

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

A1

IME I PREZIME: *Nemanja Korda*

VRIJEME POČETKA:

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1. Grafički prikazati funkciju $f(x, y) = \frac{1}{x^2 + y^2}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$?

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2. Numeričkom integracijom odrediti vrijednost $\int_0^{\pi} \frac{dx}{\sin x + 2}$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\leq 3\%$, 8 za rel. grešku $\leq 6\%$)

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3. Četverokut $ABCD$ je određen točkama $A(0,0)$, $B(2,1)$, $C(1,3)$ i $D(-1,2)$. Integriranjem mu pronađi površinu.

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4. Nađi implicitno rješenje jednadžbe $\frac{y'}{x} = \frac{\sin x}{y}$.

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5. Izračunati $\int \frac{x^2 - 2x - 2}{x^2 + x - 2} dx$.

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Ukupno:

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

~~80~~

85
kor

$$(4) \quad \frac{y'}{x} = \frac{\sin x}{y} \quad | \cdot x$$

$$\int x \sin x dx = \left| \begin{array}{l} u=x \quad du=dx \\ dv=\sin x \quad v=-\cos x \end{array} \right|$$

$$y' = \frac{1}{y} \cdot x \sin x \quad | \cdot y$$

$$= -x \cos x + \int \cos x dx$$

$$y \frac{dy}{dx} = x \sin x \quad | \cdot dx$$

$$= -x \cos x + \sin x + C$$

$$y dy = x \sin x dx \quad | \int$$

$$\int y dy = \int x \sin x dx$$

$$\frac{y^2}{2} = -x \cos x + \sin x + C \quad | \cdot 2 \quad \checkmark$$

$$y^2 = -2x \cos x + 2 \sin x + C$$

$$y = \sqrt{2 \sin x - 2x \cos x + C}$$

$$(2) \quad \int_0^{\pi} \frac{dx}{\sin x + 2}$$

k	0	$\frac{\pi}{2}$	π
$\frac{1}{\sin x + 2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$

$$S = \frac{b}{6} (t_0 + 4t_1 + t_2)$$

$$S = \frac{\pi}{6} \cdot \left(\frac{1}{2} + 4 \cdot \frac{1}{3} + \frac{1}{2} \right)$$

$$S = \frac{\pi}{6} \cdot \frac{7}{3}$$

$$S = \frac{7\pi}{18} = \underline{\underline{1.2217}}$$

TOČNO REŠENJE 1.2092
UMTAR $\approx 1\%$

$$5. \int \frac{x^2 - 2x - 2}{x^2 + x - 2} dx$$

$$\begin{array}{r} x^2 - 2x - 2 : x^2 + x - 2 = 1 \\ -x^2 - x + 2 \\ \hline \end{array}$$

$$= \int I_1 dx - \int \frac{II_2}{x^2 + x - 2}$$

$$\boxed{-3x}$$

$$I_1 = x + C$$

$$I_2 = \int \frac{3x}{x^2 + x - 2} = \frac{A}{x-1} + \frac{B}{x+2} \quad | \quad \text{Naz}$$

$$3x = A(x+2) + B(x-1)$$

$$3x = Ax + 2A + Bx - B$$

$$A + B = 3$$

$$2A - B = 0$$

$$A = 3 - B$$

$$2A = B$$

$$\frac{1}{2}B = 3 - B$$

$$A = \frac{1}{2}B$$

$$\frac{3}{2}B = 3 \quad | : \frac{3}{2}$$

$$\boxed{A = 1}$$

$$\boxed{B = 2}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$x_{1,2} = \frac{-1 \pm 3}{2}$$

$$\boxed{x_1 = 1} \quad \boxed{x_2 = -2}$$

$$\frac{(x-1)(x+2) = x^2 + 2x - x - 2}{= x^2 + x - 2}$$

$$\int \frac{3x}{x^2 + x - 2} = \int \frac{1 dx}{x-1} + 2 \int \frac{dx}{x+2} = \ln|x-1| + 2 \ln|x+2| + C$$

$x-1 = k \Rightarrow \frac{dx}{dx} = \frac{dk}{dx} = \frac{1}{dx}$

$$\int \frac{x^2 - 2x - 2}{x^2 + x - 2} = x - \ln|x-1| - 2 \ln|x+2| + C \quad \checkmark$$

$$P_{ABC_1} = \int_1^2 \overline{BC} - \overline{AB} = \int_1^2 (-2x + 5 - \frac{1}{2}x) dx = -2 \int_1^2 x dx + 5 \int_1^2 dx - \frac{1}{2} \int_1^2 x dx$$

$$= -2 \cdot \frac{x^2}{2} + 5x - \frac{1}{2} \cdot \frac{x^2}{2} = -x^2 + 5x - \frac{1}{4}x^2 \Big|_1^2 = (-(4+0) - 1) - (-(1+5) - \frac{1}{4})$$

$$= 5 - \frac{15}{4} = \frac{5}{4}$$

$$P_{ABC_2} = \int_0^1 \overline{AC} - \overline{AB} = \int_0^1 (3x - \frac{1}{2}x) dx = 3 \int_0^1 x dx - \frac{1}{2} \int_0^1 x dx$$

$$= 3 \cdot \frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2} = \frac{3}{2}x^2 - \frac{1}{4}x^2 \Big|_0^1 = (\frac{3}{2} - \frac{1}{4}) - (0 - 0)$$

$$= \frac{5}{4}$$

$$P_{ABC_{uk}} = \frac{5}{4} + \frac{5}{4} = \frac{5}{2}$$

$$P_{ABCO} = \frac{5}{2} + \frac{5}{2} = 5$$

① $f(x,y) = \frac{1}{x^2+y^2}$

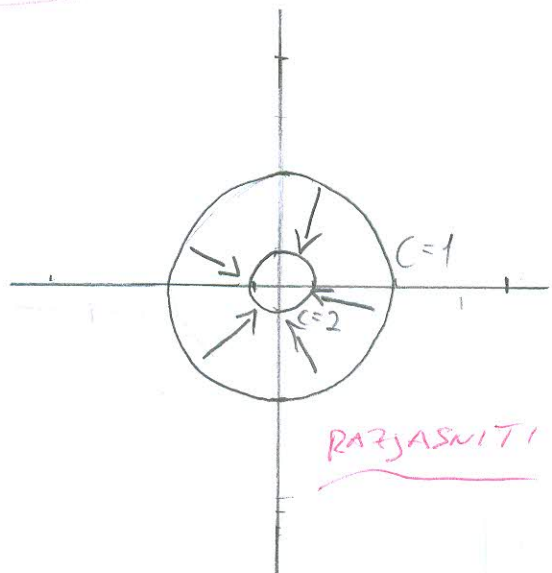
$D_f = x^2+y^2 > 0$

LIMES POSTOJI ZATO ŠTO SU $(x,y) \rightarrow (0,0)$ RUB DOMENE \times

RAZINSKE KRIVICE

$C=1 \quad \frac{1}{x^2+y^2} = \frac{1}{2}$

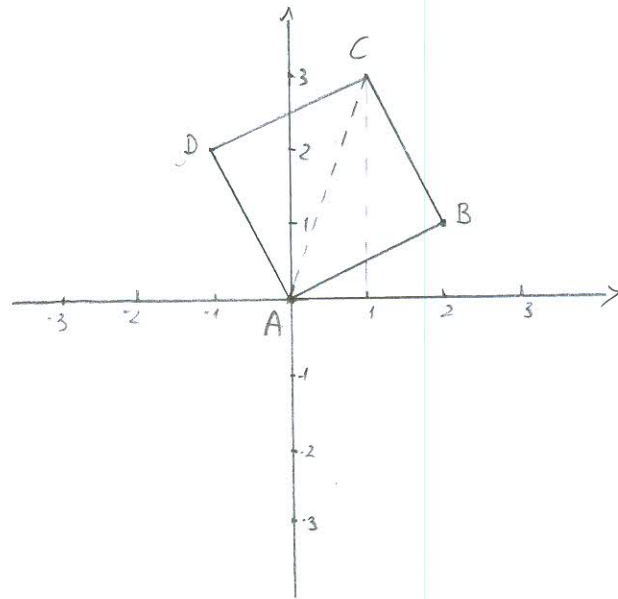
$C=2 \quad \frac{1}{x^2+y^2} = \frac{1}{8}$



3. A(0,0)
 B(2,1)
 C(1,3)
 D(-1,2)

$$(y - y_1)(x_2 - x_1) = (x - x_1)(y_2 - y_1)$$

Nemajni korda
~~A korda~~



$$\overline{AB} = (y - 0)(2 - 0) = (x - 0)(1 - 0)$$

$$2y = x$$

$$y = \frac{1}{2}x$$

$$\overline{AC} = (y - 0)(1 - 0) = (x - 0)(3 - 0)$$

$$y = 3x$$

$$\overline{AD} = (y - 0)(-1 - 0) = (x - 0)(2 - 0)$$

$$-y = 2x$$

$$y = -2x$$

$$\overline{BC} = (y - 1)(1 - 2) = (x - 2)(3 - 1)$$

$$-y + 1 = 2x - 4$$

$$-y = 2x - 5$$

$$y = -2x + 5$$

$$\overline{CD} = (y - 3)(-1 - 1) = (x - 1)(2 - 3)$$

$$-2y + 6 = -x + 1$$

$$-2y = -x - 5$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

$$P_{ACD_1} = \int_{-1}^0 \overline{CD} - \overline{AD} = \int_{-1}^0 \left(\frac{1}{2}x + \frac{5}{2} + 2x \right) dx = \frac{1}{2} \int_{-1}^0 x dx + \frac{5}{2} \int_{-1}^0 dx + 2 \int_{-1}^0 x dx$$

$$= \frac{1}{2} \cdot \frac{x^2}{2} + \frac{5}{2}x + 2 \cdot \frac{x^2}{2} = \frac{1}{4}x^2 + \frac{5}{2}x + x^2 \Big|_{-1}^0 = (0 + 0 + 0) - \left(\frac{1}{4} - \frac{5}{2} + 1 \right)$$

$$P_{ACD_2} = \int_0^1 \overline{CD} - \overline{AC} = \int_0^1 \left(\frac{1}{2}x + \frac{5}{2} - 3x \right) dx = \frac{1}{2} \int_0^1 x dx + \frac{5}{2} \int_0^1 dx - 3 \int_0^1 x dx$$

$$= \frac{1}{2} \cdot \frac{x^2}{2} + \frac{5}{2}x - 3 \cdot \frac{x^2}{2} = \frac{1}{4}x^2 + \frac{5}{2}x - \frac{3}{2}x^2 \Big|_0^1 = \left(\frac{1}{4} + \frac{5}{2} - \frac{3}{2} \right) - (0) = \frac{5}{4}$$

$$P_{ACD_{\text{un}}} = \frac{5}{2}$$

$$= \frac{5}{4}$$

