješenje koje zadovoljava ODJ $y'' + 4y = 0$, uz uvjete $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje. 10

3. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. 20

4. Zadano je $f(x) = \dfrac{2x^2 + x + 2}{x^2 - 1}$. Odrediti $\int_3^2 f(x)\,\mathrm{d}x$. X

5. Istražiti domenu i ekstreme funkcije $f(x, y) = xy - x^3 - y^2$.

Ukupno: 50

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α ($\alpha \neq 0$)	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ ($\alpha > 0$)	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x ($\alpha > 0$)	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

① $\cancel{\int_2^{+\infty} \frac{dx}{1-x^2} =}$

NAPRAVLJENA JE PROVJERA ZA RJEŠENJE
$y(x) = \sin(2x)$
POSTUPAK ZADATKA NEDOSTAJE.

② b) $(y_P)' = C_1(-\sin(2x))\cdot 2 + C_2\cos(2x)\cdot 2$

$y_P' = -2C_1\sin(2x) + 2C_2\cos(2x)$

$0 = C_1\cos(0) + C_2\sin(0)$

$0 = C_1$

$2 = -2C_1\sin(0) + 2C_2\cos(0)$

$2 = 2C_2 \to C_2 = 1$

$y_P = \sin(2x)$

PROVJERA: $y_P = \sin 2x$

$0 = \sin(2\cdot 0)$

$0 = 0$

$y_P = \cos(2x)\cdot 2$

$2 = \cos(0)\cdot 2$

$2 = 2$

10

(4) $\int_0^2 \frac{2x^2+x+2}{x^2-1}dx = \int_0^2 \frac{2x^2+x+2}{x^2-1} = \int_0^2 2dx + \int_0^2 \frac{x+4}{x^2-1}dx$

POGREŠNE GRANICE

$(2x^2+x+2) : (x^2-1) = 2$

$\frac{(2x^2-2)}{x+4}$

$x+4 = A(x+1)+B(x-1)$

$x+4 = Ax+A+Bx-B$

$x+4 = x(A+B)+(A-B)$

$A+B=1 \quad 2A=5 /:2 \quad B=1-\frac{5}{2}$

$A-B=4 \quad A=\frac{5}{2} \quad B=-\frac{3}{2}$

$\int_0^2 \frac{x+4}{x^2-1}dx = \frac{A}{x-1}+\frac{B}{x+1} =$

$= \int_0^2 \frac{\frac{5}{2}}{x-1}dx + \int_0^2 \frac{-\frac{3}{2}}{x+1}dx = 0$ X

(3) POVRŠINA: $x+y^2=6$ i $x+y+1=0$

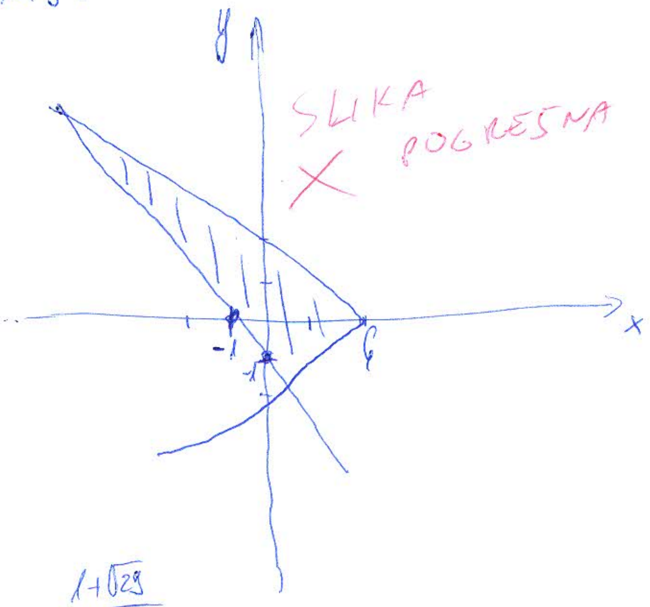
$x+y^2=6$

$\frac{x+y=1}{x=-y^2+6}$

x	6	0
y	0	$\pm\sqrt{6}$

$y=-x-1$

x	0	-1
y	-1	0



SLIKA POGREŠNA X

Sjecište:

$-y-1=-y^2+6$

$y^2-y-7=0$

$y_1=\frac{1+\sqrt{29}}{2}$

$y_2=\frac{1-\sqrt{29}}{2}$

$P = \int_{\frac{1-\sqrt{29}}{2}}^{\frac{1+\sqrt{29}}{2}} (-y^2+6+y+1)\,dy =$

$= -\frac{y^3}{3}\Big|_{\frac{1-\sqrt{29}}{2}}^{\frac{1+\sqrt{29}}{2}} + \frac{y^2}{2}\Big|_{\frac{1-\sqrt{29}}{2}}^{\frac{1+\sqrt{29}}{2}} + 7y\Big|_{\frac{1-\sqrt{29}}{2}}^{\frac{1+\sqrt{29}}{2}} =$

$= -\frac{8\sqrt{29}}{3} + \frac{\sqrt{29}}{2} + 7\sqrt{29}$

$= \frac{29\sqrt{29}}{6}$ ✓

20

DINO MARKOV
BROJ INDEKSA: 0269075721

(1) $\int_2^{+\infty} \frac{dx}{1-x^2} = \lim_{a \to \infty} \int_2^a \frac{dx}{1-x^2} = \lim_{a \to \infty} \left[\frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| \Big|_2^a \right] =$

$= \lim_{a \to \infty} \frac{1}{2} \left[\ln \left| \frac{1+a}{1-a} \right| \right] - \ln \left[\left| \frac{1+2}{1-2} \right| \right]$

$= \frac{1}{2} \lim_{a \to \infty} \ln \left| \frac{1+a}{1-a} \right| - \frac{1}{2} \ln 3$

$= \frac{1}{2} \ln \lim_{a \to \infty} \left| \frac{1+a}{1-a} \right| - \frac{1}{2} \ln 3$

$= \frac{1}{2} \ln 1 - \frac{1}{2} \ln 3 = -\frac{1}{2} \ln 3$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{B}1$ **IME I PREZIME:** JOSIP PREDOVAN **VRIJEME POČETKA:** **MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):** 17-1-0126-2012	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. ~~20~~ 20

2. Koliko iznosi $\displaystyle\int_2^{+\infty} \frac{dx}{1-x^2}$?

3. Pronaći:

   (a) opće rješenje diferencijalne jednadžbe $\sqrt[3]{x}\,y\,y' = 1 - x^2$. Na kraju provjeri rješenje.

   (b) partikularno rješenje koje zadovoljava ODJ $y'' + 4y = 0$, uz uvjete $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje.

4. Zadano je $f(x) = \dfrac{2x^2 + x + 2}{x^2 - 1}$. Odrediti $\int_3^2 f(x)\,dx$. ✗

5. Istražiti domenu i ekstreme funkcije $f(x, y) = xy - x^3 - y^2$. ✗

Ukupno: ~~40~~ 20 Kos

f	$\frac{df}{dx}$
x^α ($\alpha \neq 0$)	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ ($\alpha > 0$)	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x ($\alpha > 0$)	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
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Tablica nekih integrala		
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

ZAMJENA VARIJABLI:

① $x + y^2 = 6 \Rightarrow y = -x^2 + 6$

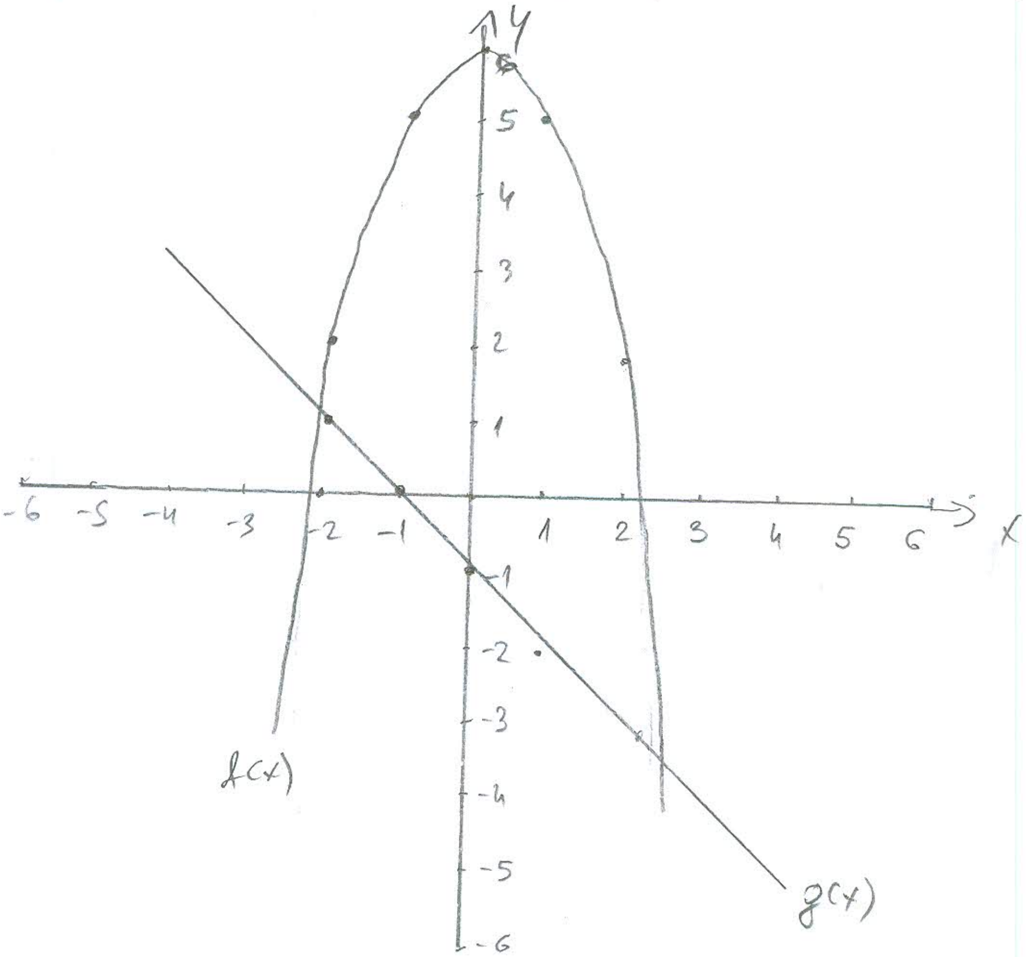
$x + y + 1 = 0 \Rightarrow y = -x - 1$

$f(x): y = -x^2 + 6$

x	-2	-1	0	1	2
	2	5	6	5	2

$g(x): y = -x - 1$

x	-2	-1	0	1	2
	1	0	-1	-2	-3



$-x^2+6=-x-1$

$-x^2+6+x+1=0$

$-x^2+x+7=0$

$x_{1,2}=\dfrac{-1\pm\sqrt{1^2-4\cdot(-1)\cdot 7}}{2\cdot(-1)}$

$x_1=-2,19$

$x_2=3,19$

$$P=\int_{-2,19}^{3,19}-x^2+6-(-x-1)$$

$$P=\int_{-2,19}^{3,19}-x^2+6-x+1$$

$$P=\int_{-2,19}^{3,19}-x^2-x+7$$

$$P=-\frac{x^3}{3}-\frac{x^2}{2}+7x\Big|_{-2,19}^{3,19}$$

$$P=\left(-\left(\frac{3,19}{3}\right)^3-\frac{3,19^2}{2}+7\cdot 3,19\right)-\left(-\frac{(-2,19)^3}{3}-\frac{(-2,19)^2}{2}+7(-2,19)\right)$$

$P=16,42+14,22$

$P=20,64$ //

JOSIP PREDOVAN

(2) $$\int_2^{+\infty} \frac{dx}{1-x^2} = \lim_{a\to+\infty} \int_2^{a} \frac{dx}{1-x^2}$$

$$= \lim_{a\to+\infty} \left(\frac{1}{2\cdot 1} \ln \left| \frac{1+x}{1-x} \right| \right) \Bigg|_2^a = \lim_{a\to+\infty} \frac{1}{2} \ln \left| \frac{1+a/a}{1-a/a} \right| - \frac{1}{2}$$

$$\ln \left| \frac{1+2}{1-2} \right| = \frac{1}{2} \ln \left| 1 \right| - \frac{1}{2} \ln \left| -3 \right|$$

$$= \frac{1}{2} \ln|1| - \frac{1}{2} \ln|-3|$$

$$= -0,549$$

(5) $f(x,y) = xy - x^3 - y^2$

$$\frac{\partial f}{\partial x} = y - 3x^2 \qquad \frac{\partial f}{\partial y} = x - 2y$$

$$\frac{\partial^2 f}{\partial x^2} = -6x \qquad \frac{\partial^2 f}{\partial y^2} = -2$$

$$\frac{\partial^2 f}{\partial x y} = 1 \qquad \frac{\partial^2 f}{\partial y x} = 1$$

STACIONARNE TOČKE

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$$

$y - 3x^2 = 0$ $\qquad$ $x - 2y = 0$

$-3x^2 + y = 0$ $\qquad$ $-2y = -x \,/\cdot(-2)$

$-3x^2 + 2x = 0$ $\qquad$ $y = 2x$

DRUGA STACIONARNA TOČKA?

$T(0,0)$

$$\Delta = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix}$$

$\Delta = 0\cdot(-2) - 1\cdot 1 = -1$

$\Delta < 0$ $\boxed{\text{SEDLO}}$

(4) $\int_3^2 \frac{2x^2+x+2}{x^2-1}\,dx$

$x_{1,2} = \frac{0 \pm \sqrt{0^2-4\cdot 1\cdot(-1)}}{2\cdot 1}$ $x_1 = 1$ $x_2 = -1$

$\boxed{\text{PREKID U } x = 1}$

OVAJ INTEGRAL NE POSTOJ. DIVERGIRA!!!

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{B}1$

IME I PREZIME: DANIEL ŠOŠA **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$. $\emptyset$

2. Koliko iznosi $\displaystyle\int_2^{+\infty} \frac{\mathrm{d}x}{1-x^2}$? ×

3. Pronaći:

   (a) opće rješenje diferencijalne jednadžbe $\sqrt[3]{x}\, y\, y' = 1 - x^2$. Na kraju provjeri rješenje. ×

   (b) partikularno rješenje koje zadovoljava ODJ $y'' + 4y = 0$, uz uvjete $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje. ×

4. Zadano je $f(x) = \dfrac{2x^2 + x + 2}{x^2 - 1}$. Odrediti $\int_3^2 f(x)\,\mathrm{d}x$. ×

5. Istražiti domenu i ekstreme funkcije $f(x, y) = xy - x^3 - y^2$. 5

Ukupno: 5

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
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$\tan x$	$\frac{1}{\cos^2 x}$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3. a) $\sqrt[3]{x}\; yy' = 1 - x^2$

$x^{\frac{1}{3}} yy' = 1 - x^2$

$x^{\frac{1}{3}} y \frac{dy}{dx} = 1 - x^2 \;/\, dx$

$x^{\frac{1}{3}} y\, dy = dx - x^2 dx \;/ : x^{\frac{1}{3}}$

$y\,dy = \dfrac{dx - x^2 dx}{x^{\frac{1}{3}}} \;/\int$

$\int y\,dy = \int \dfrac{dx}{x^{\frac{1}{3}}} - \int \dfrac{x^2}{x^{\frac{1}{3}}} dx$

$\dfrac{y^2}{2} = \ln|x^{\frac{1}{3}}| - \dfrac{3x^{\frac{8}{3}}}{8} + C$

$\int \dfrac{dx}{x^{\frac{1}{3}}} = \ln|x^{\frac{1}{3}}| + c$

$\int \dfrac{x^2}{x^{\frac{1}{3}}} dx = \int x^{\frac{5}{3}} = \dfrac{x^{\frac{8}{3}}}{\frac{8}{3}} = \dfrac{3x^{\frac{8}{3}}}{8}$

$y^2 = \dfrac{\ln x^{\frac{1}{3}}}{2} - \dfrac{3x^{\frac{8}{3}}}{16} + C \;/\sqrt{\ }$

$y = \sqrt{\dfrac{\ln x^{\frac{1}{3}}}{2} - \dfrac{3x^{\frac{8}{3}}}{16}} + C$

3 b) $y'' + 4y = 0$ UVJETI $y(0)=0$ $y'(0)=2$

$r^2 + 4r = 0$ X

$r(r+4) = 0$

$r_1 = 0$

$r_2 = -4$

$y_{(H)} = C_1 e^{r_1 x} + C_2 e^{r_2 x} = C_1 e^0 + C_2 e^{-4x} = C_1 + C_2 e^{-4x}$

$y_{(H)} = C_1 + C_2 e^{-4x}$ $x=0, y=0$ $y'_{(H)} = -4 C_2 e^{-4x}$

$2 = -4 C_2 e^0$

$2 = -4 C_2$

$C_2 = \frac{2}{-4} = -\frac{1}{2}$

$0 = C_1 + C_2 \cdot e^0$

$0 = C_1 + C_2$

$0 = C_1 - \frac{1}{2}$

$-C_1 = -\frac{1}{2}$

$C_1 = \frac{1}{2}$

KONAČNO RJEŠENJE:

$y_{(H)} = \frac{1}{2} - \frac{1}{2} C_2 e^{-4x}$ X

PROVJERA?

5. $f(x,y) = xy - x^3 - y^2$

DOMENA

$D_{f(x,y)} = \mathbb{R}^2$ ✓

5

EKSTREMI

$\frac{\partial f}{\partial x} = y - 3x^2$

$\frac{\partial f}{\partial y} = x - 2y$

$y - 3x^2 = 0 \;/\cdot 2$

$-2y + x = 0$

$2y - 6x^2 = 0$

$-2y + x = 0$

$-5x^2 = 0$ X

$x = 0$

$-2y + x = 0$

$-2y + 0 = 0$

$-2y = 0$

$y = 0$

$T(0,0)$

$\frac{\partial^2 f}{\partial x} = -6x$

$\frac{\partial^2 f}{\partial y} = 1$

$\frac{\partial^2 f}{\partial x} = 1$ $\frac{\partial^2 f}{\partial y} = -2$

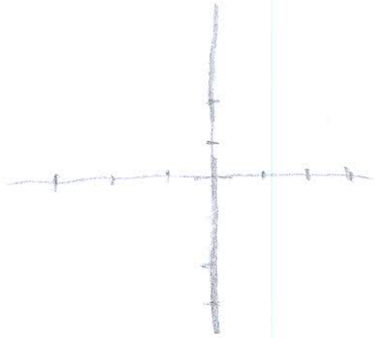
$\Delta = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = -1 < 0$ NEMA EKSTREMA $T(0,0)$

NIJE EKSTREM

1. $x+y^2=6$

DANIEL ŠOŠA

$x+y+1=0$



$x+y^2-6=x+y+1$

$\cancel{x}+y^2-6-\cancel{x}-y-1=0$

$y^2-y-7=0$

$y_{1,2}=\dfrac{1\pm\sqrt{1-4\cdot1\cdot(-7)}}{2}=\dfrac{1\pm 5.39}{2}=$ $y_1=-2.195$, $y_2=3.195$

$\int (x+y^2-6)-(x+y+1)=\int_{-2.2}^{3.2} x+y^2-6-x-y-1=\dfrac{x^2}{2}+\dfrac{y^3}{3}-6x-\dfrac{x^2}{2}-\dfrac{y^2}{2}-x\Big|_{-2.2}^{3.2}$

$=$

$\emptyset$

2. $\int_2^{+\infty}\dfrac{dx}{1-x^2}=\int\dfrac{dx}{1-\sqrt{x^2}}=\int\dfrac{dx}{1-x}=\dfrac{1}{2}\ln\left|\dfrac{1+x}{1-x}\right|+C$

$=\dfrac{1}{2}\ln\left|\dfrac{1+x}{1-x}\right|\Big|_2^{+\infty}=\infty-\left(\dfrac{1}{2}\ln|-3|\right)=\infty-0.55=+\infty$ X

4 $\int_2^3\dfrac{2x^2+x+2}{x^2-1}=\int 2dx+\int\dfrac{x+4}{x^2-1}dx=2x+8\ln|x^2-1|\Big|_2^3=22.64-12.79=9.85$

X

$2x^2+x+2 : x^2-1=2$

$2x^2-2$

$x+4$

$\int\dfrac{x+4}{x^2-1}=\int\dfrac{\frac{1}{2}(2x+4)}{x^2-1}=4\int\dfrac{x}{x^2-1}dx$ $\left|\begin{array}{l}x^2-1=t\\2x\,dx=dt\\dx=\frac{dt}{2x}\end{array}\right|$ $=4\int\dfrac{x\cdot\frac{dt}{2x}}{t}=\dfrac{\frac{dt}{2}}{\frac{t}{1}}=\dfrac{dt}{2t}=2\ln|t|$

$8\ln(x^2-1)+C$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{B}1$

Popunjava nastavnik
Broj ↓ bodova

Ime i prezime: STIPE BREULJ. **Vrijeme početka:** 17:30

Matični broj studenta (iznad slike u indeksu): 17-1-0192-2013

1. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$.

2. Koliko iznosi $\displaystyle\int_2^{+\infty} \frac{dx}{1-x^2}$? ∅

3. Pronaći:
   (a) opće rješenje diferencijalne jednadžbe $\sqrt[3]{x}\, y\, y' = 1 - x^2$. Na kraju provjeri rješenje.
   (b) partikularno rješenje koje zadovoljava ODJ $y'' + 4y = 0$, uz uvjete $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje. ×

4. Zadano je $f(x) = \dfrac{2x^2 + x + 2}{x^2 - 1}$. Odrediti $\int_3^2 f(x)\,dx$. ∅

5. Istražiti domenu i ekstreme funkcije $f(x, y) = xy - x^3 - y^2$.

Ukupno: ∅

f	$\frac{df}{dx}$
x^α ($\alpha \neq 0$)	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ ($\alpha > 0$)	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x ($\alpha > 0$)	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

STIPE BREULJ

2. $\int_2^{+\infty} \frac{dx}{1-x^2} = \int_2^{+\infty} \frac{dx}{\sqrt{1^2}-x^2} = \frac{1}{2\sqrt{1^2}} \ln \left| \frac{\sqrt{1^2}+x}{\sqrt{1^2}-x} \right| = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|$

$\left[\frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| \right]_2^{+\infty} = \cancel{0}$

$\lim_{x\to+\infty} \frac{1+x}{1-x} = -1$

$\lim_{x\to+\infty} \ln \left| \frac{1+x}{1-x} \right| = \ln\left(\lim_{x\to+\infty} \frac{1+x}{1-x}\right)$

$= \ln|-1| = \ln 1 = 0$

3. a) $\sqrt[3]{x}\, y\, y' = 1 - x^2$ $\quad \cancel{0}$

3.

b) $y'' + 4y = 0 \qquad y(0) = 0 \qquad y'(0) = 2$

$r^2 + 4r = 0$ ✗

$r_{1,2} = \frac{-4 \pm \sqrt{16-0}}{2}$

$r_{1,2} = \frac{-4 \pm 4}{2}$

$r_{1,2} = -4, 0$

$y_H = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

$y_H = c_1 e^{-4x} + c_2 e^{0x}$

$y_H = c_1 e^{-4x} + c_2$

$0 = c_1 e^{-4\cdot 0} + c_2$

$0 = c_1 + c_2$

$c_1 = -c_2 \qquad c_2 = \frac{1}{2}$

$y_H' = -4 c_1 e^{-4x}$

$2 = -4 c_1 e^{-4\cdot 0}$

$2 = -4 c_1$

$c_1 = -\frac{1}{2}$

$y = -\frac{1}{2} e^{-4x} + \frac{1}{2}$ ✗

3. PROVJERA:

$$y = -\frac{1}{2}e^{-4x} + \frac{1}{2}$$

$$0 = -\frac{1}{2}e^{-4\cdot 0} + \frac{1}{2}$$

$$0 = -\frac{1}{2} + \frac{1}{2}$$

$$0 = 0 \quad \checkmark$$

$$y' = -4 \cdot \left(-\frac{1}{2}\right) e^{-4x}$$

$$y' = 2e^{-4x}$$

$$2 = 2e^{-4\cdot 0}$$

$$2 = 2 \quad \checkmark$$

NIJE PROVJERENA ODJ

1. $\int_3^2 \frac{2x^2+x+2}{x^2-1}\,dx =$

$$2\int\frac{x^2dx}{x^2-1} + \int\frac{xdx}{x^2-1} + 2\int\frac{dx}{x^2-1} = *$$

#1 #2 #3

$$\#1 \quad \int x\cdot\frac{xdx}{x^2-1} = \left[\begin{array}{ll} u=x & dv=\frac{xdx}{x^2-1} \Big/ \int \\ du=dx & v=\int\frac{xdx}{x^2-1} = \left[\begin{array}{l} x^2-1=t \,/' \\ 2xdx=dt \\ xdx=\frac{dt}{2}\end{array}\right] = \\ & =\frac{1}{2}\int\frac{dt}{t} = \frac{1}{2}\ln|t| = \underbrace{\frac{1}{2}\ln|x^2-1|}_{\#2}\end{array}\right] =$$

$$= x\cdot\frac{1}{2}\ln|x^2-1| - \int\frac{1}{2}\ln|x^2-1|\,dx$$

$$\#1)\ 2\int\frac{x^2-1+1}{x^2-1}dx = 2\int\frac{x^2-1}{x^2-1}dx + 2\int\frac{dx}{x^2-\sqrt{1}^2}$$

$$= 2x + \cancel{2}\cdot\frac{1}{\cancel{2}}\ln\left|\frac{x-1}{x+1}\right|$$

$$= 2x + \underbrace{\ln\left|\frac{x-1}{x+1}\right|}_{\#3}$$

$$* = \left.\left|2x + \ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\ln|x^2-1| + \ln\left|\frac{x-1}{x+1}\right|\right|\right|_3^2$$

$$= \left(4 + \ln\left|\frac{1}{3}\right| + \frac{1}{2}\ln|3| + \ln\left|\frac{1}{3}\right|\right) - \left(6 + \ln\left|\frac{1}{2}\right| + \frac{1}{2}\ln|8| + \ln\left|\frac{1}{2}\right|\right)$$

$$= \left(4 - \frac{11}{10} + \frac{1}{2}\cdot\frac{11}{10} - \frac{11}{10}\right) - \left(6 - \frac{7}{10} + \frac{1}{2}\cdot\frac{21}{10} - \frac{7}{10}\right)$$

$$= \left(\frac{80-22+11+22}{20}\right) - \left(\frac{360-42+63-42}{60}\right) = \frac{91}{20} - \frac{339}{60} = \frac{273-339}{60}$$

0

$$= -\frac{66}{60} = -1\frac{6}{60}$$

OSTATAK ?

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{B}1$	**Popunjava nastavnik** Broj ↓ bodova
Ime i prezime: IVAN MARDETKO **Vrijeme početka:** 17:30	
Matični broj studenta (iznad slike u indeksu): 17-1-0216-2013; 0269081944	

1. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$.

2. Koliko iznosi $\int\limits_2^{+\infty} \frac{dx}{1 - x^2}$?

3. Pronaći:

   (a) opće rješenje diferencijalne jednadžbe $\sqrt[3]{x}\, y\, y' = 1 - x^2$. Na kraju provjeri rješenje.

   (b) partikularno rješenje koje zadovoljava ODJ $y'' + 4y = 0$, uz uvjete $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje.

4. Zadano je $f(x) = \dfrac{2x^2 + x + 2}{x^2 - 1}$. Odrediti $\int_3^2 f(x)\,dx$.

5. Istražiti domenu i ekstreme funkcije $f(x, y) = xy - x^3 - y^2$.

Ukupno: $\emptyset$

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
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$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha + 1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
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IVAN MARDETKO

5. $f(x,y) = xy - x^3 - y^2$

$\frac{\partial f}{\partial x} = y - 3x^2$

$\frac{\partial f}{\partial y} = x - 2y$

$\frac{\partial f}{\partial x} = 0$

$\frac{\partial f}{\partial y} = 0$

$y - 3x^2 = 0 \,/ : (-3)$

$x - 2y = 0 \,/ : (-6)$

$-\frac{y}{3} + x^2 = 0 \Rightarrow x^2 = \frac{y}{3}$

$-\frac{x}{6} + \frac{y}{3} = 0$

$-\frac{x}{6} - x^2 = 0 \,/ \cdot (-6)$

$x + 6x^2 = 0$

$x(1 + 6x) = 0$

$x_1 = 0 \qquad 1 + 6x = 0$

$6x = -1$

$x_2 = -\frac{1}{6}$

$0 - 2y = 0$

$-2y = 0$

$y_1 = 0$

$-\frac{1}{6} - 2y = 0$

$-2y = \frac{1}{6} \,/ : (-2)$

$y_2 = -\frac{1}{12}$

$T_0(0,0)$ ✓

$T_0\left(-\frac{1}{6}, -\frac{1}{12}\right)$ ✗

NE ZADOVOLJAVA $y - 3x^2 = 0$

$\frac{\partial^2 f}{\partial x^2} = -6x$

$\frac{\partial^2 f}{\partial y^2} = -2$

$\frac{\partial^2 f}{\partial x \partial y} = 1$

$\frac{\partial^2 f}{\partial x^2}(T_0) = -6 \cdot 0 = 0$

$\frac{\partial^2 f}{\partial x^2}(T_0) = -6 \cdot \left(-\frac{1}{6}\right) = 1$

✗

$\Delta = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} = 1 \cdot (-2) - (1 \cdot 1) =$

$= -2 - 1 = -3$

$\Delta < 0, \frac{\partial^2 f}{\partial x \partial y} > 1,$

NEMA EKSTREMA

2. $\int_{2}^{+\infty} \frac{dx}{1-x^2} = \int_{2}^{+\infty} \frac{dx}{1^2-x^2} = \left[\frac{1}{2\cdot 1} \ln \left|\frac{1+x}{1-x}\right|\right]_{2}^{+\infty}$

$1-x^2 \neq 0$ $\quad$ $D_1: \mathbb{R} \setminus \{-1, 1\}$

$-x^2 \neq -1 / \cdot(-1)$

$x^2 \neq 1 / \sqrt{\ }$

$x \neq \pm 1$

$= \lim_{b \to +\infty} \left[\frac{1}{2} \ln \left|\frac{1+b}{1-b}\right|\right]_{2}^{b}$

IVAN MARDETKO

5. $f(x)=\frac{2x^2+x+2}{x^2-1}$

$$\int_3^2\left(\frac{2x^2+x+2}{x^2-1}\right)dx=\int_3^2\left(2\cdot\frac{x+4}{x^2-1}\right)dx=2\int_3^2 dx+\int_3^2\frac{x+4}{x^2-1}dx$$

$(2x^2+x+2):(x^2-1)=2$

$\underline{-2x^2 \quad -2}$

$x+4$

$$=2\left[x\right]_3^2+\left[(x^2-1)1-\int 1\cdot 2x\,dx\right]_3^2=2\cdot\left[2-3\right]+\left[x^2-1-2\int x\,dx\right]_3^2$$

$$\left[\begin{matrix} u=x^2-1 & dv=\int x+4\,dx \\ du=2x\,dx & v=1\end{matrix}\right]$$

$$=2\cdot(-1)+\left[x^2-1-2\cdot\frac{x^2}{2}\right]_3^2=-2+\left[2^2-1-2\cdot\frac{2^2}{2}-\left(3^2-1-2\cdot\frac{3^2}{2}\right)\right]$$

$$=-2+\left[4-1-2\cdot\frac{4}{2}-\left(9-1-2\cdot\frac{9}{2}\right)\right]=-2+\left[4-1-4-(9-1-9)\right]=-2+0=-2$$

IVAN

1. $x+y^2=6$

$x+y+1=0$

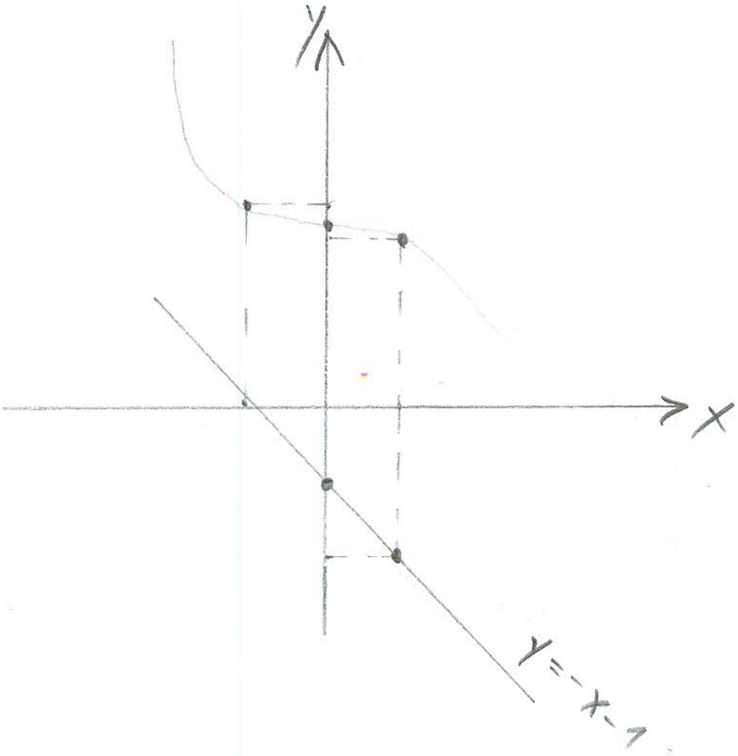
$y^2=6-x \,/\sqrt{\ }$

$y=\sqrt{6-x}$

x	-1	0	1
y	$2{,}65$	$2{,}45$	$2{,}24$

$y=-x-1$

x	0	1
y	-1	-2



?

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B1

IME I PREZIME: LUKA ČALUŠIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-270193-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati površinu područja omeđenog krivuljama $x + y^2 = 6$ i $x + y + 1 = 0$.

2. Koliko iznosi $\displaystyle\int_2^{+\infty} \frac{dx}{1-x^2}$? ×

3. Pronaći:

   (a) opće rješenje diferencijalne jednadžbe $\sqrt[3]{x}\, y\, y' = 1 - x^2$. Na kraju provjeri rješenje.

   (b) partikularno rješenje koje zadovoljava ODJ $y'' + 4y = 0$, uz uvjete $y(0) = 0$ i $y'(0) = 2$. Na kraju provjeri rješenje.

4. Zadano je $f(x) = \dfrac{2x^2 + x + 2}{x^2 - 1}$. Odrediti $\int_3^2 f(x)\,dx$. ×

5. Istražiti domenu i ekstreme funkcije $f(x, y) = xy - x^3 - y^2$. ×

Ukupno: ∅

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
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~~(1) $x+y^2=6 \quad x+y+1=0$~~

(4) $f(x) = \dfrac{2x^2+x+2}{x^2+1}$

$2x^2+x+2 : x^2+1 = 2$
$-(2x^2+0+2)$
x

$\dfrac{\cancel{2x^2+x+2}\ x}{x^2+1} = \dfrac{Ax+B}{x^2+1} \quad / \cdot x^2+1$

$x = Ax + B$

$A = 1$

$B = 0$

$\displaystyle\int_3^2 2\frac{x}{x^2+1}dx = 2\int_3^2 \frac{x}{x^2+1}dx = \begin{bmatrix} t = x^2+1 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{bmatrix}$

$\displaystyle 2\int_3^2 \frac{\cancel{x}}{t}\cdot\frac{dt}{2\cancel{x}} = 2\int_3^2 \frac{1}{2}\frac{dt}{t} = \frac{\cancel{2}}{\cancel{2}}\int_3^2 \frac{dt}{t} =$

$= \ln|x^2+1|\Big|_3^2 = \ln(2^2+1) - \ln(3^2+1) =$

$= \ln 5 - \ln 10 = -0{,}693147180\underline{6}$ //

$$f(x,y) = xy - x^3 - y^2$$

$$D = \mathbb{R} \times \mathbb{R}$$

5

$$\frac{\delta f}{\delta x} = y - 3x^2 \qquad \frac{\delta^2 f}{\delta x \delta y} = 1$$

$$\frac{\delta f}{\delta y} = x - 2y \qquad \frac{\delta^2 f}{\delta y \delta x} = 1$$

$$\frac{\delta^2 f}{\delta^2 x} = -6x$$

$$\frac{\delta^2 f}{\delta^2 y} = -2$$

STACIONARNE TOČKE ?

∅

MANJE OD 0

$$\Delta = \begin{vmatrix} -6 & 1 \\ 1 & -2 \end{vmatrix} = +12 - 1 = 11 \qquad 11 > 0$$

FUNKCIJA IMA LOKALNE MAKSIMUME

(2)
$$\int_2^{+\infty} \frac{dx}{1-x^2} = \int \frac{dx}{1-x^2} = \left[\frac{1}{2} \ln\left|\frac{1+x}{1-x}\right|\right] + C = \Big|_2^{+\infty}$$

$$= \frac{1}{2} \ln\left|\frac{1+\infty}{1-\infty}\right| - \left(\frac{1}{2} \underset{\Downarrow\, N/P}{\ln} \left|\frac{1+2}{1-2}\right|\right) = N/P$$