MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ A2

IME I PREZIME: GORAN PAVLIČEVIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
17-2-0452-2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti integrale:

   (a) $\int \frac{1+2x^2}{x^2(1+x^2)}\,\mathrm{d}x$,

   (b) $\int_0^2 \frac{2x}{x^2-1} = ?$.

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2. Nekom metodom numeričke integracije procijeniti vrijednost $\int_0^1 3\sqrt{1-x^3}\,\mathrm{d}x$, s relativnom greškom manjom od 10%.

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3. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.

X

4. Riješiti: $y' + 2xy = x - 3$.

X

5. Grafički prikazati funkciju $f(x, y) = \ln(x - y + 2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.

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Ukupno: 57

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

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1)
a) $\int \frac{1+2x^2}{x^2(1+x^2)}\,dx = \int \frac{1+x^2+x^2}{x^2(1+x^2)}\,dx = \int \frac{1+x^2}{x^2(1+x^2)}\,dx + \int \frac{x^2}{x^2(1+x^2)}\,dx$

$= \int \frac{1}{x^2}\,dx + \int \frac{1}{1+x^2}\,dx = \int x^{-2}\,dx + \int \frac{1}{1+x^2}\,dx =$

$= \frac{x^{-1}}{-1} + \frac{1}{1}\arctan\frac{x}{1} + C = -\frac{1}{x} + \operatorname{arctg} x + C$ ✓

b) $\int_0^2 \frac{2x}{x^2-1}\,dx$

Domena: $x \neq 1$

$\lim_{a\to 1^-} \int_0^a \frac{2x}{x^2-1}\,dx = \lim_{a\to 1^-} \left(\ln|a^2-1| - \ln|0-1|\right) = -\infty - \ln|1| = -\infty$ //

$\int \frac{2x}{x^2-1}\,dx \left[\begin{matrix} t = x^2-1 \,/' \\ dt = 2x\,dx \end{matrix}\right] = \int \frac{dt}{t} = \ln|t| + C = \ln|x^2-1| + C$

$\lim_{b\to 1^+} \int_b^2 \frac{2x}{x^2-1}\,dx = \lim_{b\to 1^+} \left(\ln(4-1) - \ln|b^2-1|\right) =$

$= \ln 3 - (-\infty) = +\infty$

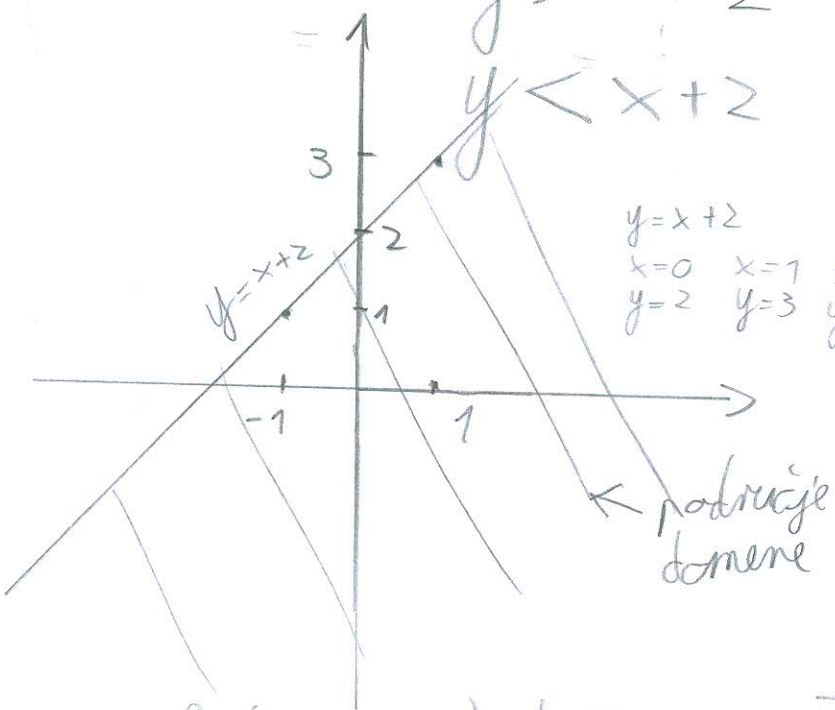
$\int_0^2 \frac{2x}{x^2-1}\,dx = -\infty + \infty =$ **N/P** ✓

5) $f(x,y) = \ln(x-y+2)$

Domena: $x-y+2>0$

$-y > -x-2$

$y < x+2$



$y = x+2$

$x=0 \quad y=2$

$x=1 \quad y=3$

$x=-1 \quad y=1$

← področje domene

$c = \ln(x-y+2) \mid e$

$e^c = x-y+2$

$y = x - e^c + 2$

$f(x, x-e^c+2) =$

$= \ln(x - x + e^c - 2 + 2) = c$

PRIJEDLOSTI

$c_1 = 1 \Rightarrow y = x+2-e \Rightarrow y = x+2-e$

$c_2 = -1 \Rightarrow y = x+2-\frac{1}{e} \Rightarrow y = x+2-\frac{1}{e}$

$c_3 = \ln 2 \Rightarrow y = x+2-2 \Rightarrow y = x$

$c_4 = \ln 3 \Rightarrow y = x+2-3 \Rightarrow y = x-1$

$x=0 \quad x=1$

$y = -0{,}71 \quad y = 0{,}29$

$x=0 \quad x=1$

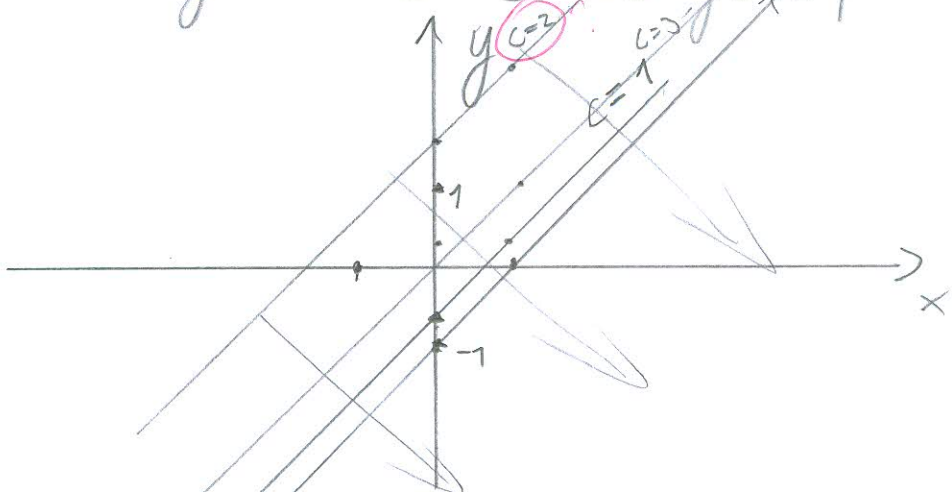
$y = 1{,}63 \quad y = 2{,}63$

$x=0 \quad x=1$

$y=0 \quad y=1$

$x=0 \quad x=1$

$y=-1 \quad y=0$



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(2) $\int_0^1 3\sqrt{1-x^3}\,dx$

k	0	1	2
x_k	0	1/2	1
f_k	3	2,8062	0
$S=\frac{d}{6}(f_0+4f_1+f_2)$	$S=\frac{1}{6}(3+4\cdot 2{,}8062+0)=2{,}3708$		

✓

(3) $y'' - y = (-x+1)\cdot e^{0x}$ GORAN PAVLIČEVIĆ

1) $y'' - y = 0$

$r^2 - 1 = 0$

$(r-1)(r+1) = 0$

$r_1 = 1$

$r_2 = -1$

$y_H = C_1 \cdot e^x + C_2 \cdot e^{-x}$

2) $f(x) = (-x+1)\cdot e^{0x}$

$y_p = Ax + B$

$y' = A$ $\boxed{A = 0}$

$y'' = 0$

$\boxed{y = C_1 e^x + C_2 \cdot e^{-x} + A}$ ✗

$y' = C_1 \cdot e^x - C_2 \cdot e^{-x}$

$0 = C_1 - C_2$ $\boxed{C_1 = C_2 = 1}$

$C_1 = -1$ $C_2 = -1$