MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}1$ **IME I PREZIME:** Nemanja Korda **VRIJEME POČETKA:** **MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):** 0269076510	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Grafički prikazati funkciju $f(x,y) = \dfrac{1}{x^2+y^2}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$? ~~10~~ 5

2. Numeričkom integracijom odrediti vrijednost $\int\limits_0^{\pi} \dfrac{dx}{\sin x + 2}$. (bodovanje: 20 za rel. grešku$\leq$ 1%, 15 za rel. grešku $\lessapprox$ 3%, 8 za rel. grešku $\lessapprox$ 6%) 20

3. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(2,1)$, $C(1,3)$ i $D(-1,2)$. Integriranjem mu pronađi površinu. 20

4. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$. 20

5. Izračunati $\int \dfrac{x^2-2x-2}{x^2+x-2}\, dx$. 20

Ukupno: ~~90~~ 85 Kor

f	$\frac{df}{dx}$
x^{α} $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_{\alpha} x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4.
$$\frac{y'}{x} = \frac{\sin x}{y} \Big| \cdot x$$

$$\int x \sin x\,dx = \begin{vmatrix} u = x & du = dx \\ dv = \sin x & v = -\cos x \end{vmatrix}$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

$$y' = \frac{1}{y} \cdot x \sin x \Big| \cdot y$$

$$y\frac{dy}{dx} = x \sin x \Big| \cdot dx$$

$$y\,dy = x \sin x\,dx \Big| \int$$

$$\int y\,dy = \int x \sin x\,dx$$

$$\frac{y^2}{2} = -x\cos x + \sin x + C \Big| \cdot 2$$

$$y^2 = -2x\cos x + 2\sin x + C$$

$$y = \sqrt{2\sin x - 2x\cos x + C}$$

2.
$$\int_0^{\pi} \frac{dx}{\sin x + 2}$$

k	0	$\frac{\pi}{2}$	π
$\frac{1}{\sin x+2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$

$$S = \frac{d}{6}\left(f_0 + 4f_1 + f_2\right)$$

$$S = \frac{\pi}{6} \cdot \left(\frac{1}{2} + 4 \cdot \frac{1}{3} + \frac{1}{2}\right)$$

$$S = \frac{\pi}{6} \cdot \frac{7}{3}$$

$$S = \frac{7\pi}{18} = \quad = 1.2217$$

TOČNO RJEŠENJE 1.2092

UNUTAR $\approx 1\%$

Nemanja Koroka

5. $\int \frac{x^2-2x-2}{x^2+x-2}\,dx$

$\cancel{x^2}-2x-\cancel{2} : x^2+x-2 = \boxed{1}$

$-\cancel{x^2}-x+\cancel{2}$

$\boxed{-3x}$

$= \overset{I_1}{\int dx} - \overset{I_2}{\int \frac{3x}{x^2+x-2}}$

$I_1 = x + C$

$I_2 = \int \frac{3x}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2} \quad \Big|\ \text{Naz}$

$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$

$x_{1,2} = \frac{-1 \pm 3}{2}$

$\boxed{x_1 = 1}$ $\boxed{x_2 = -2}$

$(x-1)(x+2) = x^2+2x-x-2 = x^2+x-2$

$3x = A(x+2) + B(x-1)$

$3x = Ax + 2A + Bx - B$

$A + B = 3$ $\qquad 2A - B = 0$

$A = 3 - B$ $\qquad 2A = B$

$\frac{1}{2}B = 3 - B$ $\qquad A = \frac{1}{2}B$

$\frac{3}{2}B = 3 \ \Big|\ : \frac{3}{2}$ $\qquad \boxed{A = 1}$

$\boxed{B = 2}$

$\int \frac{3x}{x^2+x-2} = \int \frac{1\,dx}{x-1} + 2\int \frac{dx}{x+2} = \ln|x-1| + 2\ln|x+2| + C$

$x-1=t$ $\quad \int \frac{dt}{t}$

$dx=dt$

$\int \frac{x^2-2x-2}{x^2+x-2} = x - \ln|x-1| - 2\ln|x+2| + C$

$$P_{ABC_1} = \int_1^2 \overline{BC} - \overline{AB} = \int_1^2 \left(-2x + 5 - \frac{1}{2}x\right)dx = -2\int_1^2 x\,dx + 5\int_1^2 dx - \frac{1}{2}\int_1^2 x\,dx$$

$$= -\cancel{2}\frac{x^2}{\cancel{2}} + 5x - \frac{1}{2}\cdot\frac{x^2}{2} = -x^2 + 5x - \frac{1}{4}x^2 \Big|_1^2 = (-4+10-1) - \left(-1+5-\frac{1}{4}\right)$$

$$= 5 - \frac{15}{4} = \frac{5}{4}$$

$$P_{ABC_2} = \int_0^1 \overline{AC} - \overline{AB} = \int_0^1 \left(3x - \frac{1}{2}x\right)dx = 3\int_0^1 x\,dx - \frac{1}{2}\int_0^1 x\,dx$$

$$= 3\cdot\frac{x^2}{2} - \frac{1}{2}\cdot\frac{x^2}{2} = \frac{3}{2}x^2 - \frac{1}{4}x^2 \Big|_0^1 = \left(\frac{3}{2} - \frac{1}{4}\right) - (0-0)$$

$$= \frac{5}{4}$$

$$P_{ABC_{uk}} = \frac{5}{4} + \frac{5}{4} = \frac{5}{2}$$

$$P_{ABCD} = \frac{5}{2} + \frac{5}{2} = 5$$

✓

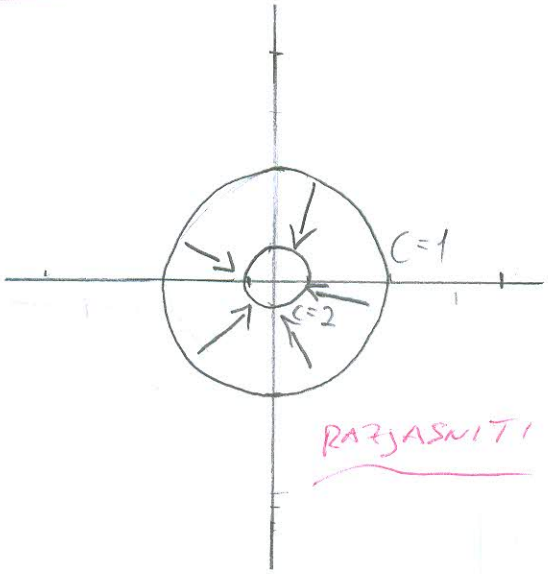
1. $f(x,y) = \frac{1}{x^2+y^2}$ $\quad \boxed{D_f = x^2+y^2 > 0}$?

LIMES POSTOJI ZATO ŠTO SU $(x,y)\to(0,0)$ RUB DOMENE ✗

RAZINSKE KRIVULJE

$c=1 \quad \frac{1}{x^2+y^2} = \frac{1}{2}$

$c=2 \quad \frac{1}{x^2+y^2} = \frac{1}{8}$



RAZJASNITI!

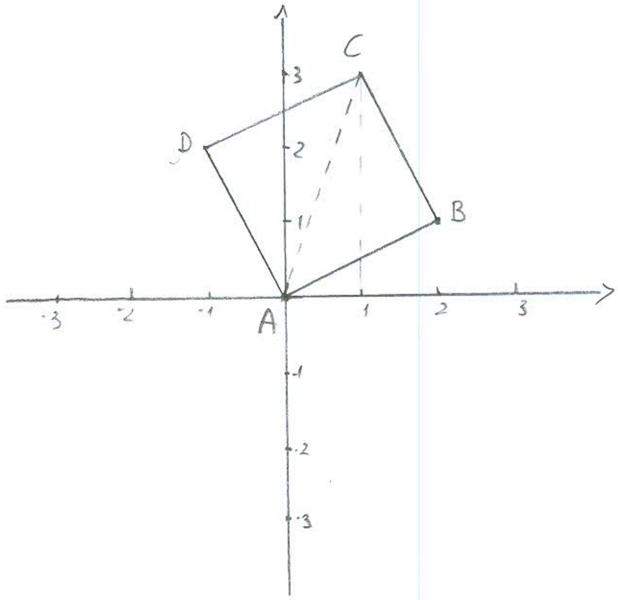
③ $A(0,0)$ $\quad (y-y_1)(x_2-x_1) = (x-x_1)(y_2-y_1)$ Nemanja Kordu

~~N Kordu~~

$B(2,1)$

$C(\overset{x_1}{1},\overset{y_1}{3})$

$D(\overset{x_2}{-1},\overset{y_2}{2})$



$\overline{AB} = (y-0)(2-0) = (x-0)(1-0)$

$2y = x$

$\boxed{y = \frac{1}{2}x}$

$\overline{AC} = (y-0)(1-0) = (x-0)(3-0)$

$\boxed{y = 3x}$

$\overline{AD} = (y-0)(-1-0) = (x-0)(2-0)$

$-y = 2x$

$\boxed{y = -2x}$

$\overline{BC} = (y-1)\underbrace{(1-2)}_{-1} = (x-2)(3-1)$

$-y+1 = 2x-4$

$-y = 2x-5$

$\boxed{y = -2x+5}$

$\overline{CD} = (y-3)\underbrace{(-1-1)}_{-2} = (x-1)\underbrace{(2-3)}_{-1}$

$-2y+6 = -x+1$

$-2y = -x-5$

$\boxed{y = \frac{1}{2}x + \frac{5}{2}}$

$P_{ACD_1} = \int_{-1}^{0} \overline{CD} - \overline{AD} = \int_{-1}^{0}\left(\frac{1}{2}x + \frac{5}{2} + 2x\right)dx = \frac{1}{2}\int_{-1}^{0} x\,dx + \frac{5}{2}\int_{-1}^{0} dx + 2\int_{-1}^{0} x\,dx$

$= \frac{1}{2}\cdot\frac{x^2}{2} + \frac{5}{2}x + \cancel{2}\cdot\frac{x^2}{\cancel{2}} = \frac{1}{4}x^2 + \frac{5}{2}x + x^2 \Big|_{-1}^{0} = (0+0+0) - \left(\frac{1}{4} - \frac{5}{2} + 1\right) = \frac{5}{4}$

$P_{ACD_2} = \int_{0}^{1} \overline{CD} - \overline{AC} = \int_{0}^{1}\left(\frac{1}{2}x + \frac{5}{2} - 3x\right)dx = \frac{1}{2}\int_{0}^{1} x\,dx + \frac{5}{2}\int_{0}^{1} dx - 3\int_{0}^{1} x\,dx$

$= \frac{1}{2}\cdot\frac{x^2}{2} + \frac{5}{2}x - 3\cdot\frac{x^2}{2} = \frac{1}{4}x^2 + \frac{5}{2}x - \frac{3}{2}x^2 \Big|_{0}^{1} = \left(\frac{1}{4} + \frac{5}{2} - \frac{3}{2}\right) - (0) = \frac{5}{4}$

$P_{ACD_{uk}} = \frac{5}{2}$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}1$

Popunjava nastavnik Broj ↓ bodova

Ime i prezime: JURE FRŽOP **Vrijeme početka:**

Matični broj studenta (iznad slike u indeksu):

17-2-0269-2013

1. Grafički prikazati funkciju $f(x,y)=\dfrac{1}{x^2+y^2}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$? Ø

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3. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(2,1)$, $C(1,3)$ i $D(-1,2)$. Integriranjem mu pronađi površinu. 20

4. Nađi implicitno rješenje jednadžbe $\frac{y'}{x}=\frac{\sin x}{y}$. 20

5. Izračunati $\displaystyle\int\frac{x^2-2x-2}{x^2+x-2}\,\mathrm{d}x$. ×

Ukupno: 40

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
α^x $(\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x+C$	$\int\frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int\frac{\mathrm{d}x}{a^2-x^2}=\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$
$\int x^\alpha\,\mathrm{d}x=\frac{x^{\alpha+1}}{\alpha+1}$, $\alpha\neq-1$	$\int\tan x\,\mathrm{d}x=-\ln\lvert\cos x\rvert+C$	$\int\frac{\mathrm{d}x}{x^2-a^2}=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int\frac{\mathrm{d}x}{x}=\ln\lvert x\rvert+C$	$\int\cot x\,\mathrm{d}x=\ln\lvert\sin x\rvert+C$	$\int\frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}}=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int e^x\,\mathrm{d}x=e^x+C$	$\int\frac{\mathrm{d}x}{\cos^2 x}=\tan x+C$	$\int\frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C$	$\int\frac{\mathrm{d}x}{\sin^2 x}=-\cot x+C$	$\int\frac{\mathrm{d}x}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int\sin x\,\mathrm{d}x=-\cos x+C$	$\int\sqrt{x^2\pm a^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int\cos x\,\mathrm{d}x=\sin x+C$	$\int\sqrt{a^2-x^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

4.) $\frac{y'}{x} = \frac{\sin x}{y}$

$$\int x \sin x \, dx = \left| \begin{matrix} x = u & /' \\ dx = du & \end{matrix} \right. \quad \begin{matrix} dv = \sin x & /\int \\ v = \int \sin x & \\ v = -\cos x & \end{matrix}$$

$yy' = x \sin x$

$$u \cdot v - \int v \cdot du$$

$y \frac{dy}{dx} = x \sin x \; / \cdot dx$

$$= x \cdot (-\cos x) - \int -\cos x \, dx$$

$$= x \cdot (-\cos x) + \int \cos x \, dx$$

$y \, dy = x \sin x \, dx \; /\int$

$$= x \cdot (-\cos x) + \sin x + C$$

$\int y \, dy = \int x \sin x \, dx$

$\frac{y^2}{2} = x(-\cos x) + \sin x + C$ ✓

$y^2 = 2\left[x(-\cos x) + \sin x + C\right]$

$y = \sqrt{2\left[x(-\cos x) + \sin x + C\right]}$

$x_1 = 1 \quad x_2 = -2$

5.) $\int \frac{x^2 - 2x - 2}{x^2 + x - 2} \, dx = \qquad \frac{x^2 - 2x - 2}{x^2 + x - 2} \, dx = \frac{A}{x+1} + \frac{B}{x-2} \; / (x^2 + x - 2)$ ✗

$x^2 - 2x - 2 = A(x-2) + B(x+1)$

$x^2 - 2x - 2 = Ax - 2A + Bx + B$

$x^2 - 2x - 2 = x(A+B) + (-2A + B)$

$$\underbrace{\int \frac{dx}{x+1}}_{I_1} - \underbrace{\int \frac{2}{x-2}}_{I_2} = \ln|x+1| - 2\ln|x-2| + C$$

$A + B = -2 \; / \cdot (-1)$

$-2A + B = -2 \qquad 0 + B = -2$

$-A - B = 2 \qquad B = -2$

$-2A + B = -2$

$-A = 0$

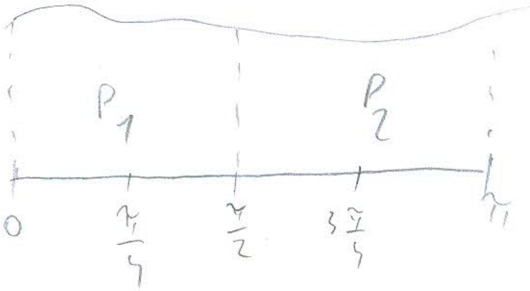
$I_1 = \int \frac{dx}{x+1} = \left| \begin{matrix} x+1 = t & /' \\ dx = dt & \end{matrix} \right| = \int \frac{dt}{t} = \ln|x+1| + C$

$I_2 = \dots = -2\ln|x-2| + C$

JURE FRZOP

2.) $\int_0^{\pi} \frac{dx}{\sin x + 2}$

$$S = \frac{d}{6}\left(f_0 + 4f_1 + f_2\right)$$



$f_0 = 1.0998$
$f_1 = 2.8677$
$f_2 = -2.4029$

$$S = \frac{\frac{\pi}{2}}{6}\left(1.0998 + 4 \cdot 2.8677 - 2.4029\right)$$

$$S = 0.2618 \cdot (10.1677)$$

$$S_1 = 2.6619$$

$f_0 = -2.4029$
$f_1 = -1.0669$
$f_2 = -1.0998$

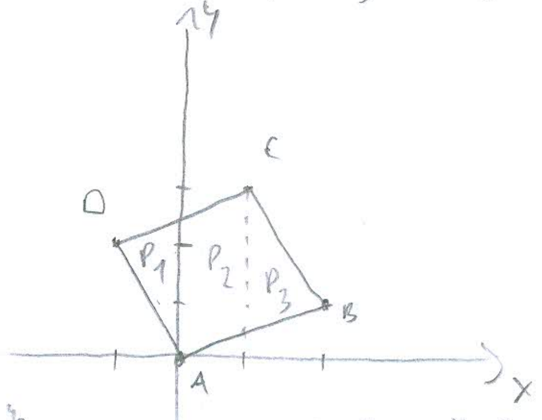
$$S = \frac{\frac{\pi}{2}}{6}\left(-2.4029 + 4(-1.0669) - 1.0998\right)$$

$$S = 0.2618(-7.7703)$$

$$S = -2.0343$$

$$S = S_1 + S_2 = 2.6619 - 2.0343 = \boxed{0.6276}$$

3.) A(0,0) B(2,1) C(1,3) D(-1,2) JURE FRŽOP



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$A(\overset{x_1}{0},\overset{y_1}{0})\ B(\overset{x_2}{2},\overset{y_2}{1})$ $\quad$ $B(\overset{x_1}{2},\overset{y_1}{1})\ C(\overset{x_2}{1},\overset{y_2}{3})$ $\quad$ $C(\overset{x_1}{1},\overset{y_1}{3})\ D(\overset{x_2}{-1},\overset{y_2}{2})$

AB

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{1-0}{2-0}(x-0)$$

$$y = \frac{1}{2}x$$

BC

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{3-1}{1-2}(x-2)$$

$$y - 1 = -\frac{2}{1}(x-2)$$

$$y - 1 = -2x + 4$$

$$y = -2x + 5$$

CD

$$y - 3 = \frac{2-3}{-1-1}(x-1)$$

$$y - 3 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

$D(\overset{x_1}{-1},\overset{y_1}{2})\ A(\overset{x_2}{0},\overset{y_2}{0})$

DA

$$y - 2 = \frac{0-2}{0+1}(x+1)$$

$$y - 2 = -2x - 2$$

$$y = -2x$$

$$P_1 = \int_{-1}^{0}\left[\left(\frac{1}{2}x + \frac{5}{2}\right) - (-2x)\right] = \int_{-1}^{0}\left[\frac{1}{2}x + \frac{5}{2} + 2x\right]$$

$$= \int_{-1}^{0}\left[\frac{5}{2}x + \frac{5}{2}\right] = \left[\frac{5}{2}\cdot\frac{x^2}{2} + \frac{5}{2}x\right]\Bigg|_{-1}^{0} = \left[\frac{5x^2}{4} + \frac{5}{2}x\right]\Bigg|_{-1}^{0} = 0 - \left(-\frac{5}{4}\right) = \frac{5}{4}$$

$$P_2 = \int_{0}^{1}\left[\left(\frac{1}{2}x + \frac{5}{2}\right) - \left(\frac{1}{2}x\right)\right] = \int_{0}^{1}\left[\cancel{\frac{1}{2}x} + \frac{5}{2} - \cancel{\frac{1}{2}x}\right] = \int_{0}^{1}\left[\frac{5}{2}\right] = \frac{5}{2}x\Bigg|_{0}^{1} = \frac{5}{2}$$

$$P_3 = \int_{1}^{2}\left[(-2x+5) - \left(\frac{1}{2}x\right)\right] = \int_{1}^{2}\left[-2x + 5 - \frac{1}{2}x\right] = \int_{1}^{2}\left[-\frac{5}{2}x + 5\right] = \left[-\frac{5x^2}{4} + 5x\right]\Bigg|_{1}^{2}$$

$$= 5 - 3.75 = \frac{5}{4}$$

$$P = P_1 + P_2 + P_3 = \frac{5}{4} + \frac{5}{2} + \frac{5}{4} = 5$$

$\boxed{\text{POVRŠINA JE 5}}$

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Ime i prezime: Mathias Grizelj **Vrijeme početka:** **Matični broj studenta (iznad slike u indeksu):**	

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3. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(2,1)$, $C(1,3)$ i $D(-1,2)$. Integriranjem mu pronađi površinu. ×

4. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$. $\varnothing$

5. Izračunati $\displaystyle\int \frac{x^2-2x-2}{x^2+x-2}\,\mathrm{d}x$. $\varnothing$

Ukupno: 20

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

$(x+\frac{1}{2})^2 = x^2+x+\frac{1}{4}$

$\frac{1}{4}+2 = \frac{1+8}{4}$

$2+\frac{1}{4} = \frac{8+0}{4}$

$-3x - \frac{1}{2}x = \frac{-6x-1}{2} = -\frac{7x}{2}$

$\frac{5}{2} - 3 = \frac{5-6}{2} = -\frac{1}{2}$

$2 + \frac{1}{2} = \frac{\quad}{2}$

$\frac{1}{2}+\frac{3}{1} = \frac{1+6}{2} = \frac{7}{2}$

$2\ \frac{1}{4}$ $\frac{9}{4}$ $\frac{1}{4}$ $\frac{7}{4}$

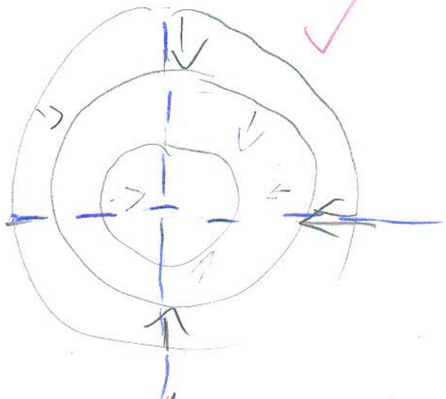
Mathias Grizelj

MATHIAS GRIZELJ

KODOMENA ✓

1. $f(x,y) = \frac{1}{x^2+y^2}$, $x^2+y^2 \neq 0$

$y,x \in \mathbb{R}^2 \setminus \{0\}$



$c=1 \Rightarrow \frac{1}{x^2+y^2} = 1 \;/\; x^2+y^2$

$1 = x^2+y^2$

$x^2+y^2=1$

$c=2 \Rightarrow \frac{1}{x^2+y^2} = 2 \;/\cdot x^2+y^2$

$1 = 2x^2+y^2$

$2x^2+y^2=1$

$x^2+y^2=\frac{1}{2}$

$c=3 \Rightarrow \frac{1}{x^2+y^2} = 3 \;/\; x^2+y^2$

$1 = 3x^2+y^2$

$3x^2+y^2=1$

$x^2+y^2=\frac{1}{3}$

✗

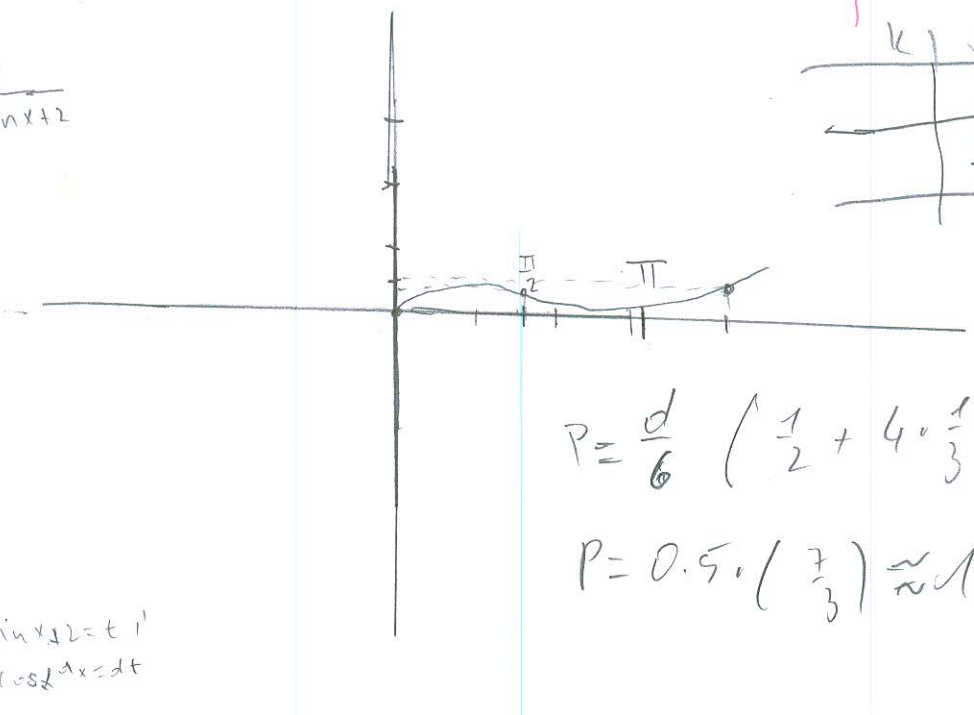
~~13~~

12

Limes postoji zato što imamo asimptote vertikalne i horizontalne te funkcija u $T(0,0)$ ne postoji.

2. $\int_0^{\pi} \frac{dx}{\sin x+2}$

k	0	1	2
	0	$\frac{\pi}{2}$	π
	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$



$P = \frac{d}{6}\left(\frac{1}{2} + 4\cdot\frac{1}{3} + \frac{1}{2}\right)$

$P = 0.5\cdot\left(\frac{7}{3}\right) \approx 1.16$

$\int_0^{\pi} = \left\{ \begin{array}{l} \sin x+2 = t \\ \cos x\, dx = dt \end{array} \right.$

$\int_0^{\pi} = \frac{dt}{t} = \ln|t| = \ln|\sin x+2|$

$= \ln|\sin\pi+2| - \ln|\sin 0+2|$

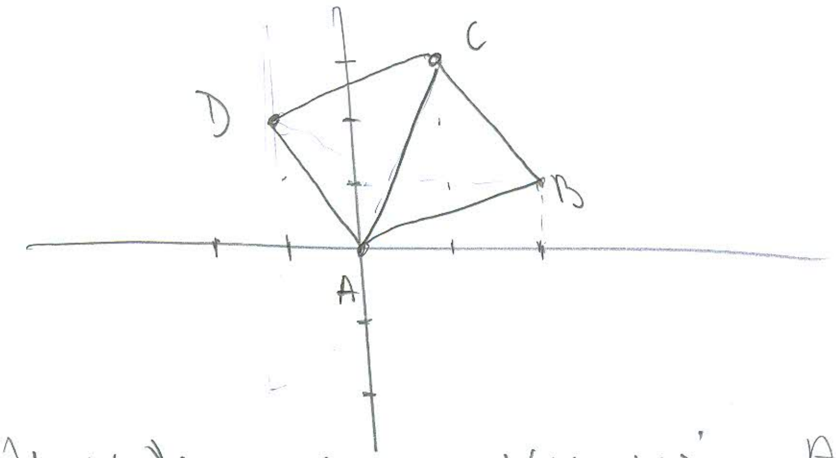
$= 0.6 - 0.6 = 0$

Umjesto mene..

REL. GREŠKA $\approx 4\%$

8

3.



$(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$ $\quad A(0,0), B(2,1), C(1,3), D(-1,2)$

AB

$(y-0)(2-0)=(x-0)(1$

$2y=x$

$y=\frac{x}{2}=\frac{1}{2}x$

BC

$(y-1)(1-2)=(x-2)(3-1)$

$(y-1)\cdot(-1)=(x-2)(2)$

$-y+1=2x-4$

$-y=2x-5$

$y=-2x+5$

CD

$(y-3)(-1-1)=(x-1)(2-3)$

$(y-3)(-2)=(x-1)(-1)$

$-2y+6=-x+1$

$-2y=-x-5$

$-y=\frac{-x-5}{2} \quad /(-1)$

$y=\frac{1}{2}x+\frac{5}{2}$

DA

$(y-2)(0+1)=(x-0)(2-0)$

$y-2=2x$

$y=2x+2$

AC

$(y-0)(-1)=(x-1)(3)$

$-y=3x-3$

$y=-3x+3$

$$\int_0^2 (AC-AB)+BC-AB\,dx+\int_{-1}^0 CD-DA+CD-AC\,dx$$

$$=\left[\int_0^2\left(-3x+3-\frac{1}{2}x\right)dx+\int_0^2\left(-2x+5-\frac{1}{2}x\right)dx\right]+\left[\int_{-1}^0\left(\frac{1}{2}x+\frac{5}{2}-2x+2\right)dx+\int_{-1}^0\left(\frac{1}{2}x+\frac{5}{2}+3x-3\right)dx\right]$$

$$\left[\int_0^2 -\frac{7x}{2}+3\,dx+\int_0^2 -\frac{3}{2}x+5\,dx\right]+\left[\int_{-1}^0 -\frac{1}{2}x+\frac{5}{2}\,dx+\int \frac{7}{2}x-\frac{1}{2}\,dx\right]$$

$$\left[-\frac{7}{2}\frac{x^2}{2}+3x\right]_0^2+\left[-\frac{3}{2}\frac{x^2}{2}+5x\right]_0^2+\left[-\frac{1}{2}\frac{x^2}{2}+\frac{5}{2}x\right]_{-1}^0+\left[\frac{7}{2}\frac{x^2}{2}-\frac{1}{2}x\right]_{-1}^0$$

$$-7+6-3+10+\left(-\frac{1}{4}+\left(\frac{7}{4}+\frac{1}{2}\right)\right)$$

$$6 \quad +2=8 \,//$$

MATHIAS GRIZELJ

$$\frac{y'}{x} = \frac{\sin x}{y} \quad / xy$$

$y' \cdot y = \sin x^2$

5. $(x^2 - 2x - 2) : (x^2 + x - 2) = 1 - \dfrac{3x}{x^2 + x - 2}$

$-(x^2 + x - 2)$

$-3x$

$(x+2)^2 = x^2 + 2x + 4$

$(x+2)^2$

$(x + \frac{1}{2})^2 = x^2 + x + \frac{1}{4}$

$$\frac{(x+2)^2 - 6}{(x + \frac{1}{2})^2 - \frac{9}{4}}$$

0

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}6$

IME I PREZIME: LUKA BAČIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0354-2014

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Numeričkom integracijom odrediti vrijednost $\int\limits_0^\pi \frac{\mathrm{d}x}{\sin x + 2}$. (bodovanje: 20 za rel. grešku$\leq$ 1%, 15 za rel. grešku $\lessapprox$ 3%, 8 za rel. grešku $\lessapprox$ 6%) Ø
2. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(2,1)$, $C(1,3)$ i $D(-1,2)$. Integriranjem mu pronađi površinu. ×
3. Grafički prikazati funkciju $f(x,y) = \frac{1}{x^2+y^2}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$? 12
4. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$.
5. Izračunati $\int \frac{x^2-2x-2}{x^2+x-2}\,\mathrm{d}x$. ×

Ukupno: 12

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
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$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

LUKA BAČIĆ 17-2-0354-2014

5. $\int \frac{x^2-2x-2}{x^2+x-2}\,dx =$

$$(x^2-2x-2):(x^2+x-2) = 1 - \frac{3x}{x^2+x-2}$$

$$\underline{-x^2-x\;2}$$

$$-3x$$

$$\frac{-1 \pm \sqrt{1-4\cdot 1\cdot(-2)}}{2} = \frac{-1 \pm 3}{2} = \begin{cases} x_1 = -2 \\ x_2 = 1 \end{cases}$$

$$\underbrace{\int 1\,dx}_{I_1} - \underbrace{\int \frac{3x}{x^2+x-2}\,dx}_{I_2} =$$

$I_1 \rightarrow \int dx = x$

$I_2 \rightarrow \int \frac{3x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1} \quad / \cdot (x+2)(x-1)$

$$3x = A(x-1) + B(x+2)$$

$$3x = Ax - A + Bx + 2B$$

$\rightarrow x^0:\ -A + 2B = 0$

$x^1:\ A + B = 3$

$A = 3 - B$

$\rightarrow -3 + B + 2B = 0$

$-3 + 3B = 0$

$\boxed{B = 1}$

$A + 1 = 3$

$\boxed{A = 2}$

$$\int \frac{3x}{(x+2)(x-1)} = \int \frac{2}{x+2}\,dx + \int \frac{1}{x-1}\,dx$$

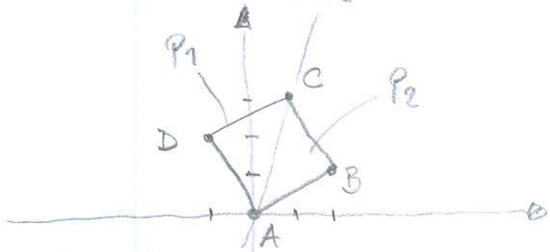
$$= 2\ln|x+2| + \ln|x-1| + C$$

$$\int \frac{x^2-2x-2}{x^2+x-2} = x - 2\ln|x+2| + \ln|x-1| + C$$

2. $A(0,0)$ $B(2,1)$ $C(1,3)$ $D(-1,2)$

$x_1\ y_1$ $x_2\ y_2$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



$p_{AB} \rightarrow y - 0 = \frac{1-0}{2-0}(x-0) \rightarrow y = \frac{1}{2}x$

$p_{BC} \rightarrow y - 1 = \frac{3-1}{1-2}(x-2) \rightarrow y - 1 = -2x + 4 \rightarrow y = -2x + 5$

$p_{CD} \rightarrow y - 3 = \frac{2-3}{-1-1}(x-1) \rightarrow y - 3 = \frac{1}{2}x - \frac{1}{2} \rightarrow y = \frac{1}{2}x + \frac{5}{2}$

$p_{DA} \rightarrow y - 0 = \frac{2-0}{-1-0}(x-0) \rightarrow y = -2x$

$$P_1 \rightarrow \int_{-1}^{1} \frac{1}{2}x + \frac{5}{2} + 2x = \int_{-1}^{1}\left(\frac{5}{2}x + \frac{5}{2}\right)dx = \frac{5}{2}\cdot\frac{x^2}{2} + \frac{5}{2}x = \frac{5x^2}{4} + \frac{5}{2}x \Big|_{-1}^{1}$$

$$= \frac{15}{4} + \frac{5}{4} = 5$$

$$P_2 \rightarrow \int_{1}^{2} -2x + 5 - \frac{1}{2}x = \int_{1}^{2}\left(-\frac{5}{2}x + 5\right)dx = -\frac{5}{2}\frac{x^2}{2} + 5x = -\frac{5x^2}{4} + 5x \Big|_{1}^{2}$$

$$= 5 - \frac{15}{4} = \frac{5}{4}$$

$$P = P_1 + P_2 = 5 + \frac{5}{4} = \frac{25}{4}$$

(4.) $\frac{y'}{x} = \frac{\sin x}{y} \;/\cdot xy$ LUKA BAČIĆ 17-2-0354-2014

$yy' = x\sin x$

$y\frac{dy}{dx} = x\sin x \;/\cdot dx$

$\int y\,dy = \int x\sin x\,dx \rightarrow \left\{ \begin{matrix} u = x & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{matrix} \right\}$

$\frac{y^2}{2} = -x\cos x + \int \cos x\,dx$

$\frac{y^2}{2} = -x\cos x + \sin x$

$y^2 = -2x\cos x + 2\sin x$

$\boxed{\frac{y^2}{2} = -x\cos x + \sin x + C \quad \text{ZA } C \in \mathbb{R}}$

(3) $f(x,y) = \frac{1}{x^2+y^2}$ $D(f) = \{\mathbb{R}^2 / x^2+y^2 \neq 0\}$ KODOMENA $\rightarrow [0, +\infty\rangle$

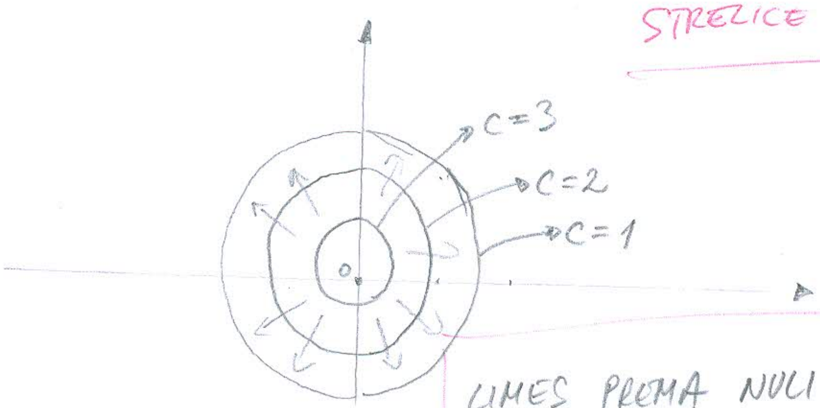
x

$c=1 \longrightarrow \frac{1}{x^2+y^2} = 1 \;/\cdot(x^2+y^2) \rightarrow x^2+y^2 = 1$

$c=2 \longrightarrow \frac{1}{x^2+y^2} = 2 \rightarrow 2x^2+2y^2 = 1 \rightarrow x^2+y^2 = \frac{1}{2}$

$c=3 \longrightarrow \frac{1}{x^2+y^2} = 3 \rightarrow 3x^2+3y^2 = 1 \rightarrow x^2+y^2 = \frac{1}{3}$

STRELICE POGREŠNE!



12

LIMES PREMA NULI POSTOJI ZATO JER SE KRUŽNICA PRIBLIŽAVA PREMA NULI.

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}6$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MARINA BEŠKER **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0193-2012	

1. Numeričkom integracijom odrediti vrijednost $\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2}$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) ✗

2. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(2,1)$, $C(1,3)$ i $D(-1,2)$. Integriranjem mu pronađi površinu. ✗

3. Grafički prikazati funkciju $f(x,y) = \frac{1}{x^2+y^2}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?

4. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$.

5. Izračunati $\int \frac{x^2-2x-2}{x^2+x-2}\,\mathrm{d}x$. ✗

Ukupno: Ø

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

1. $\int_0^{\pi} \frac{dx}{\sin x + 2} = \left| \begin{matrix} u = \sin x + 2 \,|' & dv = dx \,|\int \\ du = \cos x & v = \int dx \\ & v = x + C \end{matrix} \right| =$

$= \sin x \cdot x - \int_{}^{\pi} \cos x \cdot dx$

$= \sin x^2 - \sin x + x \Big|_0^{\pi}$

$= \sin x + x \Big|_0^{\pi}$

$\sin(3.14) + 3.14 - \sin(0) + 0 = 3.14$

$\int_0^{\pi} \frac{dx}{\sin x + 2} = \int_0^{\pi} \frac{dx}{\sin x} + \int_0^{\pi} \frac{dx}{2} =$

$=$

5. $\int \frac{x^2 - 2x - 2}{x^2 + x - 2} = \int \frac{x^2}{x^2} - \int \frac{2x}{x} + \int \frac{2}{2} + C$

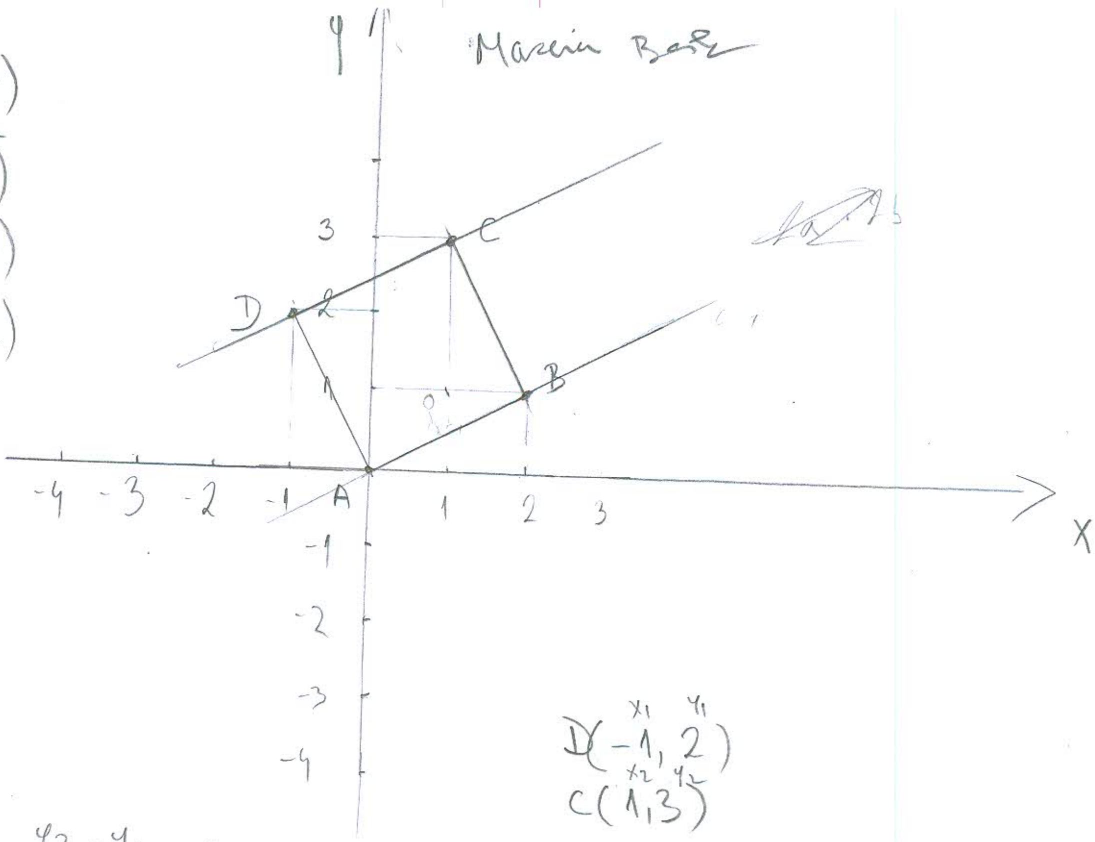
Marcin Bar...

2. $A\overset{x_1\ y_1}{(0,0)}$

$B\overset{x_2\ y_2}{(2,1)}$

$C(1,3)$

$D(-1,2)$



$A\overset{x_1\ \ y_1}{(0,\ 0)}$

$B\overset{x_2\ \ y_2}{(2,\ 1)}$

$p_{AB} \ldots\ y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$

$y - 0 = \frac{1-0}{2-0} \cdot (x - 0)$

$y_1 = \frac{1}{2}x$ //

$D\overset{x_1\ \ y_1}{(-1,\ 2)}$

$C\overset{x_2\ y_2}{(1,3)}$

$p_{DC} \ldots\ y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 2 = \frac{3-2}{1+1}(x+1)$

$y - 2 = \frac{1}{2}(x+1)$

$y = \frac{1}{2}x + \frac{3}{2} + 2$

$y = \frac{1}{2}x + \frac{7}{2}$ //

$a = 0$
$b = 3$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}6$

Ime i prezime: **Vrijeme početka:** 17:30

Matični broj studenta (iznad slike u indeksu):

NIKOLA MILETIĆ 17-2-0265-2013 17.06.2015.

Popunjava nastavnik

Broj ↓ bodova

1. Numeričkom integracijom odrediti vrijednost $\int_0^{\pi} \frac{dx}{\sin x + 2}$. (bodovanje: 20 za rel. grešku$\leq$ 1%, 15 za rel. grešku $\lessapprox$ 3%, 8 za rel. grešku $\lessapprox$ 6%)

2. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(2,1)$, $C(1,3)$ i $D(-1,2)$. Integriranjem mu pronađi površinu.

3. Grafički prikazati funkciju $f(x,y) = \frac{1}{x^2+y^2}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?

4. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$.

5. Izračunati $\int \frac{x^2-2x-2}{x^2+x-2}\, dx$.

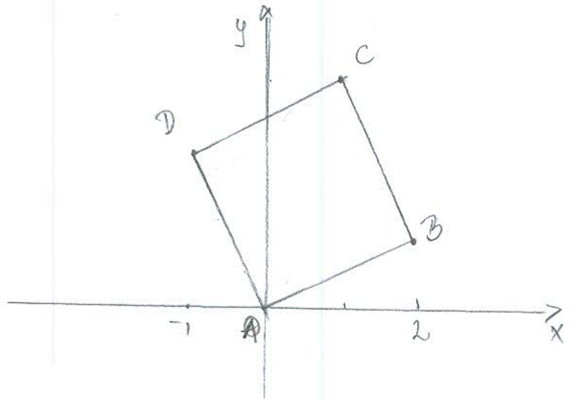
Ukupno: Ø

f	$\frac{df}{dx}$
x^{α} $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_{\alpha} x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^{\alpha}\, dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

(2) $A(0,0)$ $B(2,1)$ $C(1,3)$ $D(-1,2)$



$P_1 = \int_{-1}^{0}$ $\quad P_2 = \int_{0}^{2}$

$P_1 = \int_{-1}^{0} \overline{DC} - \overline{DA}$

$\overline{DC} \Rightarrow D(\underset{x_1}{-1}, \underset{y_1}{2})$ $C(\underset{x_2}{1}, \underset{y_2}{3})$

$(y_2 - y_1)(x - x_1) = (x_2 - x_1)(y - y_1)$

$(3-2)(x+1) = (1-2)(y-2)$

$1(x+1) = -1(y-2)$

$x + 1 = -y - 2$

$-y = x + 1 + 2 \quad / \cdot (-1)$

$y = -x - 3$

$\overline{DA} \Rightarrow D(\overset{x_1}{-1}, \overset{y_1}{2}) \quad A(\overset{x_2}{0}, \overset{y_2}{0})$

$(y_2 - y_1)(x - x_1) = (x_2 - x_1)(y - y_1)$

$(0 - 2)(x + 1) = (0 + 1)(y - 2)$

$-2(x+1) = 1(y-2)$

$-2x - 2 = y - 2$

$y = -2x \cancel{-2+2}$

$y = -2x$

$P_1 = \int_{-1}^{0} [(-x-3) - (-2x)] dx$

$P_1 = \int_{-1}^{0} (-x - 3 + 2x) dx = \int_{-1}^{0} (x - 3) dx$

$P_1 = \int_{-1}^{0} x dx^{*} - \int_{-1}^{0} 3 dx^{**} = \left[\frac{1}{2}x^2 - 3x\right]_{-1}^{0}$

$P_1 = \left[\frac{1}{2} \cdot (-1)^2 - 3 \cdot (-1)\right] - \left[\frac{1}{2} \cdot 0^2 - 3 \cdot 0\right]$

$P_1 = \frac{1}{2} \cdot 1 + 3 \Rightarrow \frac{7}{2}$

* $\int x dx = \frac{1}{2}x^2$

** $\int 3 dx = 3\int dx = 3x$

$P_2 = \int_0^2 \overline{DC} - \overline{AB} = \int_0^2 [(-x-3) - 2x] dx$

$P_2 = \int_0^2 (-3x - 3) dx = \int_0^2 -3x dx^{\bullet} - \int_0^2 3 dx^{\bullet\bullet}$

$P_2 = \left[-\frac{3}{2}x^2 - 3x\right]_0^2$

$P_2 = \left(-\frac{3}{2}(0)^2 - 3 \cdot 0\right) - \left(-\frac{3}{2}(2)^2 - 3 \cdot 2\right)$

$P_2 = \left(-\frac{3}{2} \cdot 4 - 6\right) \Rightarrow P_2 = -(-12)$

$P_2 = 12$

$\overline{DC} \Rightarrow y = -x - 3$

$\overline{AB} \quad A(\overset{x_1}{0}, \overset{y_1}{0}) \quad B(\overset{x_2}{2}, \overset{y_2}{1})$

$(y_2 - y_1)(x - x_1) = (x_2 - x_1)(y - y_1)$

$(2 - 0)(x - 0) = (1 - 0)(y - 0)$

$2x = y$

$^{\bullet}$ $\int -3x dx = -3\int x dx = -3 \cdot \frac{1}{2}x^2 = -\frac{3}{2}x^2$

$^{\bullet\bullet}$ $\int 3 dx = 3\int dx = 3x$

$P = P_1 + P_2 = \frac{7}{2} + 12 = \frac{31}{2} = 15\frac{1}{2}$

(1) $\int_0^{\pi} \frac{dx}{\sin x + 2}$

SIMPSONOVO PRAVILO

$P = \frac{d}{6}(f_0 + 4f_1 + f_2) \Big|_0^{\pi}$

$a = \sin x$

$b = 2$

$d = b - a = 2 - \sin x$

$x_1 = \frac{a+b}{2} = \frac{\sin x + 2}{2}$

$P = \frac{2 - \sin x}{6}\left(\cos x + 4 \cdot \frac{\cos x}{4} + 2\right) \Big|_0^{\pi}$

$P = \left[\frac{2 - \sin \pi}{6}(\cos \pi + \cos \pi + 2)\right] - \left[\frac{2 \sin 0}{6}(\cos 0 + \cos 0 + 2)\right]$

$a = x_0$

$f_0 = f(x_0) = (\sin)' = \cos x$

$P = 0$

$f_2 = f(b) = b \Rightarrow 2$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{F}6$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: TONI STOŠIĆ VRIJEME POČETKA:	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 57817-2009	

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2. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(2,1)$, $C(1,3)$ i $D(-1,2)$. Integriranjem mu pronađi površinu. ✗
3. Grafički prikazati funkciju $f(x,y) = \dfrac{1}{x^2+y^2}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?
4. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$.
5. Izračunati $\int \dfrac{x^2-2x-2}{x^2+x-2}\,dx$. ✗

Ukupno: $\emptyset$

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1. $\int_0^{\pi} \frac{dx}{\sin x + 2}$

$\int \frac{dx}{\sin x + 2} = \ln|\sin x + 2| + C$ ✗

$= \ln|\sin x + 2| \Big|_0^{\pi}$

$= \ln|\sin \pi + 2| - (\ln|\sin 0 + 2|)$

$= \ln 2 - \ln 2 = 0$

5.

$$\int \frac{x^2-2x-2}{x^2+x-2}$$

$$= x - \int \frac{3x}{x^2+x-2}$$

$$= x - \int \frac{0}{x+2}dx + \int \frac{1}{x-1}dx = x - \int \frac{dx}{x+2} + 1\int \frac{dx}{x-1} =$$

$$= x - \ln|x+2| + \ln|x-1| + C$$

$$(x^2-2x-2) : (x^2+x-2) = 1$$

$$\underline{-x^2 \mp x \pm 2}$$

$$-3x$$

$$\int \frac{x^2-2x-2}{x^2+x-2} = \int 1 - \frac{3x}{x^2+x-2} = \int 1dx - \int \frac{3x}{x^2+x-2} = x - \int \frac{3x}{x^2+x-2}$$

$$ax^2+bx+c = a(x-x_1)(x-x_2)$$

$$(x+2)(x-1)$$

$$x^2+x-2=0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1-4\cdot 1\cdot(-2)}}{2}$$

$$x = \frac{-1 \pm \sqrt{9}}{2}$$

$$x_1 = \frac{-1-3}{2} = -2$$

$$x_2 = \frac{-1+3}{2} = 1$$

$$\frac{3x}{x^2+x-2} = \frac{3x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1} \Big/ (x+2)(x-1)$$

$$3x = A(x-1) + B(x+2)$$

$$3x = Ax - A + Bx + 2B$$

$$3x = (A+B)x - A + 2B$$

$$A+B=3$$

$$-A+2B=0$$

$$3B = 3$$

$$B = \frac{3}{3} = 1$$

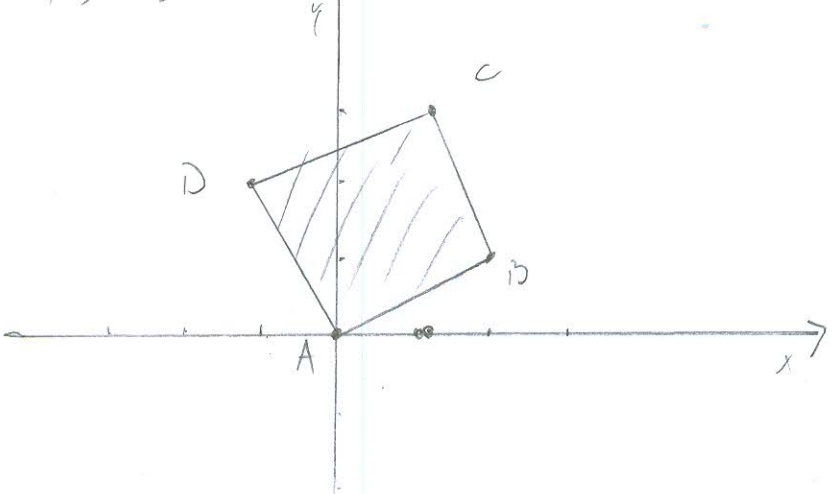
$$A+3=3$$

$$A=3-3$$

$$A=0$$

TONI STOJIĆ

2. ABCD četverokut



$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$A(0,0)$ $B(2,1)$
$x_1\ y_1$ $x_2\ y_2$

$(2-0)(y-0)=(1-0)(x-0)$

$2y=x$

$\overline{AB} \ldots y=\frac{1}{2}x$

$B(2,1)$ $C(1,3)$
$x_1\ y_1$ $x_2\ y_2$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(1-2)(y-1)=(3-1)(x-2)$

$-y+1=2x-4$

$-y=2x-4-1$

$-y=2x-5 \ /\cdot(-1)$

$\overline{BC} \ldots y=-2x+5$

$C(1,3)$ $D(-1,2)$
$x_1\ y_1$ $x_2\ y_2$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(-1-1)(y-3)=(2-3)(x-1)$

$-2y+6=-x+1$

$-2y=-x+1-6 \ /:(-2)$

$\overline{CD} \ldots y=\frac{1}{2}x-\frac{1}{2}+3=$

$y=\frac{1}{2}x+\frac{5}{2}$

$D(-1,2)$ $A(0,0)$
$x_1\ y_1$ $x_2\ y_2$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(0+1)(y-2)=(0-2)(x+1)$

$y-2=-2x-2$

$y=-2x-2+2$

$\overline{DA} \ldots y=-2x$

$$P=\int_{-1}^{0}\frac{1}{2}x+\frac{5}{2}-(-2x)+\int_{0}^{1}\frac{1}{2}x+\frac{5}{2}-\left(\frac{1}{2}x\right)+\int_{1}^{2}\left(-2x+5-\left(\frac{1}{2}x\right)\right)=$$

$$P=\int_{-1}^{0}\frac{1}{2}x+\frac{5}{2}+2x+\int_{0}^{1}\frac{1}{2}x+\frac{5}{2}-\frac{1}{2}x+\int_{1}^{2}-2x+5-\frac{1}{2}x$$

$$P=\left(\frac{1}{2}\cdot\frac{x^2}{2}+\frac{5}{2}x+2\cdot\frac{x^2}{2}\Big|_{-1}^{0}\right)+\left(\frac{1}{2}\cdot\frac{x^2}{2}+\frac{5}{2}x-\frac{1}{2}\cdot\frac{x^2}{2}\Big|_{0}^{1}\right)+\left(-2\cdot\frac{x^2}{2}+5x-\frac{1}{2}\cdot\frac{x^2}{2}\Big|_{1}^{2}\right)$$

$$P=\left(\frac{1}{4}\cdot x^2+\frac{5}{2}x+x^2\Big|_{-1}^{0}\right)+\left(\frac{1}{4}x^2+\frac{5}{2}x-\frac{1}{4}x^2\Big|_{0}^{1}\right)+\left(-x^2+5x-\frac{1}{4}x^2\Big|_{1}^{2}\right)$$

→

$$P = \left(\frac{1}{4}\cdot 0^2 + \frac{5}{2}\cdot 0 + 0^2 - \left(\frac{1}{4}\cdot(-1)^2 + \frac{5}{2}\cdot(-1) + (-1)^2\right)\right) +$$

$$\left(\frac{1}{4}\cdot 1^2 + \frac{5}{2}\cdot 1 - \frac{1}{4}\cdot 1^2 - \left(\frac{1}{4}\cdot 0^2 + \frac{5}{2}\cdot 0 - \frac{1}{4}\cdot 0^2\right)\right) +$$

$$\left(-2^2 + 5\cdot 2 - \frac{1}{4}\cdot 2^2 - \left(-1^2 + 5\cdot 1 - \frac{1}{4}\cdot 1^2\right)\right)$$

$$P = 0 - \left(\frac{1}{4} - \frac{5}{2} + 2\right) +$$

$$\frac{5}{2} - (0) +$$

$$13 - \left(1 + 6 - \frac{1}{4}\right.$$

$$P = 0 - \frac{1}{4} + \frac{5}{2} - 2 + \frac{5}{2} - 0 + 13 - 1 - 6 + \frac{1}{4} = \frac{0 - 1 + 10 - 4 + 10 - 0 + 52 - 4 - 24 + 1}{4}$$

$$P = \frac{40}{4} = 10$$

$$\underline{P = 10}$$

×