MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 32

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Antonijo Knežević **VRIJEME POČETKA:** 09:49

MATIČNI BROJ STUDENTA: 57672 **USTMENI ISPIT KOD NASTAVNIKA:** Uglešić. N.

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$y'''(t) + 3\,y'(t) = t^2, \quad y(0)' = y''(0) = 0,\; y(0) = 2.$$

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 9$, ravninom $z = -4$ i parabolom $z = x^2 + y^2$. Napomena: tijelo koje se traži sadrži ishodište. 20

3. Izračunaj integral funkcije $f(x, y, z) = x$ u dijelu prostora omeđenog plohama $x = z^2$, $x = 4$, $y = -1$ i $y = 4 + x$. ~~20~~

4. Izračunati površinu plohe u obliku stošca koja odgovara eksplicitnoj jednadžbi $x^2 + y^2 = \frac{z^2}{2}$ gdje je $0 \le z \le 4$. 20

5. Primjenom Stokesove formule izračunati $\oint_{\partial S} x\,dx + (x + y)\,dy + (x + y + z)\,dz$ ako je $S = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + \cancel{20} \le 3,\ z = x$ ~~20~~

Ukupno: 40

$\boxed{1.}$ $y'''(t)+3y'(t)=t^2, \quad y(0)'=y''(0)=0, \quad y(0)=2$

$$\left(s^3F(s)-s^2\underbrace{f(0)}_{=2}-s\underbrace{f'(0)}_{=0}-\underbrace{f''(0)}_{=0}+3\cdot\left(sF(s)-\underbrace{f(0)}_{=2}\right)\right)=\frac{2}{s^3}$$

$$s^3F(s)-2s^2+3sF(s)-2=\frac{2}{s^3}$$

$$F(s)\left(s^3+3s\right)=2s^2+2+\frac{2}{s^3}=\frac{2s^5+2s^3+2}{s^3}$$

$$F(s)=\frac{2s^5+2s^3+2}{s^4(s^2+3)}$$

$$\frac{2s^5+2s^3+2}{s^4(s^2+3)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{D}{s^4}+\frac{Es+F}{s^2+3}$$

$$=\frac{As^5+3\cdot As^3+Bs^4+3Bs+Cs^3+3Cs+Ds^2+3D+Es^5+Fs^4}{(s^2+3)s^4}$$

$A + E = 2$
$B + F = 0$
$3A + C = 2$
$3B + D = 0 \Rightarrow$
$3C + \cancel{} = 0 \Rightarrow$
$3D = 2 \Rightarrow$

$F = \frac{2}{9}$

$A = \frac{2}{3}$

$B = \frac{-D}{3} = -\frac{2}{9}$

$C = 0$

$D = \frac{2}{3}$

$E = 2 - A = 2 - \frac{2}{3} = \frac{4}{3}$

VIDI FESTINI

$$F(s) = \frac{2}{3} \cdot \frac{1}{3} - \frac{2}{9} \cdot \frac{1}{s^2} + \frac{2}{3} \cdot \frac{1}{s^4} + \frac{\frac{4}{3} s + \frac{2}{9}}{s^2+3}$$

$$f(t) = \frac{2}{3} - \frac{2}{9} \cdot \frac{1}{t} + \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{t^3} + \frac{4}{3} \cdot \cos 3t + \frac{2}{9} \cdot \frac{1}{\sqrt{3}} \sin\sqrt{3}t$$

$$f(t) = \frac{2}{3} - \frac{2}{9} \cdot \frac{1}{t} + \frac{1}{9} \frac{1}{t^3} + \frac{4}{3} \cos 3t + \frac{2}{9\sqrt{3}} \sin\sqrt{3}t \quad \times$$

PROVJERA ?

2) $x^2 + y^2 = 9$

$z = -4$

$z = x^2 + y^2$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ z = z \end{cases}$$

$z \in [-4, r^2]$

$\varphi \in [0, 2\pi]$

$r \in [0, 3]$

$x^2 + y^2 = r^2\cos^2\varphi + r^2\sin^2\varphi = r^2$

$J_\phi = r$

$$V = \int_0^{2\pi}\int_0^3\int_{-4}^{r^2} r \, dz\, dr\, d\varphi = \checkmark$$

$$= \int_0^{2\pi}\int_0^3 rz \Big|_{-4}^{r^2} dr\, d\varphi = \int_0^{2\pi}\int_0^3 (r^3 + 4r)\, dr\, d\varphi =$$

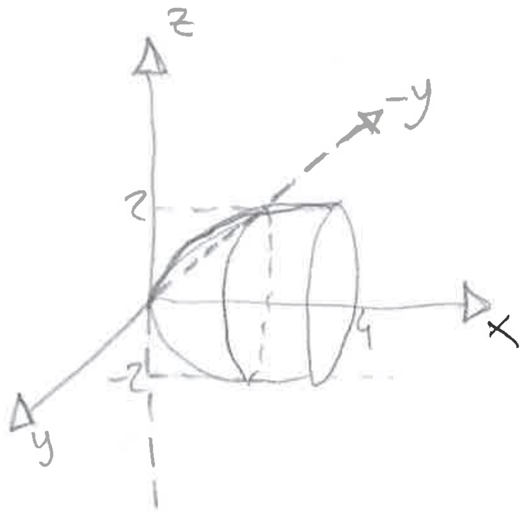
$$= \int_0^{2\pi} \left(\frac{r^4}{4} + 4 \cdot \frac{r^2}{2}\right)\Big|_0^3 d\varphi = \int_0^{2\pi} \left[\left(\frac{81}{4} + 18\right) - 0\right] d\varphi =$$

$$= \int_0^{2\pi} \frac{153}{4} d\varphi = \left(\frac{153}{4}\varphi\right)\Big|_0^{2\pi} = \frac{153}{2}\pi$$

3.

$f(x,y,z)=x$

$x=z^2$
$x=4$ $\Big\}\ x\in[0,4]$
$y=-1$
$y=4+x$



VIDI FEŠTINI

$$\iiint_V x\,dV=\int_{-2}^{2}\int_{0}^{z^2}\int_{-1}^{4+x} x\,dy\,dx\,dz=$$

$$=\int_{-2}^{2}\int_{0}^{z^2}(xy)\Big|_{-1}^{4+x}dx\,dz=\int_{-2}^{2}\int_{0}^{z^2}(x^2+4x+x)\,dx\,dz$$

$$=\int_{-2}^{2}\left(\frac{1}{3}x^3+2x^2\right)\Big|_{0}^{z^2}dz=\int_{-2}^{2}\left(\frac{1}{3}z^6+2z^4\right)dz$$

$$=\left(\frac{1}{3}\cdot\frac{1}{7}z^7+2\cdot\frac{1}{5}\cdot z^5\right)\Big|_{-2}^{2}=2\cdot\left(\frac{1}{21}\cdot 128+\frac{1}{10}\cdot 32\right)$$

$$=\frac{256}{21}+\frac{32}{5}$$

Antonijo Knežević

4. $x^2+y^2=\frac{z^2}{2}$

$0 \le z \le 4$

$$\left.\begin{array}{l} x = r\sin\varphi \\ y = r\cos\varphi \end{array}\right\} \Rightarrow r^2 = \frac{z^2}{2}$$

$\Rightarrow r \in [0, \sqrt{8}]$ ✓

$z = \sqrt{2x^2+2y}$

Parametrizacija

$$r(x,y) = \begin{pmatrix} x \\ y \\ \sqrt{2x^2+2y^2} \end{pmatrix} \Rightarrow \partial_x r = \begin{pmatrix} 1 \\ 0 \\ \frac{2x}{\sqrt{2x^2+2y^2}} \end{pmatrix} \quad \partial_y r = \begin{pmatrix} 0 \\ 1 \\ \frac{2y}{\sqrt{2x^2+2y^2}} \end{pmatrix}$$

Normala:

$$\vec{n} = \partial_x r + \partial_y r = \begin{pmatrix} 1 \\ 0 \\ \frac{2x}{\sqrt{2x^2+2y^2}} \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ \frac{2y}{\sqrt{2x^2+2y^2}} \end{pmatrix}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & \frac{2x}{\sqrt{2x^2+2y^2}} \\ 0 & 1 & \frac{2y}{\sqrt{2x^2+2y^2}} \end{vmatrix} = \left(-\frac{2x}{\sqrt{2x^2+2y^2}}, -\frac{2y}{\sqrt{2x^2+2y^2}}, 1 \right)$$

$$\Rightarrow \|\vec{n}\| = \sqrt{\frac{4x^2}{2x^2+2y^2} + \frac{4y^2}{2x^2+2y^2} + 1} = \sqrt{3}$$

$$P = \iint_S 1\,dS = \iint_D \sqrt{3}\,dx\,dy = \int_0^{2\pi}\int_0^{\sqrt{8}} \sqrt{3}\,r\,dr\,d\varphi = ✓$$

$$= \int_0^{2\pi} \left(\sqrt{3}\, r^2 \Big|_0^{\sqrt{8}} \right) d\varphi = \int_0^{2\pi} 8\sqrt{3}\,d\varphi = 16\sqrt{3}\,\pi$$

5 (1 Dio)

$$\oint_{\partial S} x\,dx + (x+y)\,dy + (x+y+z)\,dz$$

$$S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 \leq 3,\ z = x^3$$

$$f = \begin{pmatrix} x \\ x+y \\ x+y+z \end{pmatrix} \qquad \operatorname{rot} f = \begin{pmatrix} \partial x \\ \partial y \\ \partial z \end{pmatrix} \times \begin{pmatrix} x \\ x+y \\ x+y+z \end{pmatrix}$$

$$= (1, 0, 1) \quad 0$$

DALJE...

Antonija Knežević

5 (2 DIO)

$$\oint_{\partial S} x\,dx + (x+y)\,dy + (x+y+z)\,dz = \iint \operatorname{rot} f\, dS =$$

$$= \iint_S \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} dS$$

PARAMETRIZACIJA.

$$r(x,y) = \begin{pmatrix} x \\ y \\ x \end{pmatrix} \Rightarrow \partial_x r = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \partial_y r = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{u} = \partial_x \vec{r} \times \partial_y \vec{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = (-1, 0, 1)$$

$$\cancel{\iint_D \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} dD =}$$

$$\|\vec{u}\| = \sqrt{2}$$

$$\iint_D \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} u \; dxy = \iint_D \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix} dx\,dy$$

$$= \iint_D \left(-\frac{\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2} \right) dx\,dy = 0$$

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: JOSIP FESTINI

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA:

USTMENI ISPIT KOD NASTAVNIKA: prof. Uglešić

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Ukupno: 40

① $y'''(t) + 3y'(t) = t^2 \qquad y(0)' = y''(0) = 0 \qquad y(0) = 2$

$s^3 Y(s) - s^2 y(0) - s\,y'(0) - y''(0) + 3(sY(s) - y(0)) = \frac{2}{s^3}$

$s^3 Y(s) - 2s^2 - 0 - 0 + 3sY(s) - 6 = \frac{2}{s^3}$

$s^3 + 3s = \frac{2}{s^3} + 2s^2 + 6$

$s^3 + 3s = \frac{2 + 2s^5 + 6s^3}{s^3} <> \dfrac{\frac{2+2s^5+6s^3}{s^3}}{\frac{s^3+3s}{1}} \to \frac{2+2s^5+6s^3}{s^3(s^3+3s)}$

$Y(s) = \frac{2s^5 + 6s^3 + 2}{s^4(s^2+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{E+Fs}{s^2+3} \quad / \cdot s^4(s^2+3)$

$2s^5 + 6s^3 + 2 = As^3(s^2+3) + Bs^2(s^2+3) + Cs(s^2+3) + D(s^2+3) + (E+Fs)s^4$

$= As^5 + 3As^3 + Bs^4 + 3Bs^2 + Cs^3 + 3Cs + Ds^2 + 3D + Es^4 + Fs^5$

$2 = A + F$
$0 = B + E$ ✓
$6 = 3A + C$ ✓
$0 = 3B + D$ ✓
$0 = 3C \Rightarrow 3C = 0 \Rightarrow \boxed{C = 0}$
$2 = 3D \Rightarrow 3D = 2 \quad \boxed{D = \frac{2}{3}}$

$3B + D = 0$
$3B = 0 - D$
$3B = 0 - \frac{2}{3}$
$B = \dfrac{-\frac{2}{3}}{\frac{3}{1}} \Rightarrow \boxed{B = -\frac{2}{9}}$

$B + E = 0$
$E = 0 - B$
$\boxed{E = +\frac{2}{9}}$

$3A + C = 6$
$3A = 6 - C$
$A = \frac{6}{3}$
$\boxed{A = 2}$

$A + F = 2$
$F = 2 - A$
$F = 2 - 2$
$\boxed{F = 0}$

JOSIP FETTINI

$A = 2$
$B = -\frac{2}{9}$
$C = 0$
$D = \frac{2}{3}$
$E = \frac{2}{9}$
$F = 0$

$$\frac{2s^5+6s^3+2}{s^4(s^2+3)} \quad \frac{2}{s} + \frac{-\frac{2}{9}}{s^2} + \frac{\frac{2}{3}}{s^4} + \frac{\frac{2}{9}}{s^2+3}$$

provjera

za 1
$$\frac{2+6+2}{1(1+3)} = 2 - \frac{2}{9} + \frac{2}{3} + \frac{2}{36}$$

$$\frac{10}{4} = \frac{72-8+24+2}{36}$$

$$\frac{10}{4} = \frac{90}{36} = \frac{10}{4}$$

$$\boxed{\frac{10}{4} = \frac{10}{4}} \text{ OK.}$$

$$y(t) = 2\mathcal{L}^{-1}\left[\frac{1}{s}\right] - \frac{2}{9}\mathcal{L}^{-1}\left[\frac{1}{s^2}\right] + \frac{2}{3}\mathcal{L}^{-1}\left[\frac{1}{s^4}\right] + \frac{2}{9}\mathcal{L}^{-1}\left[\frac{1}{s^2+3}\right]$$

$$y(t) = 2 - \frac{2}{9}t + \frac{2}{3}\frac{t^3}{6} + \frac{2}{9}\cdot\frac{1}{\sqrt{3}}\sin\sqrt{3}t$$

$$y(t) = 2 - \frac{2}{9}t + \frac{1}{3}\cdot\frac{t^3}{3} + \frac{2}{9}\cdot\frac{1}{\sqrt{3}}\sin\sqrt{3}t$$

$$= 2 - \frac{2}{9}t + \frac{t^3}{9} + \frac{2}{9}\cdot\frac{1}{\sqrt{3}}\sin\sqrt{3}t \quad / \; y(0) = 2$$

$$\frac{1}{s^2+\sqrt{3}^2} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{s^2+\sqrt{3}^2}$$

$y'(t) = -\frac{2}{9} + \frac{1}{3}t^2 + \frac{2}{9}\cos(\sqrt{3}t)$ $\quad y'(0) = 0$

$y''(t) = \frac{2}{3}t - \frac{2\sqrt{3}}{9}\sin(\sqrt{3}t)$ $\quad y''(0) = 0$

$y'''(t) = \frac{2}{3} - \frac{2}{3}\cos(\sqrt{3}t)$

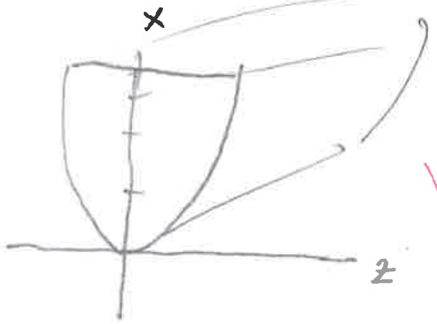
$$y''' + 3y' = \left[\cancel{\tfrac{2}{3}} - \tfrac{2}{3}\cos(\sqrt{3}t)\right] + 3\left[-\cancel{\tfrac{2}{9}} + \tfrac{1}{3}t^2 + \tfrac{2}{9}\cos(\sqrt{3}t)\right]$$
$$= t^2$$

✓

20

JOSIP FEŠTINI

③ $f(x,y,z) = x \qquad x = z^2, \; x = 4, \; y = -1, \; y = 4+x$



$$\int_{-2}^{2}\int_{z^2}^{4}\int_{-1}^{4+x} x \; dy \; dx \; dz$$

$$\int_{-2}^{2}\int_{z^2}^{4}\int_{-1}^{4+x} x \, dy \, dx \, dz = \int_{-2}^{2}\int_{z^2}^{4} \left| x\cdot\frac{y^2}{2}\right|_{-1}^{4+x}$$

$$= \int_{-2}^{2}\int_{z^2}^{4} x\cdot\frac{(4+x)^2}{2} - x\cdot\frac{(-1)}{2} = \int_{-2}^{2}\int_{z^2}^{4} x\cdot\frac{(16+8x+x^2)}{2} - \frac{-x}{2}$$

$$= \int_{-2}^{2}\int_{z^2}^{4} \frac{16x+8x^2+x^3+x}{2} \, dx \, dz = \int_{-2}^{2} \left| 8\frac{x^2}{2} + 4\frac{x^3}{3} + \frac{1}{2}\frac{x^4}{4} + \frac{1}{2}\frac{x^2}{2} \right|_{z^2}^{4} dz$$

$$\int_{-2}^{2} \left| 4x^2 + \frac{4x^3}{3} + \frac{x^4}{8} + \frac{x^2}{4} \right|_{z^2}^{4} dz = \int_{-2} \left| \frac{96x^2+32x^3+3x^4+6x^2}{24} \right|_{z^2}^{4}$$

$$= \int_{-2}^{2} \frac{1536+2048+768+96}{24} - \frac{96z^4+32z^5+3z^6+6z^4}{24}$$

$$= \int_{-2}^{2} \frac{4448}{24} - \frac{3z^6+32z^5+102z^4}{24} = \int_{-2} \frac{1112}{6} - \frac{3z^6+32z^5+102z^4}{24} \, d$$

$$= \int_{-2}^{2} \frac{556}{3} - \frac{3z^6+32z^5+102z^4}{24} \, dz = \left| \frac{556}{3}z - 8\frac{z^7}{7} + \frac{4}{3}\frac{z^6}{6} + \frac{34}{8}\frac{z^5}{5} \right|_{-2}^{2}$$

$$= \frac{1112}{3} - \frac{1024}{7} + \frac{256}{18} + \frac{1088}{40} - \left[-\frac{1112}{3} - \left(-\frac{1024}{7}\right) + \frac{256}{18} - \frac{1088}{40} \right)$$

NASTAVAK 3. JOSIP FESTINI

$$\frac{1112}{3} - \frac{1024}{7} + \cancel{\frac{256}{18}} + \frac{1088}{40} + \frac{1112}{3} - \frac{1024}{7} - \cancel{\frac{256}{18}} + \frac{1088}{40}$$

$$= \frac{311360 - 122880 + 22848 + 311360 - 122880 + 22848}{840}$$

$$= \frac{422656}{840} = 503.16$$

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IME I PREZIME: Branimir Pijaca **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: 17-2-0086-2011 **USTMENI ISPIT KOD NASTAVNIKA:**	

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Ukupno: 20

3) $f(x,y,z) = x$ $\quad x = z^2 \quad x = 4 \quad y = -1 \quad i \quad y = 4+x$

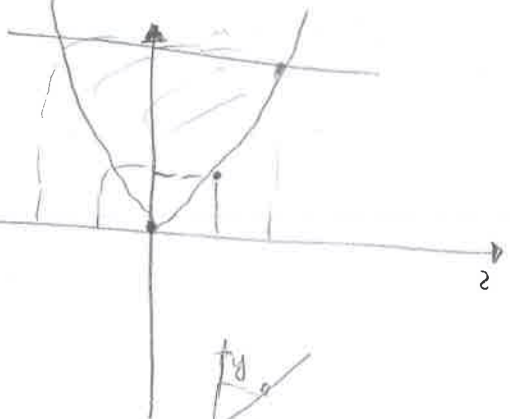
$z \in [-2, 2]$ $\quad z^2 = 4,\ z = \sqrt{4},\ z = \mp 2$

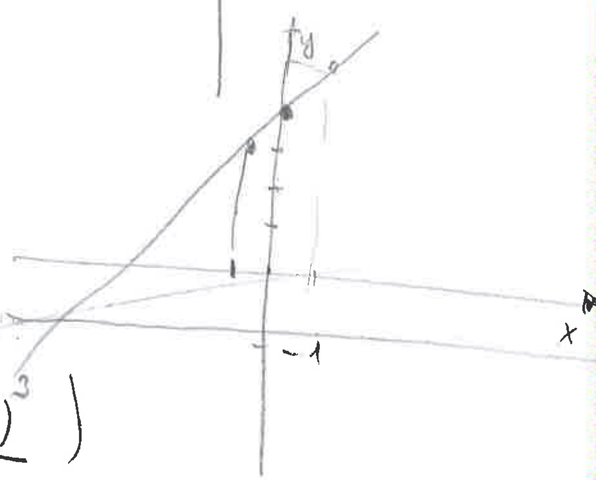
$x \in [z^2, 4]$

$y \in [-1, 4+x]$

z	0	1	2	-1	-2
$x=z^2$	0	1	4	1	4

x	0	-1	1
$y=4+x$	4	3	5





$$\int_{-2}^{2}\int_{z^2}^{4}\int_{-1}^{4+x} x \, dy\, dx\, dz$$

$$\int_{-2}^{2}\int_{z^2}^{4} xy \Big|_{-1}^{4+x} dx\, dz = \int_{-2}^{2}\int_{z^2}^{4} \left(x(4+x) - (-x)\right) dx\, dz$$

$$\int_{-2}^{2}\int_{z^2}^{4} (4x + x^2 + x)\, dx\, dz = \int_{-2}^{2}\int_{z^2}^{4} (5x + x^2)\, dx\, dz$$

$$\int_{-2}^{2} 5\frac{x^2}{2} + \frac{x^3}{3} \Big|_{z^2}^{4} = \int_{-2}^{2} \frac{5 \cdot 16}{2} + \frac{64}{3} - \left(\frac{5(z^2)^2}{2} + \frac{(z^2)^3}{3}\right)$$

$$\int_{-2}^{2} \left(40 + \frac{64}{3} - \frac{5z^4}{2} - \frac{z^5}{3}\right) dz = \int_{-2}^{2} \left(\frac{184}{3} - \frac{5}{2}z^4 - \frac{1}{3}z^5\right) dz$$

$$= \frac{184}{3}z - \frac{5}{2}\frac{z^5}{5} - \frac{1}{3}\frac{z^6}{6} \Big|_{-2}^{2} = \frac{184 \cdot 2}{3} - \frac{5}{2}\cdot\frac{32}{5} - \frac{1}{3}\cdot\frac{64}{6} - \left(-\frac{184 \cdot 2}{3} + \frac{5}{2}\cdot\frac{(-2)}{5} + \frac{1}{2}\right.$$

$$x^2+y^2=\frac{z^2}{2}$$

$$0 \le z \le 4$$

r	1	0	-1
$z=\frac{r}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$

r	
$z=2r$	

$$2x^2+2y^2=z^2$$

$$2z\,\partial z=4x\,\partial x$$

$$\frac{\partial z}{\partial x}=\frac{4x}{2z}=\frac{2x}{z}$$

$$2z\,\partial z=4y\,\partial y$$

$$\frac{\partial z}{\partial y}=\frac{4y}{2z}=\frac{2y}{z}$$

$$P=\sqrt{1+\left(\frac{2x}{z}\right)+\left(\frac{2y}{z}\right)^2}=\sqrt{1+\frac{4x^2}{z^2}+\frac{4y^2}{z^2}}$$

$$=\sqrt{\frac{z^2+4x^2+4y^2}{z^2}}=\sqrt{\frac{z^2+4(x^2+y^2)}{z^2}}=\sqrt{\frac{z^2+4r}{z^2}}$$

$$=\sqrt{\frac{z^2+4\cdot 4}{z^2}}=\sqrt{\frac{z^2+16}{z^2}}$$

$$\int_0^{2\pi}\int_0^{4}$$

③ $f(x,y,z)=x$ $x=z^2$ $x=4$ $y=-1$ $y=4+x$ Branimir Ryuca

$x\in[z^2,4]$

$y\in[-1,4+x]$

$z\in[-2,2]$

$$\int_{-2}^{2}\int_{z^2}^{4}\int_{-1}^{4+x} x\,dy\,dx\,dz$$

$$\int_{-2}^{2}\int_{z^2}^{4} xy\Big|_{-1}^{4+x} dx\,dz = \int_{-2}^{2}\int_{z^2}^{4}(4x+x^2+x)\,dx\,dz = \int_{-2}^{2}\int_{z^2}^{4}(5x+x^2)\,dx\,dz$$

$$\int_{-2}^{2} 5\frac{x^2}{2}+\frac{x^3}{3}\Big|_{z^2}^{4} dz = \int_{-2}^{2} 40+\frac{64}{3}-\left(\frac{5}{2}z^4+\frac{1}{3}z^5\right)$$

$$=\int_{-2}^{2}\left(\frac{184}{3}-\frac{5}{2}z^4-\frac{1}{3}z^5\right)dz = \frac{184}{3}z-\frac{5}{2}\frac{z^5}{5}-\frac{1}{3}\frac{z^6}{6}\Big|_{-2}^{2}$$

$$=\frac{368}{3}-16-\frac{32}{9}-\left(-\frac{368}{3}+16-\frac{32}{9}\right)$$

$$=\frac{368}{3}-16-\cancel{\frac{32}{9}}+\frac{368}{3}-16+\cancel{\frac{32}{9}} = \frac{640}{3}$$

(2) $x^2+y^2=9$

$r^2=9$

$r=\sqrt{9}$

$r=3$

$z=-4$

$$\int_0^{2\pi}\int_0^3\int_0^{r^2} r\,dz\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^3 rz\Big|_0^{r^2}\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^3 r^3\,dr\,d\varphi = \int_0^{2\pi}\frac{r^4}{4}\Big|_0^3$$

$$=\int_0^{2\pi}\frac{81}{4}\,d\varphi = \frac{81}{4}\varphi\Big|_0^{2\pi}$$

$$=\frac{81}{2}\pi$$

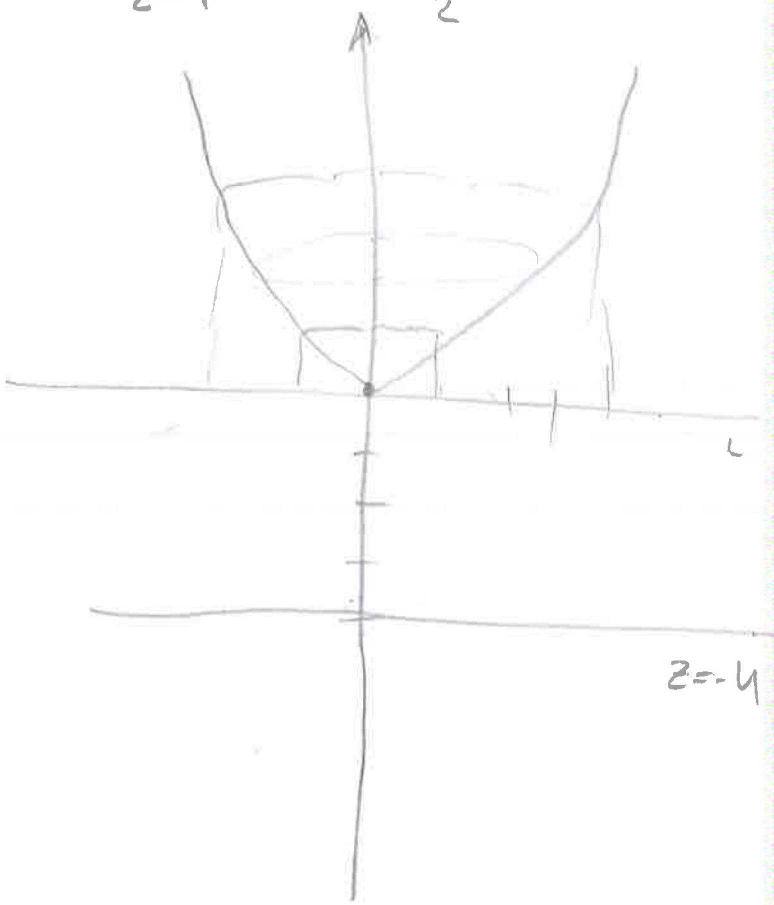
$z=x^2+y^2$

$x^2+y^2=z$

$r^2=z$

$z=r^2$

r	0	-1	1	2
$z=r^2$	0	1	1	4



$$y'''(t)+3y'(t)=t^2 \qquad y(0)'=y(0)''=0 \qquad y(0)=2$$

$$s^3Y(s)-s^2Y(0)-sy'(0)-y''(0)+3\left(sY(s)-y(0)\right)=\frac{2!}{s^3}$$

$$s^3Y(s)-2s^2 \cancel{-0+0}+3sY(s)-\cancel{6}=\frac{2}{s^3}$$

$$s^3Y_s+3sY(s)=\frac{2}{s^3}+2s^2+6$$

$$Y(s)(s^3+3s)=\frac{2+2s^5+6s^3}{s^3} \qquad Y(s)=\frac{\frac{2+2s^5+6s^3}{s^3}}{\frac{s^3+3s}{1}}=\frac{2s^5+6s^3+2}{s^3s(s^2+3)}$$

$$\frac{2s^5+6s^3+2}{s^4(s^2+3)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{D}{s^4}+\frac{Es+F}{(s^2+3)} \qquad / s^4(s^2+3)$$

$$2s^5+6s^3+2=As^3(s^2+3)+Bs^2(s^2+3)+Cs(s^2+3)+D(s^2+3)+(Es+F)s^4$$

$$2s^5+6s^3+2=As^5+3As^3+Bs^4+3Bs^2+Cs^3+3Cs+Ds^2+3D+Es^5+Fs^4$$

$$2s^5+6s^3+2=(A+E)s^5+(B+F)s^4+(3A+C)s^3+(3B+D)s^2+(3C)s+3D$$

$(A+E)=2$

$B+F=0$

$3A+C=6$

$3B+D=0$

$3C=0$

$3D=2$

$C=0$

$D=\frac{2}{3}$

$A=2$

$E=0$

$B=\frac{-2}{9}$

$F=\frac{2}{9}$

$$f(s)\; 2\frac{1}{s}-\frac{2}{9}\frac{1}{s^2}+\frac{2}{3}\frac{3!}{3s^4}+\frac{2}{9}\frac{5}{s^2+\sqrt{3}^2}$$

$$f(t)=2-\frac{2}{9}t+\frac{2}{9}t^3+\frac{2}{9}\sin(\sqrt{3}t)$$

$$f(t)'=\frac{-2}{9}+\frac{6}{9}t^2+\frac{2}{9}\cos(\sqrt{3}t)\cdot\sqrt{3}$$

$$f(t)''=\frac{12}{9}-\frac{2\sqrt{3}}{9}\sin(\sqrt{3}t)\cdot\sqrt{3}=$$

$3A+0=6$

$A=\frac{6}{3}=2$

$3B+\frac{2}{3}=0$

$3B=-\frac{2}{3}$

$B=\frac{-2}{9}$

$y'''(t)+3y'(t)=t^2$ $\quad y'(0)=y''(0)=0 \quad y(0)=2$

$s^3Y(s)-s^2y(0)-sy'(0)-y''(0)+3sY(s)-3y(0)=\frac{2}{s^3}$

$s^3Y(s)-2s^2+3sY(s)-6=\frac{2}{s^3}$

$Y_s(s^3+3s)=\frac{2}{s^3}+2s^2+6 \qquad Y(s)(s^3+3s)=\frac{2+2s^5+6s^3}{s^3}$

$=\frac{\frac{6s^3+2s^5+2}{s^3}}{\frac{s^3+3s}{1}}=\frac{6s^3+2s^5+2}{s^3\cdot s(s^2+3)}=\frac{6s^3+2s^5+2}{s^4(s^2+3)}$

$\frac{6s^3+2s^5+2}{s^4(s^2+3)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{D}{s^4}+\frac{Es+F}{s^2+3)} \quad / s^4(s^2+3)$

$6s^3+2s^5+2=As^3(s^2+3)+Bs^2(s^2+3)+Cs(s^2+3)+D(s^2+3)+(Es+F)s^4$

$6s^3+2s^5+2=As^5+3As^3+Bs^4+3Bs^2+Cs^3+3Cs+Ds^2+3D+Es^5+Fs^4$

$6s^3+2s^5+2=(A+E)s^5+(B+F)s^4+(3A+C)s^3+(3B+D)s^2+(3C)s+3D$

$A+E=2 \qquad D=\frac{2}{3} \quad C=0 \quad A=2$

$B+F=0 \qquad E=0 \quad B=-\frac{2}{9} \quad F=\frac{2}{9}$

$3A+C=6$

$3B+D=0 \qquad F(s)=2\frac{1}{s}-\frac{2}{9}\frac{1}{s^2}+0+\frac{2}{3\cdot 3}\frac{3!}{s^{n+1}}+\frac{2}{9\sqrt{3}}\frac{\sqrt{3}}{s^2+\sqrt{3}^2}$

$3C=0$

$3D=2$

$f(t)=2-\frac{2}{9}t+\frac{2}{9}t^3+\frac{2}{9\sqrt{3}}\sin\sqrt{3}t=2$ ✓

$f'(t)=-\frac{2}{9}+\frac{6}{9}t^2+\frac{2}{9\sqrt{3}}\cos\sqrt{3}\cdot\sqrt{3}=0$ ✗ ✓

$f''(t)=\frac{\cancel{12}\,4}{\cancel{9}\,3}t-\frac{2\sqrt{3}}{9\sqrt{3}}\sin(\sqrt{3}\cdot\overset{t}{0})\cdot\sqrt{3}=0$ ✓

$f'''(t)=\frac{4}{3}-\frac{2}{3}\cos(\sqrt{3}t)$

$f'''+3f'=\frac{4}{3}-\frac{2}{3}\cos(\sqrt{3}t)-3\frac{2}{9}+3\cdot\frac{2}{3}t^2+\frac{2}{9_3}\cos(\sqrt{3}t)$

VIDI FESTINI

Branimir Pijoag

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!! 32

IME I PREZIME: JOSIP MARIĆ **VRIJEME POČETKA:** 09 00 h

MATIČNI BROJ STUDENTA: 17-2-0227-2012 **USTMENI ISPIT KOD NASTAVNIKA:** N. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + 3\,y'(t) = t^2, \quad y(0)' = y''(0) = 0,\ y(0) = 2.$$

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 9$, ravninom $z = -4$ i parabolom $z = x^2 + y^2$. Napomena: tijelo koje se traži sadrži ishodište. 20

3. Izračunaj integral funkcije $f(x, y, z) = x$ u dijelu prostora omeđenog plohama $x = z^2$, $x = 4$, $y = -1$ i $y = 4 + x$. 20

4. Izračunati površinu plohe u obliku stošca koja odgovara eksplicitnoj jednadžbi $x^2 + y^2 = \frac{z^2}{2}$ gdje je $0 \le z \le 4$. 20

5. Primjenom Stokesove formule izračunati $\oint_{\partial S} x\,dx + (x+y)\,dy + (x+y+z)\,dz$ ako je $S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 \le 3,\ z = x$ 20

Ukupno: $\varnothing$

1. $y'''(t) + 3y'(t) = t^2 \qquad y(0)' = y''(0) = 0,\ y(0) = 2$

$$s^3 y(t) - s^2 y(t) - s^1 y'(t) - y''(t) + 3s^1 y(t) - 3y(t) = t^2$$

$$t^2 = \frac{2}{s^3}$$

$$2s^3 - 2s^2 - 6s - 6y = \frac{2}{s^3}$$

$$2s(s^2 - s - 3) - 6y = \frac{2}{s^3}$$

$$2s(s^2 - s - 3) - 6y = \frac{2}{s^3} \quad \Big| : (2s(s^2-s-3)-6)$$

$$\cancel{-6y = \frac{2}{s^3} - 2s(s^2-s-3)} \quad \cancel{| : -6}$$

$$\cancel{y = \frac{\frac{2}{s^3} - 2s(s^2-s-3)}{\frac{6}{1}}}$$

$$y = \frac{\frac{2}{s^3}}{2s(s^2-s-3)-6} = \frac{2}{2s^4(s^2-s-3) - 6s^3}$$

$$\frac{2}{2s^4(s^2-s-3) - 6s^3} = \frac{A}{2s^4(s^2-s-3)} + \frac{Bs+C}{-6s^3} \quad \Big| \cdot 2s^4(s^2-s-3)-6s^3$$

$$2 = -6As^3 + 2s^4(s^2-s-3)\cdot Bs + 2s^4(s^2-s-3)\cdot C$$

$$2 = -6As^3 + 2Bs^5(Bs^3 - Bs^2 - 3Bs) + 2Cs^4(Cs^2 - Cs - 3C)$$

$s^5 \rightarrow 2B$

$s^4 \rightarrow 2C$

$s^3 \rightarrow -6A + B = 0 \quad 6A = -B \Rightarrow 6A = -3 \,/ :6 \Rightarrow A = -\frac{3}{6} = -\frac{1}{2}$

$s^2 \rightarrow -B + C = 0 \quad -B = -C \Rightarrow -B = -3B \,/ :(B) \quad B = 3$

$s \rightarrow -3B - C = 0 \quad -C = -3B \,/ \cdot(-1)$

$C = 3B = 3\cdot 3 = 9$

$$y = \frac{-\frac{1}{2}}{\underline{2s^4(s^2-s-3)}} + \frac{3s+9}{-6s^3} = \frac{1}{6s^4(s^2-s-3)} + \frac{3s+9}{-6s^3}$$

1

0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ 32 **Ime i prezime:** TOMISLAV GLAVAN **Vrijeme početka:** 09:49 h **Matični broj studenta:** 17-0115-2011 **Ustmeni ispit kod nastavnika:** UGLEŠIĆ	Popunjava nastavnik Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + 3\,y'(t) = t^2, \quad y(0)' = y''(0) = 0,\; y(0) = 2.$$

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 9$, ravninom $z = -4$ i parabolom $z = x^2 + y^2$. Napomena: tijelo koje se traži sadrži ishodište. 20

3. Izračunaj integral funkcije $f(x, y, z) = x$ u dijelu prostora omeđenog plohama $x = z^2$, $x = 4$, $y = -1$ i $y = 4 + x$. 20

4. Izračunati površinu plohe u obliku stošca koja odgovara eksplicitnoj jednadžbi $x^2 + y^2 = \frac{z^2}{2}$ gdje je $0 \leq z \leq 4$. 20

5. Primjenom Stokesove formule izračunati $\oint_{\partial S} x\,dx + (x+y)\,dy + (x+y+z)\,dz$ ako je $S = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 \leq 3,\ z = x$ 20

Ukupno: Ø

$$y'''(t) + 3y'(t) = t^2, \quad y(0) = y''(0) = 0, \; y(0) = 2$$

Ø

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 32

IME I PREZIME: ANDRO KLARIN

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA:

USTMENI ISPIT KOD NASTAVNIKA: Dr. sc. NIKICA UGLEŠIĆ

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + 3\,y'(t) = t^2, \quad y(0)' = y''(0) = 0,\, y(0) = 2.$$

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4. Izračunati površinu plohe u obliku stošca koja odgovara eksplicitnoj jednadžbi $x^2 + y^2 = \frac{z^2}{2}$ gdje je $0 \leq z \leq 4$. 20
5. Primjenom Stokesove formule izračunati $\oint_{\partial S} x\,dx + (x + y)\,dy + (x + y + z)\,dz$ ako je $S = \{(x, y, z) \in \mathbb{R}^3 \,|\, x^2 + y^2 \leq 3,\, z = x$ 20

Ukupno: 0

(1.) $y'''(t) + 3y'(t) = t^2$

$y(0)' = y''(0) = 0$

$y(0) = 2$

$s^3 y(s) - s^2 y(0) - s y'(0) - y''(0) + 3(s y(s) - y(0)) =$

$s^3 y(s) - s^2 \cdot 2 - s \cdot 0 - 0 + 3 s y(s) - 3 \cdot 0 =$

$s^3 y(s) - 2s^2 + 3 s y(s) =$

$s^3 y(s) + 3 s y(s) = 2s^2$

$y(s)(s^3 + 3s) =$

0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 32

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: DINO CVITAN **VRIJEME POČETKA:** 9 45

MATIČNI BROJ STUDENTA: 17-2-0068-2010 **USTMENI ISPIT KOD NASTAVNIKA:** NIKICA UGLEŠIĆ

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$y'''(t) + 3\,y'(t) = t^2, \quad y(0)' = y''(0) = 0,\, y(0) = 2.$$

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 9$, ravninom $z = -4$ i parabolom $z = x^2 + y^2$. Napomena: tijelo koje se traži sadrži ishodište. 20

3. Izračunaj integral funkcije $f(x, y, z) = x$ u dijelu prostora omeđenog plohama $x = z^2$, $x = 4$, $y = -1$ i $y = 4 + x$. 20

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5. Primjenom Stokesove formule izračunati $\oint_{\widehat{\partial S}} x\,\mathrm{d}x + (x+y)\,\mathrm{d}y + (x+y+z)\,\mathrm{d}z$ ako je $S = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + 2\,$ 20 $\leq 3,\; z = x$

Ukupno: 0

1. $y'''(t) + 3y'(t) = t^2, \; y(0)' = y''(0), \; y(0) = 2$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ 32	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ANTONIO ŠEKULA **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: 17-2-0025-2010 **USMENI ISPIT KOD NASTAVNIKA:**	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + 3\,y'(t) = t^2, \quad y(0)' = y''(0) = 0,\ y(0) = 2.$$

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 9$, ravninom $z = -4$ i parabolom $z = x^2 + y^2$. Napomena: tijelo koje se traži sadrži ishodište. 20

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Ukupno: 0

(1.)

$$y'''(t) + 3y'(t) = t^2$$

$y'(0) = 0$

$y''(0) = 0$

$y(0) = 2$

$$y'''(t) \to s^3 Y(s) - s^2 \underbrace{y(0)}_{2} - s\underbrace{y'(0)}_{0} - \underbrace{y''(0)}_{0}$$

$$s^3 Y(s) - 2s^2$$

$$3y'(t) \to sY(s) - \underbrace{y(0)}_{2}$$

$$3sY(s) - 6$$

$$t^2 \to \frac{2}{s^{2+1}} = \frac{2}{s^3}$$

$$s^3 Y(s) - 2s^2 + 3sY(s) - 6 = \frac{2}{s^3}$$

$$Y(s)\underbrace{(s^3 + 3s)}_{s(s^2+3)} = \frac{2}{s^3} + 2s^2 + 6$$

$$= \frac{2 + 2s^5 + 6s^3}{s^3 \cdot s(s^2+3)}$$

$$\frac{2}{s^3 \cdot s(s^2+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s} + \frac{Es+F}{s^2+3} \Big/ \cdot s^3 \cdot s(s^2+3)$$

$$A(s^5+3s^3) + B(s^4+3s^2) + C(s^3+3s) + D(s^5+3s^3)$$

$$Es^5 + Fs^4$$

$$As^5 + 3As^3 + Bs^4 + 3Bs^2 + Cs^3 + 3Cs + Ds^5 + 3Ds^3$$

$$+Es^5 + Fs^4$$

$A + D + E = 0$

$B + F = 0$

$3A + C + 3D = 0$

$3B + 3D \; 0$

$3C = 0$

$F = 0$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ 32	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: MARINO ŽUBČIĆ **Vrijeme početka:**	
Matični broj studenta: 17-2-0216-2012 **Usmeni ispit kod nastavnika:** prof. Uglešić	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + 3\,y'(t) = t^2, \quad y(0)' = y''(0) = 0,\ y(0) = 2.$$

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Ukupno: 0