MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}2$

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: GORAN PAVLIČEVIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0452-2014

1. Riješiti integrale:

   (a) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}\,\mathrm{d}x,$ 10

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2-1} = ?.$ 10

2. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1-x^3}\,\mathrm{d}x$, s relativnom greškom manjom od 10%. 20

3. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. X

4. Riješiti: $y' + 2xy = x - 3$. X

5. Grafički prikazati funkciju $f(x, y) = \ln(x - y + 2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. 17

Ukupno: 57

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

Goran Pavličević

1)
a) $\int \frac{1+2x^2}{x^2(1+x^2)}dx = \int \frac{1+x^2+x^2}{x^2(1+x^2)}dx = \int \frac{\cancel{1+x^2}}{x^2\cancel{(1+x^2)}}dx + \int \frac{\cancel{x^2}}{\cancel{x^2}(1+x^2)}dx$

$= \int \frac{1}{x^2}dx + \int \frac{1}{1+x^2}dx = \int x^{-2}dx + \int \frac{1}{1+x^2}dx =$

$= \frac{x^{-1}}{-1} + \frac{1}{1}\arctan\frac{x}{1} + C = -\frac{1}{x} + \operatorname{arctg} x + C$

b) $\int_0^2 \frac{2x}{x^2-1}dx$

Domena: $x \neq 1$

$\lim_{a\to 1^-}\int_0^a \frac{2x}{x^2-1}dx = \lim_{a\to 1^-}\left(\ln|a^2-1| - \ln|0-1|\right) = -\infty - \ln|1|$
$= -\infty$

$\int \frac{2x}{x^2-1}dx = \left[\begin{matrix} t = x^2-1 \; /' \\ dt = 2x\,dx \end{matrix}\right] = \int \frac{dt}{t} = \ln|t| + C = \ln|x^2-1| + C$

$\lim_{b\to 1^+}\int_b^2 \frac{2x}{x^2-1}dx = \lim_{b\to 1^+}\left(\ln(4-1) - \ln|b^2-1|\right) =$

$= \ln|3| - (-\infty) = +\infty$

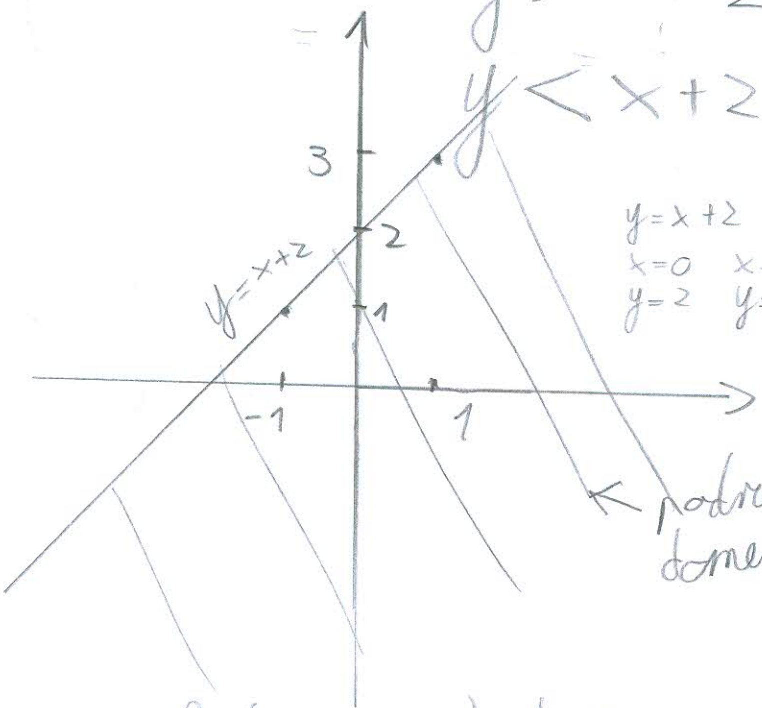
$\int_0^2 \frac{2x}{x^2-1}dx = -\infty + \infty = \boxed{N/P}$

5) $f(x,y) = \ln(x-y+2)$

Domena: $x-y+2>0$

$-y > -x-2$

$y < x+2$



$y=x+2$

$x=0 \quad x=1 \quad x=-1$

$y=2 \quad y=3 \quad y=1$

VRIJEDNOSTI ?

$f(x, x-e^c+2) =$

$= \ln(x-x+e^c-2+2) = c$

$c = \ln(x-y+2) \mid e$

$e^c = x-y+2$

$y = x - e^c + 2$

$c_1 = 1 \Rightarrow y = x+2-e \Rightarrow y = x+2-e$

$c_2 = -1 \Rightarrow y = x+2-\frac{1}{e} \Rightarrow y = x+2-\frac{1}{e}$

$c_3 = \ln 2 \Rightarrow y = x+2-2 \Rightarrow y = x$

$c_4 = \ln 3 \Rightarrow y = x+2-3 \Rightarrow y = x-1$

$x=0 \quad x=1$

$y=-0{,}71 \quad y=0{,}29$

$x=0 \quad x=1$

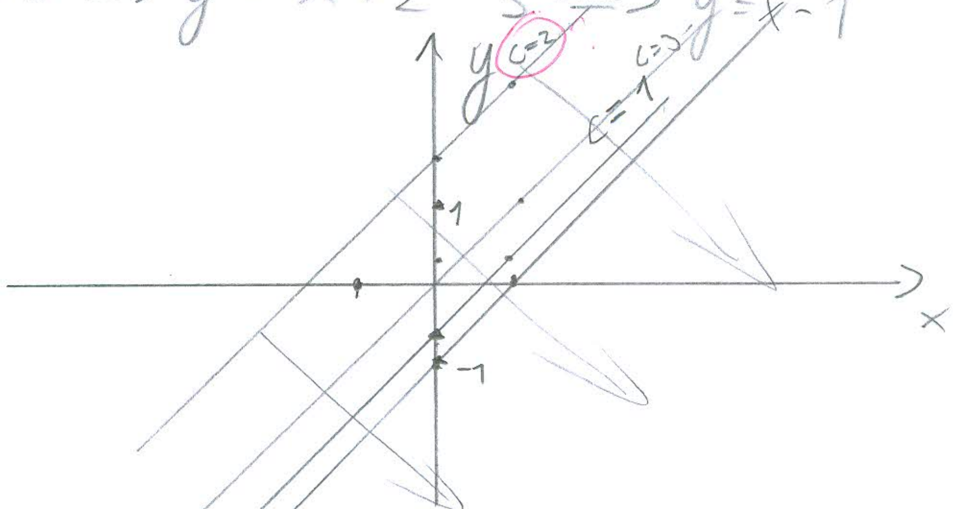
$y=1{,}63 \quad y=2{,}63$

$x=0 \quad x=1$

$y=0 \quad y=1$

$x=0 \quad x=1$

$y=-1 \quad y=0$



17

GORAN PAVLIČEVIĆ

(2) $\int_0^1 \sqrt[3]{1-x^3}\, dx$

k	0	1	2
x_k	0	1/2	1
f_k	3	2,8062	0
$S=\frac{d}{6}(f_0+4f_1+f_2)$	$S=\frac{1}{6}(3+4\cdot 2{,}8062+0)=\boxed{2{,}3708}$		

③ $y'' - y = (-x+1)\cdot e^{0x}$ GORAN PAVLIČEVIĆ

1) $y'' - y = 0$

$r^2 - 1 = 0$

$(r-1)(r+1) = 0$

$r_1 = 1$

$r_2 = -1$

$y_H = C_1 \cdot e^x + C_2 \cdot e^{-x}$

2) $f(x) = (-x+1)\cdot e^{0x}$

$y_P = Ax + B$

$y' = A$ $\boxed{A=0}$

$y'' = 0$

$$\boxed{y = C_1 e^x + C_2 \cdot e^{-x} + A}$$

$y' = C_1 \cdot e^x - C_2 \cdot e^{-x}$

$0 = C_1 - C_2$ $\boxed{C_1 = C_2 = 1}$

$C_1 = -1$ $C_2 = -1$

GORAN PAVLIČEVIĆ

4) $y' + 2xy = x - 3$

$$y(x) = e^{-\int p(x)\,dx}\left[c + \int g(x)\cdot e^{\int p(x)\,dx}\,dx\right]$$

$$y(x) = e^{-\int 2x\,dx}\left[c + \int (x-3)\cdot e^{\int 2x\,dx}\,dx\right] =$$

$$= e^{-2\cdot\frac{x^2}{2}}\left[c + \int (x-3)\, e^{2\cdot\frac{x^2}{2}}\,dx\right] =$$

$$= e^{-x^2}\left[c + \int (x-3)\, e^{x^2}\,dx\right] =$$

$$= e^{-x^2}\left[c + (x-3)\cdot e^{x^2} - \int e^{x^2}\,dx\right]$$

$$\boxed{y(x) = e^{-x^2}\left[c + (x-3)\cdot e^{x^2} - e^{x^2}\right]}$$

$$\boxed{y(x) = e^{-x^2}\left[c + x e^{x^2} - 4e^{x^2}\right]}$$

$$\int (x-3)\, e^{x^2}\,dx =$$

$$= \begin{bmatrix} u = x-3 \;/' & du = dx \\ dv = e^{x^2} \;/\int & v = e^{x^2} \end{bmatrix}$$

$$= (x-3)\, e^{x^2} - \int e^{x^2}\,dx$$

$$y(x) = ce^{-x^2} + x - 4$$

PROVJERA:

$$\underbrace{c\cdot(-2x)\,e^{-x^2} + 1 + 2x\,ce^{-x^2} + 1 \ldots}_{2x+2} \neq x-3$$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: KRISTIAN DUŠEVIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

DOPUNJAVA NASTAVNIK D2 Broj ↓ bodova

1. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. 0
2. Riješiti: $y' + 2xy = x - 3$. 3
3. Grafički prikazati funkciju $f(x, y) = \ln(x - y + 2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. X
4. Riješiti integrale:

   (a) $\displaystyle\int \frac{1 + 2x^2}{x^2(1 + x^2)} dx$,

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2 - 1} = ?$. X

5. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1 - x^3}\, dx$, s relativnom greškom manjom od 10%. 10

Ukupno: 13

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a + x}{a - x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha + 1}, \ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x - a}{x + a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

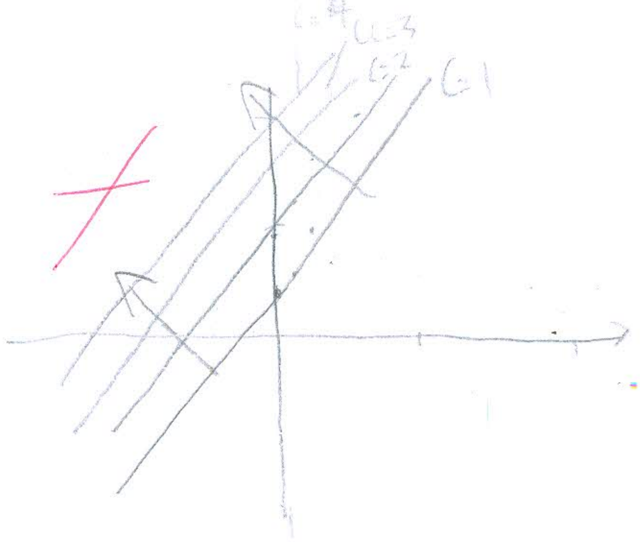
KRISTIAN DUJEVIĆ

3. $f(x,y) = \ln(x-y+2)$

$x - y + 2 > 0$

$y < x + 2$ ✓

$D_{f} = \{(x,y) \in \mathbb{R}^2 : y < x+2\}$



$c = 1$ $\ln(x-y+2) = 1$

$x - y + 2 = e^1$

$x + 2 - e^1 = y$

$c = 2$

$x + 2 + e^2 = y$

$c = 3$

$x + 2 + e^3 = y$

3

	0	1	2	3	
c=1	4.71	5.71	6.71	7.71	
c=2	9.38	10.38	11.38		
c=3					

4. a)

$$\int \frac{1+2x^2\,dx}{x^2(1+x^2)} = \int \frac{2x^2\,dx}{x^2(1+x^2)} + \int (x^2 \cdot \ldots)^{-1} \quad \left\{ \begin{array}{l} u = (1+x^2)^{-1} \quad dv = x^{-2} \\ du = -(1+x^2)^{-2} 2x\,dx \quad v = -x^{-1} \end{array} \right\}$$

$$= \arctan x - \frac{1}{1+x^2 \cdot 2x^2} - \int (1+x^2)^{-2} \cdot 2x \quad \left\{ \begin{array}{l} u = (1+x^2)^{-2} \cdot 2x \\ du = 1+x^2 \end{array} \right.$$

$$= \arctan x - \frac{1}{1+x^2 \cdot 2x^2} -$$

2.

$y' + 2xy = x - 3$

$y' + 2xy = 0$

$y' = -2xy$

$\frac{y'}{y} = -2x$

$\frac{\frac{dy}{dx}}{y} = -2x$

$\frac{dy}{y} = -2x\,dx$

$\int \frac{dy}{y} = -2 \int x$

$\ln|y| = -x^2 + \ln|c|$

$y = e^{-x^2} \cdot C(x)$

$y' = -e^{-x^2} \cdot 2x \cdot C(x) + C'(x) \cdot e^{-x^2}$

$\cancel{-e^{-x^2} \cdot 2x \cdot C(x)} - C'(x) \cdot e^{-x^2} \cancel{+ 2x \cdot e^{-x^2} \cdot C(x)} = x - 3$

$-C'(x) \cdot e^{-x^2} = x - 3$

$-C'(x) = \frac{x-3}{e^{-x^2}}$

$C'(x) = -\frac{x-3}{e^{-x^2}}$

$\frac{dc}{dx} = -\frac{x-3}{e^{-x^2}}$

$dc = -\frac{x-3}{[illegible]}$

KRISTIAN DUŠEVIĆ

$$\int dC(x) = \int \frac{-x-3}{e^{-x^2}}\,dx$$

$$C(x) = \int \frac{-x}{e^{-x^2}}\,dx - \int \frac{3}{e^{-x^2}}\,dx$$

$$C(x) = -\int \frac{x\,dx}{e^{-x^2}} = \left\{ \begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ dx = \frac{dt}{2x} \end{array} \right\} - 3\int e^{x^2} dx$$

$$C(x) = -\int \frac{x\,dt}{2x \cdot e^{-t}} - 3\int e^{x^2}$$

$$C(x) = -\frac{1}{2} e^{t} -$$

$$C(x) = -\frac{1}{2} e^{t} - 3$$

RJEŠENJE ?

Ø

KRISTIAN BUSENK

4.b) $\int_0^2 \frac{2x}{x^2-1}$

$x^2-1=0$

$x^2 \neq 1$

$x \neq \pm 1$

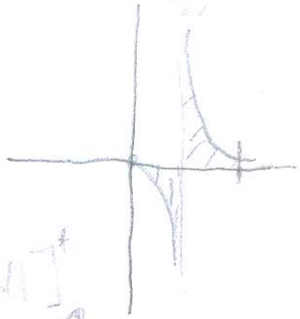
0	0.5	1	1.5	2
0	-1.33		2.4	

$x^2-1=t$

$dx = \frac{dt}{2x}$

$$\lim_{a \to 1^-} \int_0^a \frac{2x}{x^2-1} + \lim_{a \to 1^+} \int_b^2 \frac{2x}{x^2-1}$$

$$\lim_{a \to 1^-} \int_0^a \frac{2x \, dt}{2x \cdot t} + \lim_{b \to 1^+} \int_b^2 \frac{2x \cdot dt}{2x \cdot t}$$



$$\lim_{a \to 1^-} \left[\ln|t|\right]_0^a + \lim_{b \to 1^+} \left[\ln|t|\right]_b^2$$

$$\lim_{a \to 1^-} \left[\ln|x^2-1|\right]_0^a + \lim_{b \to 1^+} \left[\ln|x^2-1|\right]_b^2$$

$$\underbrace{0 - \cancel{\infty} + \cancel{\infty}}_{=N/P} + 1.09 = 1.09$$ X

5.

$$\int_0^1 3\sqrt{1-x^2}$$

$$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$$

$f_0 = 3$

$f(0.5) = 2.806$

$f(1) = 0$

$$S = \frac{1}{6}(3 + 4 \cdot 2.8 + 0)$$

$$S = \frac{1}{6}(14.224)$$

$$S = 2.37$$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}2$

IME I PREZIME: IVAN GAČINA **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
17-1-0075-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti integrale:

   (a) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}\,\mathrm{d}x$,

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2-1} = ?$.

2. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1-x^3}\,\mathrm{d}x$, s relativnom greškom manjom od 10%.

3. Riješiti $y''-y=-x+1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x=0$, $y=0$, $y'=0$. Provjeri rješenje.

4. Riješiti: $y'+2xy=x-3$.

5. Grafički prikazati funkciju $f(x,y)=\ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.

Ukupno: 0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
α^x $(\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha\neq-1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|+C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\|+C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\|+C$
$\int e^x\,\mathrm{d}x = e^x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x+C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a}+C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x+C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right)+C$
$\int \sin x\,\mathrm{d}x = -\cos x+C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int \cos x\,\mathrm{d}x = \sin x+C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

$$\frac{-b\pm\sqrt{b^2-4AC}}{2}$$

$$y=-x^2\cdot c \;/ :c$$
$$\frac{y}{c}=-x^2 \;/ \cdot 4$$

$$\frac{0\pm\sqrt{0+4}}{2}$$

$$\frac{\sqrt{4}}{2} \quad \frac{2}{2} \quad 1$$

5/ $f(x,y) = \ln(x-y+2)$

DOMENA $[\mathbb{R}]$ SVI REALNI BROJEVI

$\ln > 0$

$x-y+2 > 0$

$x-y > -2$

IVAN GAĆINA

1] a)

$$\int \frac{1+2x^2}{x^2(1+x^2)}dx = \int \frac{1}{x^2(1+x^2)}dx + 2\int \frac{x^2}{x^2(1+x^2)}dx$$

$$\cancel{\ }\ = 2\int \frac{dx}{1+x^2} = 2\int \frac{dx}{\sqrt{1}^2+x^2} = 2\cdot \frac{1}{\sqrt{1}} \operatorname{arctg} \frac{x}{\sqrt{1}} + C$$

$$\Delta = \int \frac{dx}{x^2(1+x^2)} = \dots \left| \frac{A}{x^2} + \frac{B}{x} + \frac{Cx+D}{1+x^2} \right| \cdot x^2(1+x^2)$$

$$A(1+x^2) + B\cdot x(1+x^2) + x^2(Cx+D) = 1$$

$$A + Ax^2 + Bx + Bx^3 + Cx^3 + Dx^2 = 1$$

Uz
$x^3 \Rightarrow B + C = 1$
$x^2 \Rightarrow A + D = 1$
$x^1 \Rightarrow B = 1$
$x^0 \Rightarrow A = 1$

$A = 1$
$B = 1$
$C \Rightarrow 1 + C = 1 \quad C = 0$
$D = \quad 1 + D = 1 \quad D = 0$

$$\int \frac{dx}{x^2} + \int \frac{dx}{x} \Rightarrow \ln|x| + C$$

$$\int x^{-2}dx$$

$$\Rightarrow \frac{x^{-1}}{1} + C$$

Rješenje

$$2\cdot \frac{1}{\sqrt{1}} \operatorname{arctg} \frac{x}{\sqrt{1}} + \frac{x^{-1}}{-1} + \ln|x| + C$$

1] b)

$$\int_0^2 \frac{2x}{x^2-1}\,dx \Rightarrow \int_0^2 2x \cdot \frac{1}{x^2-1}$$

$u = 2x$ $du = 2\,dx$

$dv = \frac{1}{x^2-1}$ $v = \int \frac{1}{x^2-1}\,dx$

$\int dv = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + c$

$u \cdot v + \int v\,du$

NEPRAVI INTEGRAL

$$2x \cdot \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + \int \frac{2}{x^2-1}\,dx$$

$$\Rightarrow 2x \cdot \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + 2 \cdot \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + c$$

$$\left[x \cdot \ln\left|\frac{x-1}{x+1}\right| + \ln\left|\frac{x-1}{x+1}\right|\right]_0^2$$

$\varnothing$

$$\left[2 \cdot \ln\left|\frac{1}{3}\right| + \ln\left|\frac{1}{3}\right|\right] - \left[0 + \ln\left|\frac{-1}{1}\right|\right]$$

$= -3,29$

$\boxed{4}$ $y' + 2xy = x - 3$

LINEARNA ODJ 1. REDA

$y' + 2xy = 0$

$y' = -2xy \quad /:y$

$\frac{\frac{dy}{dx}}{y} = -2x \quad / \cdot dx$

$\frac{dy}{y} = -2x\,dx \quad / \int$

$\int \frac{dy}{y} = \int -2x\,dx$

$\ln|y| = -2\frac{x^2}{2} + C$

$\ln|y| = -x^2 + C \quad /^{EXP} \quad [e^{-x^2+C}$

$y = -x^2 \cdot C$

$y_x = C \cdot x$

$y_x = -\frac{y}{x^2} \cdot x$

$y(x) = -\frac{y}{x}$

$y(x)' = -\left(\frac{x - y}{x^2}\right)$

$y(x)' = \frac{x+y}{x^2} \Rightarrow \frac{1}{x} + \frac{y}{x^2}$

$y_x = C$

$C = -\frac{y}{x^2}$

$\left(\frac{1}{x} + \frac{y}{x^2}\right) + 2x \cdot (v_x \cdot x) = x - 3$

RJEŠENJE?

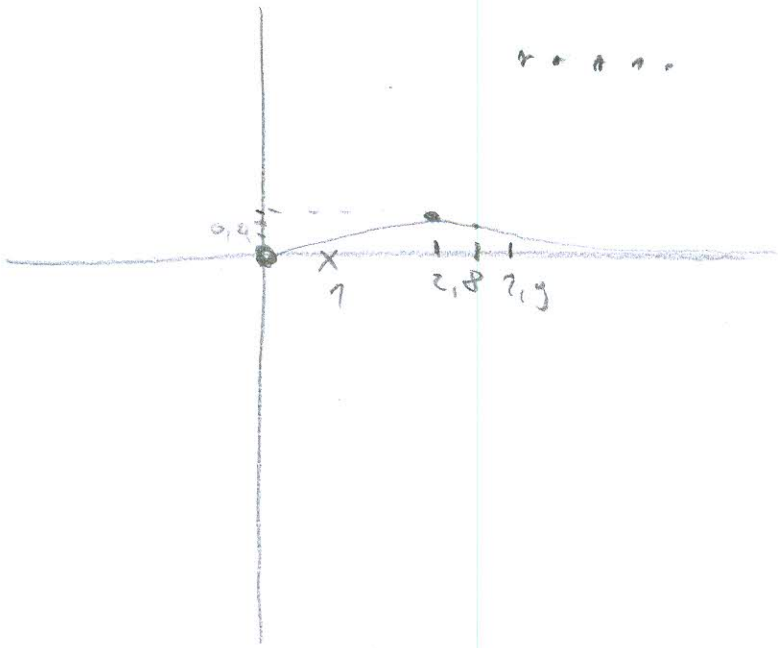
$\frac{x+y}{x^2} + \frac{2x}{1} \quad (v_x \cdot x$

$\boxed{2}$

IVAN GAĆINA

$$\int_0^1 3\sqrt{1-x^2}\,dx$$

SIMPSON $\frac{d}{6}\left(f_0 + 4f_1 + f_2\right)$



$$\frac{1}{6}\left(0 + 4\cdot 0{,}5 + 1\right)$$

$$\frac{3}{6} = \frac{1}{2}$$

IVAN GAĆINA

MATEMATIKA II

3

$y'' - y = -x + 1$

$r^2 - 1 = 0$

$r^2 = +1$

$r = \pm\sqrt{1} \quad r = 1$

$$\boxed{Y_H = e^{x}(c_1 x + c_2)}$$

$-x = e^{\alpha x}(P_{m(x)} \cos_{(\beta x)} + \cancel{Q_{n(x)} \sin_{(\beta x)}})$

$\alpha = 0 \quad \beta = 0$

$P_{m(x)} = -x \quad m = 1$

$Q_{n(x)} =$ NEVAŽAN $\quad n = N/P$

$k = 0 \neq 0i$

$k = 0$

$N = 1$

$y_p = x^k \cancel{e^{\alpha x}}(S_{N(x)} \cos_{(\beta x)} + \cancel{T_{N(x)} \sin_{(\beta x)}})$

$y_p = Ax + B \qquad y_p' = A \qquad y_p'' = 0$

$0 - (Ax + B) = -x$

$-Ax - B = -x$

$-B = 0 \qquad -A = -1$

$A = 1$

$B = 0$

$$\boxed{y_{p_1} = x}$$

y_{p_2}

$+1 = e^{\alpha x}(P_{m(x)} \cos_{(\beta x)} + Q_{n(x)} \sin_{(\beta x)})$

$\alpha = 0 \quad \beta = 0 \quad P_{m(x)} = +1 \quad m = 0 \quad N = 0 \quad k = 0$

$y_{p_2} = \cancel{x^k} \cdot \cancel{e^{\alpha x}}(S_{N(x)} \cos_{(\beta x)} + \cancel{T_{N(x)} \sin_{(\beta x)}})$

$y_{p_2} = A \qquad y_{p_2}' = 0 \qquad y_{p_2}'' = 0$

$0 - (A) = +1$

$-A = 1$

$A = -1$

$$\boxed{y_{p_2} = -1}$$

$$y(x) = y_H + y_{P_1} + y_{P_2}$$

$$= e^{x}(c_1 x + c_2) + x - 1$$

$\boxed{x=0,\ y=0,\ y'=0}$

$$y(x)' = [e^{x}]\cdot(c_1 x + c_2) + e^{x}\cdot c_1 + 1$$

$y_x \Rightarrow x=0, y=0 \}$ $\quad 0 = \cancel{e^0}^{1}(c_1\cdot 0 + c_2) + \cancel{0}^{-1} - 1$

$0 = c_2 - 1 \qquad \boxed{c_2 = 1}$

$y(x)' \Rightarrow y'=0,\ x=0 \}$ $\quad 0 = \cancel{e^0}^{1}\cdot(\cancel{c_1\cdot 0} + 1) + \cancel{e^0}^{1}\cdot c_1 + 1$

$0 = 1\cdot c_1 + 1 \qquad \boxed{c_1 = -1}$

$$\boxed{y(x) = e^{x}(-x+1) + x - 1}$$

$y_x' = e^{x}(-x+1) - e^{x} + 1$

$y_x'' = e^{x}(-x+1) - e^{x} - e^{x}$

PROVJERA $\quad y(x)'' = e^{x}\cdot(c_1 x + c_2) + e^{x}\cdot c_1 + e^{x}\cdot c_1 +$

$$\left(e^{x}(c_1 x + c_2) + e^{x}\cdot c_1 + e^{x}\cdot c_1\right) - \left(e^{x}(c_1 x + c_2) + x - 1\right) = -x + 1$$

$$\cancel{e^{x}(c_1 x + c_2)} + e^{x}\cdot c_1 + e^{x} c_1 - \cancel{e^{x}(c_1 x + c_2)} - x + 1 = -x + 1$$

$c_1 =$

$$\left(e^{x}(-x+1) - e^{x} - e^{x}\right) - \left(e^{x}(-x+1) + x - 1\right) = x - 1$$

$$\cancel{e^{x}}(-x+1) \cancel{-e^{x}} - \cancel{e^{x}} - \cancel{e^{x}}(-x+1) - x + 1 = x - 1 \qquad [x=0]$$

$$\cancel{-1+1} - 1 - 1 - 1 - 1 + 1 - 1 + 1 = 1 - 1$$

...

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A2

IME I PREZIME: LOVRE BUBALO **VRIJEME POČETKA:** 17:35

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0389-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti integrale:

   (a) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}\mathrm{d}x$, $\emptyset$

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2-1} = ?$. X

2. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1-x^3}\,\mathrm{d}x$, s relativnom greškom manjom od 10%. X

3. Riješiti $y''-y=-x+1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x=0$, $y=0$, $y'=0$. Provjeri rješenje.

4. Riješiti: $y'+2xy=x-3$.

5. Grafički prikazati funkciju $f(x,y)=\ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. X

Ukupno: $\emptyset$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$	$\int\frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int\tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert+C$	$\int\frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int\frac{\mathrm{d}x}{x} = \ln\lvert x\rvert+C$	$\int\cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert+C$	$\int\frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int e^x\,\mathrm{d}x = e^x+C$	$\int\frac{\mathrm{d}x}{\cos^2 x} = \tan x+C$	$\int\frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a}+C$	$\int\frac{\mathrm{d}x}{\sin^2 x} = -\cot x+C$	$\int\frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right)+C$
$\int\sin x\,\mathrm{d}x = -\cos x+C$	$\int\sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int\cos x\,\mathrm{d}x = \sin x+C$	$\int\sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

1b

$$\int_0^2\frac{2x}{x^2-1}dx = \int_0^2 2x\,dx + \int_0^2\frac{1}{x^2-1}dx = 2\cdot\left[\frac{x^2}{2}\right]_0^2 + \frac{1}{2}\left[\ln\left|\frac{x-1}{x+1}\right|\right]_0^2 =$$

$$= 2\cdot\left[\frac{4}{2}-\frac{0}{2}\right] + \frac{1}{2}\left[-1{,}0986-0\right] = 4-0{,}5493 = \underline{\underline{3{,}4507}}$$ X

1 a $\int \frac{1+2x^2}{x^2(1+x^2)}\,dx = \int \frac{1}{x^2}$

$$\frac{1+2x^2}{x^2(1+x^2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{(1+x^2)} \quad / \, x^2(1+x^2)$$

$$1+2x^2 = Ax(1+x^2) + B(1+x^2) + (Cx+D)x^2$$

$$1+2x^2 = Ax + Ax^3 + B + Bx^2 + Cx^3 + Dx^2 \qquad \boxed{A=-1} \quad \boxed{B=1}$$

$$2x^2 = Ax + Ax^3 + Bx^2 + Cx^3 + Dx^2 \quad / : x$$

$$2x = A + Ax^2 + \underset{=1}{B}x + Cx^2 + Dx$$

$$2x - x = A + Ax^2 + Cx^2 + Dx$$

LOVRE BUBALO

5 $f(x,y) = \ln(x-y+2)$

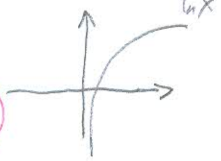
$x-y+2 > 0$

$x-y > -2$

$\underline{x > -2+y}$

$\underline{y > -2-x}$

$K = \langle 0, +\infty \rangle$



jer funkcija ln ne može imati 0 niti manje od nje, a za sve pozitivne je valjana.

$\frac{df}{dx} = \frac{1}{x-y+2} \cdot 1$

$\frac{df}{dy} = \frac{1}{x-y+2} \cdot (-1) = -\frac{1}{x-y+2}$

RAZINSKE KRIVULJE

$c=1$

$\ln(x-y+2) = 1$

$e = x-y+2$

$x-y = e-2$

$x-y = 0{,}7183$

$c=2$

$\ln(x-y+2) = 2$

$x-y+2 = e^2$

$x-y = e^2-2$

$x-y = 5{,}3891$

SKICA

(2) $\int_0^1 3\sqrt{1-x^3}\,dx = \lim_{x\to 1}\left[3\sqrt{1-x^3}\right]_0^1 = \lim_{x\to 1}\left[3\sqrt{1-1} - 3\sqrt{1-0}\right] = 0-3=-3$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}2$

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Marko Milaković

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

12-2-0146-2011

1. Riješiti integrale:

   (a) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}\,dx$,

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2-1} = ?$.

2. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1-x^3}\,dx$, s relativnom greškom manjom od 10%.

3. Riješiti $y''-y=-x+1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x=0$, $y=0$, $y'=0$. Provjeri rješenje.

4. Riješiti: $y'+2xy=x-3$.

5. Grafički prikazati funkciju $f(x,y)=\ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.

Ukupno: ∅

f	$\frac{df}{dx}$
x^α $(\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
α^x $(\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq -1$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

MARKO MILOLOVIĆ

① a) $\int \frac{1+2x^2}{x^2(1+x^2)}$

$x^2=0$ $\quad$ $1+x^2=0$

$x_1=0$ $\quad$ $x^2=1/\sqrt{}$

$x_2=1$

$\frac{1+2x^2}{x^2(1+x^2)} = \frac{A}{x^2} + \frac{B}{(1+x^2)} \quad / \cdot [x^2(1+x^2)]$

$1+2x^2 = A(1+x^2) + B x^2$

$\cancel{za\ x=0 \Rightarrow 1 = 0 + 1B}$ $\quad$ $\cancel{\boxed{1=B}}$

za $x=0 \Rightarrow 1 = 1A + 0$

$\boxed{1=A}$

$\cancel{za\ x=1 \Rightarrow 3 = 2A + 2B}$

za $x=1 \Rightarrow 3 = 2A + B$

$3 = 2\cdot 1 + B$

$3 = 2 + B \quad /\cdot 2$

$\boxed{\frac{3}{2} = B}$

$= \int \frac{1}{x^2} + \frac{3}{2}\cdot\frac{1}{(1+x^2)}\,dx$

$= \int x^{-2} + \frac{3}{2}\cdot\frac{1}{(1+x^2)}$

$= \frac{x^{-1}}{-1} + \frac{3}{2}\cdot \ln|1+x^2|$

$= \frac{1}{x} + \frac{3}{2}\cdot \ln|1+x^2| + C$

② $\int_0^2 \frac{2x}{x^2-1} = \left| \begin{array}{l} x^2-1 = t \\ 2x\,dx = dt \end{array} \right| \quad \left. \begin{array}{l} x=2 \Rightarrow t = 3 \\ x=0 \Rightarrow t=-1 \end{array} \right|$

$\int_{-1}^{3} \frac{dt}{t} = \ln|t| \Big|_{-1}^{3} = \ln|3| - \ln|-1| + C$

③ $y'' - y = -x + 1 \qquad x=0,\ y=0 \qquad y'=0$

$r^2 - r = 0$

$r_{1,2} = \frac{1 \pm \sqrt{1}}{2}$

$r_1 = \frac{1+1}{2} = 1 \qquad y_H = C_1 e^{1x} + C_2 e^{0x}$

$r_2 = \frac{1-1}{2} = 0$

$f(x) = -x + 1 \qquad \alpha = 0 \qquad \beta = 0 \qquad N = 1$

$\underline{\underline{k = 0}}$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓ A2

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Ivan-Maximilian Šimović **VRIJEME POČETKA:** 17:30

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0107-2011

1. Riješiti integrale:

   (a) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}\,\mathrm{d}x$, ✗

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2-1} = ?$. ✗

2. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1-x^3}\,\mathrm{d}x$, s relativnom greškom manjom od 10%.

3. Riješiti $y''-y=-x+1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x=0$, $y=0$, $y'=0$. Provjeri rješenje. ∅

4. Riješiti: $y'+2xy=x-3$.

5. Grafički prikazati funkciju $f(x,y)=\ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.

Ukupno: ∅

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha\neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha>0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
$\alpha^x\ (\alpha>0)$	$\alpha^x\ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int\frac{\mathrm{d}x}{a^2-x^2}=\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$
$\int x^\alpha\,\mathrm{d}x=\frac{x^{\alpha+1}}{\alpha+1},\ \alpha\neq-1$	$\int\tan x\,\mathrm{d}x=-\ln\lvert\cos x\rvert+C$	$\int\frac{\mathrm{d}x}{x^2-a^2}=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int\frac{\mathrm{d}x}{x}=\ln\lvert x\rvert+C$	$\int\cot x\,\mathrm{d}x=\ln\lvert\sin x\rvert+C$	$\int\frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}}=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int e^x\,\mathrm{d}x=e^x+C$	$\int\frac{\mathrm{d}x}{\cos^2 x}=\tan x+C$	$\int\frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C$	$\int\frac{\mathrm{d}x}{\sin^2 x}=-\cot x+C$	$\int\frac{\mathrm{d}x}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int\sin x\,\mathrm{d}x=-\cos x+C$	$\int\sqrt{x^2\pm a^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]+C$	
$\int\cos x\,\mathrm{d}x=\sin x+C$	$\int\sqrt{a^2-x^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$	

✗

1) a) $\displaystyle\int\frac{1+2x^2}{x^2(1+x^2)}dx = \int\frac{1}{x^2+x^4}+\int\frac{2\cancel{x^2}}{\cancel{x^2}+x^4}dx = \int x^{-2}+x^{-4}+2\int x^{-4}\,dx =$

$\displaystyle=\frac{x^{-1}}{-1}+\frac{x^{-3}}{-3}+\frac{2x^{-3}}{-3}+C = -\frac{x^{-1}}{1}+\frac{3x^{-3}}{-3}+C$

5) $\int_0^2 \frac{2x}{x^2-1}\,dx = \left| \begin{matrix} x^2-1=t \,/\,' \\ 2x\,dx=dt \end{matrix} \middle| \begin{matrix} x=2 \to t=3 \\ x=0 \to t=-1 \end{matrix} \right| = \int_{-1}^{3} \frac{dt}{t} = \ln|t| \Big|_{-1}^{3} =$

$= \ln|x^2-1| \Big|_{-1}^{3} =$

$\cancel{= \ln|3^2-1| - \ln|(-1)^2-1| =}$

$\lim_{a \to -1^+} \int_a^3 \frac{dt}{t} = \lim_{a \to -1^+} (\ln|t|) \Big|_a^3 = \lim_{a \to -1^+} (\ln|x^2-1|) \Big|_a^3 = \infty$

divergira

3) $y'' - y = -x + 1 \quad x=0 \quad y=0 \quad y'=0$

$r^2 - 1 = 0$

$r_1 = -1$

$r_2 = 1 \qquad \alpha=0 \quad \beta=0 \quad k=0 \quad N=1$

$y_H = C_1 e^{-1} + C_2 e$

$g(0) = -x+1$
$= 0+1$
$= 1$

$y_p = Ax + B$

$y_p' = A \to 0$

$y_p'' = 0$

$e^{-1}\left((Ax+B)\cdot\cos(\alpha) - (Cx+D)\cdot\sin(\beta)\right) =$

$= e^{-1} - e^{-1} = 0$

$0 - Ax + B = 0$

uz x: $-A = 1$

$\boxed{A=-1}$

bez x: $\boxed{B=0}$

$0 - Ax + B = -x + 1$

$0 + 1 + 0 = -0 + 1$

$1 = 1$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}2$

IME I PREZIME: JOSIP GAUTA **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0383-2014

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti integrale:

   (a) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}\mathrm{d}x,$

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2-1} = ?.$

2. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1-x^3}\,\mathrm{d}x$, s relativnom greškom manjom od 10%.

3. Riješiti $y''-y=-x+1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x=0$, $y=0$, $y'=0$. Provjeri rješenje.

4. Riješiti: $y'+2xy=x-3$.

5. Grafički prikazati funkciju $f(x,y)=\ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.

Ukupno:

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x\ln\alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln\alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

(2) $\int_0^1 3\sqrt{1-x^3}\,dx \Rightarrow$

$\frac{\partial f}{\partial x} = 6 \frac{1}{\sqrt{1-x^3}} \cdot (-3x^2) \Rightarrow -\frac{18x^2}{\cancel{\sqrt{1-x^2}}} \Rightarrow -\frac{18x^2}{1-x} \Rightarrow$

$S = \frac{d}{6}\left(f_0 + 4f_1 + f_2\right)\Big|_0^1$

$S = \frac{3}{6}(0 + 2 + 1$

$S = \frac{1}{2}(3)$

$S = 1.6666667$

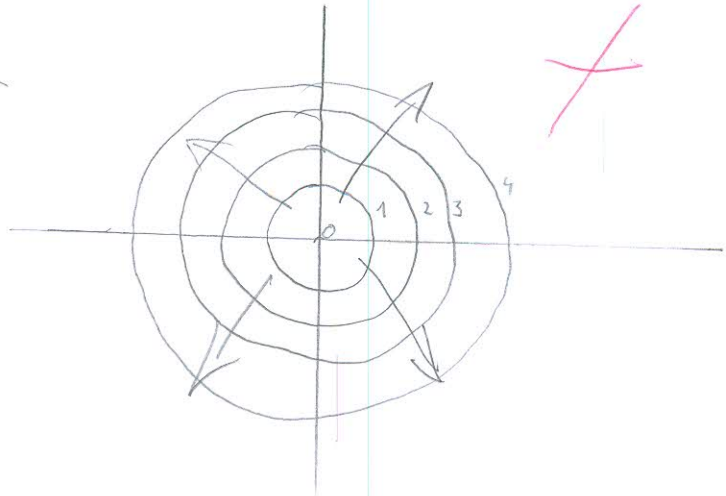
Josip Gaula.

(5) $f(x,y) = \ln(x-y+2)$

$$\frac{\partial f}{\partial x} = \frac{1}{(x-y+2)} \cdot 1$$

$$D f = \mathbb{R} \cdot \mathbb{R}$$

$$D f : \mathbb{R}^2$$



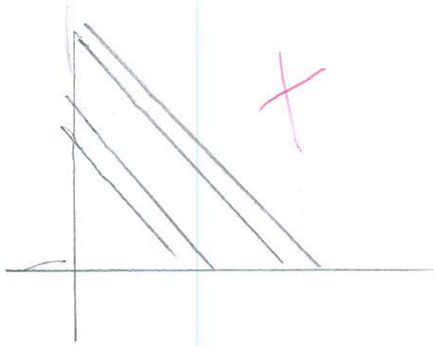
$$\frac{\partial f}{\partial y} = -\frac{1}{x-y+2} \cdot 1$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{(x-y+2)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = -\frac{1}{x-y+2} \Rightarrow -\frac{1}{y+2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{-1}{(x-y+2)^2}$$

$$\frac{\partial^2 f}{\partial y \partial x} = -\frac{1}{y+2}$$



$\ln(x-y+2) = 0$

$\ln(x-y+2) = 1$

Josip Gauta.

(1) $\int \frac{1+2x^2}{x^2(1+x^2)} dx \Rightarrow \int 1+2\frac{1}{1+x^2} \Rightarrow \int 1+\frac{2}{1+x^2}$

$x^2=-1 /\sqrt{}$ Df: R

$2x^2+1^2 : x^2(1+x^2) = 2$

$x=\pm\sqrt{-1}$

$-2x^2$

1

$\int 1dx + 2\int \frac{1}{1+x^2}dx \Rightarrow \int 1dx + 2\int 1 \arctan x^2 \Rightarrow 1[x] + 1[x] \arctan[x^2]$

$= 1+1+\arctan 1 \Rightarrow 2.785398163$ //

b) $\int_0^2 \frac{2x}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)} / (x-1)(x+1)$

$x^2-1=0$

$x^2=1/\sqrt{}$

Df $\setminus R\{-1,1\}$ $x=\pm 1$

$= A(x-1) + B(x+1) \Rightarrow$

$A(x-1)=0 / :A$ $B(x+1)=0 / :B$

$\Rightarrow Ax - A = 0 \Rightarrow 0$

$Bx + B = 0 \Rightarrow 0$

$x-1=0$

$x=1$

$x+1=0$

$x=-1$

$Ax - A = 0 \Rightarrow$

$\int \frac{1}{x+1} - \frac{1}{x-1} \Rightarrow \int x-1-(x+1)dx \Rightarrow \int \cancel{x}-1-\cancel{x}-1\,dx$

$\Rightarrow \int -2\,dx \Rightarrow -2$ //

Josip Ganta

(3). $y'' - y = -x + 1 \quad x = 0,\ y = 0,\ y' = 0$

$r^2 - 1 = -x + 1 \Rightarrow r^2 - x = 2 \Rightarrow r^2 - x - 2 = 0$

$r^2 - r = 0$

$b_{1,2} = \frac{1 \pm \sqrt{9}}{2} \Rightarrow \frac{1 \pm 3}{2} \Rightarrow b_1 = 2$

$b_2 = 1$

$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$r_{1,2} = \frac{1 \pm \sqrt{1}}{2 \cdot 1} \Rightarrow r_{1,2} = \frac{1 \pm 1}{2} \Rightarrow r_1 = 1,\ r_2 = 0$

$-x + 1 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1}}{2 \cdot 0} \Rightarrow x_{1,2} = 0$

$\emptyset$

(4). $y' + 2xy = x - 3$

$r + 2x = x - 3$

$r + x = -3$

$r = -3 - x \Rightarrow r = -3$ //

$x_{1,2} = \frac{+1 \pm \sqrt{1 - \dots}}{0} \Rightarrow 0$

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓

DOPUNJAVA NASTAVNIK

D2

Broj ↓ bodova

IME I PREZIME: LUKA GULAN

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

1. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.
2. Riješiti: $y' + 2xy = x - 3$.
3. Grafički prikazati funkciju $f(x, y) = \ln(x - y + 2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.
4. Riješiti integrale:

   (a) $\displaystyle\int \frac{1 + 2x^2}{x^2(1 + x^2)} \mathrm{d}x,$

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2 - 1} = ?.$

5. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1 - x^3}\, \mathrm{d}x$, s relativnom greškom manjom od 10%.

Ukupno: ∅

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a + x}{a - x} \right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha + 1}$, $\alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x - a}{x + a} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4.) b) $\displaystyle\int_0^2 \frac{2X}{X^2 - 1} = \begin{bmatrix} t = X^2 - 1 \\ dt = 2X \end{bmatrix} = \int_0^2 \frac{dt}{t} = \ln|t| = \ln|X^2 - 1| = \ln|2^2 - 1| - \ln|0^2 - 1| = \ln|4 - 1|$

$- \ln|0 - 1| = \ln 3 - \ln 1 = 1.098612289$

4.) a) $\int \frac{1+2x^2}{x^2(1+x^2)}\,dx = \left[\begin{array}{l} t = x^2(1+x^2) \\ dt = 1+2x^2 \end{array}\right] = \frac{dt}{t} = \frac{1+4x}{1+x^4} = \ln|t|$

3.) $f(x,y) = \ln(x-y+2)$

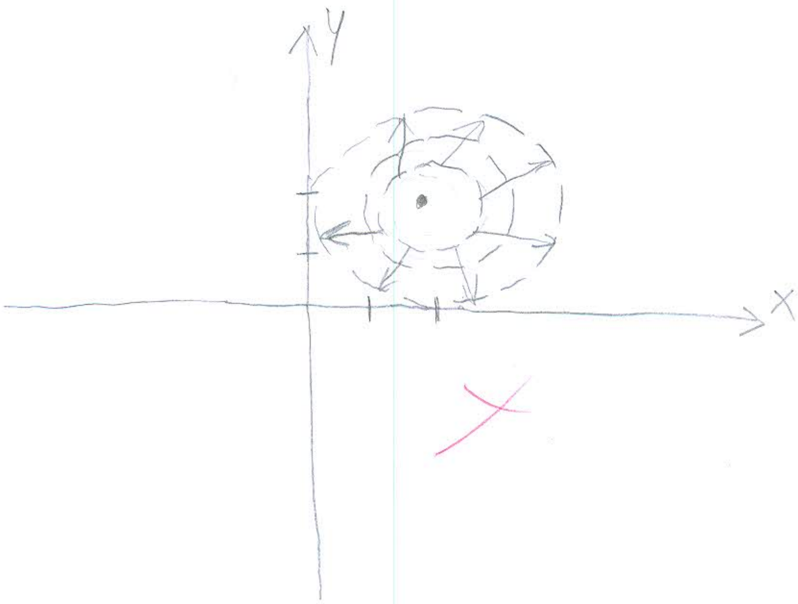
$\ln(x-y+2) = 0$

$x - y = 0$

$x = y$

$e^c = c$

$Df = \mathbb{R}$



MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

DOPUNJAVA NASTAVNIK

D2

Broj ↓ bodova

IME I PREZIME: Josip Janković

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0099-2011

1. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.
2. Riješiti: $y' + 2xy = x - 3$.
3. Grafički prikazati funkciju $f(x, y) = \ln(x - y + 2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.
4. Riješiti integrale:
   (a) $\int \frac{1 + 2x^2}{x^2(1 + x^2)} dx$,
   (b) $\int_0^2 \frac{2x}{x^2 - 1} = ?$.
5. Nekom metodom numeričke integracije procijeniti vrijednost $\int_0^1 3\sqrt{1 - x^3}\, dx$, s relativnom greškom manjom od 10%.

Ukupno: ∅

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

5. $\int_0^1 3\sqrt{1-x^3}\, dx \quad \left| \begin{array}{l} t = 1 - x^3 \\ dt = -3x^2\, dx \\ dx = \frac{dt}{-3x^2} \end{array} \right.$

$$\int_0^1 3\sqrt{t} \cdot \frac{dt}{-3x^3} = -1\int_0^1 t^{\frac{1}{2}} \frac{1}{-x^3} dt = -1\int_0^1 t^{\frac{1}{2}} \cdot (-x^{-3}) \cdot dt = -1 \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \cdot (-1) \cdot \left(\frac{-x^{-2}}{2}\right)\Bigg|_0^1 =$$

$$-1 \cdot \frac{(\sqrt{1-x^3})^{\frac{3}{2}}}{\frac{3}{2}} \cdot (-1) \cdot \left(\frac{-x^2}{2}\right)\Bigg|_0^1 = -\frac{2}{3} \cdot 0 + 0 \cdot \frac{1}{2} = 0$$

4.

a) $\int \frac{1+2x^2}{x^2(1+x^2)} dx$

Josip Janković

b) $\int_0^2 \frac{2x}{x^2-1} = \left| \; t = x^2-1 \right.$

$\int_0^2 \frac{2x}{t} = \int_0^2 2x \cdot t^{-1} = \frac{2x^2}{2} \cdot t \Big|_0^2 = \frac{2x^2}{2} \cdot (x^2-1) = 0 \cdot (-1) + 4 \cdot 3 = 12$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓	DOPUNJAVA NASTAVNIK D2 Broj ↓ bodova

IME I PREZIME: MATIJA ŠEGARIĆ **VRIJEME POČETKA:** 17:30

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
17-1-0283-2014 0269092378

1. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. Ø
2. Riješiti: $y' + 2xy = x - 3$.
3. Grafički prikazati funkciju $f(x, y) = \ln(x - y + 2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.
4. Riješiti integrale:

   (a) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}\,\mathrm{d}x$,

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2-1} = ?$. 1.099 X

5. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1-x^3}\,\mathrm{d}x$, s relativnom greškom manjom od 10%. P = 7.411 X:

Ukupno: Ø

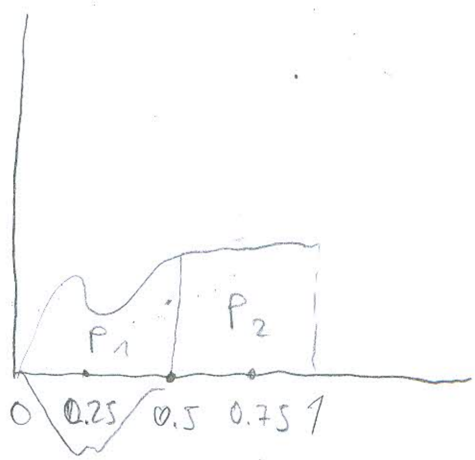
f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α ($\alpha \neq 0$)	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ ($\alpha > 0$)	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x ($\alpha > 0$)	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

(5) $\int_0^1 3\sqrt{1-x^3}\,dx$

MATIJA SEGARIC



$d = 0.5$

SIMPSONOVA FORMULA

$P = \frac{d}{6}\left(f_0 + 4f_1 + f_2\right)$

$P_1 = ?$

$d_1 = 0.5$

$f_0(0) = 3$

$f_1(0.25) = 2.976$

$f_2(0.5) = 2.806$

$P_1 = \frac{0.5}{2}\left(3 + 4 \cdot 2.976 + 2.806\right)$

$P_1 = 4.428$ ×

$P_2 = ?$

$d = 0.5$

$f_0(0.5) = 2.806$

$f_1(0.75) = 2.281$

$f_2(1) = 0$

$P_2 = \frac{0.5}{2}\left(2.806 + 4 \cdot 2.281 + 0\right)$

$P_2 = 2.983$ ×

$P_U = P_1 + P_2$

$P_U = 4.428 + 2.983$

$P_U = 7.411$ ×

4. a) $\int \frac{1+2x^2}{x^2(1+x^2)}\,dx =$

$= \int \frac{1+2x^2}{x^2+x^4}\,dx = \underbrace{\int \frac{1}{x^2+x^4}\,dx}_{I_1} + \underbrace{\int \frac{2x^2}{x^2+x^4}\,dx}_{I_2}$

$\underbrace{\int \frac{1}{x^2(1+x^2)}\,dx}_{I_1} + \underbrace{\int \frac{2x^2}{x^2(1+x^2)}}_{I_2} =$

$x_1 = -1, \quad x_2 = 3$

$I_1 = \int \frac{1}{x^2+(1+x^2)}\,dx = \left| \begin{array}{l} t = 1+x^2 \\ dt = 2x\,dx \quad /2 \\ \frac{dt}{2} = x\,dx \end{array} \right.$

MATIJA SEGARIC

4.b)

b) $\int_0^2 \frac{2x}{x^2-1}dx = \left| \begin{array}{l} t = x^2-1 \\ dt = 2x\,dx \end{array} \right.$ $t_1 = -1$, $t_2 = 3$.

$\int_{-1}^{3} \frac{1}{t}\,dt = \ln|t| \Big|_{-1}^{3} =$

F(B) − F(A)

$= \ln|3| - \ln|-1| = 1.099 - 0$

$= 1.099$ //

I$_1$

4.a) $\underline{1+2x^2}$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

DOPUNJAVA NASTAVNIK

D2

Broj ↓ bodova

IME I PREZIME: MISLAV ROGOZNICA

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0345-2013

1. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.
2. Riješiti: $y' + 2xy = x - 3$.
3. Grafički prikazati funkciju $f(x, y) = \ln(x - y + 2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.
4. Riješiti integrale:

   (a) $\displaystyle\int \frac{1 + 2x^2}{x^2(1 + x^2)} \mathrm{d}x,$ ✗

   (b) $\displaystyle\int_0^2 \frac{2x}{x^2 - 1} = ?.$ ✗

5. Nekom metodom numeričke integracije procijeniti vrijednost $\displaystyle\int_0^1 3\sqrt{1 - x^3}\, \mathrm{d}x$, s relativnom greškom manjom od 10%. ✗

Ukupno: ∅

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
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$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln \lvert \cos x \rvert + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \cot x \, \mathrm{d}x = \ln \lvert \sin x \rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

4. b) $\displaystyle\int_0^2 \frac{2x}{x^2-1} dx =$

$f(x) = \frac{2x}{x^2-1}$

$x^2 - 1 \neq 0$

$x^2 \neq 1 \quad /\sqrt{\ }$

$x \neq \sqrt{1}$

$D(f) = \mathbb{R}^2 \setminus \{1\}$

INTEGRAL JE NEPRAV JER [0,2] NIJE U DOMENI

$$\int \frac{2x}{x^2-1} dx = \begin{bmatrix} t = x^2 - 1 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{bmatrix} = \int \frac{2x}{t} \cdot \frac{dt}{2x} = \int \frac{dt}{t} =$$

$$= \ln|t| = \ln|x^2 - 1|$$

RJEŠENJE?

1

$$\int_0^2 \frac{2x}{x^2-1}\,dx = \lim_{a\to 1^-} \ln|x^2-1| \Big|_0^a + \lim_{b\to 1^+} \ln|x^2-1| \Big|_b^2 =$$

$$= \lim_{a\to 1^-} \left(\ln|a^2-1| - \ln|0^2-1| \right) + \lim_{b\to 1^+} \left(\ln|2^2-1| - \ln|b^2-1| \right) =$$

$$= \left(\ln|0| - \ln|-1| \right) + \left(\ln|4-1| - \ln|0| \right) =$$

$$= \infty - 0 + 1{,}0986 - \infty = \boxed{1{,}0986}$$

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4. a) $\int \frac{1+2x^2}{x^2(1+x^2)}\,dx = \int \frac{1+2x^2}{x^4+x^2}\,dx = \left[\begin{array}{l} t = x^4+x^2 \\ dt = 4x^3+2x\,dx \\ dx = \frac{dt}{4x^3+2x} \end{array} \right] =$

$$= \int \frac{1+2x^2}{t} \cdot \frac{dt}{4x^3+2x} = \int \frac{\cancel{1+2x^2}}{t} \cdot \frac{dt}{2x\cancel{(2x^2+1)}} =$$

$$= \int \frac{dt}{t \cdot 2x} = \frac{1}{2} \int \frac{dt}{t} \cdot$$

---

4. a) $\int \frac{1+2x^2}{x^2(1+x^2)}\,dx = \int \frac{A}{x^2}\,dx + \int \frac{B}{(1+x)}\,dx + \int \frac{C}{(1-x)}\,dx =$

$1+2x^2 = Ax^2 + Bx + C$

$A = 2$

$B = 0$

$C = 1$

$$\int \frac{2}{x^2}\,dx + \int \frac{0}{1+x}\,dx + \int \frac{1}{(1-x)}\,dx = \left[\begin{array}{l} t = 1-x \\ dt = -1\,dx \\ dx = \frac{dt}{-1} \end{array} \right] =$$

$$= 2\int x^{-2}\,dx + 0 + \int \frac{1}{t} \cdot (-dt) =$$

$$= 2 \cdot \frac{x^{-1}}{-1} - \ln|t| = \frac{-2}{x} - \ln|1-x|$$

5.) $\int_0^1 3\sqrt{1-x^3}\,dx = \left[\begin{array}{l} t = 1-x^3 \\ dt = -3x^2\,dx \\ dx = -\frac{dt}{3x^2} \end{array}\right]$

$f(x) = 3\sqrt{1+x^3}$

$1-x^3 > 0$

$-x^3 > -1 / \cdot(-1)$

$x^3 > 1 / \sqrt[3]{}$

$x > \sqrt[3]{1}$

$x > 1$

$D(f) = \mathbb{R}$

INTEGRAL NIJE NEPRAVI JER JE $[0,1] \in \mathbb{R}$

$\int 3\sqrt{1-x^3}\,dx = \left[\begin{array}{l} t = 1-x^3 \\ dt = -3x^2\,dx \\ dx = -\frac{dt}{3x^2} \end{array}\right] = \int 3\sqrt{t} \cdot x =$

$= 3\int t^{\frac{1}{2}}$

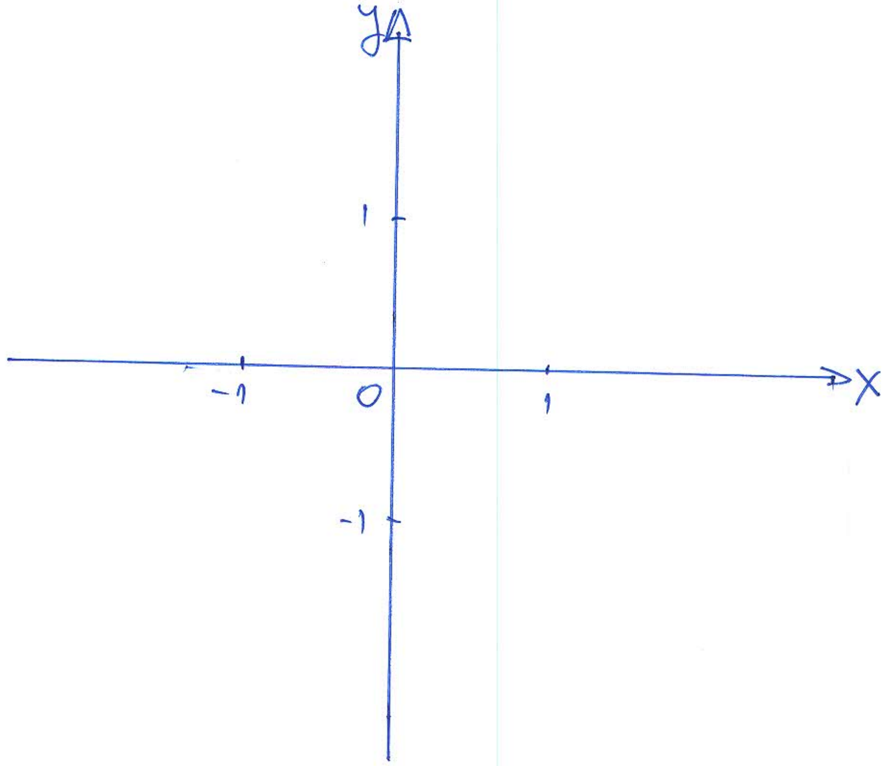
×

MISLAV ROGOZNICA
17-2-0345-2013

3. $f(x;y) = \ln(x-y+2)$

DOMENA:

$\left.\begin{matrix} x>0 \\ y>0 \end{matrix}\right\} \mathcal{D}(f) = \{(x,y)\mathbb{R} : x>0,\ y>0\}$



$f(x,y) = \ln(x-y+2)$

$f(0,0) = \ln(0-0+2)$

$f(0,0) = \ln 2$

$f(0,0) = 0,693$