

MATEMATIKA 2: Ispit se održava sukladno pravilima koja su vam pročitana. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

85

IME I PREZIME: Loure Štokić

VRIJEME POČETKA: 17:30

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0221-2014

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1. Riješiti diferencijalnu jednadžbu: $y'' - y' = e^x + 1$.

2. $\int_1^3 \frac{dx}{x^2 - 2x + 4} = ?$

3. $\int_0^\pi \frac{dx}{\sin x + 2} = ?$

4. Izračunati:

(a) tangencijalnu ravninu plohe $z = \sin(x^2y)$ u točki $(2, 1, \sin(4))$.

(b) površinu lika omeđenog pravcem $y = x + 1$ i parabolom $y = x^2 + 3x - 2$.

5. Numeričkom integracijom odrediti vrijednost $\int_0^2 \sin(x^2) dx$. (bodovanje: 20 za rel. grešku $\leq 1\%$, 15 za rel. grešku $\approx 3\%$, 8 za rel. grešku $\approx 6\%$)

X
20

20

X

0

Ukupno:

40

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x \sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2}) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

$$\textcircled{2} \int_1^3 \frac{dx}{x^2 - 2x + 4} = \quad x_{1,2} = \frac{2 \pm \sqrt{4 - 16}}{2} \notin \mathbb{R}$$

Nema singulariteta u intervalu $[1, 3]$ stoga možemo riješiti ps Newton-Leibnizovoj formuli.

$$= \left. \begin{aligned} &= (x)^2 - \underbrace{2 \cdot x \cdot ?}_{2x} + 4 \\ &= (x)^2 - 2 \cdot x \cdot 1 + \underbrace{1^2 + ?}_4 \\ &= (x)^2 - 2 \cdot x \cdot 1 + 1^2 + 3 \\ &= (x - 1)^2 + 3 \end{aligned} \right\} \Rightarrow \int \frac{dx}{(x-1)^2 + (\sqrt{3})^2} = \frac{1}{\sqrt{3}} \arctan\left(\frac{x-1}{\sqrt{3}}\right) + C$$

$$\Rightarrow \left[\frac{1}{\sqrt{3}} \arctan\left(\frac{x-1}{\sqrt{3}}\right) \right]_1^3 = \frac{1}{\sqrt{3}} \arctan\left(\frac{3-1}{\sqrt{3}}\right) - \frac{1}{\sqrt{3}} \arctan\left(\frac{1-1}{\sqrt{3}}\right) \quad \overset{0}{\rightarrow}$$

$$= \frac{\sqrt{3}}{3} \arctan\left(\frac{2\sqrt{3}}{3}\right) \approx 0.495 \quad \checkmark$$

(1) $y'' - y' = e^x + 1$

$$r^2 - r = 0$$

$$r(r-1) = 0 \Rightarrow r_1 = 0, r_2 = 1$$

$$r_1 \neq r_2 \in \mathbb{R}$$

$$y_H(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

a) $e^x = e^{\alpha x} (P_m \cos(\beta x) + \underbrace{Q_n \sin(\beta x)}_{\text{relativ}})$

$$\alpha = 1, \beta = 0, m = 0, k = 1$$

$$y_1(x) = x^k \cdot e^{\alpha x} (S_M \cos(\beta x))$$

$$y_1(x) = x \cdot e^x \cdot A$$

~~$$e^x = e^x \cdot x \cdot A$$~~

$$x \cdot A = 0 \Rightarrow A = 0$$

$$y_1(x) = 0$$

b) $1 = e^{\alpha x} (P_m \cos(\beta x) + Q_n \sin(\beta x))$

$$\alpha = 0, \beta = 0, m = 0, k = 1$$

$$1 = x \cdot e^{0x} (A)$$

$$1 = x \cdot A \Rightarrow A = 0$$

$$y_2(x) = 0$$

$$y(x) = y_H(x) + y_1(x) + y_2(x)$$

$$y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

X

③. $\int_0^{\pi} \frac{dx}{\sin x + 2} =$ Nema singulariteta toka da nije neprovi integral.

$$\Rightarrow \int \frac{dx}{\sin x + 2} = \left\{ \begin{array}{l} t = \tan\left(\frac{x}{2}\right) \\ \sin x = \frac{2t}{1+t^2} \\ dx = \frac{2dt}{1+t^2} \end{array} \right\}$$

$$= \int \frac{2dt}{\frac{2t}{1+t^2} + 2} = \int \frac{2dt}{\frac{2t + 2 + 2t^2}{1+t^2}} = \int \frac{2dt}{2(t^2 + t + 1)}$$

$$= \int \frac{dt}{t^2 + t + 1} \left\{ \begin{array}{l} = (t)^2 + 2 \cdot t \cdot \frac{1}{2} + 1 \\ = (t)^2 + 2 \cdot t \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \frac{3}{4} \\ = \left(t + \frac{1}{2}\right)^2 + \frac{3}{4} \end{array} \right\}$$

$$= \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{\frac{\sqrt{3}}{2}} \cdot \arctan\left(\frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$$

$$= \left[\frac{2\sqrt{3}}{3} \arctan\left(\frac{2\sqrt{3}\left(t + \frac{1}{2}\right)}{3}\right) \right]$$

$$\Rightarrow \left[\frac{2\sqrt{3}}{3} \arctan\left(\frac{2\sqrt{3}\left(\tan\left(\frac{x}{2}\right) + \frac{1}{2}\right)}{3}\right) \right]_0^{\pi}$$

Pojavljuje se problem pri izboru restitucija $t = \tan\left(\frac{x}{2}\right)$
 $\pi \Rightarrow \tan\left(\frac{\pi}{2}\right) = N/P$
 zbog je ova nepravilni integral.

$$\lim_{\alpha \rightarrow \pi^-} \left[\frac{2\sqrt{3}}{3} \arctan \left(\frac{2\sqrt{3}(\tan(\frac{\alpha}{2}) + \frac{1}{2})}{3} \right) \right]_{\alpha=0}^{\alpha}$$

$$= \frac{2\sqrt{3}}{3} \arctan \left(\frac{2\sqrt{3}(\tan(\frac{\pi}{2}) + \frac{1}{2})}{3} \right)$$

$$- \frac{2\sqrt{3}}{3} \arctan \left(\frac{2\sqrt{3}(\tan(\frac{0}{2}) + \frac{1}{2})}{3} \right)$$

$$= \frac{2\sqrt{3}}{3} \arctan(+\infty) - \frac{2\sqrt{3}}{3} \arctan\left(\frac{\sqrt{3}}{3}\right)$$

$$= \frac{2\sqrt{3}}{3} \cdot \frac{\pi}{2} - \frac{2\sqrt{3}}{3} \cdot \frac{\pi}{6} = \frac{\pi\sqrt{3}}{3} - \frac{\pi\sqrt{3}}{9} \approx 1.21 //$$



(4. b) $f(x) = x + 1$, $g(x) = x^2 + 3x - 2$

$f(x)$	x
1	0
0	-1
-1	-2
2	1

$g(x)$	x
-2	0
2	1
-4	-1
8	2
-4	-2

Točka sjecišta:

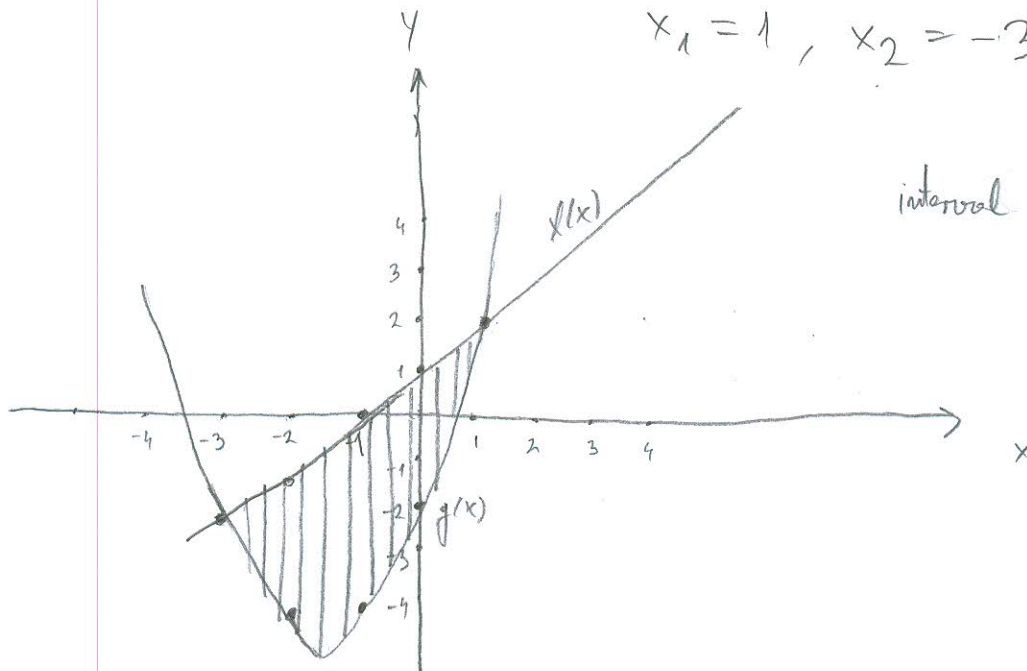
$$x + 1 = x^2 + 3x - 2$$

$$x^2 + 2x - 3 = 0$$

$$x_{1/2} = \frac{-2 \pm \sqrt{4 + 12}}{2}$$

$$x_1 = 1, x_2 = -3$$

interval $[-3, 1]$



$$P = \int_{-3}^1 (f(x) - g(x)) dx = \int_{-3}^1 (x + 1 - x^2 - 3x + 2) dx = \int_{-3}^1 (-x^2 - 2x + 3) dx$$

$$\Rightarrow -\int x^2 dx - 2\int x dx + 3\int dx = -\frac{x^3}{3} - 2\frac{x^2}{2} + 3x + C$$

$$\Rightarrow \left[\frac{x^3}{3} - x^2 + 3x \right]_{-3}^1 = -\frac{(1)^3}{3} - (1)^2 + 3 \cdot 1 - \left(-\frac{(-3)^3}{3} - (-3)^2 + 3 \cdot (-3) \right)$$

$$= -\frac{1}{3} - 1 + 3 - 9 + 9 + 9 = \frac{22}{3} \approx 7.33 //$$

Positivna vrijednost
pozitivna.

