MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 14

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: LOVRE BUBALO **VRIJEME POČETKA:** 9:30

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0389-2014

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 2x - 2}$ i skicirati graf. — 20 graf — 10

2. Riješiti: $\frac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!* — 12+3

3. Odrediti konvergenciju reda $\sum_n \frac{n^2}{2^n}$. — 15

4. Odrediti tok funkcije $f(x) = \frac{x^2+2}{x^2-4}$ i skicirati graf. — 20 graf

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. — 6+6+3

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-4x}$, u točki gdje je $x = 0$. — 15

Ukupno: 45

(2)

$\frac{z+i}{z-i} = \overline{2+3i}$ $\quad \{ z = x+yi \;\; \bar{z} = x-yi \}$

$\frac{x+yi+i}{x+yi-i} = 2-3i \;/\cdot(x+yi-i)$

$x+yi+i = 2x + 2yi - 2i - 3xi + 3y - 3$

Re ... $x = 2x - 3 + 3y$

Im ... $y+1 = 2y - 2 - 3x$

$x + 3y = 3 \Rightarrow -\frac{3}{5} + 3y = 3$

$3x - y = -3 \;/\cdot 3$

$3y = 3 + \frac{3}{5}$

$x + 3y = 3$

$y = \frac{6}{5}$

$9x - 3y = -9$

$10x = -6 \;/:10$

$x = -\frac{3}{5}$

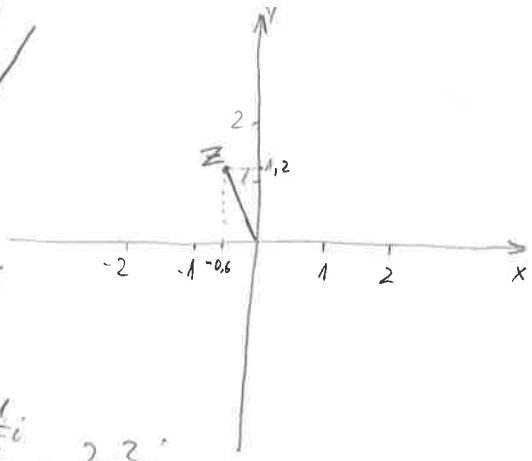
$x = -\frac{3}{5} = -0{,}6$

$y = \frac{6}{5} = 1{,}2$

$z = -\frac{3}{5} + \frac{6}{5}i$ ✓

PROVJERA:

$\frac{-\frac{3}{5} + \frac{6}{5}i + i}{-\frac{3}{5} + \frac{6}{5}i - i} = 2-3i$



$\frac{-\frac{3}{5} + \frac{11}{5}i}{-\frac{3}{5} + \frac{1}{5}i} \cdot \frac{-\frac{3}{5} - \frac{1}{5}i}{-\frac{3}{5} - \frac{1}{5}i} = 2-3i$

$\frac{\frac{9}{25} + \frac{3}{25}i - \frac{33}{25}i + \frac{11}{25}}{\frac{9}{25} + \frac{1}{25}} = 2-3i$

$\frac{\frac{4}{5} - \frac{6}{5}i}{\frac{2}{5}} = 2-3i \;/\cdot\frac{2}{5}$

$\frac{4}{5} - \frac{6}{5}i = \frac{4}{5} - \frac{6}{5}i$ ✓

④ $f(x) = \frac{x^2+2}{x^2-4}$

DOMENA: $x^2-4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

$D(f) = \mathbb{R} \setminus \{-2, 2\}$

NULTOČKE

$\frac{x^2+2}{x^2-4} = 0 \quad / \cdot (x^2-4)$

$x^2+2 = 0$

$x^2 = -2$

NEMA IH!!!

V.A

$\lim_{x \to -2} \frac{4+2}{4-4} = \frac{6}{0} = +\infty$ → $x = -2$

$\lim_{x \to 2} \frac{4+2}{4-4} = \frac{6}{0} = +\infty$ → $x = 2$

→ V.A

HA.

→ D. H.A.

$\lim_{x \to \infty} \frac{x^2+2 \,/ :x^2}{x^2-4 \,/ :x^2} = \frac{1+0}{1-0} = 1$

$y = 1$ → H.A

→ L.H.A.

$\lim_{x \to -\infty} \frac{x^2+2 \,/ :x^2}{x^2-4 \,/ :x^2} = \frac{1}{1} = 1$

$f'(x) = \frac{2x(x^2-4) - (x^2+2) \cdot 2x}{(x^2-4)^2}$

$f'(x) = \frac{2x(x^2-4-x^2-2)}{(x^2-4)^2}$

$f'(x) = \frac{-12x}{(x^2-4)^2}$

$f'(x) = 0$

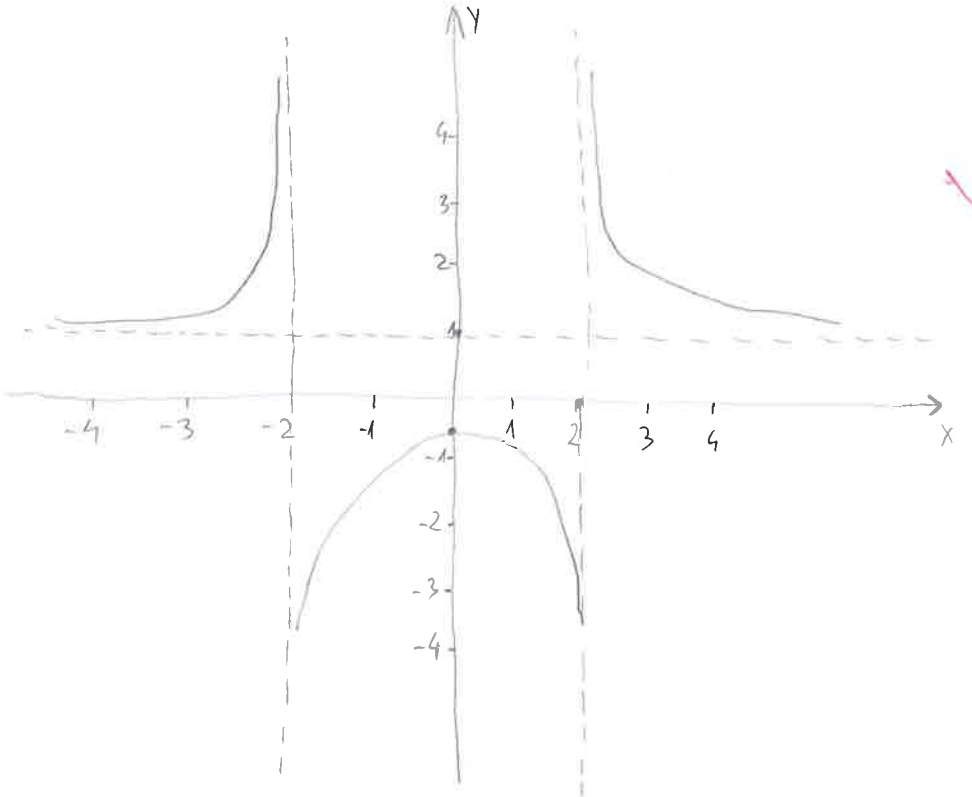
$\frac{-12x}{(x^2-4)^2} = 0 \quad / \cdot (x^2-4)^2$

$-12x = 0$

$x = 0 \qquad f(0) = \frac{0+2}{0-4} = -\frac{1}{2}$

MIN$(0, -\frac{1}{2})$

	$-\infty$	-2		0		2	$+\infty$
$f'(x)$	+	+	+	0	−	−	−
$f(x)$	↗	↗	↗	MIN	↘	↘	↘



LOVRE BUBALO

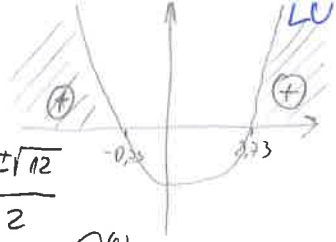
① $f(x) = x + \sqrt{x^2-2x-2}$

DOMENA:

$x^2-2x-2 \geq 0$

$x^2-2x-2=0$

$x_{1,2} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2}$



$x_1 = 2,73$

$x_2 = -0,73$

$D(f) = \langle -\infty, -0,73] \cup [2,73, +\infty \rangle$

[V.A.]

→nema

[H.A]

→D.H.A

$$\lim_{x\to\infty} \left(x+\sqrt{x^2-2x-2}\right) \cdot \frac{x-\sqrt{x^2-2x-2}}{x-\sqrt{x^2-2x-2}} = \lim_{x\to\infty} \frac{x^2-x^2+2x+2 \,/:x}{x-\sqrt{x^2-2x-2}\,/:x} = \frac{2+0}{1-\sqrt{1-0-0}} = \frac{2}{0} \Rightarrow \text{NEMA D.H.A}$$

→L.H.A

$$\lim_{x\to-\infty} \frac{2x+2}{x-\sqrt{x^2-2x-2}} = \lim_{x\to-\infty} \frac{-2x+2\,/:x}{-x-\sqrt{x^2+2x-2}\,/:x} = \frac{-2+0}{-1-\sqrt{1+0-0}} = 1$$

$y=1 \rightarrow$ L.H.A.

[K.A.] $y = kx + l$ $\quad k = \lim_{x\to\infty} \frac{f(x)}{x}$ $\quad l = \lim_{x\to\infty} [f(x) - kx]$

D.K.A

$$k = \lim_{x\to\infty} \frac{x+\sqrt{x^2-2x-2}\,/:x}{x\,/:x} = \frac{1+\sqrt{1-0-0}}{1} = 2$$

$k=2$

$$l = \lim_{x\to\infty} \left[x+\sqrt{x^2-2x-2} - 2x\right] = \lim_{x\to\infty} \left[\sqrt{x^2-2x-2} - x\right] \cdot \frac{\sqrt{x^2-2x-2}+x}{\sqrt{x^2-2x-2}+x} = \lim_{x\to\infty} \frac{x^2-2x-2-x^2}{\sqrt{x^2-2x-2}+x} =$$

$$= \lim_{x\to\infty} \frac{-2x-2\,/:x}{\sqrt{x^2-2x-2}+x\,/:x} = \frac{-2-0}{\sqrt{1-0-0}+1} = \frac{-2}{2} = -1$$

$l=-1$

$y = 2x-1 \rightarrow$ K.A

L.K.A

$$k = \lim_{x\to-\infty} \frac{-x+\sqrt{x^2+2x-2}\,/:x}{-x\,/:x} = \frac{-1+1}{-1} = \frac{0}{-1} \Rightarrow \text{NEMA L.K.A}$$

$$f'(x) = 1 + \frac{1}{2\sqrt{x^2-2x-2}} \cdot (2x-2) = 1 + \frac{2x-2}{2\sqrt{x^2-2x-2}} = 1 + \frac{2(x-1)}{2\sqrt{x^2-2x-2}} = 1 + \frac{x-1}{\sqrt{x^2-2x-2}}$$

$f'(x)=0$

$1+\frac{x-1}{\sqrt{x^2-2x-2}}=0 \;/\cdot\sqrt{x^2-2x-2}$

$\sqrt{x^2-2x-2}+x-1=0$

$\sqrt{x^2-2x-2}=1-x\;/^2$

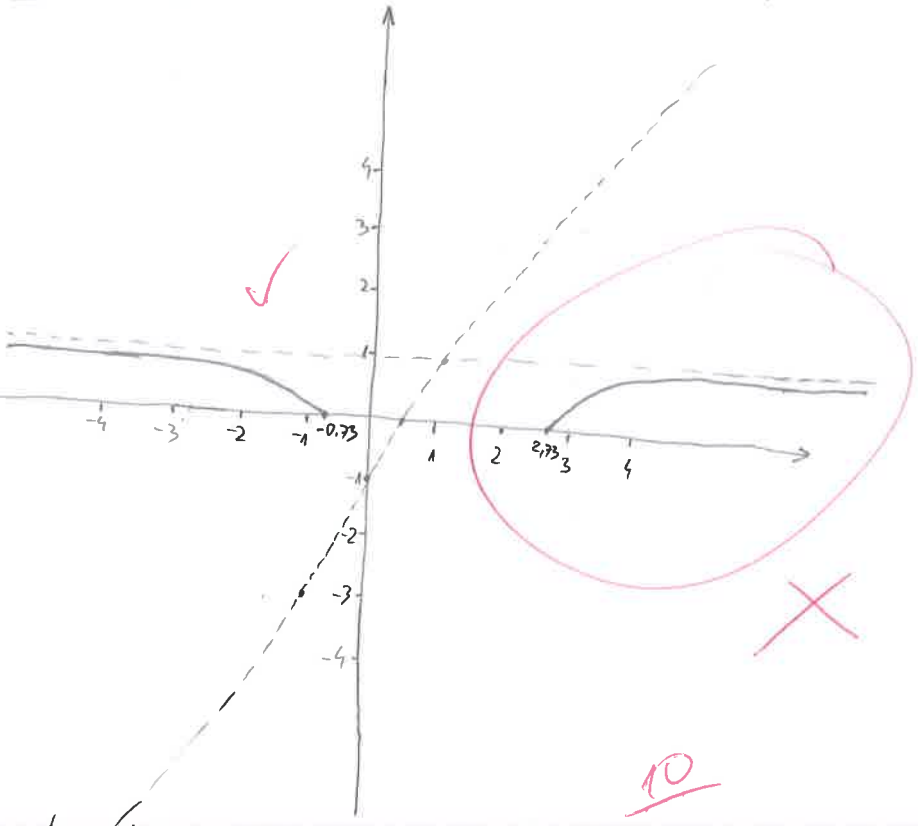
$x^2-2x-2=1-2x+x^2$

$-2=1$ ⇒ NEMA EKSTREMA

	$-\infty$	0,73	2,73	∞
$f'(x)$	−		+	
$f(x)$	↘		↗	

$y=2x-1$

x	y
0	-1
1	1
-1	-3
2	3
$\frac{1}{2}$	0



✓ ✗

10

⑥ $f(x)=e^{x^2-4x}$

$x_0=0$

$f'(x)=e^{x^2-4x}$ ✗

$f'(x_0)=e^{0^2-4\cdot0}=e^0=1$

$y-y_0=f'(x_0)(x-x_0)$

$y-1=1\cdot(x-0)$

$y=x-1$ ✓

⑤ $f(x)=\sqrt{x+2}+\sqrt{4-x}$

$f'(x)=\frac{1}{2\sqrt{x+2}}\cdot1+\frac{1}{2\sqrt{4-x}}\cdot(-1)$

$f'(x)=\frac{1}{2\sqrt{x+2}}-\frac{1}{2\sqrt{4-x}}$

$\frac{1}{2\sqrt{x+2}}-\frac{1}{2\sqrt{4-x}}=0\;/\cdot[2\sqrt{x+2}\,/\,2\sqrt{4-x}]$

$2\sqrt{4-x}-2\sqrt{x+2}=0$

$2\sqrt{4-x}=2\sqrt{x+2}\;/^2$

$4(4-x)=4(x+2)\;/:4$

$4-x-x-2=0$

$-2x=-2$

$x=1$

$y=3,46$?

0

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POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: PETAR DOBRIĆ **VRIJEME POČETKA:** 9:20

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

0269078611

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5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. — 6+6+3

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-4x}$, u točki gdje je $x = 0$. — 15

Ukupno: 23

(6.) $f(x) = e^{x^2-4x}$ $\quad x_0 = 0$ $\quad y_0 = e^{0^2-4\cdot 0} = e^0 = 1$ $\quad T(0,1)$

$t \dots \quad y - y_0 = f'(0)(x - x_0)$

$f'(x) = e^{x^2-4x} \cdot (x^2-4x)' = e^{x^2-4x} \cdot (2x-4) = 2xe^{x^2-4x} - 4e^{x^2-4x}$

$f'(0) = 2 \cdot 0 \cdot e^{0^2-4\cdot 0} - 4 \cdot e^{0^2-4\cdot 0} = -4$

$y - 1 = -4(x - 0)$

$y - 1 = -4x$

$t \dots \quad y = -4x + 1$ ✓

(3.) $\sum_n \frac{n^2}{2^n} = \frac{\frac{(n+1)^2}{2^{n+1}}}{\frac{n^2}{2^n}} = \frac{\frac{n^2+2n+1}{2^{n+1}}}{\frac{n^2}{2^n}} = \frac{2n(n^2+2n+1)}{n^2 \cdot 2^{n+1}} = \frac{2n^3+4n^2+2n}{2n^{n+3}}$

$\frac{a_{n+1}}{a_n}$

$= \frac{2n(n^2-2n+2)}{2n^{n+2}} = \frac{n^2-2n+2}{2n^{n+2}}$?

(4) $f(x) = x + \sqrt{x^2-2x-2}$

1) DOMENA

$x^2-2x-2 \geq 0$

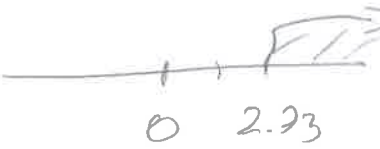
$x^2-2x-2=0 \quad a=1,\ b=-2,\ c=-2$

$x_{1,2} = \frac{2 \pm \sqrt{4+8}}{2}$

$x_{1,2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$

$x_1 = \frac{2+2\sqrt{3}}{2} = 2.73$

$x_2 = \frac{2-2\sqrt{3}}{2} = -0.73$



...$x \in [2.73, +\infty)$

2) NUL TOCKA

$x + \sqrt{x^2-2x-2} = 0 \ /^2$

$x^2 + x^2 - 2x - 2 = 0$

$2x^2 - 2x - 2 = 0 \ /:2$

$x^2 - x - 1 = 0 \quad a=1,\ b=-1,\ c=-1$

$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2}$

$x_1 = \frac{1+\sqrt{5}}{2} = 1.62$ nema N.T.

$x_2 = \frac{1-\sqrt{5}}{2} = -0.62$

3) ASIMPTOTE

V. A

$\lim_{x \to 2.73^+} x + \sqrt{x^2-2x-2} = +\infty$

$x = 2.73$ s desne

H. A.

$\lim_{x \to +\infty} x + \sqrt{x^2-2x-2} = \frac{x}{x} + \sqrt{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{2}{x^2}}$

$= 1 + 1 = 2$

$y = 2$ s desne

nema kose

4) PARNOST

$f(-x) = -x + \sqrt{(-x)^2 - 2(-x) - 2}$

$f(-x) = -x + \sqrt{x^2 + 2x - 2}$

ni parna ni neparna

5) I PER. (STOC. RAST)

$f'(x) = x' + \left(\sqrt{x^2-2x-2}\right)' = 1 + \left[\frac{1}{2\sqrt{x^2-2x-2}} \cdot (x^2-2x-2)'\right] = 1 + \left(\frac{2x-2}{2\sqrt{x^2-2x-2}}\right)$

$= 1 + \frac{2(x-1)}{2\sqrt{x^2-2x-2}} = \frac{(x-1)+(\sqrt{x^2-2x-2})}{\sqrt{x^2-2x-2}} = \frac{x-1+\sqrt{x^2-2x-2}}{\sqrt{x^2-2x-2}}$

$x - 1 + \sqrt{x^2-2x-2} = 0 \ /^2$

$x^2 - 1 + x^2 - 2x - 2 = 0$

$2x^2 - 2x - 3 = 0 \quad a=2,\ b=-2,\ c=-3$

$x_{1,2} = \frac{-2 \pm \sqrt{4+24}}{4}$

$x_{1,2} = \frac{-2 \pm \sqrt{28}}{4} \quad x_{1,2} = \frac{-2 \pm 2\sqrt{7}}{4}$

$x_1 = \frac{-2+2\sqrt{7}}{4} = 0.82$

$x_2 = \frac{-2-2\sqrt{7}}{4} = -1.82$

LOMA m i M

6) II DER. (100.INF.)

$$f'(x) = \frac{x-1+\sqrt{x^2-2x-2}}{\sqrt{x^2-2x-2}}$$

$$f''(x) = \frac{\left(x-1+\sqrt{x^2-2x-2}\right)' \cdot \sqrt{x^2-2x-2} - \left(x-1+\sqrt{x^2-2x-2}\right) \cdot \left(\sqrt{x^2-2x-2}\right)'}{\left(\sqrt{x^2-2x-2}\right)^2}$$

$$f''(x) = \frac{1+\frac{2x-2}{2\sqrt{x^2-2x-2}} \cdot \sqrt{x^2-2x-2} - \left(x-1+\sqrt{x^2-2x-2} \cdot \frac{2x-2}{2\sqrt{x^2-2x-2}}\right)}{x^2-2x-2}$$

$$f''(x) = \frac{1+\frac{2(x-1)}{2\sqrt{x^2-2x-2}} \cdot \sqrt{x^2-2x-2} - \left(x-1+\sqrt{x^2-2x-2} \cdot \frac{2(x-1)}{2\sqrt{x^2-2x-2}}\right)}{x^2-2x-2}$$

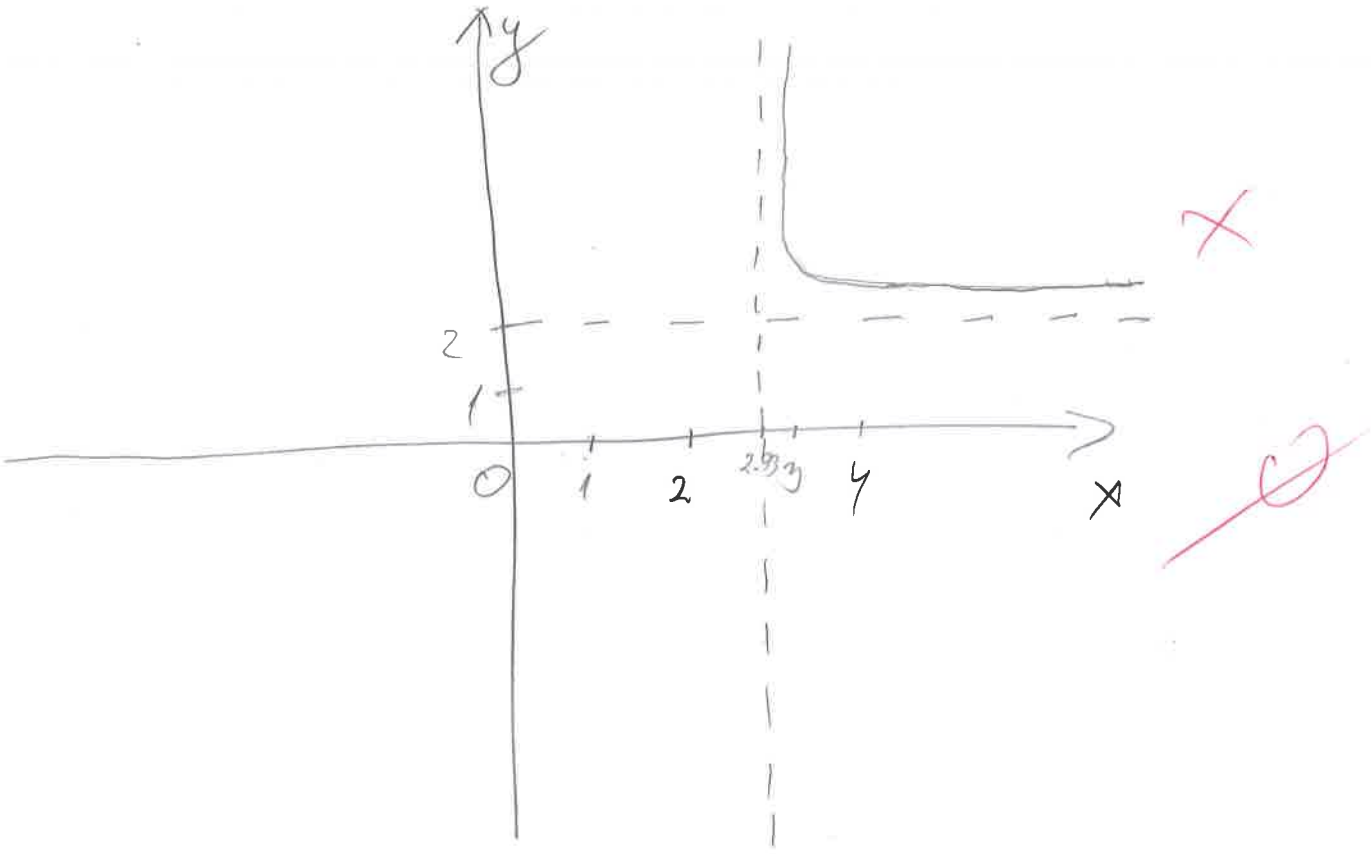
$$f''(x) = \frac{1+\frac{(x-1)\left(\sqrt{x^2-2x-2}\right)}{\sqrt{x^2-2x-2}} - \left(x-1+\frac{(x-1)\left(\sqrt{x^2-2x-2}\right)}{\sqrt{x^2-2x-2}}\right)}{x^2-2x-2}$$

$$f''(x) = \frac{1+\frac{(x-1)\cdot\left(\sqrt{x^2-2x-2}\right)}{\sqrt{x^2-2x-2}} - x+1-\frac{(x-1)\cdot\left(\sqrt{x^2-2x-2}\right)}{\sqrt{x^2-2x-2}}}{x^2-2x-2}$$

$$f''(x) = \frac{\frac{(x-1)\cdot\sqrt{x^2-2x-2}+\left(\sqrt{x^2-2x-2}\right)}{\sqrt{x^2-2x-2}} - \frac{(x-1)\cdot\left(\sqrt{x^2-2x-2}\right)+\left(\sqrt{x^2-2x-2}\right)-x\left(\sqrt{x^2-2x-2}\right)}{\sqrt{x^2-2x-2}}}{x^2-2x-2}$$

$$f''(x) = \frac{\frac{(x-1)\cdot\left(\sqrt{x^2-2x-2}\right)+\left(\sqrt{x^2-2x-2}\right)}{\sqrt{x^2-2x-2}} \cdot \left[1-\left(x-\sqrt{x^2-2x-2}\right)\right]}{x^2-2x-2}$$

$$\frac{(x-1)\cdot\left(\sqrt{x^2-2x-2}\right)+\left(\sqrt{x^2-2x-2}\right)}{\sqrt{x^2-2x-2}} \cdot \left[1-x\left(\sqrt{x^2-2x-2}\right)\right] = 0$$



(2) $\frac{z+i}{z-i} = \overline{2+3i}$

$i^0 = 1$
$i^1 = i$
$i^2 = -1$

$z_1 + i = \overline{2+3i}$

$z_1 = 2-3i-i$

$z_1 = 2-4i$

$z_2 - i = 2-3i$

$z_2 = 2-3i+i$

$z_2 = 2-2i$

(4) $f(x) = \frac{x^2+2}{x^2-4}$

1) DOMENA

$x^2 - 4 \neq 0$

$x^2 \neq 4 \mid \sqrt{}$

$x \neq \pm 2$

$D_f: x \in [-2, 2]$

2) NULTOČKE

$x^2 + 2 = 0$

$x^2 = 2 \mid \sqrt{}$

$x = \pm\sqrt{2}$

$x_1 = \sqrt{2} \approx 1.41$

$x_2 = -\sqrt{2} \approx -1.41$

N.T $(1.41, 0)$
$(-1.41, 0)$

3) ASIMPTOTE

VA

$\lim_{x \to 2^+} \frac{x^2+2}{x^2-4} = -\infty$

$x = -2$ slijeva ne postoji

$\lim_{x \to 2^-} \frac{x^2+2}{x^2-4} = -\infty$

$x = 2$ slijeva

H. A

$\lim_{x \to \pm\infty} \frac{x^2+2}{x^2-4} : x^2 = \frac{\frac{x^2}{x^2} + \frac{2}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = \frac{1}{1} = 1$

$y = 1$ s lijeva i desna nema kos

4) I DERIVACIJA (STAC. TOČKE)

$f'(x) = \frac{(x^2+2)' \cdot (x^2-4) - (x^2+2)(x^2-4)'}{(x^2-4)^2}$

$f'(x) = \frac{2x(x^2-4) - (x^2+2)2x}{(x^2-4)^2}$

$f'(x) = \frac{2x(x^2-4) - 2x(x^2+2)}{(x^2-4)^2}$

$2x(x^2-4) - 2x(x^2+2) = 0$

$2x^3 - 8x - 2x^3 - 4x = 0$

$-12x = 0$

$x = 0$

$(0, -\frac{1}{2})$

$y = \frac{0^2+2}{0^2-4} = \frac{2}{-4} = -\frac{1}{2}$

	-2		-1		0		1		2
f'		+		+	0	−		−	
f		↗		↗	max	↘		↘	

3) II DER. (TOČKE INF)

$$f'(x) = \frac{-12x}{(x^2-4)^2}$$

$$f''(x) = \frac{(-12x)' \cdot (x^2-4)^2 - (-12x)(x^2-4)^2}{(x^2-4)^4} = \frac{-12(x^2-4)^2 + 12x \cdot 2(x^2-4)}{(x^2-4)^4}$$

$$\cdot (x^2-4)' = \frac{12(x^4-8x^2+16) + 24x(x^2-4)2x}{(x^2-4)^4} =$$

$$= \frac{12x^4 - 96x^2 + 192 + 48x(x^2-4)}{(x^2-4)^4} = \frac{12x^4 - 96x^2 + 192 + 48x^3 - 192x}{(x^2-4)^4}$$

$$= \frac{12x^4 + 48x^3 - 96x^2 - 192x + 192}{(x^2-4)^4}$$

$$12x^4 + 48x^3 - 96x^2 - 192x + 192 = 0 \quad / :12$$

$$x^4 + 4x^3 - 8x^2 - 16x + 16 = 0$$

$$x^4 + 4x^3 - 8x^2 - 16x = -16$$

$$x(x^3 + 4x^2 - 8x - 16) = -16$$

$x_1 = -16$

$$x^3 + 4x^2 - 8x - 16 = 0$$

$$x^3 + 4x^2 - 8x = 16$$

5) II DEO (TOOKE 12T.) 4. zadatak

$$f'(x) = \frac{-12x}{(x^2-4)^2} \qquad f''(x) = \frac{-12(x^2-4)^2 + 48x(x^2-4)}{(x^2-4)^4} = \frac{(x^2-4)\left(-12(x^2-4)+48x\right)}{(x^2-4)^4}$$

$$= \frac{-12x^2+48+48x}{(x^2-4)^3}$$

$-12x+48x+48=0 \;/:12$

$-x+4x+4=0$ $\quad a=-1,\ b=4,\ c=4$

$$x_{12} = \frac{-4 \pm \sqrt{16+16}}{-2}$$

$$x_1 = \frac{-4+4\sqrt{2}}{-2} = -0.83$$

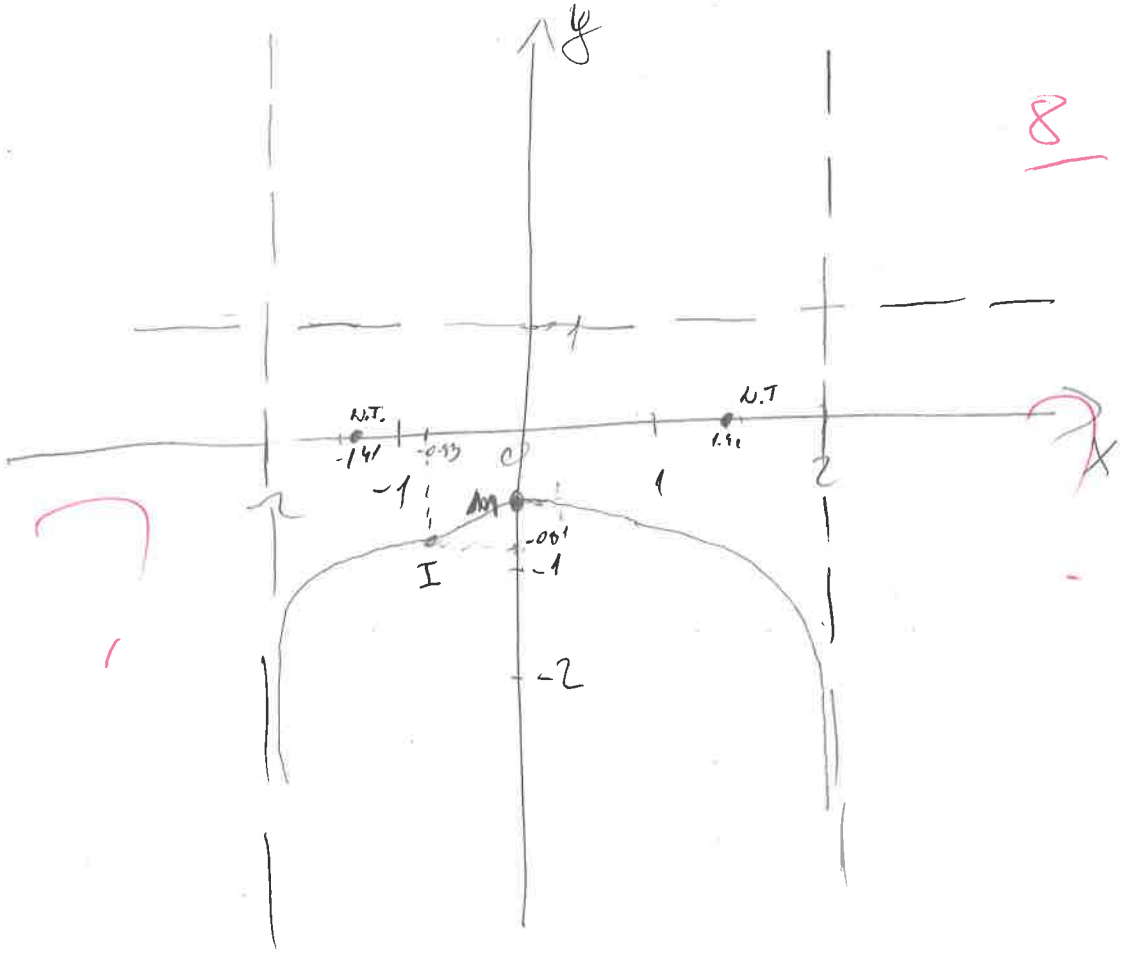
$$x_2 = \frac{-4-4\sqrt{2}}{-2} = 4.83$$

	-2		-1		-0.83		2	
f''		+		+	0	−		
f		∪		∪	I	∩		

I (-0.83, -0.81)

$$y = \frac{(-0.83)^2+2}{(-0.83)-4} = -0.81$$

8



⑤ $f(x)=\sqrt{x+2}+\sqrt{4-x}$

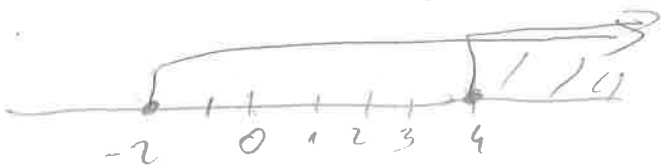
1) DOMENA

$x+2\geq 0$ $\quad 4-x\geq 0$ $\quad \Rightarrow \ldots x\in[4,+\infty]$

$x\geq -2$ $\quad x\leq 4$

2) I DER. (STAC. TOČKE)

$$f'(x)=\frac{1}{2\sqrt{x+2}}\cdot(x+2)'+\frac{1}{2\sqrt{4-x}}\cdot(4-x)'$$



$$f'(x)=\frac{1}{2\sqrt{x+2}}-\frac{1}{2\sqrt{4-x}}$$

$$f'(x)=\frac{2\sqrt{4-x}-2\sqrt{x+2}}{2\sqrt{x+2}-2\sqrt{4-x}}=\frac{2(\sqrt{4-x}-\sqrt{x+2})}{2\sqrt{x+2}-\sqrt{4-x}}\quad\frac{\sqrt{4-x}-\sqrt{x+2}}{\sqrt{x+2}-\sqrt{4-x}}$$

$\sqrt{4-x}-\sqrt{x+2}=0\ /^2$

$4-x-(x+2)=0$

$4-x-x-2=0$

$-2x=2\ /(:2)$

$x=-1$

nema min. ni max

Tok je ograničen s lijeve strane sa domenom, a s desne ide u beskonačnost tj. $\Rightarrow \ldots x\in[4,+\infty\rangle$

nema minimuma ni maksimuma

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 14

IME I PREZIME: Bruno Stipanov **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0189-2013

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 2x - 2}$ i skicirati graf. — 20 graf
2. Riješiti: $\frac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!* — 12+3
3. Odrediti konvergenciju reda $\sum_n \frac{n^2}{2^n}$. — 15
4. Odrediti tok funkcije $f(x) = \frac{x^2+2}{x^2-4}$ i skicirati graf. — 20 graf
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. — 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-4x}$, u točki gdje je $x = 0$. — 15

Ukupno: 20

4. $f(x) = \frac{x^2+2}{x^2-4}$

domena $x^2 - 4 \neq 0$

$x \neq \pm 2$

ZAŠTO?

asimptote V.A. $x = 2$, $x = -2$ M.A. $y = 1$

	$-\infty$		-2		0		$+\infty$
$f'(x)$		$+$	$\nexists$	$+$	0	$-$	$\nexists$
$f(x)$		↗		↗		↘	

ŠTO OVO ZNAČI?

$f'(x) = \frac{2x(x^2-4) - 2x(x^2+2)}{(x^2-4)^2}$

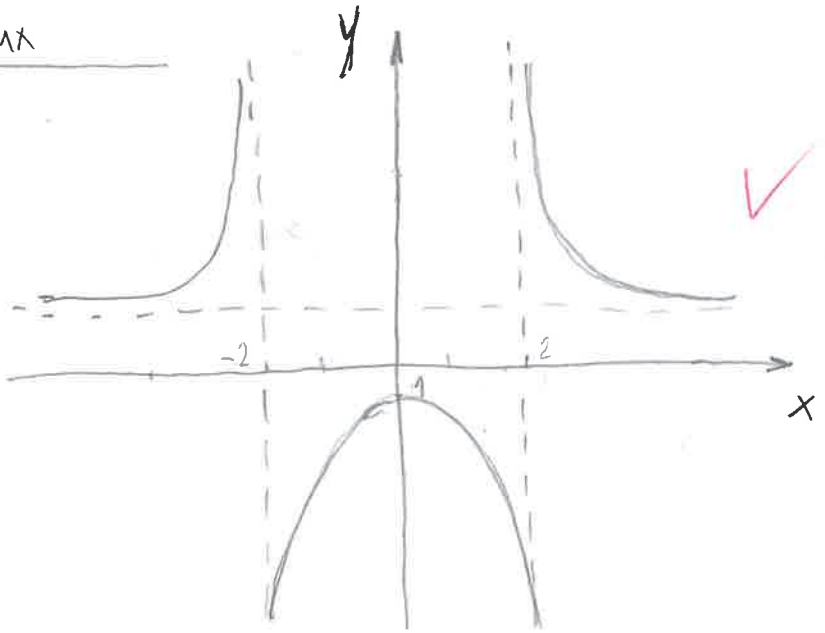
$f'(x) = \frac{2x - 8x - 2x^3 - 4x}{(x^2-4)^2}$

$= \frac{-12x}{(x^2-4)}$

$f'(x) = 0$

$-12x = 0$

$x = 0$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 14

IME I PREZIME: ANTONIO BEGIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0374-14

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 2x - 2}$ i skicirati graf. 20 graf
2. Riješiti: $\frac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!* 12+3
3. Odrediti konvergenciju reda $\sum_n \frac{n^2}{2^n}$. 15
4. Odrediti tok funkcije $f(x) = \frac{x^2+2}{x^2-4}$ i skicirati graf. 20 graf
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-4x}$, u točki gdje je $x = 0$. 15

Ukupno: 35

$y = f(x_0) + f'(x_0) \cdot (x - x_0)$

BEGIĆ ANTONIO

5. $f(x) = e^{x^2-4x}$ u točki gdje je $x = 0$

$f'(x) = e^{x^2-4x}$

$f'(x) = e^{x^2-4x} \cdot 2x - 4$

$f(x_0) = 1$

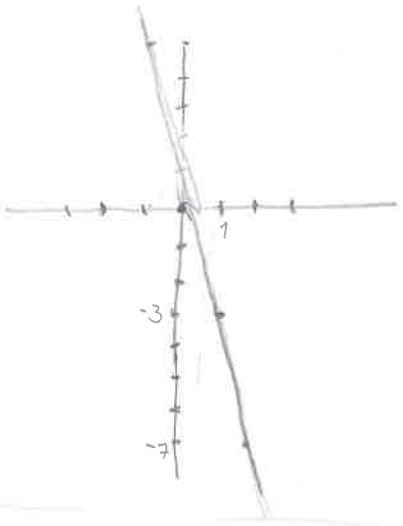
$f'(x_0) = -4$

$y = 1 - 4 \cdot (x - 0)$

$y = 1 - 4x$ ✓

$x = 1 \quad y = -3$

$x = 2 \quad y = -7$

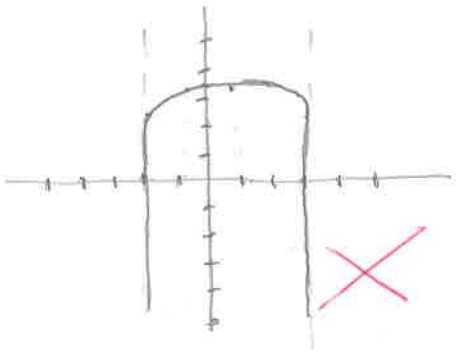


5. $f(x) = \sqrt{x+2} + \sqrt{4-x}$

$x + 2 \geq 0 \qquad 4 - x \geq 0$

$x \geq -2 \qquad -x \geq -4$

$x \leq 4$



$f'(x) = \sqrt{x+2} + \sqrt{4-x}$

$f'(x) = \frac{1}{2\sqrt{x+2}} \cdot 1 + \frac{1}{2\sqrt{4-x}} \cdot -1$

$f'(x) = \frac{1}{2\sqrt{x+2}} - \frac{1}{2\sqrt{4-x}}$

$D_f = [-2, 4] \qquad f(x_0) = 3,4$

Globalni maksimum je $f(x_0) = 3,4$ dok minimum ne postoji zbog toka funkcije, dok sam tok funkcije graniči sa vertikalnim osima ali u njima je postojan

0

4. $f(x) = \frac{x^2+2}{x^2-4}$

$\cdot x^2 - 4 \neq 0$

$x^2 = 4$

$x = \sqrt{4}$

$x = \pm 2$

V.A.

$\lim_{x \to 2} \frac{x^2+2}{x^2-4} = \frac{4+2}{4-4} = \frac{6}{0} = \pm\infty$ POSTOJI

H.A.

$\lim_{x \to \pm\infty} \frac{x^2+2}{x^2-4} \overset{L'H}{=} \frac{1+0}{1-0} = 1$ Postoji

$Df = \mathbb{R} \setminus \{2\}$

$f(x_0) = -0{,}5$

$f'(x) = \frac{x^2+2}{x^2-4} \cdot \frac{(2x)(x^2-4) - (x^2+2)(2x)}{(x^2-4)^2}$

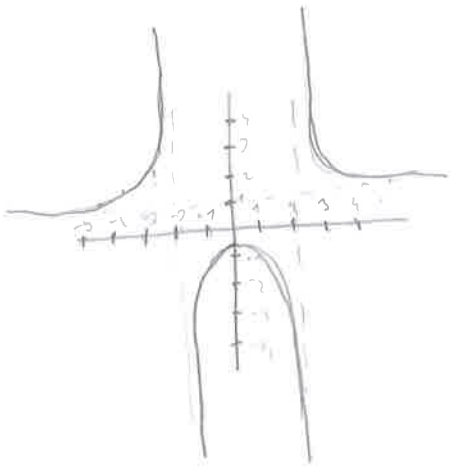
$= \frac{2x^3 - 8x - 2x^3 + 4x}{(x^2-4)^2} = \frac{-12x}{(x^2-4)^2}$

	-2	0	2
$f'(x)$	+	+	−	−
$f''x$	+	−	−	+
$f(x)$	↗	(	)	↘

$f''(x) = \frac{-12(x^2-4)^2 + 12x \cdot 2(x^2-4) \cdot 2x}{(x^2-4)^4}$

$f''(x) = \frac{[x^2-4]\left(-12(x^2-4) + 12 \cdot 2 \cdot 2x\right)}{(x^2-4)^{4}}$

$f''(x) = \frac{-12x^2 + 48 + 48x}{(x^2-4)^3}$



1. ANTONIO BEGIĆ

$f_{(x)} = x + \sqrt{x^2-2x-2}$

$f(x_0) = \text{error}$ ...

$x^2-2x-2 \geq 0$

$a=1 \quad b=-2 \quad c=-2$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{2 \pm \sqrt{4+8}}{2}$

$x_1 = 2{,}73$

$x_2 = -0{,}73$

$Df = \mathbb{R} \setminus \{2{,}73, -0{,}73\}$

NIJE GRAF NIKAKVE POZNATE FUNKCIJE

$Df \langle -\infty, -0{,}73 \rangle \cup \langle 2{,}73, +\infty \rangle$

H.A. $\lim_{x \to \pm\infty} = x + \sqrt{x^2-2x-2} = x + \sqrt{x^2-2x-2} \cdot \frac{x - \sqrt{x^2-2x-2}}{x - \sqrt{x^2-2x-2}}$

$\lim_{x \to \pm\infty} = \frac{x^2 + x^2 - 2x - 2}{x - \sqrt{x^2-2x-2}} \overset{(\infty)}{=} \frac{1+1-0-0}{0-1} = \frac{2}{0} = \pm\infty$

NE POSTOJI

K.A. $\lim_{x \to \pm\infty} = \frac{f_{(x)}}{x} = \frac{x + \sqrt{x^2-2x-2}}{x} \overset{L'H}{=} \frac{1 + \sqrt{1+0+0}}{1} = \frac{2}{1} = 2$

$b = \lim_{x \to \pm\infty} (f_{(x)} - a \cdot x) = x + \sqrt{x^2-2x-2} - 2x$

$b = x + \sqrt{x^2-2x-2} \cdot \frac{x - \sqrt{x^2-2-2}}{x - \sqrt{x^2-2-2}} - 2x = \frac{x^2 + x^2 - 2x - 2 + 4x^2}{x - \sqrt{x^2-2-2}} \overset{L'H}{=}$

$b = \frac{1+1-0-0+4}{0} = \frac{6}{0} = \infty$

K.A. $y = 2x + b$

$y = 2x + 0$

$x = 1 \quad y = 2$

$x = 2 \quad y = 4$

$f'_{(x)} = x + \sqrt{x^2-2x-2} = 1 + \frac{1}{2\sqrt{x^2-2x-2}} \cdot 2x - 2$

$= 1 + \frac{2x-2}{2\sqrt{x^2-2x-2}}$

2. $\frac{z+i}{z-i} = \overline{2+3i}$ $\quad z = x+yi$ BEGIĆ ANTONIO

$\frac{x+yi+i}{x+yi-i} = 2-3i \quad / \cdot x+yi-i$

$x+yi+i = (2-3i)(x+yi-i)$

$x+yi+i = 2x+2yi-2i-3xi-3yi^2-3i^2$

$x = 2x$

$yi+i = 2yi-2i-3xi-3yi^2-3i^2$

$x = 2x$

$x = 0$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 14

IME I PREZIME: ANTONELA KAČAN **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0391-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 2x - 2}$ i skicirati graf. 20 graf
2. Riješiti: $\frac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!* 12+3
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6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-4x}$, u točki gdje je $x = 0$. 15

Ukupno: 0

$\frac{1}{\infty} = 0 \quad \frac{1}{0} = \infty \quad \frac{0}{1} = 0$

$\frac{-b \pm \sqrt{b^2-4 \cdot a \cdot c}}{2 \cdot a}$

(6.) $f(x) = e^{x^2-4x}$ $\quad x=0$ $\quad (x_0, y_0)$ $\quad (0, 1)$

$y - y_1 = f(x_0)(x - x_0)$

$e^{0^2-4\cdot 0} = e^0 = 1$

$f(x) = e^{x^2-4x} \cdot (x^2-4x)'$

$= e^{x^2-4x} \cdot 2x - 4$

$f'(x) = f(0) = e^{0^2-4\cdot 0} \cdot 2 \cdot 0 - 4 =$

TANGENTA ?

4. $f(x) = \frac{x^2+2}{x^2-4}$

1 $x^2-4 \neq 0$

$x^2 = 4 \,/\sqrt{}$

$x = 2$

$Df(x)\ x \in R \setminus \{2\}$

1. V.A $\lim_{x \to 2} \frac{2^2+2}{2^2-4} = \frac{4+2}{4-4} = \frac{6}{0} = \infty$ ✓

H.A $y = \lim_{x \to \infty} \frac{x^2+2}{x^2-4} \begin{matrix} :x^2 \\ :x^2 \end{matrix} = \frac{1}{1} = 1$

$y = \lim_{x \to -\infty} \frac{x^2+2}{x^2-4} \begin{matrix} :x^2 \\ :x^2 \end{matrix} = \frac{1}{1} = 1$

K.A $y = kx + l$

$k = \lim_{x \to \infty} \frac{f(x)}{x} = \left[\frac{\frac{x^2+2}{x^2-4}}{\frac{x}{1}}\right] = \frac{x^2+2}{x^3-4x} \begin{matrix} \\ :x^3 \end{matrix} = \frac{0}{1}$

$= 0$ — nema

4 sa osi y $x \to 0$ $f(0) = \frac{0^2+2}{0^2-4} = -\frac{1}{2}$

$\left(0, -\frac{1}{2}\right)$

sa osi x $f(x) \to 0$ $x^2+2 = 0$

$x^2 = -2$ // nema

5) $f'(x) = \dfrac{(x^2+2)' \cdot (x^2-4) - (x^2+2) \cdot (x^2-4)}{(x^2-4)^2}$

$= \dfrac{2x \cdot (x^2-4) - (x^2+2) \cdot 2x}{(x^2-4)^2} =$

$= \dfrac{2x^3 - 8x - 2x^3 - 4x}{(x^2-4)^2} = \dfrac{-12x}{(x^2-4)^2}$

$f''(x) = \dfrac{(-12x)' \cdot (x^2-4)^2 - (-12x) \cdot (x^2-4)^2{}'}{(x^2-4)^4}$

$= \dfrac{-12 \cdot (x^2-4)^2 + 12x \cdot 2(x^2-4) \cdot 2x}{(x^2-4)^4}$

$= \dfrac{-12x^2 + 48 + 48x^2}{(x^2-4)^3}$

$= \dfrac{36x^2 + 48}{(x^2-4)^3} = \dfrac{12(3x^2+4)}{(x^2-4)^3}$

6) $f'(x) = 0$

$-12x = 0$

$x = 12 \quad (12, 1.04)$

x	-0.5	0.8	1.5	5	13
$f(x)$	$+$	$+$	$-$	$-$	$-$
	↗	MAX ↘	↘	↘	↘

$MAX\left(0, -\frac{1}{2}\right)$

ANJOLELLA KACANI

$f''(x) = 0$

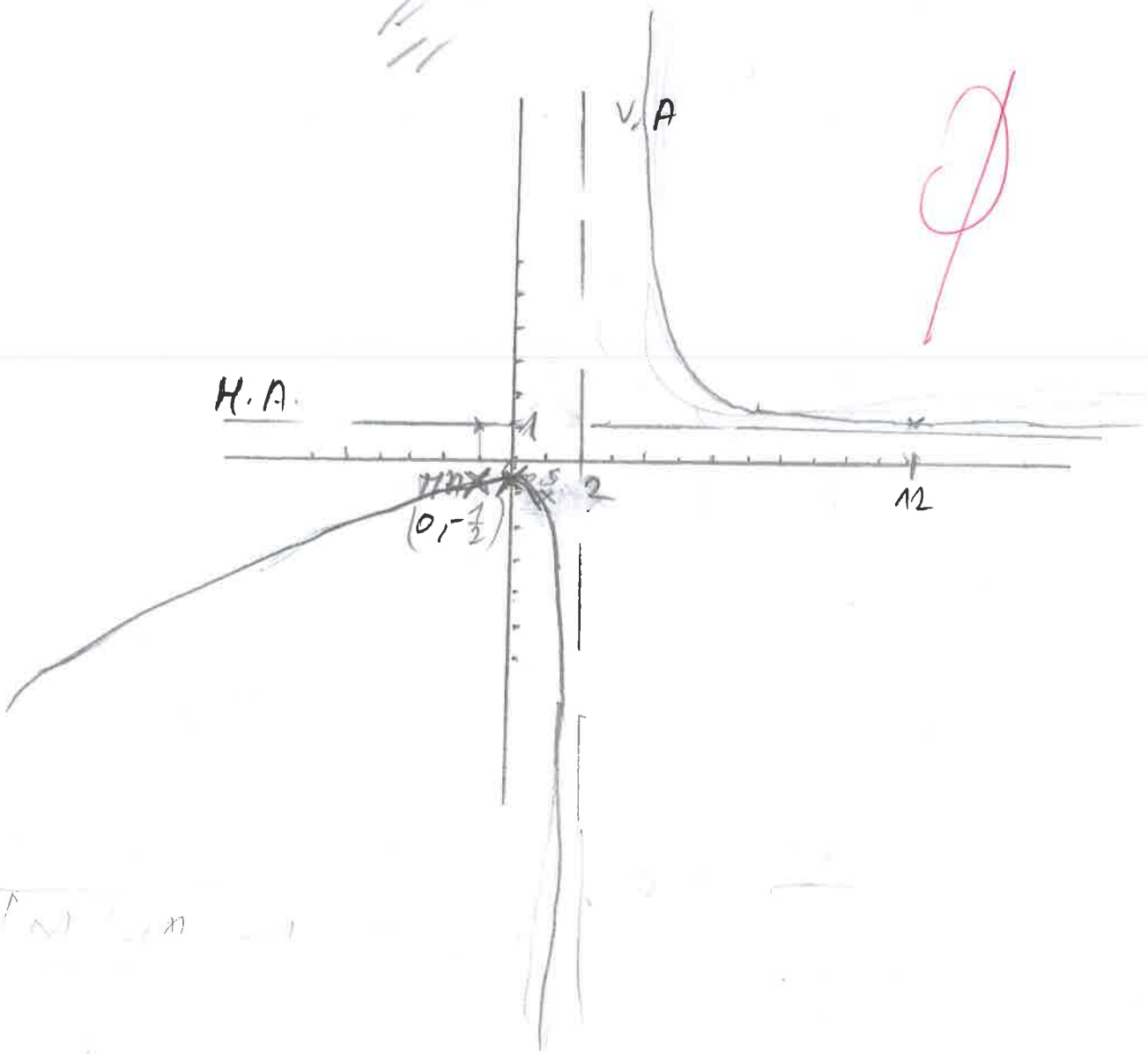
$12(3x^2+4) = 0$

$12 = 0$ // $3x^2+4 = 0$

$3x^2 = -4+3$

$x^2 = -1$ N

	$-\infty$		1		1,5		2		$+\infty$
x		-5			~~0,5~~			5	
$f''(x)$		$+$		$\|$	$-$	$\|$		$+$	
		∪			∩			∪	



① $f(x) = x + \sqrt{x^2-2x-2}$

$x^2-2x-2 \geq 0$

$x^2-2x-2=0$

$x_1 = 2.73$

$x_2 = -0.73$

	$-\infty$		-0.73		2.73		$+\infty$
x		-1		2		5	
$f(x)$		$+$	$\cup/\cap$			$+$	
x							

$D f(x) = \langle -\infty, -0.73] \cup [2.73, +\infty \rangle$

2. H.A.

$y = \lim_{x \to \infty} x + \sqrt{x^2-2x-2} \cdot \frac{x - \sqrt{x^2-2x-2}}{x - \sqrt{x^2-2x-2}} =$

$= \frac{x^2 - x^2 + 2x + 2}{x - \sqrt{x^2-2x-2}} : x = \frac{2}{1-1-2} = \frac{2}{-2} = 1$ nema

$y = \lim_{x \to -\infty} \frac{+2x-2}{-x-\sqrt{x^2+2x-2}} : x = \frac{2}{-1-1+2} = \frac{2}{0} = \infty$

SU osi y $\quad x \to 0 \quad f(0) = 0 + \sqrt{0^2-2\cdot 0-2}$

$= 0$ nema

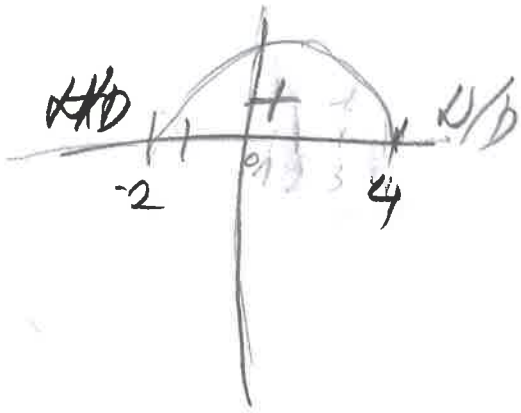
$f'(x) = (x)' \cdot (\sqrt{x^2-2x-2}) + (x) \cdot (\sqrt{x^2-2x-2})'$

$= 1 \cdot \sqrt{x^2-2x-2} + x \cdot \frac{1}{2\sqrt{x^2-2x-2}} \cdot (x^2-2x-2)'$

$= \sqrt{x^2-2x-2} + x \cdot \frac{1}{2\sqrt{x^2-2x-2}} \cdot (2x-2)$

$= \sqrt{x^2-2x-2} + x \cdot \frac{2x-2}{2\sqrt{x^2-2x-2}} = \frac{2x^2-2x}{2\sqrt{x^2-2x-2}} =$

5. $f(x) = \sqrt{x+2} + \sqrt{4-x}$



$x+2 \geq 0$ $\quad 4-x \geq 0$

$x+2=0$ $\quad 4-x=0$

$x=-2$ $\quad -x=-4$

$x=4$

$Df(x)\ [-2, 4]$

1. A.A $\quad y = \lim_{x \to \infty} \sqrt{x+2} + \sqrt{4-x} = \infty$

$y = \lim_{x \to -\infty} \sqrt{-\infty+2} + \sqrt{4+\infty} = \infty$

K.A

$f'(x) = (\sqrt{x+2})' \cdot (\sqrt{4-x}) + (\sqrt{x+2}) \cdot (\sqrt{4-x})'$

$= \frac{1}{2\sqrt{x+2}} \cdot 1 \cdot \sqrt{4-x} + (\sqrt{x+2}) \cdot \frac{1}{2\sqrt{4-x}} \cdot (-1)$

$= \frac{1\sqrt{4-x}}{2\sqrt{x-2}} + \frac{-\sqrt{x-2}}{2\sqrt{4-x}} =$

$= \frac{1\sqrt{4-x}}{2\sqrt{x-2}} + \frac{-\sqrt{x-1}}{\sqrt{4-x}}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 14

IME I PREZIME: Luka Gibić **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0172-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 2x - 2}$ i skicirati graf. — 20 graf
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6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-4x}$, u točki gdje je $x = 0$. — 15

Ukupno:

(4) $f(x) = \frac{x^2+2}{x^2-4}$

Parnost ne parnost

$f(-x) = \frac{-(x^2)+2}{-(x)^2-4} = \frac{x^2+2}{x^2-4}$ Parna

Domena

$x^2-4=0$

$x^2=4$ /√

$x=\pm 2$

Nul točke

Nemaš nul točaka jer je pod korjenom neg. broj

$\frac{x^2+2}{x^2-4} = 0 \;/\cdot(x^2-4)$

$x^2+2=0$

$x^2=-2$ /√

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 14

IME I PREZIME: ŠIME STOJAK **VRIJEME POČETKA:** 09:00 30.4.2015.

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0273-2013

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 2x - 2}$ i skicirati graf. — 20 graf
2. Riješiti: $\frac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!* — 12+3
3. Odrediti konvergenciju reda $\sum_n \frac{n^2}{2^n}$. — 15
4. Odrediti tok funkcije $f(x) = \frac{x^2+2}{x^2-4}$ i skicirati graf. — 20 graf
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. — 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-4x}$, u točki gdje je $x = 0$. — 15

Ukupno: 6

4) $f(x) = \frac{x^2+2}{x^2-4}$

DOMENA:

x^2-4 NE SMIJE BITI NULA

$x^2-4 \neq 2$

$f(x) = \frac{x^2+2}{x^2-4}$

$f'(x) = \frac{(x^2+2)' \cdot (x^2-4) - (x^2+2) \cdot (x^2-4)'}{(x^2-4)^2}$

$= \frac{2x \cdot (x^2-4) - (x^2+2) \cdot 2x}{(x^2-4)^2}$

$= \frac{2x^3 - 8x - 2x^3 + 4x}{(x^2-4)^2}$

$= \frac{-4x}{(x^2-4)^2}$

ŠIME ŠTOJAK 17-2-0273-2013

(1) $f(x) = x + \sqrt{x^2 - 2x - 2}$

DOMENA:

$x \in \mathbb{R}$

$\sqrt{x^2 - 2x - 2} \Rightarrow x \geq 0$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ 14

IME I PREZIME: JOSIP JURIČIN **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-01-18-2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 2x - 2}$ i skicirati graf. 20 graf
2. Riješiti: $\frac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!* 12+3
3. Odrediti konvergenciju reda $\sum_n \frac{n^2}{2^n}$. 15
4. Odrediti tok funkcije $f(x) = \frac{x^2+2}{x^2-4}$ i skicirati graf. 20 graf
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-4x}$, u točki gdje je $x = 0$. 15

Ukupno: 0

(4.)

$f(x) = \frac{x^2+2}{x^2-4}$

$D(f) = \mathbb{R} \setminus$

$x^2 - 4 \neq 0$

$a=1, b=0, c=-4$

$x_{1,2} = \frac{\pm\sqrt{-4 \cdot 1 \cdot (-4)}}{2}$

$x_{1,2} = \frac{\pm\sqrt{16}}{2}$

$x_{1,2} = \frac{\pm 4}{2} \rightarrow x_1 = \frac{4}{2} = 2$

$x_2 = \frac{-4}{2} = -2$

$f(x)$

$x^2 + 2 \neq 0$

$a=1, b=0, c=2$

$x_{1,2} = \frac{\pm\sqrt{-4 \cdot 1 \cdot 2}}{2} = \frac{\pm\sqrt{-8}}{2}$

NT (-2, 0)

NT (0, 2)

$f(0) = \frac{2}{-4} = -\frac{1}{2}$ $S(-\frac{1}{2}, 0)$

$\lim_{x \to 0^-} \frac{x^2+2}{x^2-4} = \frac{2}{-4} = -0.41$

$\lim_{x \to 0^+} \frac{x^2+2}{x^2-4} = -0.6$

nema V. A.

$$\lim_{x\to\pm\infty} \frac{x^2+2}{x^2-4} \;/:x^2 \overset{=}{[\tfrac{\infty}{\infty}]} \lim_{x\to\pm\infty} \frac{1+\frac{2}{x^2}}{1-\frac{4}{x^2}} = \frac{1}{1} = 1$$

$\downarrow$

O.B.A. ... $y = 1$

nema K.A.

$$f(x) = \frac{x^2+2}{x^2-4}$$

$$f'(x) = \frac{2x\cdot(x^2-4) - (x^2+2)\cdot 2x}{(x^2-4)^2} = \frac{2x^3 - 8x - 2x^3 - 4x}{(x^2-4)^2}$$

$$= \frac{-12x}{(x^2-4)^2}$$

$$f''(x) = \frac{-12\cdot(x^2-4)^2 + 12x\cdot(x^4+8x^2+16)}{(x^2-4)^4}$$

$$= \frac{-12\cdot(x^4+16) + 12x\cdot(x^4+8x^2+16)}{(x^2-4)^4}$$

$$= \frac{-12x^4 - 192 + 12x^4 + 96x^3 + 192}{(x^2-4)^4}$$

$$= \frac{12x + 96x^3}{(x^2-4)^4} =$$

$$f'(x) = \frac{-12x}{(x^2-4)^2}$$

$$f''(x) = \frac{12x + 96x^3}{(x^2-4)^4}$$

$f'(x) = 0 \longrightarrow -12x = 0$

nema stacionarne točke

	$-\infty$	-2	0	2	$+\infty$
$f'(x)$	$+$	$+$	$-$	$-$	
$f''(x)$	$-$	$+$	$+$	$-$	
$f(x)$	∩	∪	∪	∩	

$f''(x) = 0 \longrightarrow 12x + 96x^3 = 0$

$a = 96, b = 12, c = 0$

$$x_{1,2} = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 96 \cdot 0}}{2 \cdot 96}$$

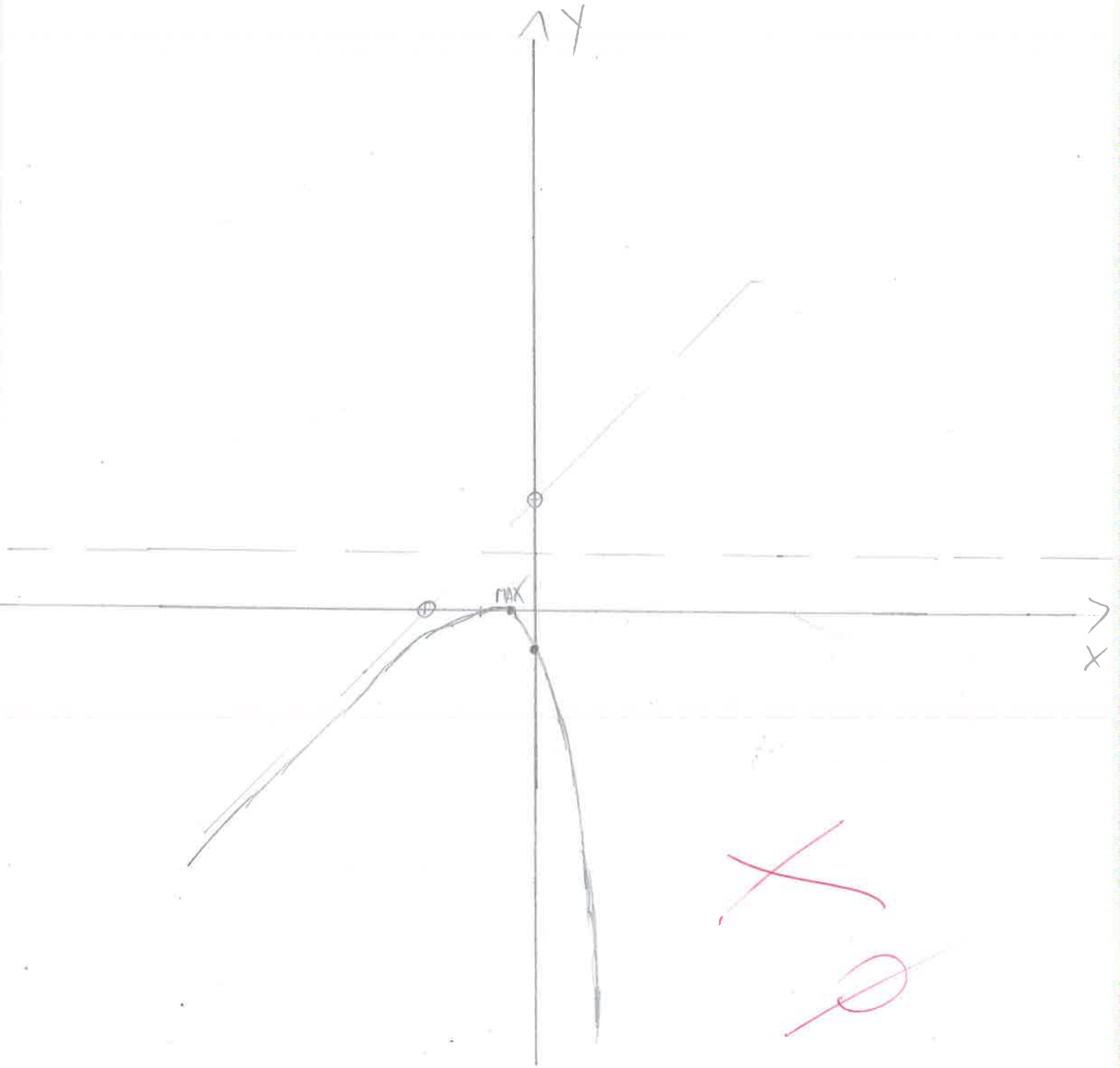
$$= \frac{-12 \pm \sqrt{144}}{192}$$

$$= \frac{-12 \pm 12}{192}$$

točke infleksije:

$$x_1 = \frac{-12+12}{192} = \boxed{0}$$

$$x_2 = \frac{-12-12}{192} = \boxed{-0.13}$$


Y
X
MAX

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 14

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: LUKA STIPANOV **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0219-2012

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 2x - 2}$ i skicirati graf. 20 graf
2. Riješiti: $\frac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!* 12+3
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5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. 6+6+3
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-4x}$, u točki gdje je $x = 0$. 15

Ukupno: 0

1. $f(x) = x + \sqrt{x^2-2x-2}$

$$f'(x) = \frac{1}{2}(x+2)^{-\frac{1}{2}} - \frac{1}{2}(4-x)^{-\frac{1}{2}} \cdot (-1)$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x+2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{4-x}}$$

$$= \frac{1}{2\sqrt{x+2}} - \frac{1}{2\sqrt{4-x}}$$

$$\frac{1}{2\sqrt{x+2}} - \frac{1}{2\sqrt{4-x}} = 0$$

$$\frac{1}{2\sqrt{x+2}} - \frac{1}{2\sqrt{4-x}} = 0 \quad / \cdot 2$$

$$\frac{1}{4(x+2)} = \frac{1}{4(4-x)}$$

$$4(4-x) = 4(x+2)$$

$$16 - 16x = 4x + 8$$

$$-16x - 4x = 8 + 16$$

$$-20x = 24$$

$$-x = 24 - 20 \qquad x = -4$$

$$-x = 4$$