MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Lovre Štokić **VRIJEME POČETKA:** 17:30

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0281-2014 0269092224

~~1.~~ Riješiti diferencijalnu jednadžbu: $y'' - y' = e^x + 1$. ✗

~~2.~~ $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ 20

~~3.~~ $\int\limits_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = ?$ 20

4. Izračunati:

(a) tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$.

~~(b)~~ površinu lika omeđenog pravcem $y = x + 1$ i parabolom $y = x^2 + 3x - 2$. ✗

5. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin(x^2)\, \mathrm{d}x$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) ~~0~~

Ukupno: 40

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, \mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, \mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

② $\int_1^3 \frac{dx}{x^2-2x+4} =$

$$x_{1,2} = \frac{2 \pm \sqrt{4-16}}{2} \cancel{\in \mathbb{R}}$$

Nema singulariteta u intervalu $[1, 3]$ stoga možemo riješiti po Newton-Leibnitzovoj formuli.

$$= \left\{ \begin{aligned} &= (x)^2 - \underbrace{2 \cdot x \cdot ?}_{2x} + 4 \\ &= (x)^2 - 2 \cdot x \cdot 1 + \underbrace{1^2 + ?}_{4} \\ &= (x)^2 - 2 \cdot x \cdot 1 + 1^2 + 3 \\ &= (x-1)^2 + 3 \end{aligned} \right\} \Rightarrow \int \frac{dx}{(x-1)^2 + (\sqrt{3})^2}$$

$$= \frac{1}{\sqrt{3}} \arctan\left(\frac{x-1}{\sqrt{3}}\right) + C$$

$$\Rightarrow \left[\frac{1}{\sqrt{3}} \arctan\left(\frac{x-1}{\sqrt{3}}\right)\right]_1^3 = \frac{1}{\sqrt{3}} \arctan\left(\frac{3-1}{\sqrt{3}}\right) - \cancel{\frac{1}{\sqrt{3}} \arctan\left(\frac{1-1}{\sqrt{3}}\right)} \rightarrow 0$$

$$= \frac{\sqrt{3}}{3} \arctan\left(\frac{2\sqrt{3}}{3}\right) \approx 0.495$$

Lovre Štokić 17.06.2015.

(1.) $y'' - y' = e^x + 1$

$r^2 - r = 0$

$r(r-1) = 0 \Rightarrow r_1 = 0,\ r_2 = 1$

$r_1 \neq r_2 \in \mathbb{R}$

$y_H(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

a) $e^x = e^{\alpha x}\left(P_m \cos(\beta x) + \underbrace{Q_n \sin(\beta x)}_{\text{nebitno}}\right)$

$\alpha = 1,\ \beta = 0,\ m = 0,\ k = 1$

$y_1(x) = x^k \cdot e^{\alpha x}\left(S_M \cos(\beta x)\right)$

$y_1(x) = x \cdot e^x \ A$

$\cancel{e^x} = \cancel{e^x} \cdot x \cdot A$

$x \cdot A = 0 \Rightarrow A = 0$

$y_1(x) = 0$

b) $1 = e^{\alpha x}\left(P_m \cos(\beta x) + Q_n \sin(\beta x)\right)$

$\alpha = 0,\ \beta = 0,\ m = 0,\ k = 1$

$1 = x \cdot e^{0x}(A)$

$1 = x \cdot A \Rightarrow A = 0$

$y_2(x) = 0$

$y(x) = y_H(x) + y_1(x) + y_2(x)$

$y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

Lovre Stokić 17.06.2015.

(3.) $\displaystyle\int_0^{\pi}\frac{dx}{\sin x+2}$ = Nema singulariteta tako da nije nepravi integral.

$$\Rightarrow \int\frac{dx}{\sin x+2} = \left\{\begin{array}{c} t=\tan\left(\frac{x}{2}\right) \\ \sin x=\frac{2t}{1+t^2} \\ dx=\frac{2dt}{1+t^2}\end{array}\right\}$$

$$=\int\frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}+2} = \int\frac{\frac{2dt}{1+t^2}}{\frac{2t+2+2t^2}{1+t^2}} = \int\frac{\cancel{2}dt}{\cancel{2}(t^2+t+1)}$$

$$=\int\frac{dt}{t^2+t+1} \left\{\begin{array}{l} =(t)^2+\underbrace{2\cdot t\cdot ?}_{t}+1 \\ =(t)^2+2\cdot t\cdot\frac{1}{2}+\underbrace{\left(\frac{1}{2}\right)^2+?}_{1} \\ =(t)^2+2\cdot t\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2+\frac{3}{4} \\ =\left(t+\frac{1}{2}\right)^2+\frac{3}{4}\end{array}\right\}$$

$$=\int\frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\sqrt{\frac{3}{4}}\right)^2} = \frac{1}{\frac{\sqrt{3}}{2}}\cdot\arctan\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$$

$$=\left[\frac{2\sqrt{3}}{3}\arctan\left(\frac{2\sqrt{3}\left(t+\frac{1}{2}\right)}{3}\right)\right.$$

$$\Rightarrow\left[\frac{2\sqrt{3}}{3}\arctan\left(\frac{2\sqrt{3}\left(\tan\left(\frac{x}{2}\right)+\frac{1}{2}\right)}{3}\right)\right]_0^{\pi}$$

Pojavljuje se problem prilikom uvrštavanja $\pi \Rightarrow \tan\left(\frac{\pi}{2}\right) = N/P$ stoga je ovo nepravi integral.

$$\lim_{\alpha \to \pi^-} \left[\frac{2\sqrt{3}}{3} \arctan\left(\frac{2\sqrt{3}\left(\tan\left(\frac{x}{2}\right) + \frac{1}{2}\right)}{3} \right) \right]_0^\alpha$$

$$= \frac{2\sqrt{3}}{3} \arctan\left(\frac{2\sqrt{3}\left(\tan\left(\frac{\pi}{2}\right) + \frac{1}{2}\right)}{3} \right)$$

$$- \frac{2\sqrt{3}}{3} \arctan\left(\frac{2\sqrt{3}\left(\tan\left(\frac{0}{2}\right) + \frac{1}{2}\right)}{3} \right)$$

$$= \frac{2\sqrt{3}}{3} \arctan(+\infty) - \frac{2\sqrt{3}}{3} \arctan\left(\frac{\sqrt{3}}{3} \right)$$

$$= \frac{\cancel{2}\sqrt{3}}{3} \cdot \frac{\pi}{\cancel{2}} - \frac{\cancel{2}\sqrt{3}}{3} \cdot \frac{\pi}{\cancel{6}_3} = \frac{\pi\sqrt{3}}{3} - \frac{\pi\sqrt{3}}{9} \approx 1.21 \;//$$

Lovre Štokić 17.06.2015.

4. b) $f(x) = x + 1$, $g(x) = x^2 + 3x - 2$

$f(x)$	x
1	0
0	-1
-1	-2
2	1

$g(x)$	x
-2	0
2	1
-4	-1
8	2
-4	-2

Točke sjecišta:

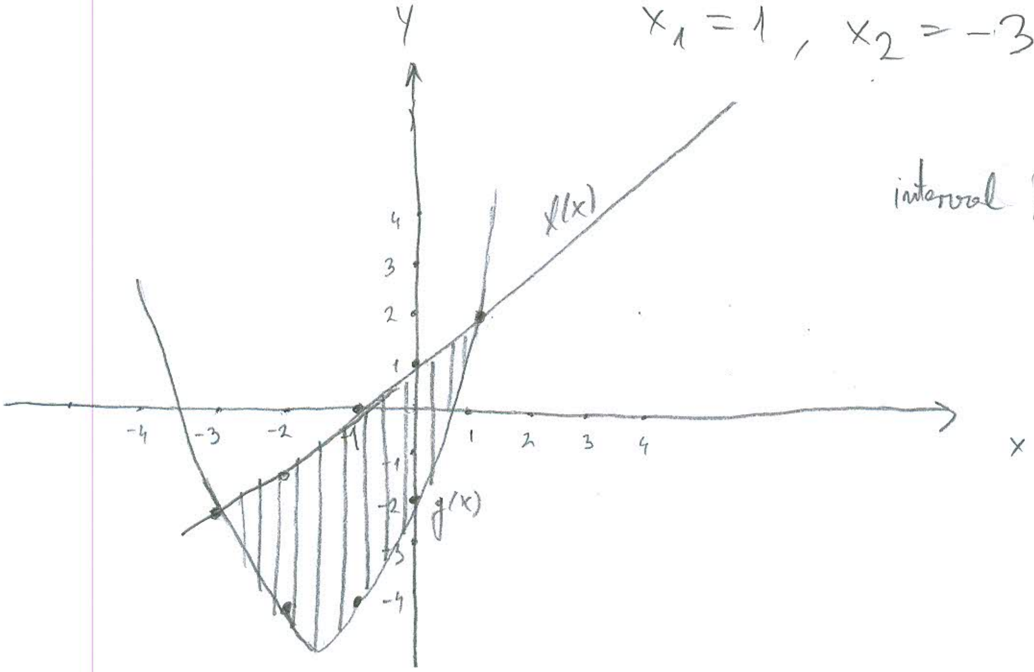
$$x + 1 = x^2 + 3x - 2$$

$$x^2 + 2x - 3 = 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4 + 12}}{2}$$

$$x_1 = 1 , \quad x_2 = -3$$

interval $[-3, 1]$



$$P = \int_{-3}^{1} (f(x) - g(x))\,dx = \int_{-3}^{1} (x + 1 - x^2 - 3x + 2)\,dx = \int_{-3}^{1} (-x^2 - 2x + 3)\,dx$$

$$\Rightarrow -\int x^2 dx - 2\int x\,dx + 3\int dx = -\frac{x^3}{3} - \cancel{2}\frac{x^2}{\cancel{2}} + 3x + C$$

$$\Rightarrow \left[-\frac{x^3}{3} - x^2 + 3x \right]_{-3}^{1} = -\frac{(1)^3}{3} - (1)^2 + 3 \cdot 1 - \left(-\frac{(-3)^3}{3} - (-3)^2 + 3 \cdot (-3) \right)$$

$$= -\frac{1}{3} - 1 + 3 - 9 \overset{+}{\cancel{-9}} + \cancel{9} = \frac{22}{3} \approx 7{,}33$$

Površina uvijek pozitivna.

5. $\int_0^2 \sin(x^2)\,dx$

k	0	1	2
x_k	0	1	2
f_k	0	0.8415	-0.7568

×

$$S = \frac{d}{6}(f_1 + 4f_2 + f_3) = \frac{2}{6}(0 + 4 \cdot 0.8415 + 0.7568) = 1.3743$$

∅

TOČNO RJEŠENJE : 0.8048

APS. GREŠKA : 0.5695

REL. GREŠKA : ≈ 70%

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{E}5$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: DORA BUŽONJA VRIJEME POČETKA: 17:30	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0307-2013.	

1. Riješiti diferencijalnu jednadžbu: $y'' - y' = e^x + 1$.
2. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ 20
3. $\displaystyle\int_0^\pi \frac{dx}{\sin x + 2} = ?$ X
4. Izračunati:
   (a) tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$. X
   (b) površinu lika omeđenog pravcem $y = x + 1$ i parabolom $y = x^2 + 3x - 2$. 10
5. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin(x^2)\,dx$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) Ø

Ukupno: 30

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

2.

$$\int_1^3 \frac{dx}{x^2-2x+4} =$$

$$x^2 - 2x + 4 \neq 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4-16}}{2} = \frac{2 \pm \sqrt{-12}}{2}$$

$D_f = \mathbb{R}$, $[1,3] \in D_f$. NIJE NEPRAVI

$$= \frac{\sqrt{3}}{3}\arctan\frac{x-1}{\sqrt{3}}\Big|_1^3 =$$

$$= \frac{\sqrt{3}}{3}\arctan\frac{3-1}{\sqrt{3}} - \frac{\sqrt{3}}{3}\arctan\frac{1-1}{\sqrt{3}} = \frac{\sqrt{3}}{3}\arctan\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{3}\arctan 0 = \frac{\sqrt{3}}{3}\cdot 0.86 - \frac{\sqrt{3}}{3}\cdot 0 =$$

$$= 0.5$$

$$\int \frac{dx}{x^2-2x+4} = \int \frac{dx}{(x^2-2x+1)-1+4} = \int \frac{dx}{(x^2-2x+1)+3} = \int \frac{dx}{(x-1)^2+(\sqrt{3})^2} = \frac{1}{\sqrt{3}}\arctan\frac{x-1}{\sqrt{3}} + C =$$

$$= \frac{\sqrt{3}}{3}\arctan\frac{x-1}{\sqrt{3}}$$

(3.) $\int_0^{\pi} \frac{dx}{\sin x+2} = 2\arctan(2\tan\frac{x}{2}+1)\Big|_0^{\pi}$ $= \sin x + 2 \neq 0$, $[0,\pi] \in D_f$, INTEGRAL NIJE NEPRAVI

$= 2\arctan(2\tan\frac{\pi}{2}+1) - 2\arctan(2\tan 1) =$ $\sin x = k\pi,\ k \in \mathbb{Z}$

$= -1.82 - 2.52 = -4.34$

$\int \frac{dx}{\sin x+2} = \left[\begin{matrix} \tan\frac{x}{2} = t & dx = \frac{2dt}{1+t^2} \\ \sin x = \frac{2t}{1+t^2} & \end{matrix} \right] = \int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}+2} c = \int \frac{\frac{2dt}{\cancel{1+t^2}}}{\frac{2t+2(1+t^2)}{\cancel{1+t^2}}} =$

$= \int \frac{2dt}{2t+2+2t^2} = 2\int \frac{dt}{2t^2+2t+2} = 2\int \frac{dt}{(2t^2+2t+1)-1+2} = 2\int \frac{dt}{(2t+1)^2+1} = 2\int \frac{dt}{(2t+1)^2+1^2} =$

$= 2 \cdot \frac{1}{1}\arctan\frac{2t+1}{1} = 2\arctan(2t+1) = 2\arctan(2\tan\frac{x}{2}+1)$

(4.) a) $z = \sin(x^2, y)$ $T(2,1,\sin(4)) = (2,1,-0.76)$ $\sin x^2 y$

$2 \to \sin x \cos x$

$z - z_0 = \partial_x f(T)(x-x_0) + \partial_y f(T)(y-y_0)$

$\partial_x f = 2\sin xy \cdot \cos x$ ×

$\partial_x f(T) = 2\sin 2 \cdot \cos 2 = -0.76$ $\cos^2 x$

$-2\cos x \sin x$

$\partial_y f = \cos x^2$ ×

$\partial_y f(T) = \cos 2^2 = \cos 4 = -0.65$

$z + 0.76 = -0.76(x-2) - 0.65(y-1)$

$z + 0.76 = -0.76x + 1.52 - 0.65y + 0.65$

$z = -0.76x - 0.65y + 1.52 + 0.65 - 0.76$

$z = -0.76x - 0.65y + 1.41$ ... JEDN. TANGENCIJALNE KRIVULJE

SJECIŠTA: DORA DUŽONJA

5) $y = x^2 + 3x - 2$

$y = x + 1$

$x^2 + 3x - 2 = x + 1$

$x^2 + 3x - x - 2 - 1 = 0$

$x^2 + 2x - 3 = 0$

$x_{1,2} = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2}$

$x_1 = \frac{-2+4}{2} = \frac{2}{2} = 1 \qquad x_2 = \frac{-2-4}{2} = \frac{\cancel{-6}^{-3}}{\cancel{2}_1} = -3$

$y_1 = 1 + 1 = 2$

$y_2 = -3 + 1 = -2$

$S_1(1, 2)$

$S_2(-3, -2)$

$y = x^2 + 3x - 2, \ a > 0, \ \cup$

$x^2 + 3x - 2 = 0$

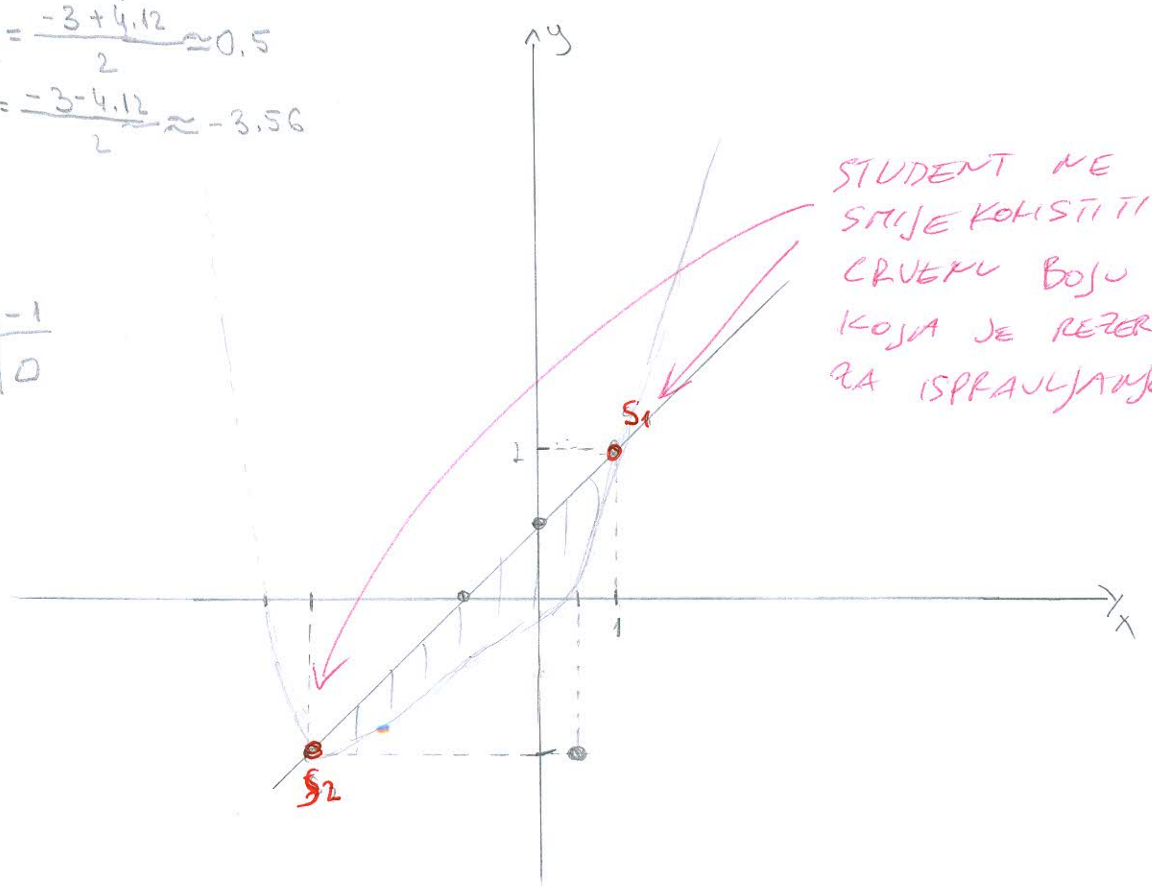
$x_{1,2} = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2} = \frac{-3 \pm 4.12}{2}$

$x_1 = \frac{-3+4.12}{2} \approx 0.5$

$x_2 = \frac{-3-4.12}{2} \approx -3.56$

$y = x + 1$

x	0	1	-1
y	1	2	0



$P = \int_{-3}^{1} \left[x + 1 - (x^2 + 3x - 2)\right] dx = \int_{-3}^{1} (x + 1 - x^2 - 3x + 2) dx = \int_{-3}^{1} (-x^2 - 2x + 3) dx =$

$= -\frac{x^3}{3} - 2 \cdot \frac{x^2}{2} + 3x \Big|_{-3}^{1} = -\frac{1}{3} - \cancel{2} \cdot \frac{1}{\cancel{2}} + 3 - \left(-\frac{(-3)^3}{3} - \cancel{2} \cdot \frac{(-3)^2}{\cancel{2}} + 3 \cdot (-3)\right) =$

$= -\frac{1}{3} - 1 + 3 - \left(\frac{\cancel{27}^{9}}{\cancel{3}_1} - 9 - 9\right) = -\frac{1}{3} + 2 - (9 - 18) = -\frac{1}{3} + 2 + 9 = -\frac{1}{3} + 11 = 10.66$ //

5.

$$\int_0^2 \sin(x^2)dx \qquad f(x) = \sin(x^2)$$

$a = 0 = x_0$

$b = 2 = x_2$

$d = b - a = 2 - 0 = 2$

$x_1 = \frac{a+b}{2} = \frac{\cancel{2}^1}{\cancel{2}_1} = 1$

$f_0 = f(x_0) = f(0) = \sin 0 = 0$

$f_1 = f(x_1) = f(1) = \sin 1 = 0.84$

$f_2 = f(x_2) = f(2) = \sin 4 = -0.76$

$$S = \frac{d}{6}(f_0 + 4f_1 + f_2) = \frac{\cancel{2}^1}{\cancel{6}_3}(0 + 4 \cdot 0.84 - 0.76) =$$

$$= \frac{1}{3} \cdot (3.36 - 0.76) = \frac{1}{3} \cdot 2.6 = 0.8666666667$$

Ø

TOČNO RJEŠENJE : 0.8048

APS. GREŠKA : 0.0619

REL. GREŠKA : ≈ 7,7 %

PREVELIKA GREŠKA

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ B3

Ime i prezime: Ante Pedišić **Vrijeme početka:**

Matični broj studenta (iznad slike u indeksu): 17-1-0114-2012

Popunjava nastavnik
Broj ↓ bodova

1. Izračunati: ×
   (a) tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$. 10
   (b) površinu lika omeđenog pravcem $y = x + 1$ i parabolom $y = x^2 + 3x - 2$.

2. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin(x^2)\,\mathrm{d}x$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$) ×

3. Riješiti diferencijalnu jednadžbu: $y'' - y' = \mathrm{e}^x + 1$. ~~0~~

4. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ 20

5. $\int\limits_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = ?$ ×

Ukupno: 30

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4. $\int_1^3 \frac{dx}{x^2-2x+4} = \int_1^3 \frac{1}{(x^2-2x+1)+3}\,dx$

$\int_1^3 \frac{1}{(x-1)^2+(\sqrt{3})^2}\,dx$ tablični integral

$\left[\frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x-1}{\sqrt{3}}\right]\Big|_1^3 = \left(\frac{1}{\sqrt{3}} \operatorname{arctg} \frac{3-1}{\sqrt{3}}\right) - \left(\frac{1}{\sqrt{3}} \operatorname{arctg} \frac{1-1}{\sqrt{3}}\right)$

$= 0{,}4548 - 0 = \underline{0{,}4548}$ ✓

5. $\int_0^\pi \frac{dx}{\sin(x)+2} = \left\{ \begin{matrix} \sin x + 2 = u \,/' & 1\,dx = dv \\ \cos x\,dx = du & x = v \end{matrix} \right\}$ ✗

$\boxed{u \cdot v - \int v\,du}$

$\int_0^\pi \frac{dx}{\sin(x)+2} = \left[(\sin(x)+2)x - \int x \cos x\,dx\right]\Big|_0^\pi$ ✗

$\int_0^\pi x \cos x\,dx = \left\{ \begin{matrix} x = u & \cos x\,dx = dv \\ dx = du & v = \sin x \end{matrix} \right\}$

$u \cdot v - \int v\,du = x \cdot \sin x - \int \sin x\,dx$

$= \left[x \cdot \sin x + \cos x\right]\Big|_0^\pi$

$= (\pi \cdot \sin \pi + \cos \pi) - (0 \cdot \sin 0 + \cos 0)$

$= -1 - 1 = \underline{\underline{-2}}$

$\int_0^\pi \frac{dx}{\sin x + 2} = \left[(\sin(x)+2) \cdot x + 2\right]\Big|_0^\pi$

$= \left[(\sin(\pi)+2) \cdot \pi + 2\right] - \left[(\sin(0)+2) \cdot 0 + 2\right] =$

$= 8{,}2832 - 2$

$= \underline{6{,}2832}$

Ante Podlišić 17-1-0114-2012 17.VI.2015.

(1.) a) $z = \sin(x^2 y)$ $T(\overset{x_0}{2}, \overset{y_0}{1}, \overset{z_0}{\sin 4})$

$\partial_x f = 2xy \cos(x^2 y)$

$f_x = \frac{\partial f}{\partial x} = \sin(2x \cdot y)$; $f_{xT} = -0,7568$

$f_y = \frac{\partial f}{\partial y} = \sin(x^2 \cdot 1)$; $f_{yT} = -0,7568$

$z - z_0 = f_{xT} \cdot (x - x_0) + f_{yT} \cdot (y - y_0)$

$z - z_0 = -0,7568 \cdot (x-2) + 0,7568 \cdot (y-1)$

$z - z_0 = -0,7568x + 1,5136 - 0,7568y + 0,7568$

$z - \sin 4 + 0,7568x - 1,5136 + 0,7568y - 0,7568 = 0$

$z - \sin(4) - 2,2704 + 0,7568x + 0,7568y = 0$ ... R_t

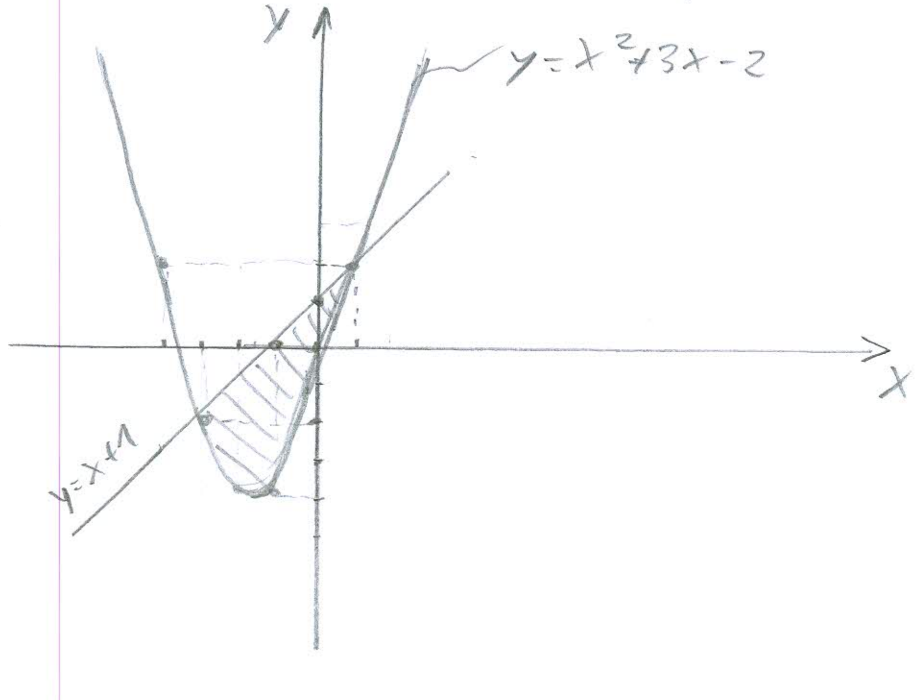
$\frac{x - x_0}{f_{xT}} = \frac{y - y_0}{f_{yT}} = \frac{z - z_0}{1}$

$\frac{x-2}{-0,7568} = \frac{y-1}{-0,7568} = \frac{z - \sin(4)}{1}$ ... R_N

b) $y = x+1$ $y = x^2 + 3x - 2$ $a > 0 \Rightarrow \cup$

x	0	1	2	3	-1
y	1	2	3	4	0

y	-1	-2	0	1	2	-3	-4
y	-4	-4	-2	2	8	-2	2



Granice integrala:

$$x^2+3x-2=x+1$$

$$x^2+3x-2-x-1=0$$

$$x^2+2x-3=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x_{1,2}=\frac{-2\pm\sqrt{4-4\cdot1\cdot(-3)}}{2\cdot1}$$

$$x_{1,2}=\frac{-2\pm\sqrt{4+12}}{2}$$

$$\boxed{x_1=1} \qquad \boxed{x_2=-3}$$

$$P=\int_{-3}^{1}\left[(x+1)-(x^2+3x-2)\right]dx$$

$$P=\int_{-3}^{1}(x+1-x^2-3x+2)\,dx$$

$$P=\left[\frac{x^2}{2}+x-\frac{x^3}{3}-3\cdot\frac{x^2}{2}+2x\right]\Bigg|_{-3}^{1}$$

$$P=\left(\frac{1^2}{2}+1-\frac{1^3}{3}-3\cdot\frac{1^2}{2}+2\cdot1\right)-\left(\frac{(-3)^2}{2}+(-3)-\frac{(-3)^3}{3}-3\cdot\frac{(-3)^2}{2}+2\cdot(-3)\right)$$

$$P=\frac{5}{3}-(-9)=\frac{5}{3}+9 \Rightarrow \boxed{P=\frac{32}{3}\approx10,66667}$$

Anto Pedišić 17-1-0114-2012 17.VI.2015.

(3.) $y'' - y' = e^x + 1$

$$\frac{\partial f}{\partial x} = e^x$$

$\varnothing$

(2.) $\int_0^2 \sin x^2 \, dx = 2 \cdot \int_0^2 \sin x \, dx$

$= \left[2 \cdot (-\cos x)\right] \Big|_0^2$

$= 0{,}83229 - (-2)$

$= \underline{2{,}83229}$

X

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3	POPUNJAVA NASTAVNIK
IME I PREZIME: Angelo Kosović **VRIJEME POČETKA:**	Broj ↓ bodova
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0264-2013.	

1. Izračunati:

   (a) tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$. X

   (b) površinu lika omeđenog pravcem $y = x + 1$ i parabolom $y = x^2 + 3x - 2$. 10

2. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin(x^2)\,\mathrm{d}x$. (bodovanje: 20 za rel. grešku$\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$)

3. Riješiti diferencijalnu jednadžbu: $y'' - y' = \mathrm{e}^x + 1$.

4. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$ 20

5. $$\int_0^{\pi} \frac{\mathrm{d}x}{\sin x + 2} = ?$$ X

Ukupno: 30

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

1. a)

$z = \sin(x^2 y)$ $\quad T(2, 1, \sin(4))$ $\quad z - z_0 = f_x(T)(x-x_0) + f_y(T)(y-y_0)$

$f_x = \cos(2xy)$ ✗

$f_y = \cos(x^2)$ ✗

$f_x(T) = \cos(2 \cdot 2 \cdot 1)$

$f_x(T) = \cos(4)$

$f_y(T) = \cos(2^2)$

$f_y(T) = \cos(4)$

✗

$z - \sin(4) = \cos(4)(x-2) + \cos(4)(y-1)$

$z - \sin(4) = x\cos(4) - 2\cos(4) + y\cos(4) - \cos(4)$

$z \sin(4) - x\cos(4) - y\cos(4) = -3\cos(4)$

4. $\int_1^3 \frac{dx}{x^2-2x+4} =$

$(x-1)^2 - 1^2 + 4$

$(x-1)^2 + 3$

$\int_1^3 \frac{dx}{(x-1)^2+3} \quad \left| \begin{array}{l} x-1=t \\ dx=dt \end{array} \right. = \int_1^3 \frac{dt}{t^2+3} = \frac{1}{\sqrt{3}} \arctan \frac{t^2}{\sqrt{3}} \Big|_1^3$

$a^2 = 3 \quad a = \sqrt{3}$

$= \frac{1}{\sqrt{3}} \arctan \frac{(x-1)^2}{\sqrt{3}} \Big|_1^3 = \frac{1}{\sqrt{3}} \arctan \frac{(3-1)^2}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}} \arctan \frac{(1-1)^2}{\sqrt{3}} \right)$

$= \frac{1}{\sqrt{3}} \arctan \frac{4}{\sqrt{3}} - \frac{1}{\sqrt{3}} \arctan$

✓

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{B}3$

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: JAKOV ZUBČIĆ **VRIJEME POČETKA:** 17:30

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

0269092107

1. Izračunati:

   (a) tangencijalnu ravninu plohe $z = \sin(x^2y)$ u točki $(2, 1, \sin(4))$.

   (b) površinu lika omeđenog pravcem $y = x + 1$ i parabolom $y = x^2 + 3x - 2$.

   10

2. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin(x^2)\, dx$. (bodovanje: 20 za rel. grešku$\leq$ 1%, 15 za rel. grešku $\lessapprox$ 3%, 8 za rel. grešku $\lessapprox$ 6%)

   X

3. Riješiti diferencijalnu jednadžbu: $y'' - y' = e^x + 1$.

   X

4. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$

5. $$\int_0^{\pi} \frac{dx}{\sin x + 2} = ?$$

Ukupno: 10

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

JAKOV ZUBČIĆ 0269092107

1. b) $y = x + 1$

x	0	1	-1	-2
y	1	2	0	-1

$y = x^2 + 3x - 2$

$f(0) = -2$

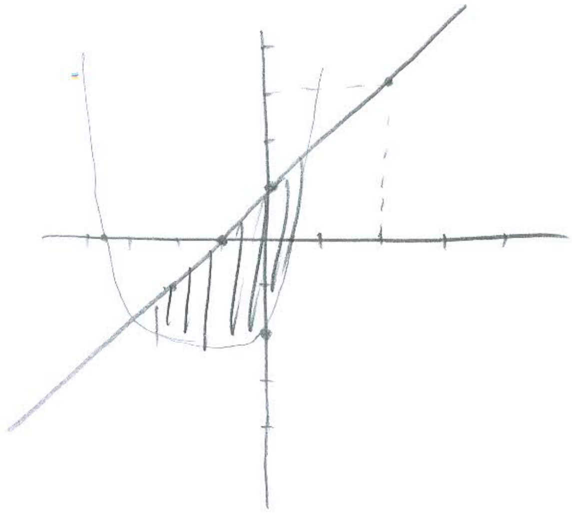
$x^2 + 3x - 2 = 0$

$x = \frac{-3 \pm \sqrt{9+8}}{2}$

$x = \frac{-3 \pm \sqrt{17}}{2}$

$x_1 = 0{,}56$

$x_2 = -3{,}56$



$x + 1 = x^2 + 3x - 2$

$0 = x^2 + 2x - 3$

$x = \frac{-2 \pm \sqrt{4+12}}{2}$

$x = \frac{-2 \pm 4}{2}$

$x_1 = 1$

$x_2 = -3$

$$\int_{-3}^{1} (x + 1 - x^2 - 3x + 2)\,dx =$$

$$\int_{-3}^{1} (-x^2 - 2x + 3)\,dx = \left[-\frac{x^3}{3} - x^2 + 3x \right]_{-3}^{1} =$$

$$-\frac{1}{3} - 1 + 3 - (\cancel{9} - 9 - 9) =$$

$$\frac{5}{3} + 9 = 10{,}66$$ ✓

4. $\int_1^3 \frac{dx}{x^2 - 2x + 4}$

$$\int \frac{dx}{(x-2)^2} = \left[\begin{array}{l} t = x - 2 \,/' \\ dt = dx \end{array} \right] = \int \frac{dt}{t^2} = \int t^{-2}\,dt = \int \frac{t^{-2+1}}{-2+1}\,dt$$

✗

$$= \frac{t^{-1}}{-1} = \frac{-1}{x-2}$$

$$\left[\frac{-1}{x-2} \right]_1^3 = \frac{-1}{3-2} - \left(-\frac{1}{1-2} \right) = \frac{-1}{1} - 1 = -2$$

JAKOV ZUBČIĆ 0269092107

$y'' - y' = e^x + 1$

$f(x) = e^x + 1$

$y'' - y' = 0$

$\lambda^2 - \lambda = 0$

$\lambda(\lambda - 1) = 0$

$\lambda_1 = 0 \quad \lambda_2 = 1$

$y_0 = C_1 \cdot e^{0 \cdot x} + C_2 \cdot e^{1 \cdot x}$

$\emptyset$

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3

IME I PREZIME: MATE RADAŠ **VRIJEME POČETKA:** 17:30

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0183-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati:
   (a) tangencijalnu ravninu plohe $z = \sin(x^2 y)$ u točki $(2, 1, \sin(4))$.
   (b) površinu lika omeđenog pravcem $y = x + 1$ i parabolom $y = x^2 + 3x - 2$.
2. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin(x^2)\,dx$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\lessapprox 3\%$, 8 za rel. grešku $\lessapprox 6\%$)
3. Riješiti diferencijalnu jednadžbu: $y'' - y' = e^x + 1$.
4. $\int_1^3 \frac{dx}{x^2-2x+4} = ?$
5. $\displaystyle\int_0^\pi \frac{dx}{\sin x + 2} = ?$

Ukupno:

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x\,dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\,dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4) $\displaystyle\int_1^3 \frac{dx}{x^2-2x+4} = \left\{ \begin{array}{l} t = x^2-2x+4 \\ dt = 2x-2 \end{array} \right\} = \int_1^3 \frac{1}{2x-2} = \frac{1}{2}\int \frac{1}{x}\,dx - \frac{1}{2}\int dx =$

$\displaystyle \frac{1}{2}\ln|x| + C - \frac{1}{2}x + C\Big|_1^3 = \left(\frac{1}{2}\ln|3| + C - \frac{1}{2}\cdot 3 + C\right) - \left(\frac{1}{2}\ln|1| + C - \frac{1}{2}\cdot 1 + C\right)$

$\displaystyle = \frac{1}{2}\ln|3| - \frac{3}{2} + \cancel{2C} - \frac{1}{2}\ln(1) + \frac{1}{2} - \cancel{2C} = 0.549 - 1.5 - 0 + \frac{1}{2}$

$= 0.549 - 1.5 + 0.5 = -0.451$

5. $\int_0^{\pi} \frac{dx}{\sin x + 2} = \int_0^{\pi} \frac{1}{\sin + 2} = \int \frac{1}{\sin x} dx + \frac{1}{2}\int dx = \frac{1}{-\cos x} + \frac{1}{2}x \Big|_0^{\pi}$

$= \left(\frac{1}{-\cos\pi} + \frac{1}{2}\pi\right) - \left(\frac{1}{-\cos(0)} + \frac{1}{2}\cdot 0\right) = (1.002 + 1.5708) + 1 =$

$= 1.5688$