MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: STIPE BRKLJAČA

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA:

USTMENI ISPIT KOD NASTAVNIKA:

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20

$$x'''(t) + 4x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 4.$$

2. Neka je kvadrat $K = \{(x, y, z) \,:\, x = 3,\ y \in [-2, 2],\ z \in [-2, 2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x ds$?. 20

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? 20

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. 20

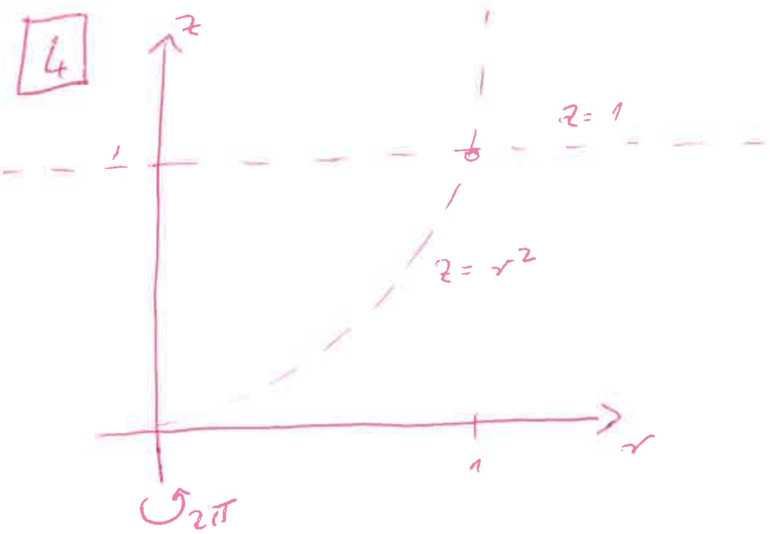
5. Neka je K kvadar $(0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3)$ i ∂K rub tog kvadra. Izračunati plošni integral 20

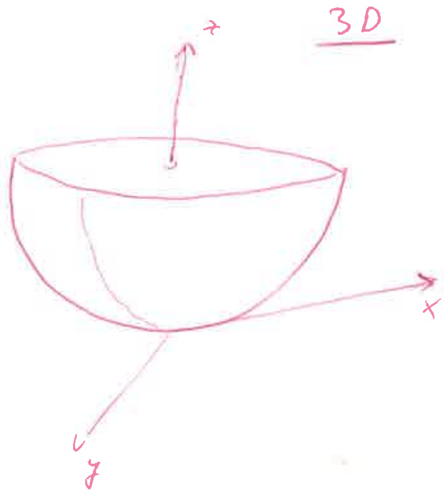
$$\iint\limits_{\widehat{\partial K}} zy\, \mathrm{d}y\mathrm{d}z + xy\, \mathrm{d}x\mathrm{d}z + x\, \mathrm{d}x\mathrm{d}y$$

Ukupno: 80

POLARNE KOORDINATE

$\boxed{4}$





3. $g = xi + yj + z^2 k$

POGREŠAN ARGUMENT

$r(t) = \begin{pmatrix} x \\ y \\ z^2 \end{pmatrix} \quad r'(t) = \begin{pmatrix} 1 \\ 1 \\ 2z \end{pmatrix}$

$\|r'(t)\| = \sqrt{1^2 + 1^2 + (2z)^2} = \sqrt{1+1+4z^2} = \sqrt{2+4z^2}$

$= \sqrt{2} + 2z$

OVISI O PUTU INTEGRACIJE

$\int_{?}^{?} = ?$ Da bi riješili integral moramo imati zadane granice, $\int \|r'(t)\| \cdot ds$ ovisi o putu integracije.

VIDI KURFIRST VUKUŠIĆ

4. $z = x^2 + y^2$, $z = 1$

$z = r^2$ $\qquad r \in [0, 1]$

$\qquad \varphi \in [0, 2\pi]$

$r = 1$ $\qquad z \in [r^2, 1]$

$$\int_0^{2\pi} \int_0^1 \int_{r^2}^1 r \, dz \, dr \, d\varphi =$$

$$\int_0^{2\pi} \int_0^1 r[1 - r^2] \, dr \, d\varphi =$$

$$\int_0^{2\pi} \int_0^1 r - r^3 \, dr \, d\varphi =$$

$$\int_0^{2\pi} \frac{r^2}{2} - \frac{r^4}{4} \Big|_0^1 d\varphi = \int_0^{2\pi} \frac{1}{2} - \frac{1}{4} \, d\varphi = \int_0^{2\pi} \frac{1}{4} \, d\varphi$$

$$= \frac{1}{4} \varphi \Big|_0^{2\pi} = \frac{2}{4} \pi = \frac{1}{2} \pi$$

5. $\iint_{\partial K} zy\,dy\,dz + xy\,dx\,dz + x\,dx\,dy$

$x \in [0,1]$
$y \in [0,2]$
$z \in [0,3]$

$$\operatorname{div} W = \begin{pmatrix} zy \\ xy \\ x \end{pmatrix}$$

$$\operatorname{div} W = \frac{\partial y}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{z} = \frac{zy}{\partial y} + \frac{xy}{\partial y} + \frac{\partial x}{\partial z} =$$

$$= 0 + x + 0 \cancel{=}$$

$$\int_0^1 \int_0^2 \int_0^3 x \; dz\,dy\,dx =$$

$$\int_0^1 \int_0^2 xz \Big|_0^3 dy\,dx = \int_0^1 \int_0^2 3x \; dy\,dx = \int_0^1 3x \cdot \int_0^2 dx =$$

$$= \int_0^1 6x\,dx = 6\frac{x^2}{2} = 6 \cdot \frac{1}{2} = 3$$

2. $x = 3 \quad y \in [-2,2] \quad z \in [-2,2]$

$$\iint_{\partial K} x\,ds =$$

$$\iint 3\,ds = \int_{-2}^{2} \int_{-2}^{2} 3\,dy\,dz = \int_{-2}^{2} 3\underbrace{[2+2]}_{4}\,dz = \int_{-2}^{2} 12\,dz =$$

$$= 12 \cdot [2+2] = 12 \cdot 4$$
$$= 48$$

VIDI RADOVČIĆ

1. $x'''(t) + 4x'(t) = t \qquad x'(0) = x''(0) = 0, \; x(0) = 4$

$$s^3F(s) - s^2\overbrace{f(0)}^{4} - s\overbrace{f'(0)}^{0} - \overbrace{f''(0)}^{0} + 4sF(s) - 4\overbrace{f(0)}^{4} = \frac{1}{s^2}$$

$$F(s)(s^3 + 4s) - 4s^2 - 16 = \frac{1}{s^2}$$

$$F(s)(s^3+4s) = \frac{1}{s^2} + 4s^2 + 16$$

$s(s^2+4)$

$$F(s)(s^3+4s) = \frac{1+4s^4+16s^2}{s^2} \quad /(s^3+4s)$$

$s(s^2+4)$

$s^2(s^3+4s)$

$s^3(s^2+4s)$

$$F(s) = \frac{\dfrac{1+4s^4+16s^2}{s^2}}{s(s^2+4)} = \frac{1+4s^4+16s^2}{s^3(s^2+4)}$$

1

$$F(s)\ \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{(s^2+4)} \quad /s^3(s^2+4)$$

$$F(s) = As^2(s^2+4) + Bs(s^2+4) + C(s^2+4) + Ds^4 + Es^3$$

$$= As^4 + 4As^2 + Bs^3 + 4Bs + Cs^2 + 4C + Ds^4 + Es^3$$

$s^4(A+D)$	4
$s^3(B+E)$	0
$s^2(4A+C)$	16
$s(4B)$	0
$C(4C) =$	1

$4C = 1$

$C = \frac{1}{4}$

$C = 1$

$B = 0 \qquad E = 0$

$4A + C = 16$

$4A + \frac{1}{4} = 16$

$4A = 16 - \frac{1}{4}$

$4A = 15/\frac{1}{4}$

$4A = \frac{63}{4} \; / : 4$

$A = \frac{63}{16}$

$16 - 1 = C$

$C = 1$

$4 = A + D$

$4 = \frac{63}{16} + D$

$D = 4 - \frac{63}{16}$

$D = \frac{1}{16}$

$$F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{(s^2+4)}$$

$A = \frac{63}{16}$

$B = 0$

$C = \frac{1}{4}$

$D = \frac{1}{16}$

$E = 0$

$$F(s) = \frac{\frac{63}{16}}{s} + \cancel{\frac{0}{s^2}} + \frac{1}{4}\frac{1}{s^3} + \frac{1}{16}\frac{s}{s^2+4}$$

$$F(s) = \frac{63}{16} + \frac{1}{4}\frac{t^2}{2} + \frac{1}{16}\cos(2t) = \frac{63}{16} + \frac{1}{8}t^2 + \frac{1}{16}\cos(2t)$$

PROVJERA:

$$F'(s) = \frac{1}{4}t - \frac{1}{8}\sin(2t) = \frac{1}{4}\cdot 0 - \frac{1}{8}\sin(2\cdot 0) = 0$$

$$\frac{t^n}{} \quad \frac{n!}{s^{n+1}} = 2\frac{1}{s^{2+1}} = \frac{1}{2}t^2$$

$$F''(s) = \frac{1}{4} - \frac{1}{4}\cos(2t) = \frac{1}{4} - \frac{1}{4}\cos(2\cdot 0) = \frac{1}{4} - \frac{1}{4} = 0$$

$$F'''(s) = \frac{1}{2}\sin(2t) = \frac{1}{2}\sin(2\cdot 0) = 0$$

$$\frac{1}{2}\sin(2t) + 4\left(\frac{1}{4}t - \frac{1}{8}\sin(2t)\right) = t$$

$$\cancel{\frac{1}{2}\sin(2t)} + \cancel{t} - \cancel{\frac{1}{2}\sin(2t)} = \cancel{t}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31

IME I PREZIME: Ines Kurfirst Valušić **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA: **USTMENI ISPIT KOD NASTAVNIKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: ~~20~~

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$$\iint\limits_{\widehat{\partial K}} zy\,dydz + xy\,dxdz + x\,dxdy$$

POGLEDATI

Ukupno: ~~60~~ ~~80~~

40 Kos

1. $x'''(t) + 4x'(t) = t \qquad x'(0) = x''(0) = 0,\ x(0) = 4$

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 4(sF(s) - f(0)) = \frac{1}{s^2}$

$s^3F(s) - 4s^2 + 4sF(s) - 16 = \frac{1}{s^2}$

$F(s)(s^3 + 4s) = \frac{1}{s^2} + 4s^2 + 16 \quad \Big/ s^3 + 4s$

$F(s)\ s(s^2+4) = \frac{1 + 4s^4 + 16s^2}{s^2} \quad \Big/ s(s^2+4)$

$\frac{1}{s(s^2+4)} \cdot \frac{1 + 4s^4 + 16s^2}{s^2} = \frac{1 + 4s^4 + 16s^2}{s^3(s^2+4)}$

$\frac{1 + 4s^4 + 16s^2}{s^3(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2+4} \quad \Big/ \cdot s^3(s^2+4)$

$1 + 4s^4 + 16s^2 = As^2(s^2+4) + Bs(s^2+4) + C(s^2+4) + (Ds+E)\cdot s^3$

$= As^4 + 4As^2 + Bs^3 + 4Bs + Cs^2 + 4C + Ds^4 + Es^3$

$4 = A + D$
$0 = B + E$
$16 = 4A + C$
$1 = 4C$
$0 = 4B$
$-4B = 0$
$-4C = 1$
$C = \frac{1}{4}$

$16 = 4A + \frac{1}{4}$
$-4A = \frac{1}{4} - 16$

$0 = B + E$
$0 = 0 + E$
$E = 0$

$4 = \frac{1}{5} + D$
$-D = \frac{1}{5} - 4$
$-D = -\frac{19}{5}$
$D = \frac{19}{5}$

$\frac{65}{16} \times \frac{s}{s^2+4}$

$F(s) = \frac{1}{16} \cdot \frac{1}{s} + \frac{1}{2\times 4} \times \frac{1}{s^3} +$

$F(t) = \frac{1}{16} + \frac{1}{8}t^2 + \frac{65}{16}\cos 2t$

VIDI BRKLJAČA

3. $g = x\,i + y\,j + z^2 k$

$$\left.\begin{aligned} g_x &= \partial_x f \\ g_y &= \partial_y f \\ g_z &= \partial_z f \end{aligned}\right\} \Rightarrow \begin{aligned} x &= \partial_x f \ / \int \\ y &= \partial_y f \ / \int \\ z^2 &= \partial_z f \ / \int \end{aligned}$$

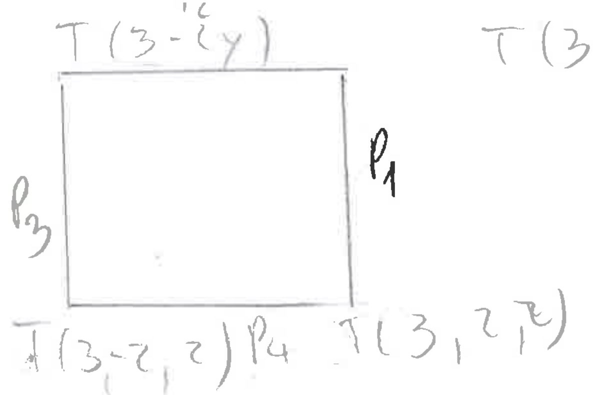
$$\left.\begin{aligned} \int x\,\partial x &= \int \partial_x f\,\partial x \\ \int y\,\partial y &= \int \partial_y f\,\partial y \\ \int z^2\,\partial z &= \int \partial_z f\,\partial z \end{aligned}\right\} \Rightarrow \begin{aligned} \frac{x^2}{2} &= f(x,y,z) \text{ Nešto što ne ovisi o } x \\ \frac{y^2}{2} &= f(x,y,z) \text{ -||- o } y \\ \frac{z^3}{3} &= f(x,y,z) \text{ -||- o } z \end{aligned}$$

$$\Rightarrow f(x,y,z) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^3}{3}$$

g je potencijalno polje $\Rightarrow$ integral ne ovisi o putu integracije

Ines Kuitist Vulušić

$x=3$



$P_1 \quad r_1 t = \begin{pmatrix} 3 \\ 2 \\ y \end{pmatrix}$

$P_2 \quad r_2 t = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$P_3 \quad r_3 t = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$

$P_4 \quad r_4 t = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$

OVO NISU PARAMETRIZACIJE

T(2,2) $\sqrt{r_1'(t)^2} = 1$

T(2,2) $\sqrt{r_2'(t)} = 1$

KAKO?

$$\int_{\partial k} x\,ds = \int_{r_1} x\,ds + \int_{r_2} x\,ds + \int_{r_3} x\,ds + \int_{r_4} x\,ds +$$

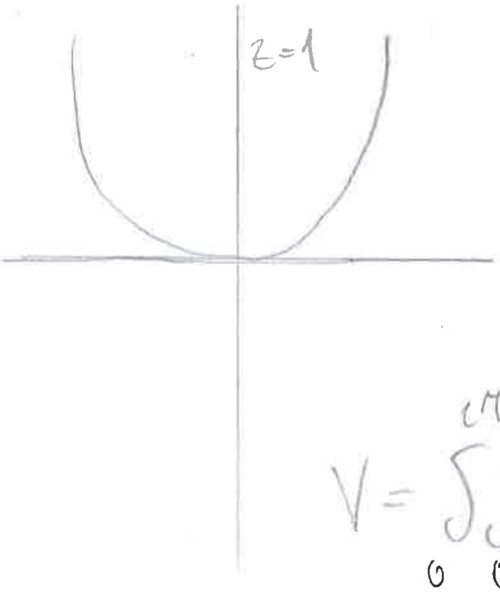
$$= \int_{-2}^{3} 3\cdot 1\,dt + \int_{-2}^{2} 3\cdot 1\,dt \int_{-2}^{2} 3\cdot 1\,dt \int_{-2}^{2} 3\cdot 1\,dt$$

$$= 4\int_{-2}^{3} 3\,dt = 4\cdot 3t \Big|_{2} = 12\cdot 2 + 12\cdot 2 = 4\cdot 12 = \underline{\underline{48}}$$

✓

NAKON KONZULTACIJA SA PROF. UGLEŠIĆEM
BODOVI NISU DODIJELJENI U 2. i 4. ZADATKU

4) $z=x^2+y^2$, $z=1$ Ines Karpist Valunsic



$r^2=1$ $r \in [0,1]$

cilindrični koordinati

$r \in (0,1)$

$\varphi \in 0,2\pi$ $z\,[r^2,1]$

$$V=\int_0^{2\pi}\int_0^1\int_{r^2}^1 1\cdot r\,dz\,dr\,d\varphi =$$

$$=\int_0^{2\pi}\int_0^1 rz\Big|_{r^2}^1 dr\,d\varphi = \int_0^{2\pi}\int_0^1 (1-r^2)\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^1 (r-r^3)\,dr\,d\varphi = \int_0^{2\pi} \frac{r^2}{2}-\frac{r^4}{4}\,d\varphi =$$

$$=\int_0^{2\pi} \frac{1}{2}+\frac{1}{4}\,d\varphi = \frac{1}{4}\Big|_0^{2\pi} = \frac{2\pi}{4} = \frac{\pi}{2}$$ ✓

5.) $$\iint_{\partial K} zy\,dy\,dz + xy\,dx\,dz + x\,dx\,dy =$$ POGREŠNO!

$g = zy\vec{i} + xy\vec{j} + x\vec{k}$

?

$$\iint_{\partial K} g\,ds = h\iiint x\,dx\,dy\,dz$$ ↖ POGREŠNO ? KAKO BODOVATI

← ISPRAVNO

$$\int_0^3\int_0^2\int_0^1 x\,dx\,dy\,dz = \int_0^3\int_0^2 \frac{x^2}{2}\Big|_0^1 dy\,dz =$$

$$\int_0^3\int_0^2 \frac{1}{2}\,dy\,dz = \int_0^3 y\Big|_0^2 dz = \int_0^3 1\,dz = 1\Big|_0^3 dz = \underline{\underline{3}}$$

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IME I PREZIME: Rikardo Radovčić **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: 17-2-0228-2012 **USTMENI ISPIT KOD NASTAVNIKA:**	

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$$\iint\limits_{\widehat{\partial K}} zy\,\mathrm{d}y\mathrm{d}z + xy\,\mathrm{d}x\mathrm{d}z + x\,\mathrm{d}x\mathrm{d}y$$

Ukupno: 40

4

$z = x^2 + y^2 \qquad z = 1 \quad , V = ?$

$z = r^2\cos^2\varphi + r^2\sin^2\varphi$

$z = r^2$

$x = r\cos\varphi$

$y = \sin\varphi$

$z = 1$

?

$$V = \int_0^{2\pi} d\varphi \int_0^{\sqrt{2}} r\,dr \cdot \int_{r^2}^{1} dt$$

$$V = 2\pi \int_0^{\sqrt{2}} r(1 - r^2)\,dr = 2\pi\left(\frac{r^2}{2} - \frac{r^3}{3}\right)\Big|_0^{\sqrt{2}}$$

$$V = 2\pi \cdot \left(\frac{2}{2} - \frac{2\sqrt{2}}{3}\right)$$

$$V = 2\pi\left(\frac{3 - 2\sqrt{2}}{3}\right)$$

X

① $x'''(t) + 4x'(t) = t \qquad x'(0) = x''(0) = 0 \qquad x(0) = 4$

$$x(t) \circ\!\!-\!\!\bullet\; y(s)$$
$$x'(t) \circ\!\!-\!\!\bullet\; sy(s) - x(0) = s\,y(s) - 4$$
$$x''(t) \circ\!\!-\!\!\bullet\; s(sy(s)-4) - x'(0) = s^2 y(s) - 4s$$
$$x'''(t) \circ\!\!-\!\!\bullet\; s(s^2 y(s) - 4s) - x''(0) = s^3 y(s) - 4s^2$$

$$s^2 y(s) - 4s^2 + 4(sy(s) - 4) = \frac{1}{s^2}$$
$$s^3 y(s) - 4s^2 + 4s\,y(s) - 16 = \frac{1}{s^2}$$
$$y(s)(s^3 + 4s) = \frac{1}{s^2} + 4s^2 + 16$$
$$y(s) = (s^3 + 4s) = \frac{1}{s^2} + 4s^2 + 16$$
$$y(s) = \frac{\frac{1}{s^2} + 4s^2 + 16}{s^3 + 4s} = \frac{1}{s^3(s^2+4)} + \frac{4(s^2+4)}{s(s^2+4)} =$$
$$= \frac{1}{s^3(s^2+4)} + \frac{4}{s}$$
$$= -\frac{1}{16}\cdot\frac{1}{s} + \frac{1}{4}\cdot\frac{1}{s^3} + \frac{1}{16}\cdot\frac{s}{s^2+4} + \frac{4}{s}$$
$$= \frac{63}{16}\cdot\frac{1}{s} + \frac{1}{8}\cdot\frac{2}{s^3} + \frac{1}{16}\cdot\frac{s}{s^2+4}$$

$$\circ\!\!-\!\!\bullet\; \boxed{x(t) = \frac{63}{16} + \frac{1}{8}t^2 + \frac{1}{16}\cdot\cos^2 t} \times$$

VIDI BRKLJAČA

$$\frac{1}{s^3(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^2} + \frac{Ds+E}{s^2+4} \Big/ \cdot\, s^3(s^2+4)$$

$$1 = As^4 + A\cdot 4s^2 + Bs^3 + 4Bs + Cs^2 + 4c + Ds^4 + Es^3$$

$0 = A + D$

$0 = B + E \Rightarrow \boxed{E = 0}$

$0 = 4A + C$

$0 = 4B \Rightarrow \boxed{B = 0}$

$1 = 4C \Rightarrow \boxed{C = \frac{1}{4}}$

$4A = -\frac{1}{4}$

$\boxed{A = -\frac{1}{16}}$

$\boxed{D = \frac{1}{16}}$

(2) $K = \{(x,y,z) : x = 3,\ y \in [-2,2],\ z \in [-2,2]\}$

$\iint_{\partial K} x\,ds = ?$

$\Gamma_1 : r_1(t) = \begin{pmatrix} 3 \\ 2 \\ t \end{pmatrix},\ t \in [-2,2]$

$x = 3$

$\sqrt{r_1'(t)^2} = 1$

$\Gamma_2 : r_2(t) = \begin{pmatrix} 3 \\ t \\ 2 \end{pmatrix},\ t \in [-2,2],\ \sqrt{r_2'(t)^2} = 1$

$\Gamma_3 : r_3(t) = \begin{pmatrix} 3 \\ -2 \\ t \end{pmatrix},\ t \in [-2,2],\ \sqrt{r_3'(t)^2} = 1$

$\Gamma_4 : r_4(t) = \begin{pmatrix} 3 \\ t \\ -2 \end{pmatrix},\ t \in [-2,2],\ \sqrt{r_4'(t)^2} = 1$ ✓

$$\iint_{\partial K} x\,ds = \int_{\partial K} x\,ds = \int_{\Gamma_1} x\,ds + \int_{\Gamma_2} x\,ds + \int_{\Gamma_3} x\,ds + \int_{\Gamma_4} x\,ds$$

$$= \int_{-2}^{2} 3\,dt + \int_{-2}^{2} 3\,dt + \int_{-2}^{2} 3\,dt + \int_{-2}^{2} 3\,dt = 4 \cdot \int_{-2}^{2} 3\,dt$$

$$= 4 \cdot 3t \Big|_{-2}^{2} = 12(2+2) = 48$$ ✓

RIKARDO RADOVČIĆ 30.04.2015.

RR

⑤ $g = zy\vec{i} + xy\vec{j} + x\vec{k}$

$$\operatorname{div} g = \frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} + \frac{\partial g_z}{\partial z} = (0 + x + 0) = x$$

$$\iint_S g\, ds = \iiint_K \operatorname{div} g\, dx\, dy\, dz =$$

$$= \int_0^3 \int_0^2 \int_0^1 x\, dx\, dy\, dz = \int_0^3 \int_0^2 \frac{x^2}{2} \Big|_0^1 dy\, dz =$$

$$= \int_0^3 \int_0^2 \frac{1}{2}\, dy\, dz = \int_0^3 \frac{1}{2} y \Big|_0^2 dz = \int_0^3 1\, dz =$$

$$= z \Big|_0^3 = 3$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31 **IME I PREZIME:** LUKA MILIN **VRIJEME POČETKA:** 09:00 **MATIČNI BROJ STUDENTA:** 17-2-0177-2012 **USTMENI ISPIT KOD NASTAVNIKA:** UGLEŠIĆA	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20

$$x'''(t) + 4x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 4.$$

2. Neka je kvadrat $K = \{(x, y, z) : x = 3,\ y \in [-2, 2],\ z \in [-2, 2]\}$. Kako preko definicije izračunati $\iint_{\partial K} x ds$?. 20

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? 20

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. 20

5. Neka je K kvadar $(0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3)$ i ∂K rub tog kvadra. Izračunati plošni integral 20

$$\iint_{\widehat{\partial K}} zy\, dydz + xy\, dxdz + x\, dxdy$$

Ukupno: 20

(1) $x'''(t) + 4x'(t) = t \qquad x'(0)=0,\ x''(0)=0,\ x(0)=4$

$x'''(t) = s^3F(s) - s^2x(0) - s x'(0) - x''(0)$

$= s^3F(s) - 4\cdot s^2$

$t = \frac{1}{s^2}$

$x'(t) = sF(s) - x(0)$

$= sF(s) - 4$

$s^3F(s) - 4s^2 + 4(sF(s) - 4) = \frac{1}{s^2}$

$s^3F(s) - 4s^2 + 4sF(s) - 16 = \frac{1}{s^2}$

$F(s)(s^3 - 4s^2 + 4s) = \frac{1}{s^2} + 16$

$F(s)(s^3 - 4s^2 + 4s) = \frac{1+16s^2}{s^2} \ /: (s^3 - 4s^2 + 4s)$

$F(s) = \frac{1+16s^2}{(s^2)(s^3-4s^2+4s)} = \frac{1+16s^2}{(s^2)(s)(s-2)(s-2)}$

$(s-2)(s-2) = s^2 - 2s - 2s + 4$

$s^2 - 4s + 4$

$(s^3 - 4s^2 + 4s)$

$s(s^2 - 4s + 4)$

$s^2 - 4s + 4 = 0$

$\frac{-b}{4ac}$

$A = 1$

$B = -4$

$C = 4$

$s = \frac{-b \pm \sqrt{4ac - b^2}}{2a}$

$s = \frac{4 \pm \sqrt{16-16}}{2} = \frac{4 \pm 0}{2} = \frac{4}{2} = 2$

$\frac{1+16s^2}{(s^3)(s-2)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-2} + \frac{E}{(s-2)^2} \ / \cdot (s^3)(s-2)^2$

$1 + 16s^2 = A(s^2)(s-2)^2 + B(s)(s-2)^2 + C(s-2)^2 + D(s^3)(s-2) + E(s^3)$

$1 + 16s^2 = As^2(s^2-4s+4) + Bs(s^2-4s+4) + C(s^2-4s+4) + Ds^3(s-2) + Es^3$

$1 + 16s^2 = As^4 - 4As^3 + 4As^2 + Bs^3 - 4Bs^2 + 4Bs + Cs^2 - 4Cs + 4C + Ds^4 - 2Ds^3 + Es^3$

$s^4 \dots\ 0 = A + D \qquad s^2 \dots\ 16 = 4A - 4B + C \qquad 1 = 4C$

$s^3 \dots\ 0 = -4A + B + E - 2D \qquad s \dots\ 0 = 4B - 4C$

④ $z = x^2 + y^2 \qquad z = 1$

$x^2 + y^2 = r^2$

$z = r^2$

$r^2 = 1$

$r = \sqrt{1}$

$r = 1$

$r \in [0,1]$

$\varphi \in [0, 2\pi]$

$z \in [r^2, 1]$

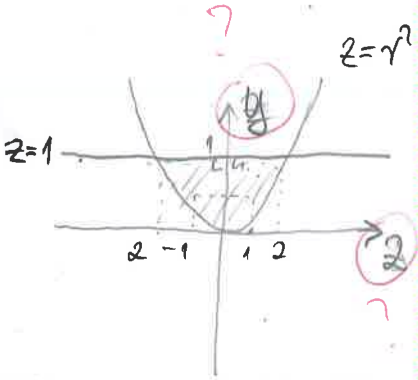
POL. KORD.

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$



$$V = \int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2} dz\,dy\,dx = \int_0^{2\pi}\int_0^1\int_{r^2}^1 r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^1 r\cdot z\Big|_{r^2}^{1} dr\,d\varphi$$

$$V = \int_0^{2\pi}\int_0^1 r - (r^3)\,dr\,d\varphi = \int_0^{2\pi}\int_0^1 r - r^3\,dr\,d\varphi = \int_0^{2\pi} \frac{r^2}{2} - \frac{r^4}{4}\Big|_0^1 d\varphi$$

$$V = \int_0^{2\pi} \frac{1}{2} - \frac{1}{4}\,d\varphi = \int_0^{2\pi} \frac{1}{4}\,d\varphi = \frac{1}{4}\varphi\Big|_0^{2\pi} = \frac{1}{4}\cdot 2\pi - \underbrace{\left(\frac{1}{4}\cdot 0\right)}_{0} = \frac{1}{2}\pi$$

⑤ $(0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3) \qquad x \in [0,1],\ y \in [0,2],\ z \in [0,3]$

$$= \iiint 2y\,dy\,dz + xy\,dx\,dz + x\,dx\,dy$$

$$= \int_0^1\int_0^2\int_0^3 2y\,dy\,dz + xy\,dx\,dz + x\,dx\,dy$$

Ø

② $K = \{(x,y,z) : x = 3 \qquad y \in [-2,2],\ z \in [-2,2] \quad ; \quad \iint_{\partial K} x\,ds = ?$

$$\int_{x_1}\int_{y_1}\int_{z_1} x\,ds = \int\int_{-2}^{2}\int_{-2}^{2} x\,ds$$

Ø

LUKA MILIN - MATEMATIKA 3

30.04.2015

① ZADATAK (NASTAVAK)

$0 = A + D$

$0 = -4A + B + E - 2D$

$16 = 4A - 4B + C$

$0 = 4B - 4C \Longrightarrow$

$1 = 4C \Rightarrow \quad 4C = 1 \ / \cdot \frac{1}{4}$

$\boxed{C = \frac{1}{4}}$

$0 = 4B - 4 \cdot \frac{1}{4}$

$0 = 4B - 1$

$-4B = -1 \ / \cdot \left(-\frac{1}{4}\right)$

$\boxed{B = \frac{1}{4}}$

$16 = 4A - 4 \cdot \frac{1}{4} + \frac{1}{4}$

$16 = 4A - 1 + \frac{1}{4}$

$-4A = -1 + \frac{1}{4} - 16$

$-4A = -\frac{67}{4} \cdot \left(-\frac{1}{4}\right)$

$\boxed{A = \frac{67}{16}}$

$0 = A + D$

$0 = \frac{67}{16} + D$

$-D = \frac{67}{16} \ / \cdot (-1)$

$\boxed{D = -\frac{67}{16}}$

$0 = -4A + B + E - 2D$

$0 = -4 \cdot \frac{67}{16} + \frac{1}{4} + E - 2 \cdot \left(-\frac{67}{16}\right)$

$0 = -\frac{67}{4} + \frac{1}{4} + E + \frac{67}{8}$

$-E = -\frac{67}{4} + \frac{1}{4} + \frac{67}{8}$

$-E = -\frac{65}{8} \ / \cdot (-1)$

$\boxed{E = \frac{65}{8}}$

$A = \frac{67}{16} \qquad C = \frac{1}{4}$

$B = \frac{1}{4} \qquad D = -\frac{67}{16}$

$E = \frac{65}{8}$

$y(s) = \frac{\frac{67}{16}}{s} + \frac{\frac{1}{4}}{s^2} + \frac{\frac{1}{4}}{s^3} + \frac{-\frac{67}{16}}{s-2} + \frac{\frac{65}{8}}{(s-2)^2}$

$y(s) = \frac{67}{16} \cdot \frac{1}{s} + \frac{1}{4} \cdot \frac{1}{s^2} + \frac{1}{4} \cdot \frac{1}{s^3} - \frac{67}{16} \cdot \frac{1}{s-2} + \frac{65}{8} \cdot \frac{1}{(s-2)^2}$

$y(t) = \frac{67}{16} + \frac{1}{4} t + \frac{1}{4} \cdot t^2 - \frac{67}{16} \cdot e^{2t} + \frac{65}{8} \cdot t e^{2t}$ ×

VIDI BRKLJAČA

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: JASMIN NEKIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA: 17-1-0058-2011 **USTMENI ISPIT KOD NASTAVNIKA:**

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20

$$x'''(t) + 4x'(t) = t, \quad x'(0) = x''(0) = 0,\, x(0) = 4.$$

2. Neka je kvadrat $K = \{(x, y, z) \,:\, x = 3,\, y \in [-2, 2]\,,\, z \in [-2, 2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x ds$?. 20

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? 20

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. ~~20~~

5. Neka je K kvadar $(0 \leq x \leq 1,\, 0 \leq y \leq 2,\, 0 \leq z \leq 3)$ i ∂K rub tog kvadra. Izračunati plošni integral ~~20~~

$$\iint\limits_{\widehat{\partial K}} zy\,\mathrm{d}y\mathrm{d}z + xy\,\mathrm{d}x\mathrm{d}z + x\,\mathrm{d}x\mathrm{d}y$$

Ukupno:

20

JASMIN NEKIĆ

(1) $x'''(t) + 4x'(t) = t \qquad x'(0) = x''(0) = 0, \quad x(0) = 4$

$$s^3 F(s) - s^2 f(0) - \underbrace{s f'(0)}_{0} - \underbrace{f''(0)}_{0} + 4(sF(s) - f(0)) = \frac{1}{s^2}$$

$$s^3 F(s) + 4sF(s) - 4s^2 - 16 = \frac{1}{s^2}$$

$$F(s)(s^3 + 4s) = \frac{1}{s^2} + 4s^2 + 16 = \frac{1 + 4s^4 + 16s^2}{s^2} \quad / : (s^3 + 4s)$$

$$F(s) = \frac{1 + 4s^4 + 16s^2}{s^2(s^3 + 4s)} = \frac{1 + 4s^4 + 16s^2}{s^3(s^2 + 4)}$$

$$1 + 4s^4 + 16s^2 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2 + 4}$$

$$= A(s^4 + 4s^2) + B(s^3 + 4s) + C(s^2 + 4) + s^3(Ds + E)$$

$$= As^4 + 4As^2 + Bs^3 + 4Bs + Cs^2 + 4C + Ds^4 + Es^3$$

$A + D = 4$
$B + E = 0$
$4A + C = 16$
$4B = 0$
$4C = 1$

$B = 0$
$C = \frac{1}{4}$
$E = 0$
$A = \frac{63}{16}$
$D = \frac{1}{16}$

$4A + \frac{1}{4} = 16$
$4A = \frac{64}{4} - \frac{1}{4}$
$4A = \frac{63}{4}$
$A = \frac{63}{16}$

$A + D = 4$
$D = \frac{64}{16} - \frac{63}{16}$
$D = \frac{1}{16}$

$$= \frac{63}{16} \cdot \frac{1}{s} + \frac{1}{4} \cdot \frac{1}{s^3} + \frac{\frac{1}{16} \cdot s}{s^2 + 4}$$

$$= \frac{63}{16} + \frac{1}{8} t^2 + \frac{1}{16} \cos(2t)$$ ✓

$\int \frac{1}{4} t - \frac{1}{8} \sin(2t) = 0 = x'(0)$

$\int \frac{1}{4} - \frac{1}{4} \cos 2(t) = 0 = x''(0)$

$= 4 = x(0)$ ✓

(4.) $z = x^2 + y^2$

$z = 1$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = r^2 = 1$

$$\int_0^{2\pi}\int_0^1 r \, dr \, d\varphi$$ ×

$$\int_0^{2\pi} \frac{r^2}{2}\Big|_0^1 d\varphi = \int_0^{2\pi} \frac{1}{2} d\varphi = \frac{1}{2}\cdot 2\pi = \pi$$

VIDI BRKYAČA

5. $$\iint_{\partial K} zy \, dy\, dz + xy \, dx\, dz + x \, dx\, dy$$

?

$$\boxed{\begin{array}{l} \frac{R(z)}{dz} - \frac{Q(y)}{dy} + \frac{R(z)}{dy} - \frac{P(x)}{dx} + \frac{Q(y)}{dy} - \frac{P(x)}{dx} \\ y - z + 0 - y + 0 - 1 \end{array}}$$

$$\int_0^1\int_0^2\int_0^3 (-z-1)\, dz\, dy\, dx = \int_0^1\int_0^2 \left(-\frac{z^2}{2} - z\right)\Big|_0^3 dy\, dx = \int_0^1\int_0^2 \frac{9}{2} - 3)\, dy\, dx =$$ ×

$$= \int_0^1 \frac{3}{2}y\Big|_0^2 dx = 3x\Big|_0^1 = 3$$

$\emptyset$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: ANTE ŠIŠAK

VRIJEME POČETKA: 09:00

MATIČNI BROJ STUDENTA: 17-2-0247-2012

USTMENI ISPIT KOD NASTAVNIKA:

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20
$$x'''(t) + 4x'(t) = t, \quad x'(0) = x''(0) = 0, \; x(0) = 4.$$
2. Neka je kvadrat $K = \{(x, y, z) : x = 3, \, y \in [-2, 2], \, z \in [-2, 2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x ds$?. 20
3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? 20
4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. 20
5. Neka je K kvadar $(0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3)$ i ∂K rub tog kvadra. Izračunati plošni integral 20
$$\iint\limits_{\widehat{\partial K}} zy \, dydz + xy \, dxdz + x \, dxdy$$

Ukupno: 20

ANIC SISAK 11/3

1. $x'''(t) + 4x'(t) = t \qquad x'(0) = x''(0) = 0, \quad x(0) = 4$

$$s^3 X(s) - s^2 x(0) - s x'(0) - x''(0) + 4 s X(s) + 4x(0) = \frac{1}{s^2}$$

$$s^3 X(s) - 4s^2 + 4s X(s) + 16 = \frac{1}{s^2}$$

$$X(s)\left(s^3 + 4s\right) = \frac{1}{s^2} + 4s^2 + 16 = \frac{4s^4 + 16s^2 + 1}{s^2}$$

$$X(s) = \frac{4s^4 + 16s^2 + 1}{s^3(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4}$$

$$4s^4 + 16s^2 + 1 = A\left(s^2(s^2+4)\right) + B\left(s(s^2+4)\right) + C(s^2+4) + \left((Ds+E)(s^3)\right)$$

$$4s^4 + 16s^2 + 1 = As^4 + 4As^2 + Bs^3 + 4Bs + Cs^2 + 4C + Ds^4 + Es^3$$

$s^4 \to \quad 4 = A + D$

$s^3 \to \quad 0 = B + E \Rightarrow \boxed{E = 0}$

$s^2 \to \quad 16 = C + 4A$

$s \to \quad 0 = 4B \Rightarrow \boxed{B = 0}$

$4C = 1 \Rightarrow \boxed{C = \frac{1}{4}}$

$4A + C = 16$

$4A = \frac{64}{4} - \frac{1}{4} = \frac{63}{4}$

$\boxed{A = \frac{63}{16}}$

$D = 4 - A$

$\boxed{D = \frac{1}{16}}$

$$X(s) = \frac{63}{16} \cdot \frac{1}{s} + \frac{1}{4} \cdot \frac{1}{s^3} + \frac{1}{16} \cdot \frac{1}{s^2+4}$$

$$X(s) \to x(t) = \frac{63}{16} + \frac{1}{4} \cdot \frac{t^2}{2} + \frac{1}{16} \cdot \frac{1}{2} \sin 2t$$

$$x(t) = \frac{63}{16} + \frac{1}{8} \cdot t^2 + \frac{1}{32} \sin 2t$$

$x(0) = 4$

$x(t) = x(0) = 4$

$$\boxed{x(t) = \frac{63}{16} + \frac{t^2}{8} + \frac{1}{32} \sin 2t}$$ ×

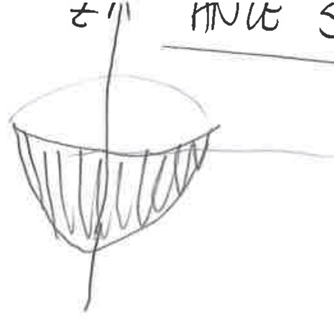
VIDI BRKYAĆA

4.

$z = x^2 + y^2$

$z = 1$

$$\begin{vmatrix} \cos\varphi & -R\sin\varphi \\ \sin\varphi & R\cos\varphi \end{vmatrix} = R$$



$x = R\cos\varphi$

$y = R\sin\varphi$

$$\int_0^{2\pi}\int_0^1\int_{R^2}^1 R\,dz\,dR\,d\varphi = \int_0^{2\pi} d\varphi \int_0^1 (R - R^3)\,dR = 2\pi\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{\pi}{2}$$

3. $g = \partial f$ je za neko skalarno polje f

$$g_x = \partial_x f \qquad x = \partial_x f \Rightarrow \int x\,dx = \int \partial_x f\,dx \Rightarrow \frac{x^2}{2} = f(x,y,z) + \left(\text{nešto neovisno o } x\right)$$

$$g_y = \partial_y f \Rightarrow \qquad y = \partial_y f \Rightarrow \int y\,dy = \int \partial_y f\,dy \Rightarrow \frac{y^2}{2} = f(x,y,z) + \left(\text{nešto neovisno o } y\right)$$

$$g_z = \partial_z f \qquad z = \partial_z f \Rightarrow \int z^2\,dz = \int \partial_z f\,dz \Rightarrow \frac{z^3}{3} = f(x,y,z) + \left(\text{nešto neovisno o } z\right)$$

Slijedi $f(x,y,z) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^3}{3}$, pa je g potencijalno polje

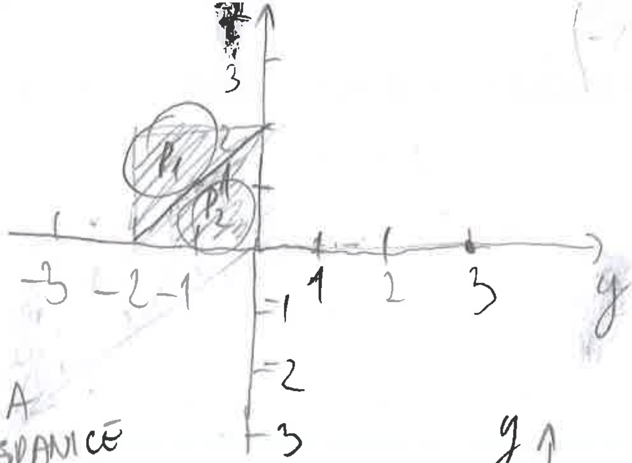
DA LI KRIVULJNI INTEGRAL OVISI O PUTU INTEGRACIJE?

2. $K=\{(x,y,z)$

$x=3$

$y\in[-2,2]$

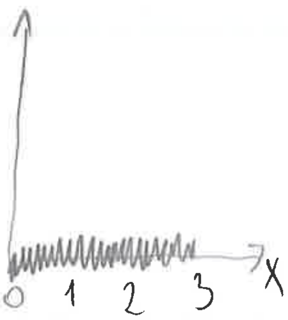
$z\in[-2,2]$



$(-2+2)=0$?

NEMA RJEŠENJA ZA OVE GRANICE JER JE $0\cdot y=0$

$$\boxed{(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)} \Rightarrow \iint_{\partial K} x\,ds$$



$P_1 \to \int_0^2\int_{-2}^{0} dy\,dz$

$P_2 \to \int_0^2\int_0$

---

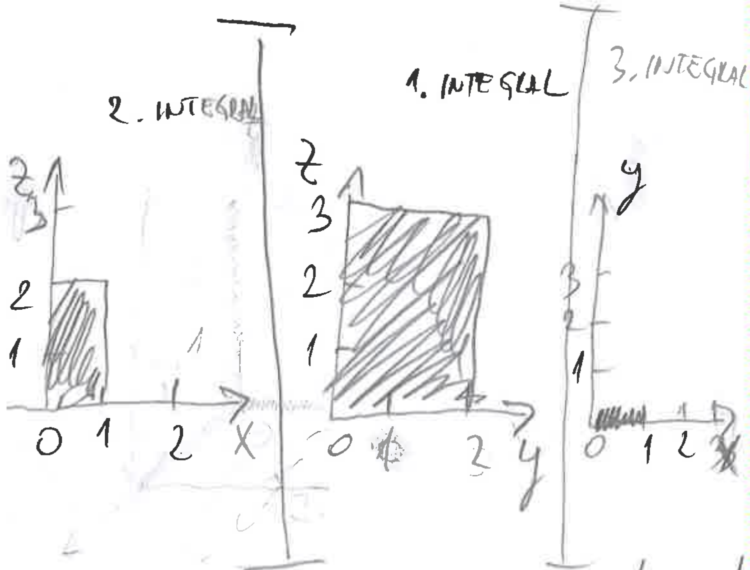
5. $$\iint_{\partial K} zy\,dy\,dz + xy\,dx\,dz + x\,dx\,dy =$$

$0\le x\le 1$

$0\le y\le 2$

$0\le z\le 3$

gdje je ∂K rub



$$=\int_0^3\int_0^2 zy\,dy\,dz + \int_0^2\int_0^1 xy\,dx\,dz + \int_0^1 x\int_0^0 dy\,dx = \frac{z^2}{2}\Big|_0^3 + \frac{y^2}{2}\Big|_0^2 + y\cdot\left(z\Big|_0^2\right) + \frac{x^2}{2}\Big|_0^1 + \frac{x^2}{2}\Big|_0^1$$

$$= 4+2+(y\cdot 2)+\frac{1}{2}+\frac{1}{2} = 6+2y+1 = \boxed{\cancel{7+2y}}$$

VIDI BRKLJAČA

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: MATE MITROVIĆ

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA:

USTMENI ISPIT KOD NASTAVNIKA: Prof. dr. sc. Nikica Uglešić

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20

$$x'''(t) + 4x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 4.$$

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4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. 20
5. Neka je K kvadar $(0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3)$ i ∂K rub tog kvadra. Izračunati plošni integral 20

$$\iint_{\partial \widehat{K}} zy\,\mathrm{d}y\mathrm{d}z + xy\,\mathrm{d}x\mathrm{d}z + x\,\mathrm{d}x\mathrm{d}y$$

Ukupno: ~~0~~ 20 Kos

1.

$$s^3X(s) - s^2x(0) - sx'(0) - x''(0) + 4(sX(s) - x(0)) = \frac{1}{s^2}$$

$$s^3X(s) - 4s^2 + 4sX(s) - 4 = \frac{1}{s^2}$$

$$X(s) = \frac{\frac{1}{s^2} + 4s^2 + 4}{s^3 + 4s} = \frac{\frac{1 + 4s^4 + 4s^2}{s^2}}{\frac{s^3+4s}{1}} = \frac{1 + 4s^4 + 4s^2}{s^2(s^3+4s)} = \frac{1 + 4s^4 + 4s^2}{s^2(s^2+4)}$$

$$s^3 + 4s = s(s^2+4)$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{s^2+4} = \frac{As^3 + 4As + Bs^2 + 4B + Cs^3 + Ds^2}{s^2(s^2+4)}$$

$$A + C = 0 \Rightarrow C = 0$$
$$B + D = 4 \Rightarrow D = \frac{15}{4}$$
$$4A = 0 \Rightarrow A = 0$$
$$4B = 1 \Rightarrow B = \frac{1}{4}$$

$$X(s) = \frac{1}{4s^2} - \frac{15}{4(s^2+4)}$$

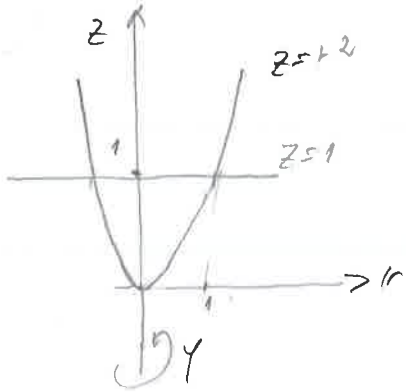
$$x(t) = \mathcal{L}^{-1}[X(s)]$$

$$x(t) = \frac{1}{4}\mathcal{L}^{-1}\left[\frac{1}{s^2}\right] - \frac{15}{4}\cdot\frac{1}{2}\mathcal{L}^{-1}\left[\frac{2}{s^2+4}\right]$$

$$x(t) = \frac{1}{4}t^2 - \frac{15}{8}\sin(2t)$$ ✗ VIDI BRKUJAČA

(4) $x^2+y^2=z$, $z=1$

$r^2=z$



$\varphi\in[0,2\pi]$

$z\in[r^2,1]$

$r\in[0,1]$

$$\int_0^{2\pi}\int_0^1\int_{r^2}^1 1\cdot r\,dz\,dr\,d\varphi$$

$$=2\pi\int_0^1 r(1-r^2)\,dr$$

$$=2\pi\int_0^1 r-r^3\,dr$$

$$=2\pi\left[\frac{r^2}{2}-\frac{r^4}{4}\right]_0^1$$

$$=2\pi\left(\frac{1}{2}-\frac{1}{4}\right)=2\pi\cdot\frac{1}{4}=\frac{1}{2}\pi=1{,}570$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31	POPUNJAVA NASTAVNIK
IME I PREZIME: AUGUSTIN PTIČAR **VRIJEME POČETKA:**	Broj ↓ bodova
MATIČNI BROJ STUDENTA: 17-1-0055-2011 **USTMENI ISPIT KOD NASTAVNIKA:**	

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20

$$x'''(t) + 4\,x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 4.$$

2. Neka je kvadrat $K = \{(x, y, z) : x = 3,\ y \in [-2, 2],\ z \in [-2, 2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x\,\mathrm{d}s$?. 20

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? 20

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. 20

5. Neka je K kvadar $(0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3)$ i ∂K rub tog kvadra. Izračunati plošni integral 20

$$\iint\limits_{\widehat{\partial K}} zy\,\mathrm{d}y\mathrm{d}z + xy\,\mathrm{d}x\mathrm{d}z + x\,\mathrm{d}x\mathrm{d}y$$

Ukupno:

20

AUGUSTIN PITICAR

1. $x'''(t) + 4x'(t) = t$

$x'(0) = x''(0) = 0$

$\boxed{x(0) = 4}$

$$x^3 F(x) - x^2 f(0) - x f'(0) - f''(0) + 4x F(x) - f(0) = \frac{1}{x^2}$$

$$x^3 F(x) + 4x F(x) = \frac{1}{x^2} + x^2 \overset{=4}{f(0)} + x \overset{=0}{f'(0)} + \overset{=0}{f''(0)} + \overset{=4}{f(0)}$$

$$F(x)\left(x^3 + 4x\right) = \frac{1}{x^2} + 4x^2 + 4$$

$$F(x)\left(x^3 + 4x\right) = \frac{1 + 4x^4 + 4x}{x^2} \Big/ \cdot \frac{1}{(x^3 + 4x)}$$

$$F(x) = \frac{1 + 4x^4 + 4x}{x^2 (x^3 + 4x)} = \frac{1 + 4x^4 + 4x}{x^2 x (x^2 + 4)} = \frac{1 + 4x^4 + 4x}{x^3 (x^2 + 4)}$$

$x^3 = 0$

$x_1 = 0$

x^2

$x^3 = 0$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx + E}{(x^2+4)}$$

$$\frac{1 + 4x^4 + 4x}{x^3 (x^2 + 4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx + E}{(x^2 + 4)} \Big/ \cdot x^3 (x^2 + 4)$$

$$1 + 4x^4 + 4x = A\left(x^2 (x^2 + 4)\right) + B\left(x (x^2 + 4)\right) + C\left(x^2 + 4\right) + (Dx + E)\left(x^3\right)$$

$$1+4x^4+4x = A(x^4+4x^2)+B(x^3+4x)+C(x^2+4)+(Dx+E)x^3$$

$$1+4x^4+4x = Ax^4+4Ax^2+Bx^3+4Bx+Cx^2+4C+Dx^4+Ex^3$$

$x=0 \rightarrow 1=0+0+0+0+0+4C+0+0$

$4C=1$

$\boxed{C=\frac{1}{4}}$

sve uz $x^2 \rightarrow 0=4A+C$

$4A=-C$

$A=-\frac{C}{4}$

$A=-\frac{\frac{1}{4}}{4}$

$\boxed{A=-\frac{1}{16}}$

sve uz $x^3 \rightarrow$

$0=B+E$

$E=-B$

$\boxed{E=-1}$

sve uz $x \rightarrow 4=4B$

$4B=4$

$\boxed{B=1}$

sve uz $x^4 \rightarrow 4=A+D$

$D=4-A$

$D=4+\frac{1}{16}$

$D=\frac{65}{16}$

$A=-\frac{1}{16}$, $B=1$, $C=\frac{1}{4}$, $D=\frac{65}{16}$, $E=-1$

$$F(x)=\frac{-\frac{1}{16}}{x}+\frac{1}{x^2}+\frac{\frac{1}{4}}{x^3}{}^{*}+\frac{\frac{65}{16}x-1}{(x^2+4)}=\frac{\frac{65}{16}x}{x^2+4}-\frac{1}{(x^2+4)}$$

$$f(t)=-\frac{1}{16}+t+\frac{1}{2}t^2+2\sin 2t\ldots$$

$*$ $\frac{\frac{1}{4}}{x^3}=8\frac{\left(\frac{1}{4}\right)}{x^{2+1}}=\frac{1}{}\frac{2}{x^3}$

$=\frac{1}{2}\frac{2}{x^3}$

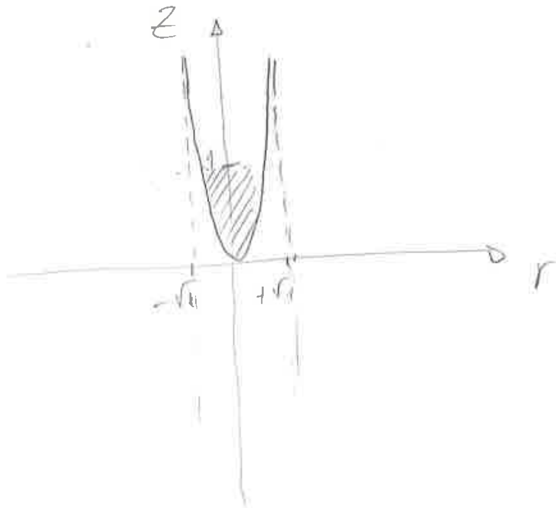
$$\boxed{f(t)=-\frac{1}{16}+t+\frac{1}{2}t^2-2\sin 2t+\frac{65}{16}\cos 2t}$$

VIDI BRKLJAČA

4. AUGUSTIN VATCAR

$z = x^2 + y^2$

$z = 1$



$z = r^2\cos^2\varphi + r^2\sin^2\varphi$

$z = r^2$

$$\int_0^{2\pi}\int_{-1}^{1}\int_{r^2}^{1} r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^{1} r\,dr\,d\varphi \; z\Big|_{r^2}^{1} = \int_0^{2\pi}\int_0^{1} r\,dr\,d\varphi \; 1 - r^2$$

$$= \int_0^{2\pi}\int_0^{1} (1-r^2)r\,dr\,d\varphi = \int_0^{2\pi}\int_0^{1} r - r^3\,dr\,d\varphi =$$

$$= \int_0^{2\pi} d\varphi \int_0^{1} r\,dr - \int_0^{1} r^3\,dr = \int_0^{2\pi} d\varphi \; \frac{r^2}{2}\Big|_0^{1} - \frac{r^4}{4}\Big|_0^{1}$$

$$= \int_0^{2\pi} d\varphi \; \frac{1}{2} - \frac{1}{4}$$

$$= \int_0^{2\pi} d\varphi \; \frac{1}{4} = \frac{1}{4}\varphi\Big|_0^{2\pi} = \frac{2\pi}{4} = \frac{1}{2}\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: MARIN MATEK **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA: 17-1-0111-12 **USTMENI ISPIT KOD NASTAVNIKA:**

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20

$$x'''(t) + 4\,x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 4.$$

2. Neka je kvadrat $K = \{(x, y, z) \,:\, x = 3,\ y \in [-2, 2]\,,\ z \in [-2, 2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x\mathrm{d}s$?. 20

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? 20

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. 20

5. Neka je K kvadar $(0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3)$ i ∂K rub tog kvadra. Izračunati plošni integral 20

$$\iint\limits_{\widehat{\partial K}} zy\,\mathrm{d}y\mathrm{d}z + xy\,\mathrm{d}x\mathrm{d}z + x\,\mathrm{d}x\mathrm{d}y$$

Ukupno: $\emptyset$

MARIN MATEK

1.

$x'''(t) + 4x'(t) = t \qquad x'(0) = x''(0) = 0, \; x(0) = 4$

$$s^3X(s) - s^2x(0) - sx'(0) - x''(0) + 4\cdot(sX(s) - x(0)) = \frac{1}{s^2}$$

$$s^3X(s) - s^2\underbrace{x(0)}_{4} - s\underbrace{x'(0)}_{0} - \underbrace{x''(0)}_{0} + 4sX(s) - 4\underbrace{x(0)}_{4} = \frac{1}{s^2}$$

$$s^3X(s) - 4s^2 + 4sX(s) - 16 = \frac{1}{s^2}$$

$$s^3X(s) + 4sX(s) = \frac{1}{s^2} + 4s^2 + 16$$

$$s^3X(s) + 4sX(s) = \frac{1 + 4s^4 + 16s^2}{s^2}$$

$$X(s)(s^3 + 4s) = \frac{4s^4 + 16s^2 + 1}{s^2}$$

$$X(s) = \frac{\frac{4s^4+16s^2+1}{s^2}}{(s^3+4s)} = \frac{4s^4+16s^2+1}{s^2(s^3+4s)} = \frac{4s^4+16s^2+1}{s^3(s^2+4)}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2+4} \quad \Big/ \cdot s^3(s^2+4)$$

$$= A(s^2(s^2+4)) + B(s(s^2+4)) + C(s^2+4) + (Ds + E)(s^3)$$

$$= A(s^4 + 4s^2) + B(s^3 + 4s) + C(s^2 + 4) + s^3Ds + Es^3$$

$$= \underline{As^4} + \underline{4As^2} + \underline{Bs^3} + \underline{4Bs} + \underline{Cs^2} + \underline{4C} + \underline{Ds^4} + \underline{Es^3}$$

$$4s^4 + 16s^2 + 1 = s^4(A + D) + s^3(B + E) + s^2(4A + C) + s(4B) + 4C$$

$A + D = 4 \qquad 4A + C = 16$

$B + E = 0 \qquad 4B = 0 \quad B = 0 \qquad 4C = 1 \quad C = \frac{1}{4} \quad \longrightarrow$

$E = 0$

$4A + C = 16$

$4A + \frac{1}{4} = 16$

$4A = \frac{63}{4}$

$A = \frac{63}{16}$ //

$A + D = 4$

$\frac{63}{16} + D = 4$

$D = \frac{1}{16}$ //

$B = 0$

$E = 0$

$C = \frac{1}{4}$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2 + 4}$$

$$= \frac{\frac{63}{16}}{s} + \frac{0}{s^2} + \frac{\frac{1}{4}}{s^3} + \frac{\frac{1}{16}s}{s^2 + 2^2}$$

$$= \frac{63}{16} \cdot \frac{1}{s} + \frac{1}{4} \cdot \left(\frac{1}{s^3} \cdot \frac{1}{s}\right) + \frac{1}{16} \cdot \frac{s}{s^2 + 2^2}$$

$$= \frac{63}{16} + \frac{1}{4} t + \frac{1}{16} \cos(2t)$$ // ×

VIDI BRKLJAČA

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Duje Mitrović VRIJEME POČETKA:	
MATIČNI BROJ STUDENTA: 17-2-0205-2012 USTMENI ISPIT KOD NASTAVNIKA: prof. Uglešić	

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20

$$x'''(t) + 4x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 4.$$

2. Neka je kvadrat $K = \{(x,y,z)\ :\ x = 3,\ y \in [-2,2]\ ,\ z \in [-2,2]\}$. Kako preko definicije izračunati $\iint_{\partial K} x ds$?. 20

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? 20

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. 20

5. Neka je K kvadar $(0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3)$ i ∂K rub tog kvadra. Izračunati plošni integral 20

$$\iint_{\widehat{\partial K}} zy\, dydz + xy\, dxdz + x\, dxdy$$

Ukupno: $\varnothing$

1) $\mathcal{L}[x'''(t)] + 4\mathcal{L}[x'(t)] = \mathcal{L}[t]$

$$s^3X(s) - s^2\underbrace{x(0)}_{4} - s\cancel{x'(0)} - \cancel{x''(0)} + 4sX(s) - 4\underbrace{x(0)}_{4} = \frac{1}{s^2}$$

$$s^3X(s) - 4s^2 + 4sX(s) - 16 = \frac{1}{s^2}$$

$$s^3X(s) + 4sX(s) = \frac{1}{s^2} + 4s^2 + 16$$

$$X(s) = \frac{\frac{1}{s^2} + 4s^2 + 16}{s^3 + 4s} = \underbrace{\frac{1}{s^3(s^2+4)}}_{I} + \underbrace{\frac{4s^2+16}{s(s^2+4)}}_{II}$$

I

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4} = 1$$

$$A[s^2(s^2+4)] + B[s(s^2+4)] + C[s^2+4] + [Ds+E]s^3 = 1$$

$$As^4 + 4As^2 + Bs^3 + 4Bs + Cs^2 + 4C + Ds^4 + Es^3 = 1$$

$$A + D = 0 \Rightarrow D = 0$$

$$4A + C = 4 \Rightarrow A = 0$$

$$B + E = 0 \Rightarrow E = 0$$

$$4B = 0 \Rightarrow B = 0$$

$$4C = 1 \Rightarrow C = \frac{1}{4}$$

$$\mathcal{L}\left[\frac{4}{s^3}\right] = \frac{1}{4}\mathcal{L}\left[\frac{1}{s^3}\right] = \frac{1}{4}\cdot\frac{1}{2}\frac{2}{s^{2+1}} = \frac{1}{8}t^2$$

1) $\frac{A}{s}+\frac{Bs+C}{s^2+4} = 4s^2+16$

$A[s^2+4]+[Bs+C]s = 4s^2+16$

$As^2+4A+Bs^2+Cs = 4s^2+16$

$A+B=4 \Rightarrow B=0$

$4A=16 \Rightarrow A=4$

$C=0$

Rezultat: $\left[\frac{1}{8}t^2+4\right]$ ×

$\mathcal{L}\left[\frac{4}{s}\right] = 4\mathcal{L}\left[\frac{1}{s}\right] = 4$

$x(0)=4$

Provjera: $\frac{1}{8}(0)^2+4=4$

$4=4$ ✓

VIDI BRKLJAČA

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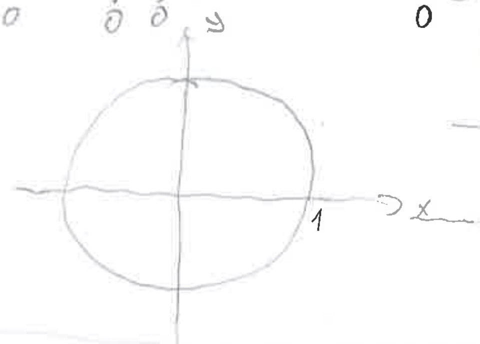
4) $z=x^2+y^2$ $\quad z=1$

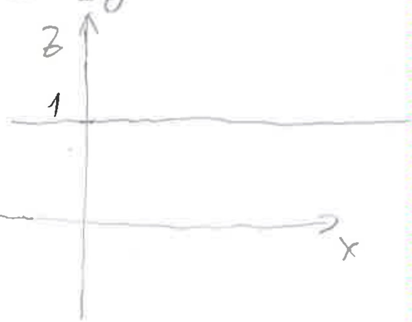
VIDI BRKLJAČA

$R=1 \quad r=1$

$V=\int_0^{2\pi}\int_0^1\int_0^1 1\, r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_0^1\left[\frac{r^2}{2}\right]_0^1 = \int_0^{2\pi}\int_0^1 \frac{1}{2}\,dz\,d\varphi = \int_0^{2\pi}\left[\frac{1}{2}z\right]_0^1 =$

$= \int_0^{2\pi}\frac{1}{2}\,d\varphi = \left[\frac{1}{2}\varphi\right]_0^{2\pi} = \pi$





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2) $\int_0^3\int_{-2}^2\int_{-2}^2 x\,ds$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: DONATO PREDOVAN **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: **USTMENI ISPIT KOD NASTAVNIKA:** prof. UGLEŠIĆ	

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20

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5. Neka je K kvadar ($0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$) i ∂K rub tog kvadra. Izračunati plošni integral 20

$$\iint_{\partial K} zy\,dydz + xy\,dxdz + x\,dxdy$$

Ukupno: $\varnothing$

1) $x'''(t) + 4x'(t) = t$ $\quad x'(0) = x''(0) = 0,\ x(0) = 4$

$= s^3 X(s) - s^2 x(0) - s x'(0) - x''(0)$

$= s^3 X(s) - 4s^2$

$= \frac{1}{s^2}$

$= sX(s) - 4$

$$= s^3 X(s) - 4s^2 + 4sX(s) - 16 = \frac{1}{s^2} \Big/ \cdot s^2$$

$$= s^5 X(s) - 4s^4 + 4s^3 X(s) - 16s^2 = 1$$

$$= X(s)(s^5 + 4s^3) = 1 + 4s^4 + 4s^2 \Big/ : s^5 + 4s^3$$

$$X(s) = \frac{1 + 4s^4 + 16s^2}{s^3(s^2+4)}$$

$$= \frac{1 + 4s^4 + 16s^2}{s^3(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4}$$

$$\frac{1+4s^4+16s^2}{s^3(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4} \quad / \cdot s^3(s^2+4)$$

$$1+4s^4+16s^2 = A\left(s^2(s^2+4)\right) + B\left(s(s^2+4)\right) + C(s^2+4) + (Ds+E)s^3$$

$$1+4s^4+16s^2 = A(s^4+4s^2) + B(s^3+4s) + Cs^2 + 4C + Ds^4 + Es^3$$

$$\underline{1} + \underline{4s^4} + 16\,\underline{s^2} = \underline{As^4} + \underline{4As^2} + \underline{Bs^3} + \underline{4Bs} + \underline{Cs^2} + \underline{4C} + \underline{Ds^4} + \underline{Es^3}$$

$$4 = A + D \Rightarrow A = 4 - D$$

$$16 = 4A + C = 16 = 4A + \frac{1}{4} \Rightarrow 4A = 16 - \frac{1}{4}$$

$$0 = B + E \Rightarrow \boxed{E = 0}$$

$$4A = \frac{64}{4} - \frac{1}{4} = \frac{63}{4}$$

$$0 = 4B \Rightarrow \boxed{B = 0}$$

$$1 = 4C \Rightarrow \boxed{C = \frac{1}{4}}$$

$$A = \frac{\frac{63}{4}}{\frac{4}{1}} = \frac{63}{16}$$

$$4 - D = A$$

$$-D = \frac{63}{16} - 4$$

$$-D = \frac{63}{16} - \frac{64}{16}$$

$$-D = -\frac{1}{16}$$

$$\boxed{D = \frac{1}{16}}$$

$$X(s) = \frac{1}{s}\cdot\frac{63}{16} + \frac{1}{s^3}\cdot\frac{1}{4} + \frac{\frac{1}{16}\ (+0)}{s^2+4}$$

$$= \frac{63}{16} + \frac{1}{4}\cdot\frac{1}{s^2}\cdot\frac{1}{s} + \frac{1}{16}\cdot\frac{2}{s^2+2^2}\cdot\frac{1}{2}$$

$$X(t) = \frac{63}{16} + \frac{1}{4}t + \frac{1}{16}\sin t\cdot\frac{1}{2}$$ ✗ VIDI BRKLJAČA

DONATO PREDOVAN

VIDI BRKLJAČA

4. $z = x^2 + y^2$, $z = 1$ ✗

$f(x,y) = z$

$$P = \iint \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dx\, dy$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

$$P = \iint \sqrt{1 + 4x^2 + 4y^2}\; r\, dr\, d\varphi$$

$$= \int_0^{2\pi} \int_0^1 \sqrt{1 + 4r^2}\; r\, dr\, d\varphi$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$dx\,dy = r\,dr\,d\varphi$

SOPSTITUCIJA

$1 + 4r^2 = t$

$r\,dr = dt$

$z = r^2\cos^2\varphi + r^2\sin^2\varphi$

$z = r^2(\cos^2\varphi + \sin^2\varphi)$

$z = r^2$

$1 = r^2 \qquad r \in [0,1]$

$$= 2\pi \int_1^5 \sqrt{t}\, dt$$

$$= 2\pi \left. \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right|_1^5$$

$$= 2\pi \left. \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right|_1^5 = 2\pi \cdot (7.45 - 0.6666)$$

$$\approx 21.31$$

=

5.

$$x \in [0,1]$$
$$y \in [0,2]$$
$$z \in [0,3]$$

$$\iint_{\partial K} zy \, dy\, dz + xy \, dx\, dz + x \, dx\, dy$$

$$= \iint_{\partial K} zy \, dy\, dz + \iint_{\partial K} xy \, dx\, dz + \iint_{\partial K} x \, dx\, dy$$

$$= \overbrace{\int_0^3 \int_0^2 zy \, dy\, dz}^{P} + \overbrace{\int_0^3 \int_0^1 xy \, dx\, dz}^{Q} + \overbrace{\int_0^2 \int_0^1 x \, dx\, dy}^{S}$$

$$P = \int_0^3 \Big|_0^2 \, dz$$

VIDI BRKLJAČA

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31 **IME I PREZIME:** JELENA MALEŠ **VRIJEME POČETKA:** 09:00 **MATIČNI BROJ STUDENTA:** 17-2-0103-2011 **USTMENI ISPIT KOD NASTAVNIKA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: ~~20~~

$$x'''(t) + 4x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 4.$$

2. Neka je kvadrat $K = \{(x,y,z) : x = 3,\ y \in [-2,2],\ z \in [-2,2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x ds$?. ~~20~~

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? ~~20~~

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. => granice ~~20~~

5. Neka je K kvadar ($0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$) i ∂K rub tog kvadra. Izračunati plošni integral ~~20~~

$$\iint\limits_{\partial K} zy\, dydz + xy\, dxdz + x\, dxdy$$

Ukupno: ~~0~~

1. $x'''(t) + 4x'(t) = t \qquad x'(0) = x''(0) = 0 \qquad x(0) = 4$

$x'''(t) = s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)$

$4x'(t) = 4(sF(s) - f(0))$

$t = \frac{1}{s^2}$

$s^3 X(s) - s^2 \underset{=4}{X(0)} - s \underset{=0}{x'(0)} - x''(0) + 4(sX(s) - x(0)) = \frac{1}{s^2}$

$s^3 X(s) - 4s - 0 - 0 + 4sX(s) - 4 \cdot 4 = \frac{1}{s^2}$

$s^3 x(s) + 4s\, x(s) - 4s - 16 = \frac{1}{s^2}$

$s^3 x(s) + 4s X(s) = \frac{1}{s^2} + 4s + 16$

$X(s)(s^3 + 4s) = \frac{1 + 4s^3 + 16s^2}{s^2} \quad / : (s^3 + 4s)$

$X(s) = \frac{1 + 4s^3 + 16s^2}{s^2(s^3 + 4s)}$

$x(s) = \frac{1 + 4s^3 + 16s^2}{s^3(s^2 + 4)}$

1. $\dfrac{\frac{63}{16}}{\frac{s}{1}} + \dfrac{0}{s^2} + \dfrac{\frac{1}{4}}{\frac{s^3}{1}} + \dfrac{-\frac{63}{16}+4}{s^2+4}$ $\rightarrow \frac{1}{16}$ =

JELENA MALEŠ

$$= \frac{63}{16}s + \frac{4}{s^3} + \frac{\frac{1}{16}}{s^2+4} = \frac{16}{s^2+4} = 4 \cdot \frac{4}{s^2+4}$$

$$= \frac{63}{16}s + 4 \cdot \frac{1}{s} \cdot \frac{1}{s^2} + 4 \cdot \frac{4}{s^2+4}$$

$$= \frac{63}{16}s + 4 \cdot \frac{1}{s} \cdot \frac{1}{s^2} + 8 \cdot \frac{2}{s^2+2^2}$$

$$x(t) = \frac{63}{16}t + 4 \cdot 1 \cdot t + 8 \cdot \sin 2t$$

✗ VIDI BRKLJAČA

$$x(t) = \frac{63}{16}t + 4t + 8\sin 2t$$

Provjera $x(t)$ $\boxed{= \frac{127}{16}t + 8\sin 2t}$ → Rj

2.

1. Nastavak JELENA MALEŠ

$$X(s) = \frac{1+4s^3+16s^2}{s^3(s^2+4)}$$

$$1+4s^3+16s^2 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4} \quad / s^3(s^2+4)$$

$$= A \cdot s^2(s^2+4) + B \cdot s(s^2+4) + C(s^2+4) + (Ds+E)(s^3)$$

$$= As^4 + 4As^2 + Bs^3 + 4Bs + Cs^2 + 4C + Ds^4 + Es^3$$

$$= 4C + (4A + C)s^2 + (B+E)s^3 + (A+D)s^4 + 4Bs$$

$4C = 1$

$C = \frac{1}{4}$

$4A + C = 16$

$4A + \frac{1}{4} = 16$

$4A = \frac{16}{1} - \frac{1}{4}$

$4A = \frac{63}{4} = \frac{63}{16}$

$\boxed{A = \frac{63}{16}}$

$B + E = 4$

$4B = 0$

$B = 0$

$B + E = 4$

$0 + E = 4$

$\boxed{E = 4}$

$A + D = 0$

$\frac{63}{16} + D = 0$

$\boxed{D = -\frac{63}{16}}$

$A = \frac{63}{16}$

$B = 0$

$C = \frac{1}{4}$

$D = -\frac{63}{16}$

$E = 4$

$$\frac{\frac{63}{16}}{\frac{s}{1}} + \cancel{\frac{0}{s^2}} + \frac{\frac{1}{4}}{\frac{s^3}{1}} + \frac{-\frac{63}{16}s}{\frac{s^2+4}{1}} + \frac{4}{s^2+4} =$$

$$= \frac{63}{16s} + \frac{1}{4s^3} \; \frac{-63s}{16(s^2+4)} + \frac{4}{s^2+4}$$

$$= \frac{63}{16} \cdot \frac{1}{s} + \frac{1}{4} \cdot \frac{1}{s} \cdot \frac{1}{s^2} - \frac{63}{16} \cdot \frac{s}{(s^2+2^2)} + 2 \cdot \frac{2}{s^2+2^2} \Rightarrow$$

$$\frac{63}{16} \cdot 1 + \frac{1}{4} \cdot 1 \cdot t - \frac{63}{16} \cos 2t + 2 \sin 2t$$

$$\frac{63}{16} + \frac{1}{4} t - \frac{63}{16} \cos 2t + 2 \sin 2t$$

$$\frac{1}{4} + \frac{63}{16} 2 \sin 2t + 4 \cos 2t$$

$$\frac{63}{16} \cdot$$

4. $z = \underbrace{x^2 + y^2}_{r^2}$ $z = 1$

JELENA MALEŠ

$x^2 + y^2 = 1$
$x^2 + y^2 = r^2 = 1^2 = \boxed{1}$

$x^2 + y^2 + z^2 = 1$
$z^2 = 1 - r^2$
$z = \pm\sqrt{1-r^2}$

$$V = \int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{+\sqrt{1-r^2}} r\,dz\,dr\,d\varphi =$$

VIDI BRKLJAČA

$$= \int_0^{2\pi}\int_0^1 rz \Big|_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} dr\,d\varphi = \int_0^{2\pi}\int_0^1 r\cdot\left(\sqrt{1-r^2} - \left(-\sqrt{1-r^2}\right)\right) dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^1 \left(r\cdot 2\sqrt{1-r^2}\right) dr\,d\varphi =$$

$$= \int_0^{2\pi} \left(\frac{2r^2}{2}\cdot\frac{\left(\sqrt{1-r^2}\right)^2}{2}\right)\Big|_0^1 d\varphi =$$

$$= \int_0^{2\pi} r^2\cdot\frac{(1-r^2)}{2}\Big|_0^1 d\varphi =$$

$$= \int_0^{2\pi} \frac{r^2 - r^4}{2}\Big|_0^1 d\varphi =$$

$$= \int_0^{2\pi} \frac{-r^2}{2}\Big|_0^1 d\varphi = \int_0^{2\pi} \frac{(-1)^2}{2}\,d\varphi = \frac{1}{2}\varphi\Big|_0^{2\pi} = \frac{1}{2}\cdot 2\pi = \frac{2}{2}\pi = \pi$$

2. $K = x, y, z$

$x = 3$

$y \in [-2, 2]$

$z \in [-2, 2]$

$$\iiint_{\partial K}$$

JELENA MATEŠ

(3) Ne ovisi? ZAŠTO?

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} x \\ y \\ z^2 \end{pmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ \frac{x}{z^2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ \frac{x}{z^2} \end{bmatrix} = \begin{bmatrix} \frac{x}{z^2} \\ \frac{y}{z^2} \\ 1 \end{bmatrix}$$

(5)
$x \in [0,1]$
$y \in [0,2]$
$z \in [0,3]$

$$\begin{array}{c} x \\ y \\ z \end{array} \begin{pmatrix} zy \\ xy \\ x \end{pmatrix} \quad \frac{\partial zy}{\partial x} + \frac{\partial xy}{\partial y} + \frac{\partial x}{\partial z} = zyx + x \quad +2x$$

$$\int_0^1 \int_0^2 \int_0^3 \underbrace{zy\,dy\,dz + xy\,dx\,dz + x\,dx\,dy}_{\text{dif. } dx\,dy\,dz}$$

$$\int_0^1 \int_0^2 \int_0^3 (zyx + x + zx)\,dz\,dy\,dx =$$

VIDI BRKYAČA

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31

POPUNJAVA NASTAVNIK

IME I PREZIME: MARTIN SEDMAK **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA: **USTMENI ISPIT KOD NASTAVNIKA:**

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: 20

$$x'''(t) + 4x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 4.$$

2. Neka je kvadrat $K = \{(x,y,z) : x = 3,\ y \in [-2,2],\ z \in [-2,2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x\,ds$?. 20

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? 20

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. 20

5. Neka je K kvadar $(0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3)$ i ∂K rub tog kvadra. Izračunati plošni integral 20

$$\iint\limits_{\widehat{\partial K}} zy\,\mathrm{d}y\mathrm{d}z + xy\,\mathrm{d}x\mathrm{d}z + x\,\mathrm{d}x\mathrm{d}y$$

Ukupno: $\cancel{0}$

3. $g = \nabla f$

$$\begin{bmatrix} g_x f = \partial_x f \\ g_y f = \partial_y f \\ g_z f = \partial_z f \end{bmatrix}$$

$x = \partial_x f / \int dx \quad f = \int x\,dx \quad f = \frac{x^2}{2}$

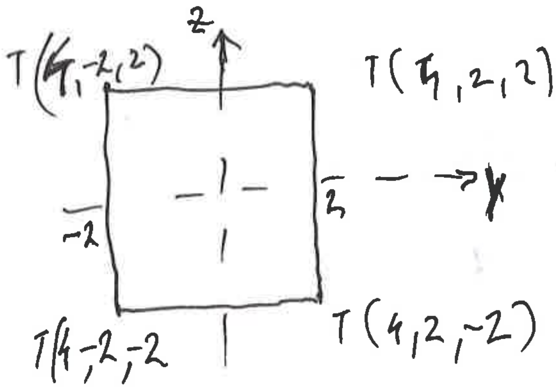
$y = \partial_y f / \int dy \quad f = \int y\,dy \quad f = \frac{y^2}{2}$

$z^2 = \partial_z f / \int dz \quad f = \int z^2 dz \quad f = \frac{z^3}{3}$

$f = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^3}{3}$ PREMA TOME g JE ~~PEEN~~ POTENCIJALNO POLJE ✓

DA LI INTEGRAL OVISI O PUTU INTEGRACIJE?

2. $K = \{(x,y,z) : x = 3, \ y \in [-2,2], \ z \in [-2,2]\}$



$r_1(t) = \begin{bmatrix} 3 \\ 2 \\ t \end{bmatrix}$ $\sqrt{r_1'(t)^2} = 1$

$r_2(t) \begin{bmatrix} 3 \\ t \\ 2 \end{bmatrix}$ $\sqrt{r_2'(t)^2} = 1$

$r_3(t) \begin{bmatrix} 3 \\ 2 \\ t \end{bmatrix}$ $\sqrt{r_3'(t)^2} = 1$

$r_4(t) \begin{bmatrix} 3 \\ t \\ -2 \end{bmatrix}$ $\sqrt{r_4'(t)^2} = 1$

$t \in [-2,2]$

$$\int_{\partial K} y\,ds = \underbrace{\int_{-2}^{2} 2 \cdot 1\,dt}_{=2} + \underbrace{\int_{-2}^{2} t\,dt}_{=0} + \underbrace{\int_{-2}^{2} -2\,dt}_{=-2} + \underbrace{\int_{-2}^{2} t\,dt}_{=0} = 0$$

4. $z = x^2 + y^2$ $\quad z = 1$ $\quad x = r\cos\varphi$

$z = r^2$ $\quad z = r^2\cos^2\varphi + r^2 \sin\varphi$ $\quad y = r\sin\varphi$

$$\int_0^{2\pi} d\varphi \int_0^{\sqrt{2}} r\,dr \int_{r^2}^{1} dz \qquad z = 1$$

V.D. BRKLJAČA

$$= 2\pi \int_0^{\sqrt{2}} r(1-r^2)\,dr = 2\pi \cdot \left(\frac{r^2}{2}\Big|_0^{\sqrt{2}} - \frac{r^3}{3}\Big|_0^{\sqrt{2}} \right) =$$

$$= 2\pi\left(1 - \frac{2\sqrt{2}}{3}\right) = 2\pi - \frac{4\pi\sqrt{2}}{3} =$$

MARTIN SEDMAK

1. $x'''(t) + 4x'(t) = t \qquad x'(0) = x''(0) = 0$

$$x(0) = 4$$

$$s^3F(s) - \underbrace{s^2 f(0)}_{4} - \underbrace{s\cancel{f'(0)}}_{0} - \underbrace{\cancel{f''(0)}}_{0} + 4sF(s) - \underbrace{4f(0)}_{4} = \frac{1}{s^2}$$

$$\cancel{s^3F(s) - 4s^2 + 4sF(s) - 16 = \frac{1}{s^2}}$$

$$\cancel{F(s)(s^3-4s) = \frac{1}{s^2} + 4s^2 + 16}$$

$$s^3F(s) - 4s^2 + 4sF(s) - 4\cdot 4 = \frac{1}{s^2}$$

$$F(s)(s^3-4s) - 4s^2 - 16 = \frac{1}{s^2}$$

$$F(s)\,s(s^2-4) - 4(s^2-4) = \frac{1}{s^2}$$

$$F(s)\,s^3(s^2-4) = \frac{1}{s^2} + 4(s^2-4)$$

$$F(s)\,s^3(s^2-4) = 4s^2(s^2-4)$$

$$F(s) = \frac{4s^4 - 8s^2}{s^3(s^2-4)}$$

↓ NASTAVAK

$$\frac{4s^4-8s^2}{s^3(s^2-4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2-4} \quad / \, s^3(s^2-4)$$

$$4s^4-8s^2 = As^2(s^2-4) + Bs(s^2-4) + C(s^2-4) + (Ds+E)(\cancel{\quad})$$

$$= \underline{As^4} - \underline{4As^2} + \underline{Bs^3} - \underline{4Bs} + \underline{Cs^2} - 4C + \cancel{Bs^3 \quad\quad} \; \underline{Ds^4} + \underline{Es^3}$$

$$s^4(A+D) + s^3(B+E) + s^2(\cancel{\quad})(-4A+C) + s(-4B) - 4C$$

$4 = A+D \qquad 0 = B+E \qquad -8 = -4A+C) \qquad -4B=0 \qquad -4C=0$

$\boxed{B=0 \qquad C=0}$

$4 = 2+D \qquad 0 = 0+E \qquad -8 = -4A$

$\boxed{2=D} \qquad \boxed{0=E} \qquad -2 = -A$

$\boxed{A=2}$

$$\frac{2}{s} + \frac{0}{s^2} + \frac{0}{s^3} + \frac{2s}{s^2-4}$$

$$2 \cdot L^{-1}\left[\frac{1}{s}\right] + 2 \cdot L^{-1}\left[\frac{s}{s^2-2^2}\right]$$

VIDI BRKLJAČA

$x(t) = 2 + 2\cos 2t$ ×

$x(0) = 2 + 2 \cdot 1$

$x(0) = 4$ ✓

MARTIN SEDMAK

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31

IME I PREZIME: MARIO IVANAC **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA: 17-1-0096-2011 **USTMENI ISPIT KOD NASTAVNIKA:** Prof. N. Uglešić

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: ~~20~~

$$x'''(t) + 4x'(t) = t, \quad x'(0) = x''(0) = 0, \; x(0) = 4.$$

2. Neka je kvadrat $K = \{(x,y,z) \,:\, x = 3, \, y \in [-2,2], \, z \in [-2,2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x ds$?. 20

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? ~~20~~

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. ~~20~~

5. Neka je K kvadar $(0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 3)$ i ∂K rub tog kvadra. Izračunati plošni integral 20

$$\iint\limits_{\widehat{\partial K}} zy \, dydz + xy \, dxdz + x \, dxdy$$

Ukupno: ~~0~~

1)

$$x'''(t) + 4x'(t) = 6, \quad x'(0) = x''(0), \quad x(0) = 4$$

$$s^3F(s) - s^2F(0) - sF'(0) - F''(0) + 4(sF(s) - F(0)) = 4$$

$$s^3F(s) - (s^2 \cdot 4) - (\cancel{s0}) - 0 + 4(sF(s) - 4) = 4$$

$$s^3F(s) - 4s^2 + 4F(s) - 16 = 4$$

$$s^3F(s) + 4F(s) = 4s^2 + 4 + 16$$

$$F(s)(s^3 + 4) = 4s^2 + 20$$

$$s \cdot s^2 + 4 = 4s^2 + 20$$

$$4s^2 + 20 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{4} \Big/ 4s^2 + 20$$

$$= A(4s + 20) + B(4 + 20) + C(s^2 + 20)$$

$$= A4s + 20A + 24B + Cs^2 + 20$$

$\boxed{C = 4}$

$\boxed{A = 1}$

$\boxed{B = 0}$

$$F(s) = \frac{1}{s} + \cancel{\frac{0}{s^2}} + \frac{1}{4}$$

$$= 1 + \frac{1}{4} = \frac{5}{4}$$

×

VIDI BRKLJAČA

4) $z = x^2 + y^2$, $z = 1$

Volumen nije moguće izračunati ! ×

3) $g = x\vec{i} + y\vec{j} + z^2\vec{k}$

$$r(t) = \begin{pmatrix} \cos x \\ \sin x \\ z^2 \end{pmatrix} \qquad r'(t) = \begin{pmatrix} -\sin x \\ \cos x \\ 2z \end{pmatrix}$$

$$\|r'(t)\| = \sqrt{(-\sin x)^2 + (\cos x)^2 + 2z} = \sqrt{\underbrace{\sin^2 x + \cos^2 x}_{1} + 2z} = \sqrt{1 + 2z}$$

Ne ovisi o putu integracije

ZAŠTO ?

0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 31

IME I PREZIME: Antun Žanetić

VRIJEME POČETKA: 09:00

MATIČNI BROJ STUDENTA: 17-2-0169-2012

USTMENI ISPIT KOD NASTAVNIKA:

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju koja zadovoljava sljedeće uvjete: ~~20~~

$$x'''(t) + 4\,x'(t) = t, \quad x'(0) = x''(0) = 0,\ x(0) = 4.$$

2. Neka je kvadrat $K = \{(x, y, z) \,:\, x = 3,\ y \in [-2, 2]\,,\ z \in [-2, 2]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x\mathrm{d}s$?. 20

3. Da li krivuljni integral u vektorskom polju $g = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ ovisi o putu integracije? ~~20~~

4. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 1$. ~~20~~

5. Neka je K kvadar ($0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$) i ∂K rub tog kvadra. Izračunati plošni integral ~~20~~

$$\iint\limits_{\widehat{\partial K}} zy\,\mathrm{d}y\mathrm{d}z + xy\,\mathrm{d}x\mathrm{d}z + x\,\mathrm{d}x\mathrm{d}y$$

Ukupno: Ø

Antun Žanetić

1. $x'''(t)+4x'(t)=t \quad ; \quad x'(0)=0,\ x''(0)=0,\ x(0)=4$

$$x'''(t)+4x'(t)=t \ / \mathcal{L}$$

$$\mathcal{L}\left[x'''(t)\right]+4\mathcal{L}\left[x'(t)\right]=\mathcal{L}(t)$$

$$\left[s^3X(t)-s^2x(0)-s\cdot x'(0)-x''(0)\right]+4\left[s\cdot X(t)-x(0)\right]=\frac{1}{s^2}$$

$$s^3X(t)-s^2x(0)-s\cdot x'(0)-x''(0)+4sX(t)-4x(0)=\frac{1}{s^2}$$

$$X(t)\cdot(s^3+4s)-4s^2-16=\frac{1}{s^2}$$

$$X(t)\ (s^3+4s)=\frac{1}{s^2}+4s^2+16 \ / \cdot\frac{1}{s^3+4s}$$

$$X(t)=\frac{1}{s^2(s^3+4s)}+\frac{4s^2+16}{s^3+4s}$$

$$X(t)=\frac{1+4s^2+16}{s^2(s^3+4s)}=\frac{17+4s^2}{s^2\cdot[s(s^2+4)]}$$ //

$$\frac{17+4s^2}{s^2\cdot[s(s^2+4)]}=\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{s^2+4} \ / \cdot s^2[s(s^2+4)]$$

$$17+4s^2=A[s^2(s^2+4)]+B[s(s^2+4)]+(Cs+D)\cdot(s^2 s)$$

$$17+4s^2=A(s^4+4s^2)+B(s^3+4s)+Cs^4+Ds^3$$

$$17+4s^2=As^4+4As^2+Bs^3+4Bs+4Cs^4+Ds^3$$

$$17+4s^2=s^4(A+4C)+s^3(B+D)+4As^2+4Bs$$

$A+4C=0$, $\quad$ $1+4C=0$

$B+D=0$ $\quad$ $0+D=0$

$4A=4 \Rightarrow \boxed{A=1}$ $\quad$ $4C=-1$

$4B=0$ $\quad$ $\boxed{C=-\frac{1}{4}}$

$\boxed{B=0}$

$\boxed{D=0}$

$$X(t) = \frac{1}{s} + 0 - \frac{\frac{1}{4}}{s^2+4}$$

$$X(t) = \frac{1}{s} - \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{2}{s^2+2^2} \quad \Big/ \mathcal{L}^{-1}$$

$$x(t) = 1 - \frac{1}{8} \sin 2t \quad //$$

$\times$ VIDI BRKLJAČA

$\boxed{5.}$ K kvadar $(0 \leq x \leq 1,\ 0 \leq y \leq 2,\ 0 \leq z \leq 3)$

Izračunati plošni integral $\iint_{\partial K} zy\,dydz + xy\,dxdz + x\,dxdy$

$$W = \begin{bmatrix} zy \\ xy \\ x \end{bmatrix} \qquad \frac{\partial W}{\partial x} = \begin{bmatrix} 0 \\ y \\ 1 \end{bmatrix}, \qquad \frac{\partial w}{\partial y} = \begin{bmatrix} z \\ x \\ 0 \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} i & j & k \\ 0 & y & 1 \\ z & x & 0 \end{bmatrix} = i \cdot \begin{vmatrix} y & 1 \\ x & 0 \end{vmatrix} - j \cdot \begin{vmatrix} 0 & 1 \\ z & 0 \end{vmatrix} + k \cdot \begin{vmatrix} 0 & y \\ z & x \end{vmatrix}$$

$$= -ix + zj - zyk$$

$$\begin{bmatrix} -x \\ z \\ -zy \end{bmatrix}$$

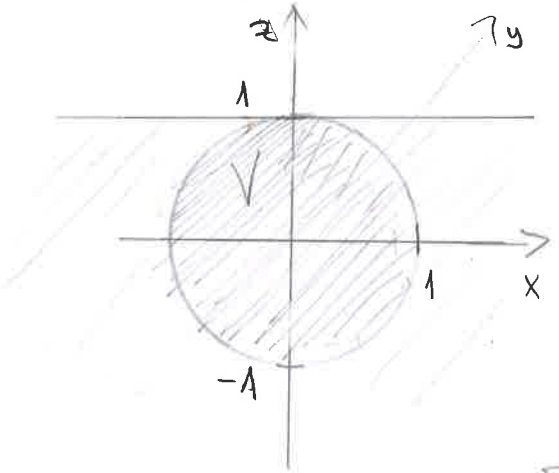
$\emptyset$

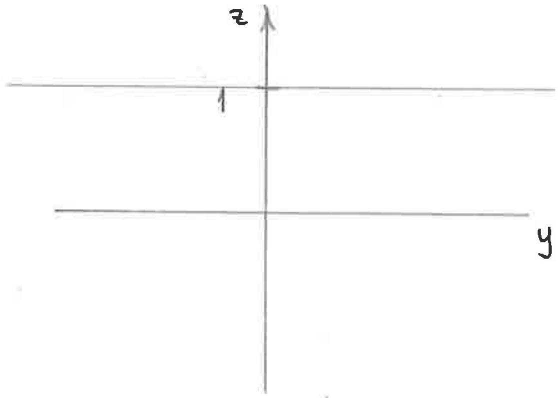
4 Volumen tijela

Plohe: $z = x^2 + y^2$

$z = 1$

$$x^2 + y^2 = 1^2 \quad \Rightarrow \quad y^2 = 1 - x^2, \quad y = \sqrt{1-x^2}$$





$x \in [-\sqrt{1-x^2}, \sqrt{1-x^2}]$

$y \in [-1, 1]$

$z \in [-1, 1]$

$$V = \int_{-1}^{1}\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1 \, dx\,dy\,dz$$

VIDI BRKLJAČA

3. $g = xi + yj + z^2 k$

PRELAZAK U POLARNE KOORDINATE

$W = \begin{bmatrix} x \\ y \\ z^2 \end{bmatrix}$

$r = \begin{bmatrix} r\sin\varphi \\ r\cos\varphi \\ z \end{bmatrix}$, $r' = [r'$

$\int_{\Gamma} W \mid dr = \int W \circ r \mid [r'$

$\emptyset$