

**MATEMATIKA 2:** Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

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IME I PREZIME: DINO MARKOV

VRIJEME POČETKA: 14<sup>35</sup>

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1. Koliko iznosi  $\int_2^{+\infty} \frac{dx}{1-x^2}$ ?

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2. Pronaći:

(a) opće rješenje diferencijalne jednačbe  $\sqrt[3]{x} y y' = 1 - x^2$ . Na kraju provjeri rješenje.

(b) partikularno rješenje koje zadovoljava ODJ  $y'' + 4y = 0$ , uz uvjete  $y(0) = 0$  i  $y'(0) = 2$ . Na kraju provjeri rješenje.

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3. Izračunati površinu područja omeđenog krivuljama  $x + y^2 = 6$  i  $x + y + 1 = 0$ .

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4. Zadano je  $f(x) = \frac{2x^2 + x + 2}{x^2 - 1}$ . Odrediti  $\int_3^2 f(x) dx$ .

X

5. Istražiti domenu i ekstreme funkcije  $f(x, y) = xy - x^3 - y^2$ .

Ukupno:

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<u>f</u>	<u>df/dx</u>
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
$e^x$	$e^x$
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 + a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 + a^2} + a^2 \ln \left( x + \sqrt{x^2 + a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$	

NAPRAVLJENA JE PROVJERA ZA RJEŠENJE

$y(x) = \sin(2x)$

POSTUPAK ZADATKA NE DOSTIJE.

~~$\int_2^{+\infty} \frac{dx}{1-x^2}$~~

2) 2)  $(4P)' = C_1(-\sin(2x)) \cdot 2 + C_2 \cos(2x) \cdot 2$   
 $4P' = -2C_1 \sin(2x) + 2C_2 \cos(2x)$   
 $0 = C_1 \cos(0) + C_2 \sin(0)$   
 $0 = C_1$   
 $2 = -2C_1 \sin(0) + 2C_2 \cos(0)$   
 $2 = 2C_2 \rightarrow C_2 = 1$   
 $4P = \sin(2x)$

PROVJERA:  $4P = \sin 2x$   
 $0 = \sin(2 \cdot 0)$   
 $0 = 0$

$4P = \cos(2x) \cdot 2$   
 $2 = \cos(0) \cdot 2$   
 $2 = 2$

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$$(4) \int_0^2 \frac{2x^2+x+2}{x^2-1} dx = \int_0^2 \frac{2x^2+x+2}{x^2-1} = \int_0^2 2 dx + \int_0^2 \frac{x+4}{x^2-1} dx$$

POGRESNE  
GRANICE

$$(2x^2+x+2) : (x^2-1) = 2$$

$$\frac{2x^2-2}{x+4}$$

$$x+4 = A(x+1) + B(x-1)$$

$$x+4 = Ax + A + Bx - B$$

$$x+4 = x(A+B) + (A-B)$$

$$A+B=1 \quad 2A=5/2 \quad B=1-\frac{5}{2}$$

$$A-B=4 \quad A=\frac{5}{2} \quad B=-\frac{3}{2}$$

$$\int_0^2 \frac{x+4}{x^2-1} dx = \frac{A}{x-1} + \frac{B}{x+1} =$$

$$= \int_0^2 \frac{5/2}{x-1} dx + \int_0^2 \frac{-3/2}{x+1} dx = 0 \quad \times$$

3) POUKAZATI:  $x+y^2=6$  i  $x+y+1=0$

$$x+y^2=6$$

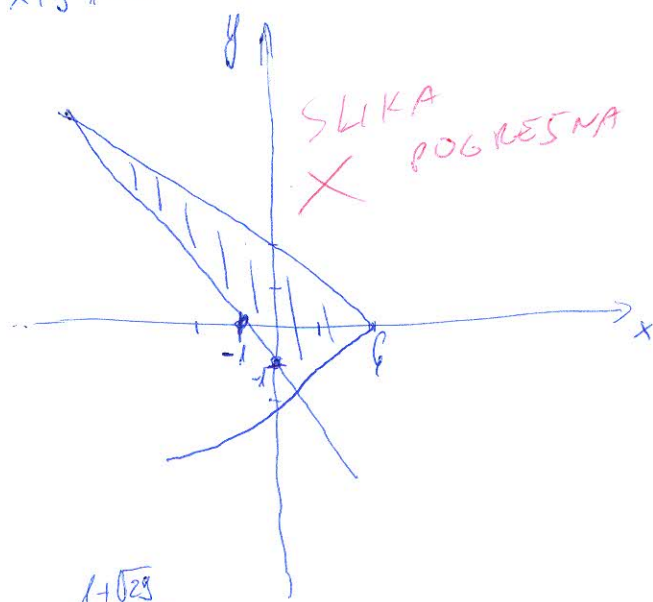
$$x+y=1$$

$$x=-y^2+6$$

x	6	0
y	0	$\pm\sqrt{6}$

$$y = -x-1$$

x	0	-1
y	-1	0



Sjecašte:

$$-y-1 = -y^2+6$$

$$y^2-y-7=0$$

$$y_1 = \frac{1+\sqrt{29}}{2}$$

$$y_2 = \frac{1-\sqrt{29}}{2}$$

$$P = \int_{\frac{1-\sqrt{29}}{2}}^{\frac{1+\sqrt{29}}{2}} (-y^2+6+y+1) dy =$$

$$= \left[ -\frac{y^3}{3} + \frac{y^2}{2} + 7y \right]_{\frac{1-\sqrt{29}}{2}}^{\frac{1+\sqrt{29}}{2}} =$$

$$= \frac{-8\sqrt{29}}{3} + \frac{\sqrt{29}}{2} + 7\sqrt{29}$$

$$= \frac{29\sqrt{29}}{6}$$

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$$\textcircled{1} \int_2^{+b} \frac{dx}{1-x^2} = \lim_{a \rightarrow \infty} \int_2^a \frac{dx}{1-x^2} = \lim_{a \rightarrow \infty} \left[ \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| \right]_2^a =$$

$$= \lim_{a \rightarrow \infty} \frac{1}{2} \left[ \ln \left| \frac{1+a}{1-a} \right| \right] - \ln \left[ \left| \frac{1+2}{1-2} \right| \right]$$

$$= \frac{1}{2} \lim_{a \rightarrow \infty} \ln \left| \frac{1+a}{1-a} \right| - \frac{1}{2} \ln 3$$

$$= \frac{1}{2} \ln \lim_{a \rightarrow \infty} \left| \frac{1+a}{1-a} \right| - \frac{1}{2} \ln 3$$

$$= \frac{1}{2} \ln 1 - \frac{1}{2} \ln 3 = -\frac{1}{2} \ln 3 \quad \checkmark$$

