MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!! 13	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ŠIME ZELENČIĆ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0370-2014 0269086384	

1. Za funkciju $f(x) = \sqrt{x^2 + 8x + 15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf

2. Odrediti tok funkcije $f(x) = \dfrac{x^2 - 2}{x^2 + 3}$ i skicirati graf. 20 graf

3. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = 2^{x^2-3}$. Posebno komentirati (ne)ograničenost. 7+7+6

4. Gaussovom metodom riješiti matrični sustav: 12+3

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

Provjeri uvrštavanjem!

5. Odrediti kada je $\dfrac{x+1}{\sqrt{x^2 - x}} + 1 > 0$. ~~15~~

6. Razvojem u Taylorov red oko nule provjeriti binomnu formulu za $f(x) = (x - 2)^4$. 15

Ukupno: 40

(1.) $f(x) = \sqrt{x^2+8x+15}$

1 DOMENA

$x^2+8x+15 \geq 0$

$x_{1,2} = \dfrac{-8 \pm \sqrt{64-4\cdot 15}}{2}$

$x_1 = -3$; $x_2 = -5$

	$-\infty$	-5		-3	$+\infty$
$x^2+8x+15$	+		−		+

$D_f = \langle -\infty, -5] \cup [-3, +\infty\rangle$

2 NULTOČKE

$f(x) = 0$

$\sqrt{x^2+8x+15} = 0 \quad /^2$

$x^2+8x+15 = 0$

$NT_1(-3, 0)$

$NT_2(-5, 0)$

3 ASIMPTOTE

$$\left.\begin{aligned} \lim_{x\to -3} \sqrt{x^2+8x+15} &= 0 \\ \lim_{x\to -5} \sqrt{x^2+8x+15} &= 0 \end{aligned}\right\} \text{NEMA V.A.}$$

$$\lim_{x\to +\infty} \sqrt{x^2+8x+15} = [+\infty] \cdot \frac{\sqrt{x^2+8x+15}}{\sqrt{x^2+8x+15}} = \frac{x^2+8x+15 \;/ :x^2}{\sqrt{x^2+8x+15} \;/ :x^2} \left[\frac{\infty}{\infty}\right]$$

$$= \lim_{x\to+\infty} \frac{1 + \overset{0}{\frac{8}{x}} + \overset{0}{\frac{15}{x^2}}}{\sqrt{\frac{1}{x^2} + \frac{8}{x^3} + \frac{15}{x^2}}} = \frac{1}{0} = \frac{1}{1} = 1 \quad y=1 \text{ JE D.H.A.}$$

$\frac{\sqrt{x^2}}{x^2} \Big|^2$ $\frac{\sqrt{8x}}{x^2} \Big|^2$ $\frac{x^2}{x^4}$ $\frac{\sqrt{x^2}}{x^4} \Big|^2$ $\frac{8x}{x^4}$ $\frac{\sqrt{x^2}}{x^2} \Big|^2$ $\frac{x^2}{x^4}$

1 EKSTREMI

$$f'(x) = \frac{1}{2\sqrt{x^2+8x+15}} \cdot \frac{(2x+8)}{1}$$

$$f''(x) = \frac{2\cdot(2\sqrt{x^2+8x+15}) - (2x+8)\cdot\left(\frac{1}{4\sqrt{x^2+8x+15}}\right)}{(2\sqrt{x^2+8x+15})^2}$$

$$f'(x) = \frac{2x+8}{2\sqrt{x^2+8x+15}} =$$

$$f''(x) =$$

	$-\infty$	-5	-3	$+\infty$
$f'(x)$	$-$	N/D	$+$	
$f(x)$	↘	N/D	↗	

$f'(x) = 0$

$2x+8=0$

$2x=-8$

$2x = \cancel{-4}$

$x=-4$ = NIJE U DOMENI

$f(-4) =$

L.HA → $\lim\limits_{x\to-\infty} \sqrt{x^2+8x+15} \left\{ \begin{matrix} x \mapsto -x \\ -\infty \mapsto +\infty \end{matrix} \right\}$

$$= \lim_{x\to+\infty} \sqrt{(-x^2)-8x+15} =$$

K.A → $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = \frac{\sqrt{x^2+8x+15}\ \ /:x}{x\ \ /:x} = \frac{1}{1}$ $\boxed{k=1}$

$$\lim_{x\to+\infty} f(x) - k\cdot x = \sqrt{x^2+8x+15} - 1\cdot x = \sqrt{x^2+8x+5} - x \cdot \frac{\sqrt{x^2+8x+5}+x}{\sqrt{x^2+8x+5}+x}$$

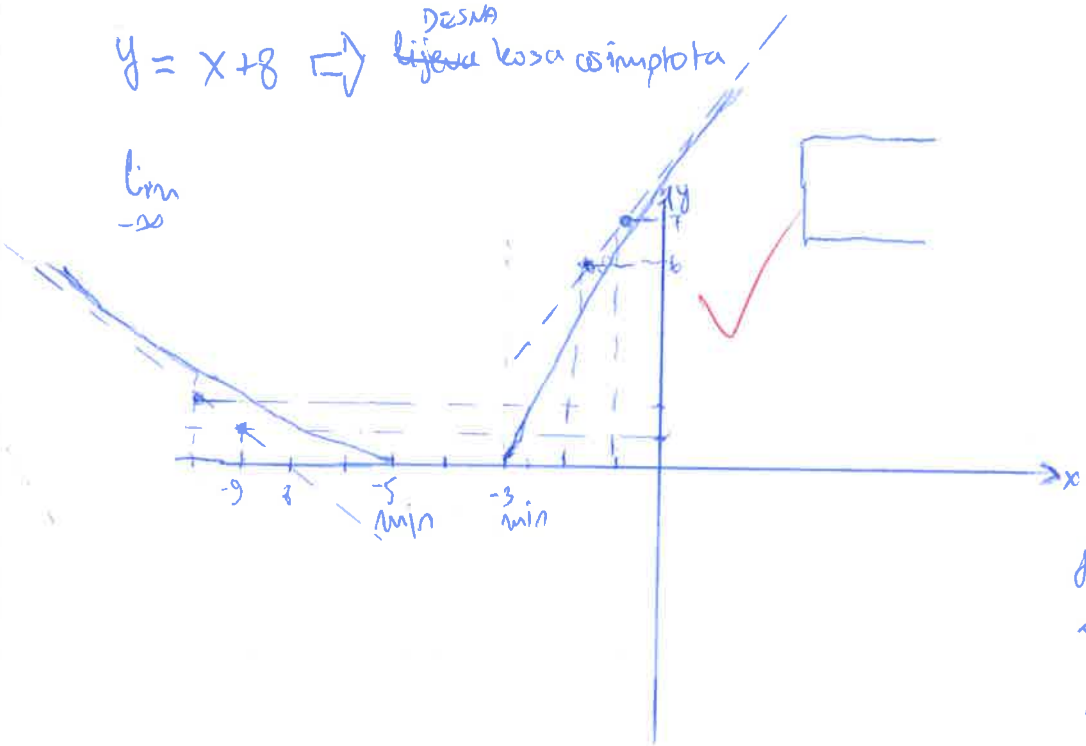
$$= \lim \frac{x^2+8x+5-x^2\ \ /:x}{\sqrt{x^2+8x+5}-x\ \ /:x} = \frac{8}{1} \qquad l=8$$

$y = x+8 \Rightarrow$ ~~lijeva~~ DESNA kosa asimptota

$\lim\limits_{-\infty}$

$\frac{\sqrt{x^2}}{x} \Big/^2 \quad \frac{x^2}{x^2}$

$\frac{\sqrt{8x}}{x} \Big/^2 \quad \frac{8x}{x^2}$



Funkcija $f(x)=\sqrt{x^2+8x+15}$ je KONVEKSNA u periodu od $\langle-\infty, 5]$ a KONKAVNA od $[-3, +\infty\rangle$

Nije omeđena odozgo ni odozdo te nije periodična.

1. NASTAVAK

ŠIME ZELENČIĆ

K.A.

$$\lim_{x\to+\infty} \frac{f(x)}{x} \begin{Bmatrix} x \to (-x) \\ -\infty \to +\infty \end{Bmatrix} = \lim_{x\to+\infty} \frac{\sqrt{x^2-8x+15}\ \ |:x}{-x\ \ |:x} = \cancel{\lim_{x\to+\infty}} = \frac{1}{-1} = -1 \quad \boxed{k=-1}$$

$$\lim_{x\to-\infty} f(x) - k\cdot x = \lim_{x\to-\infty} \sqrt{x^2-8x+15} + 1\cdot x \cdot \frac{\sqrt{x^2-8x+15}-x}{\sqrt{x^2-8x+15}-x} = \frac{\cancel{x^2}-8x+15-\cancel{x^2}\ \ |:x}{\sqrt{x^2-8x+15}-x\ \ |:x}$$

$$\begin{Bmatrix} x \to -x \\ -\infty \to +\infty \end{Bmatrix}$$

$$= \frac{-8}{1} = -8$$

$$\frac{\sqrt{x^2}}{x} = {}^2$$

$$\left(\frac{\sqrt{8x}}{x}\right)^2$$

$y = -x - 8$ je K.A.

-9, 1

-10, 2

② $f(x) = \frac{x^2-2}{x^2+3}$

ŠIME ZELENČIĆ

1. DOMENA

$x^2+3 \neq 0$

$x^2 \neq -3 \Rightarrow D_f = \mathbb{R}$

2. NULTOČKE

$x^2-2=0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$NT_1(\sqrt{2},0)$

$NT_2(-\sqrt{2},0)$

3. ASIMPTOTE

NEMA VERTIKALNIH ASIMPTOTA!

$\lim_{x\to+\infty} \frac{x^2-2}{x^2+3} = \left(\frac{+\infty}{+\infty}\right) \begin{vmatrix} :x^2 \\ :x^2 \end{vmatrix} = \frac{1}{1} = 1$ je D.H.A. ... $y=1$

$\lim_{x\to-\infty} \frac{x^2-2}{x^2+3} = \left\{\begin{matrix} -\infty \mapsto +\infty \\ x \mapsto -x \end{matrix}\right\} = \lim_{x\to+\infty} \frac{(-x)^2-2}{(-x)^2+3} = \left[\frac{+\infty}{+\infty}\right]$ L.H.A. $y=1$

4. EKSTREMI

$f'(x) = \frac{2x\cdot(x^2+3)-(x^2-2)\cdot 2x}{(x^2+3)^2}$

$f'(x) = \frac{\cancel{2x^3}+6x-\cancel{2x^3}+4x}{(x^2+3)^2} = \frac{10x}{(x^2+3)^2}$

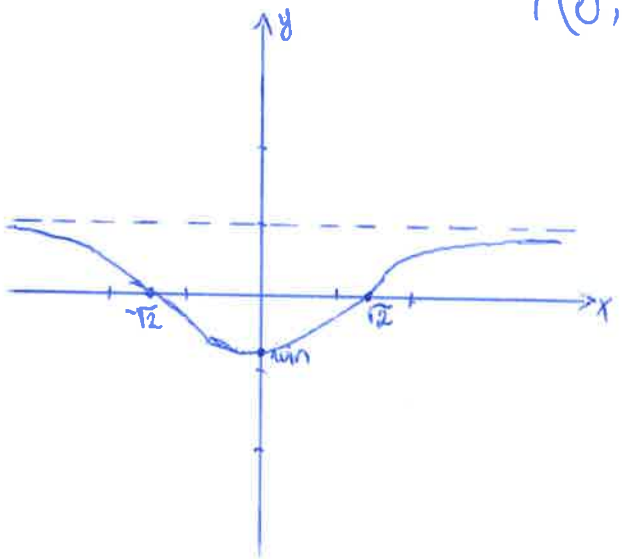
$f'(x)=0$

$10x=0$

$x=0$

$T\left(0,-\frac{2}{3}\right) \to$ minimum

	$-\infty$ … 0	0 … $+\infty$
$f'(x)$	$-$	$+$
$f(x)$	↘	↗



5. $\frac{x+1}{\sqrt{x^2-x}}+1>0$ $\quad -1, 0, 1$

$\frac{x+1}{\sqrt{x^2-x}}>1 \cdot \Big/ ^2$

$x \in R \setminus \{0,1\}$

OVAJ IZRAZ

$\frac{x^2+1}{x^2-x}>1 \Big/ \cdot (x^2-x)$ $\quad$ – x je veći od nule za svaki realni broj osim 0 i 1.

$x^2+1>\cancel{x^2}-x$

$x>-1$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 13

IME I PREZIME: HARIS AGIL **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
17-2-0126-2011

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Za funkciju $f(x) = \sqrt{x^2+8x+15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf

2. Odrediti tok funkcije $f(x) = \dfrac{x^2-2}{x^2+3}$ i skicirati graf. 20 graf

3. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = 2^{x^2-3}$. Posebno komentirati (ne)ograničenost. ~~7+7+6~~ 10

4. Gaussovom metodom riješiti matrični sustav: 12+3

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

Provjeri uvrštavanjem!

5. Odrediti kada je $\dfrac{x+1}{\sqrt{x^2-x}} + 1 > 0$. 15

6. Razvojem u Taylorov red oko nule provjeriti binomnu formulu za $f(x) = (x-2)^4$. 15

Ukupno: 40

3. $f(x) = 2^{x^2-2}$

$D(f) = \mathbb{R}$

lim (f

$\lim f(x) = +\infty$ ✓

$x \to \pm\infty$

$\lim\limits_{x\to\pm\infty} \dfrac{2^{x^2-2}}{x} \overset{L'H}{=} \lim\limits_{x\to\pm\infty} 2x \cdot 2^{x^2-2} = \infty$

LOK MAX ?
LOK MIN ✗
GLOB MIN ✗
GLOB MAX ✓ ? ✓
NEOMEĐENOST ODOZGO
OMEĐENOST ODOZDO ✓

$f'(x) = 2x \cdot 2^{x^2-2}$

$f'(x) = 0 \Rightarrow x = 0$

	$-\infty$	0	$+\infty$
$f'(x)$	$-$	$+$	
	↘	min ↗	

$f(0) = \frac{1}{4}$ ✗

Funkcija je omeđena odozdol. ✓
Ima globalni minimum $(0, \frac{1}{4})$ ✗

5. $\frac{x+1}{\sqrt{x^2-x}}+1>0$

$$\frac{x+1+\sqrt{x^2-x}}{\sqrt{x^2-x}}>0$$

$$x+1+\sqrt{x^2-x}>0$$

$$\sqrt{x^2-x}=-x-1$$

$$x^2-x=x^2+2x+1$$

$$-3x=1$$

$$\cancel{x=-\frac{1}{3}}$$

$$x^2-x>0$$

$$x(x-1)>0$$

$x=0$ $x=1$

	$-\infty$	0	1 $+\infty$
x^2-x	+	−	+

$$x \in \langle -\infty, 0 \rangle \cup \langle 1, +\infty \rangle$$

HARIS AGK

6. $f(x) = (x-2)^4$ $f(0) = 16$

$f'(x) = 4(x-2)^3$ $f'0 = -32$

$f''(x) = 12(x-2)^2$ $f''0 = 48$

$f'''(x) = 24(x-2)$ $f'''(0) = -48$

$f^{IV}(x) = 24$ $f^{IV}(0) = 24$

$f(x) = 16 - 32x + 24x^2 - 8x^3 + x^4$ ✓

BINOMNA?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 13

IME I PREZIME: MARIO NEMEŠ **VRIJEME POČETKA:** 10.00

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0269087536

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Za funkciju $f(x) = \sqrt{x^2 + 8x + 15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. ~~20 graf~~
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Ukupno: 20

1. $f(x) = \sqrt{x^2 + 8x + 15}$

$x^2 + 8x + 15 \geq 0$ $\quad D = \mathbb{R}$

$-\infty \quad -3 \quad -5 \quad \infty$

$x_1 = -3$

$x_2 = -5$

V.A. NEMA

H.A. $\lim_{x \to +\infty} \sqrt{x^2+8x+15} \;/ :x = 1$

K.A. $k = \lim_{x \to \infty} \dfrac{\sqrt{x^2+8x+15} \;/ :x}{x \;\; / :x} = 1$

$l = \lim_{x \to \infty} \sqrt{x^2+8x+15} + 1 \cdot \dfrac{\sqrt{x^2+8x+15} - 1}{\sqrt{x^2+8x+15} - 1}$

$$l = \lim_{x \to \infty} \frac{\left(\sqrt{x^2 + 8x + 15} - 1\right)^2}{\sqrt{x^2 + 8x + 5} - 1}$$

SKICA?

$2.\ f(x) = \frac{x^2-2}{x^2+3}$

MARIO NEMES

$x^2+3 \neq 0$

$D = \mathbb{R}$

$x^2 = -3$

$x = \sqrt{-3}$

V.A. NEMA

H.A $\lim\limits_{x \to \infty} \frac{x^2-2 \ /:x^2}{x^2+3 \ /:x^2} = \frac{1}{1} = 1$

K.A. $y = kx + l$

$l = \lim\limits_{x \to \infty} f(x) - k$

$k = \lim\limits_{x \to \infty} \frac{f(x)}{x}$

$k = \lim\limits_{x \to \infty} \frac{\frac{x^2-2}{x^2+3}}{\frac{x}{1}} = \lim\limits_{x \to \infty} \frac{x^2-2 \ /:x^3}{x^3+3x \ /:x^3} = \lim\limits_{x \to \infty} \frac{\frac{x^2}{x^3} - \frac{2}{x^3}}{\frac{x^3}{x^3} + \frac{3x}{x^3}} =$

$= 0$ NEMA K.A.

$f'(x) = 0$

$4x^3 + 2x = 0$

$x(4x^2+2) = 0$

$x = 0$

$4x^2+2=0$

$4x^2 = -2 \ /:2$

$2x^2 = -1 \ /:2$

$x^2 = -\frac{1}{2}$

$\cancel{x = \sqrt{-\frac{1}{2}}}$

$f'(x) = \frac{x^2-2}{x^2+3}$

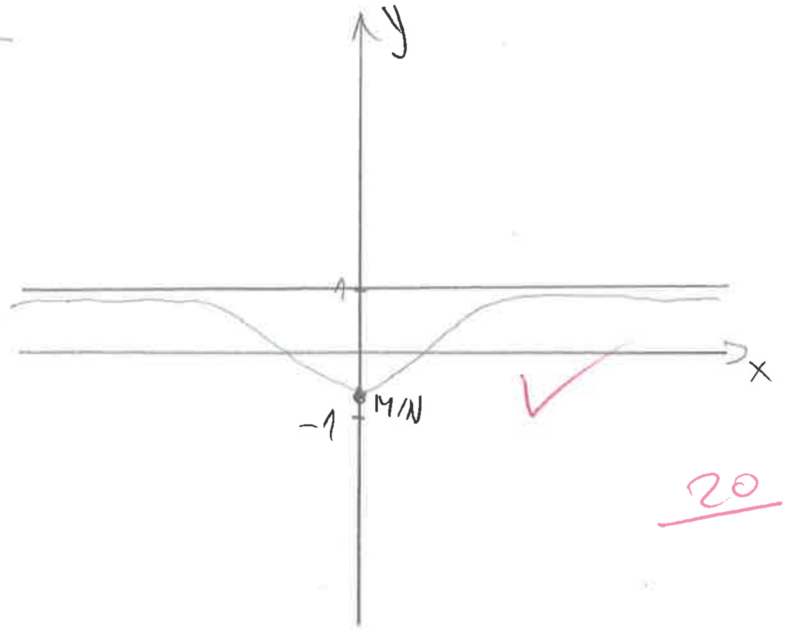
$= \frac{2x(x^2+3)+(x^2-2)\cdot 2x}{(x^2+3)^2}$

$= \frac{2x^3+6x+2x^3-4x}{(x^2+3)^2}$

$= \frac{4x^3+2x}{(x^2+3)^2}$

$-\infty$	-1	0	1	$+\infty$
	−		+	
	↘		↗	
		MIN		

MIN$(0, -0.67)$ ✓



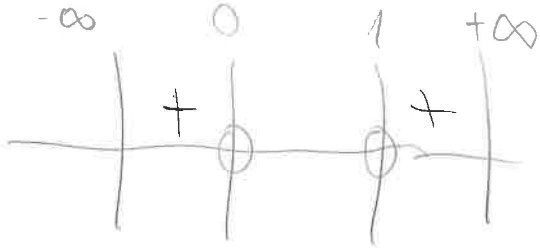
20

5. $\dfrac{x+1}{\sqrt{x^2-x}}+1>0$

$x^2-x>0$

$x_1=1$

$x_2=0$



$f(x)=\langle-\infty,0]\,[1,+\infty\rangle$

ZAŠTO ?

MARIO NEMEŠ

6. $$f(x) = f(x_0) + f'(x_0)(x-x_0) + f''(x_0)\frac{(x-x_0)^2}{2} + f'''(x_0)\frac{(x-x_0)^3}{6} + f^{IV}(x_0)\frac{(x-x_0)^4}{24}$$

$$= (x-2)^4 + 4(x-2)(x-(x-2)^4) +$$

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POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: KARLO FRANOV **VRIJEME POČETKA:** 09:00

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2 - 0311 - 2013

1. Za funkciju $f(x) = \sqrt{x^2+8x+15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf
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Ukupno: 20

(1.) $f(x) = \sqrt{x^2+8x+15}$

a) NULTOČKE

$x^2 - 8x + 15 = 0$

$x_{1,2} = -5, -3$

b) DOMENA

$x^2 - 8x + 15 = 0$

$x \in \langle -\infty, -5] \cup [-3, +\infty\rangle$

c) PARNOST

$f(-x) = \sqrt{x^2+8x+15}$

NITI PARNA NITI NEPARNA

b) ASIMPTOTE

→ VERTIKALNE NEMA

HORIZONTALNE :

$\lim\limits_{x\to+\infty} = \sqrt{x^2-8x+15} = +\infty$

KOSE

$\lim\limits_{x\to+\infty} \dfrac{\sqrt{x^2-8x+15}}{x} = 1$

$a = 1$

$\lim\limits_{x\to\infty} = \sqrt{x^2-8x+15} - x = \dfrac{\sqrt{x^2-8x+15}+x}{\sqrt{x^2-8x+15}+x} = 8$

KOSA = $y = x + 8$

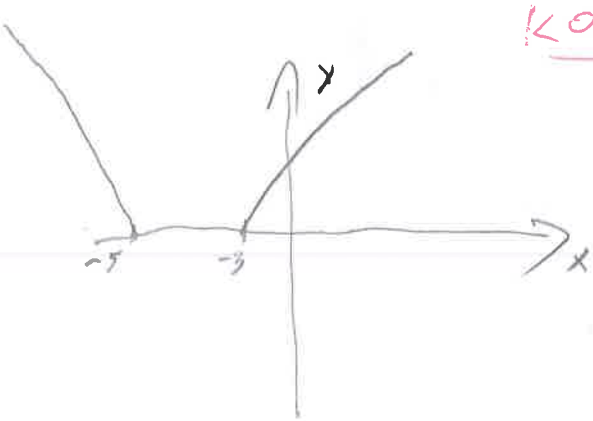
EKSTREMI ?

$$\left(\sqrt{x^2-8x+15}\right)' = \frac{2x+8}{2\sqrt{x^2-8x+15}}$$

ZBOG OBLIKA FUNKCIJE MINIMUMI SU U RUBNIM TOČKAMA

MIN (-3,0) MIN (-5,0) //

GDJE SU KOSE ASIMPTOTE ?



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 13	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: LOVRE RADOVČIĆ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0177-2013	

1. Za funkciju $f(x) = \sqrt{x^2+8x+15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. ~~20 graf~~

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Ukupno: 20

② $f(x) = \dfrac{x^2-2}{x^2+3}$

DOMENA

$x^2+3 \neq 0$

$x^2 \neq -3$

$x \neq \pm\sqrt{-3}$

$D_f: \mathbb{R}$

(NE)PARNOST

$f(-x) = \dfrac{x^2-2}{x^2+3}$ PARNA F

NULTOČKE

$\dfrac{x^2-2}{x^2+3} = 0 \,|\cdot (x^2+3)$

$x^2-2=0$

$x^2=2$

$x=\pm\sqrt{2}$

$(\sqrt{2},0)$

$(-\sqrt{2},0)$

SJECIŠTE S OSI y

$f(0) = \dfrac{0^2-2}{0^2+3} = -\dfrac{2}{3}$

$(0, -\frac{2}{3})$

ASIMPTOTE

V.A. NEMA

H.A. $\lim\limits_{x\to\infty} \dfrac{x^2-2}{x^2+3} \begin{matrix} /:x^2 \\ /:x^2 \end{matrix} = \dfrac{1}{1} = 1$

D.H.A.
L.H.A.
$y=1$

K.A.

$$k=\lim_{x\to\infty} \frac{\frac{x^2-2}{x^2+3}}{x} = \frac{x^2-2 \ /:x^3}{x^3+3x \ /:x^3} = \frac{0}{1} = 0 \qquad k=0$$

NEMA K.A.

$$l=\lim_{x\to\infty} \frac{x^2-2 \ /:x^2}{x^2+3 \ /:x^2} = \frac{1}{1} = 1$$

DERIVACIJA

$$f(x)=\left(\frac{x^2-2}{x^2+3}\right)' = \frac{(x^2-2)'(x^2+3)-(x^2-2)(x^2+3)'}{(x^2+3)^2}$$

$$= \frac{2x(x^2+3)-(x^2-2)\,2x}{(x^2+3)^2}$$

$$= \frac{2x(x^2+3-x^2+2)}{(x^2+3)^2} = \frac{10x}{(x^2+3)^2}$$

KR. TOČKE

$$\frac{10x}{(x^2+3)^2}=0 \ / \cdot (x^2+3)^2$$

$$10x=0$$

	$-\infty$ -1	0 1 $+\infty$
$f'(x)$	$-$	$+$
$f(x)$	$\searrow$	$\nearrow$

$$f(0)=\frac{0^2-2}{0^2+3}=-\frac{2}{3} \qquad \left(0,-\frac{2}{3}\right)$$

MAX

$2(x^2+3)(x^2+3)$
$(2x^2+6)(2x^3)$

DRUGA DERIVACIJA

$$f(x)=\left(\frac{10x}{(x^2+3)^2}\right)''$$

$$= \frac{(10x)'(x^2+3)^2-(10x)\left((x^2+3)^2\right)'}{(x^2+3)^4} = \frac{10(x^4+6x^2+9)-10x(4x^3+12x)}{}$$

$$= \frac{\cancel{10x^4}+60x^2+90-\cancel{40x^4}-120x^2}{(x^2+3)^4} = \frac{-30x^4-60x^2+90}{(x^2+3)^4}$$

② NASTAVAK

LOVRE RADOVČIĆ
17-1-0177-2013

$$\frac{-30x^4-60x^2+90}{(x^2+3)^4}=0 \Big/ \cdot (x^2+3)^4$$

$$-30x^4-60x^2+90=0 \Big/ \cdot(-1)$$

$$30x^4+60x^2-90$$

$$x^2=t$$

$$30t^2+60t-90=0$$

$$t_{12}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-60\pm\sqrt{60^2-4\cdot30\cdot(-90)}}{60}$$

$$=\frac{-60\pm120}{60}$$

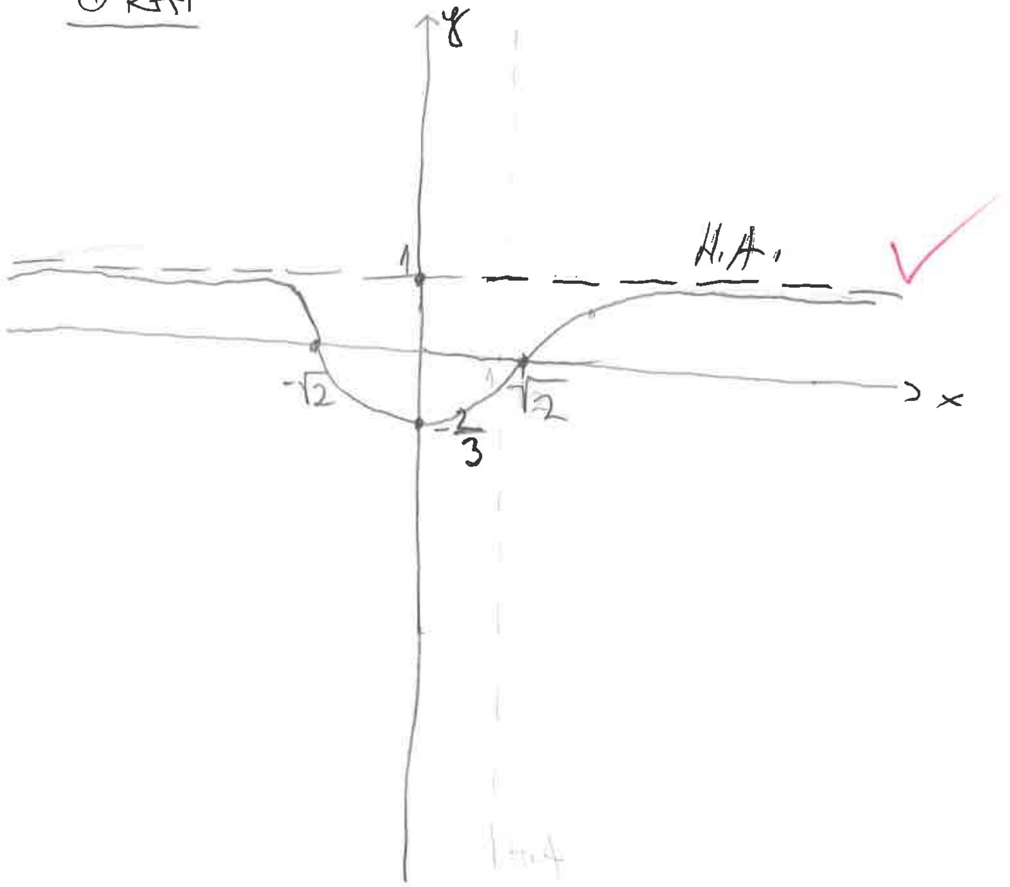
$$t_1=\frac{-60+120}{60}=1$$

$$t_2=\frac{-60-120}{60}=-3$$

KONV. | KONK.

	$-\infty$ -4	-3 0	1 2 $+\infty$
$f''(x)$	−	+	−
$f(x)$	∩	∪	∩

GRAF



1. NASTAVAK

$$f''(x)=\left(\frac{x+4}{\sqrt{x^2+8x+15}}\right)'' = \frac{(x+4)'\left(\sqrt{x^2+8x+15}\right)-(x+4)\left(\sqrt{x^2+8x+15}\right)'}{\left(\sqrt{x^2+8x+15}\right)^2}$$

$$=\frac{\sqrt{x^2+8x+15}-(x+4)\frac{1}{2\sqrt{x^2+8x+15}}\left(x^2+8x+15\right)'}{x^2+8x+15}$$

$$=\frac{\sqrt{x^2+8x+15}-(x+4)\frac{2x+8}{2\sqrt{x^2+8x+15}}}{x^2+8x+15}$$

$$=\frac{\sqrt{x^2+8x+15}-(x+4)\cdot\frac{x+4}{\sqrt{x^2+8x+15}}}{x^2+8x+15}$$

$$=\frac{\sqrt{x^2+8x+15}-\frac{x^2+4x+4x+16}{\sqrt{x^2+8x+15}}}{x^2+8x+15}$$

$$=\frac{\frac{\sqrt{x^2+8x+15}-x^2+4x+4x+16}{\sqrt{x^2+8x+15}}}{x^2+8x+15}$$

① $f(x) = \sqrt{x^2+8x+15}$

DOMENA

$x^2+8x+15 \geq 0$

$x^2+8x+15=0$

$x_{12} = \frac{-8 \pm \sqrt{8^2-4 \cdot 1 \cdot 15}}{2}$

$= \frac{-8 \pm \sqrt{64-60}}{2}$

$= \frac{-8 \pm 2}{2}$

$x_1 = \frac{-8+2}{2} = -3$

$x_2 = \frac{-8-2}{2} = -5$

	$-\infty$ -6	-5 -4	-3 -2	$+\infty$
$x^2+8x+15$	+	-	+	

D_f $\langle -\infty, -5] \cup [-3, +\infty\rangle$

(NE)PARNOST

$f(-x) = \sqrt{x^2-8x+15}$ NITI P NITI NP

NULTOČKE

$\sqrt{x^2+8x+15} = 0 \,/^2$

$x^2+8x+15=0$

$x_1 = -3 \quad x_2 = -5$

$(-3,0)$

$(-5,0)$

SJECIŠTE S OSI y

$f(0) = \sqrt{0^2+8 \cdot 0+15} = 3{,}87$

$(0, 3{,}87)$

ASIMPTOTE

V.A.

$\lim_{x \to -3} \sqrt{3^2+8\cdot(-3)+15} = 0$ NEMA V.A.

V.A.

$\lim_{x \to -5} \sqrt{5^2+6\cdot(-5)+15} = 0$

H.A.

$\lim_{x \to \infty} \sqrt{x^2+8x+15} \cdot \frac{\sqrt{x^2+8x+15}}{\sqrt{x^2+8x+15}} = \frac{\sqrt{x^2+8x+15}^{\,2}}{\sqrt{x^2+8x+15}} = \frac{x^2+8x+15 \,/ : x^2}{\sqrt{x^2+8x+15} \,/ : x^2} = \frac{1}{0}$

$= \infty$

NEMA H.A.

LOVRE RADOVČIĆ
17-1-0177-20B

D.K.A.

$$k=\lim_{x\to\infty} \frac{\sqrt{x^2+8x+15}}{x} \begin{matrix} /:x \\ /:x \end{matrix} = \frac{1}{1} = 1 \quad k=1$$

$$l=\lim_{x\to\infty} \sqrt{x^2+8x+15}-x \cdot \frac{\sqrt{x^2+8x+15}+x}{\sqrt{x^2+8x+15}+x} = \frac{\sqrt{x^2+8x+15}^2-x^2}{\sqrt{x^2+8x+15}+x} = \frac{x^2+8x+15-x^2}{\sqrt{x^2+8x+15}+x}$$

$$= \frac{8x+15}{\sqrt{x^2+8x+15}+x} \begin{matrix} /:x \\ /:x \end{matrix} = \frac{8}{2} = 4 \qquad \text{D.K.A. } y=x+4$$

L.K.A.

$$k=\lim_{x\to-\infty} \frac{\sqrt{x^2-8x+15}}{-x} \begin{matrix} /:x \\ /:x \end{matrix} = -1 \quad k=-1$$

$$l=\lim_{x\to-\infty} \sqrt{x^2+8x+15}+x \cdot \frac{\sqrt{x^2+8x+15}-x}{\sqrt{x^2+8x+15}-x} = \frac{\sqrt{x^2+8x+15}^2-x^2}{\sqrt{x^2+8x+15}-x} = \frac{x^2+8x+15-x^2}{\sqrt{x^2+8x+15}-x}$$

$$= \frac{-8x+15}{\sqrt{x^2-8x+15}+x} \begin{matrix} /:x \\ /:x \end{matrix} = \frac{-8}{2} = -4 \qquad \text{LKA } k=-x-4$$

DERIVACIJA

$$f(x)=\left(\sqrt{x^2+8x+15}\right)'$$

$$= \frac{1}{2\sqrt{x^2+8x+15}} \cdot (x^2+8x+15)'$$

$$= \frac{1}{2\sqrt{x^2+8x+15}} \cdot 2x+8 = \frac{2x+8}{2\sqrt{x^2+8x+15}} \rightarrow \frac{2(x+4)}{2\sqrt{x^2+8x+15}} = \frac{x+4}{\sqrt{x^2+8x+15}}$$

KR. TOČKE

$2x+8=0$

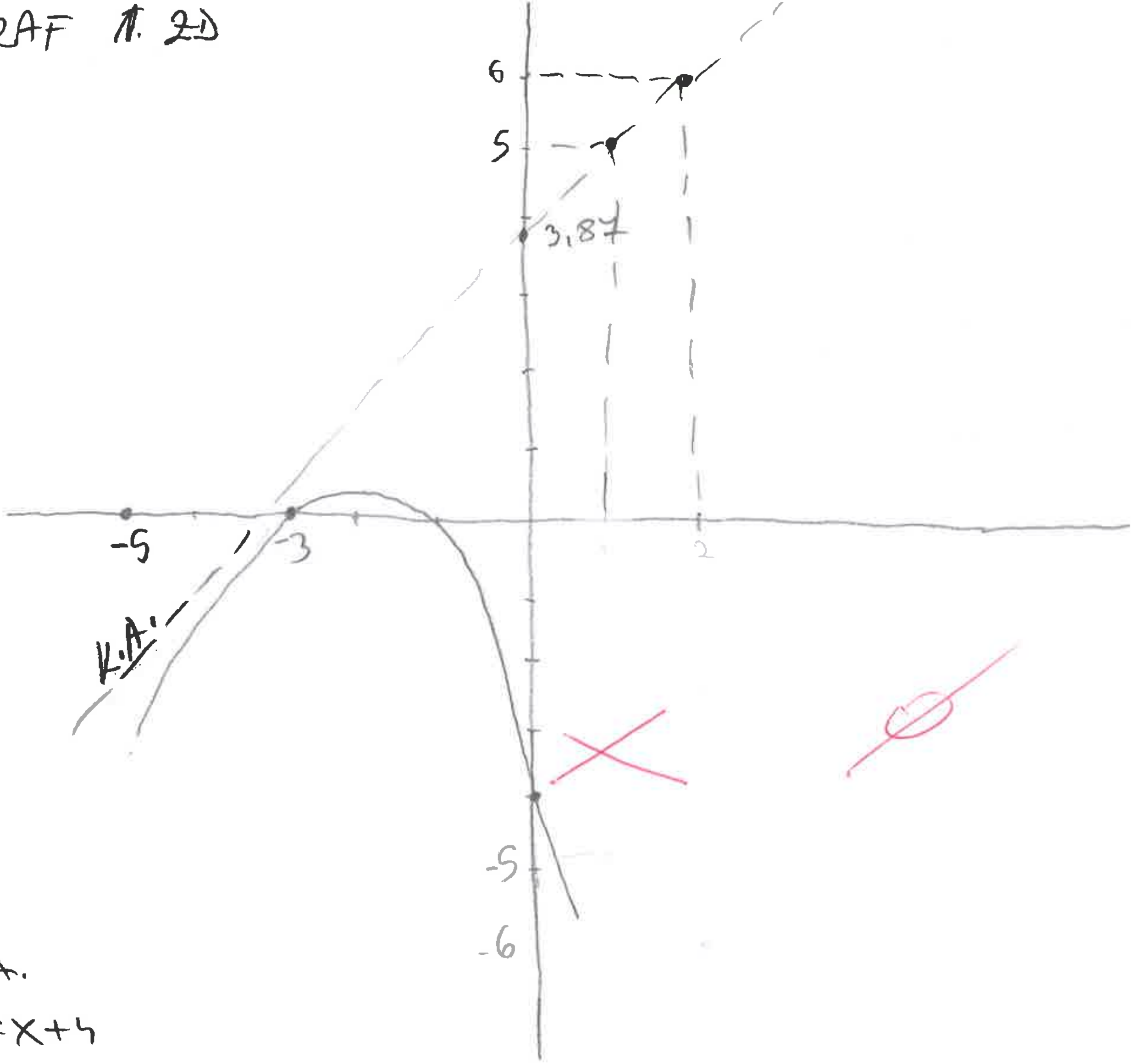
$2x=-8 \quad /:2$

$x=-4$

	$-\infty$ -5	-4 0 $+\infty$
$f'(x)$	$-$	$+$
$f(x)$	$\searrow$	$\nearrow$

$$f(-4) = \sqrt{\cancel{x}4^2+8\cdot(-4)+15} = \infty$$

GRAF 1. 2D



DK.A.

$y = x + 4$

x	1	2
y	5	6

⑤ $\frac{x+1}{\sqrt{x^2-x}}+1>0 \,/\cdot(\sqrt{x^2-x})$

LOVRE RADOVČIĆ

$x+1+1(\sqrt{x^2-x})>0 \,/^2$

$x^2+1+1\cdot(x^2-x)>0$

$x^2+1\cdot x^2-x>0$

$x^2+x^2-x>0$

$2x^2-x>0$

?

③ $f(x)=2^{x^2-3}$

DOMENA

$D_f : \mathbb{R}$

I. DER

$f(x)=(2^{x^2-3})'$

$=2^{x^2-3}\ln 2\,(x^2-3)'$

$=2^{x^2-3}\ln 2\cdot 2x$

?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 13

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: TONI GRBIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

1. Za funkciju $f(x) = \sqrt{x^2 + 8x + 15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. ~~20 graf~~
2. Odrediti tok funkcije $f(x) = \dfrac{x^2 - 2}{x^2 + 3}$ i skicirati graf. ~~20 graf~~ 6
3. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = 2^{x^2-3}$. Posebno komentirati (ne)ograničenost. 7+7+6
4. Gaussovom metodom riješiti matrični sustav: ~~12+3~~

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

Provjeri uvrštavanjem!

5. Odrediti kada je $\dfrac{x+1}{\sqrt{x^2 - x}} + 1 > 0$. 15
6. Razvojem u Taylorov red oko nule provjeriti binomnu formulu za $f(x) = (x-2)^4$. 15

Ukupno: 6

① $f(x) = \sqrt{x^2+8x+15}$

$\boxed{\text{D.P.}}$ $x^2+8x+15 \geq 0$

$$x_{1,2} = \frac{-8 \pm \sqrt{64-4\cdot 15}}{2} = \frac{-8 \pm 2}{2} = \begin{cases} x_1 = \frac{-8-2}{2} = -5 \\ x_2 = \frac{-8+2}{2} = -3 \end{cases}$$

$x_1 \geq -5$

$x_2 \geq -3$

$-5 \quad -3 \quad +\infty$

$\mathcal{D}(f)\ [-3, +\infty\rangle$

$\boxed{\text{ASIMPTOTE:}}$

V.A. - nema

D.H.A $\lim_{x\to+\infty} \sqrt{x^2+8x+15} = \infty$

L.H.A $\lim_{x\to-\infty} \sqrt{x^2+8x+15} = \infty$

NEMA. H.A

$\boxed{\text{N.T.}}$ $\sqrt{x^2+8x+15} = 0 \,/^2$

$x^2+8x+15=0$

~~$x_1 = -5$~~ NIJE N.T.

$\boxed{x_2 = -3}$

$\varnothing$

$f'(x) = \dfrac{1}{2\sqrt{\cancel{2x}+8}}$

	-3	$+\infty$
$f'(x)$	$+$	
	↗	

N.T. DERIVACIJE NEMA

2) $f(x) = \dfrac{x^2-2}{x^2+3}$ YONI GRISIC

$D(f)$ $x^2+3 \neq 0$

$x^2 \neq -3$

$D(f) \in \mathbb{R}$

N.T. $x^2-2=0$

$x^2 = 2 / \sqrt{\ }$

$x = \pm\sqrt{2}$

V.A. nema

H.A. $\lim\limits_{x\to+\infty} \dfrac{x^2-2}{x^2+3} = \dfrac{\infty}{\infty} \overset{L'H}{=} \lim\limits_{x\to+\infty} \dfrac{2x}{2x} = \dfrac{\infty}{\infty} \overset{L'H}{=} \lim\limits_{x\to+\infty} \dfrac{2x}{2x} = \dfrac{2}{2} = 1$

$\lim\limits_{x\to-\infty} \dfrac{x^2-2}{x^2+3} = \dfrac{-\infty}{-\infty} \overset{L'H}{=} \lim \dfrac{2x}{2x} = \dfrac{-\infty}{-\infty} \overset{L'H}{=} \lim\limits_{x\to-\infty} \dfrac{-2}{-2} = 1$

$f'(x) = \dfrac{2x(x^2+3) - (x^2-2)(2x)}{(x^2+3)^2} = \dfrac{\cancel{2x^3}+6x-\cancel{2x^3}-4x}{(x^2+3)^2} = \dfrac{2x}{(x^2+3)^2}$

$2x=0$

$x=0$

	$-\infty$	$-\sqrt{2}$	0	$\sqrt{2}$	$+\infty$
$f'(x)$	$-$	$-$	$+$	$+$	
	↘	↘	↗	↗	

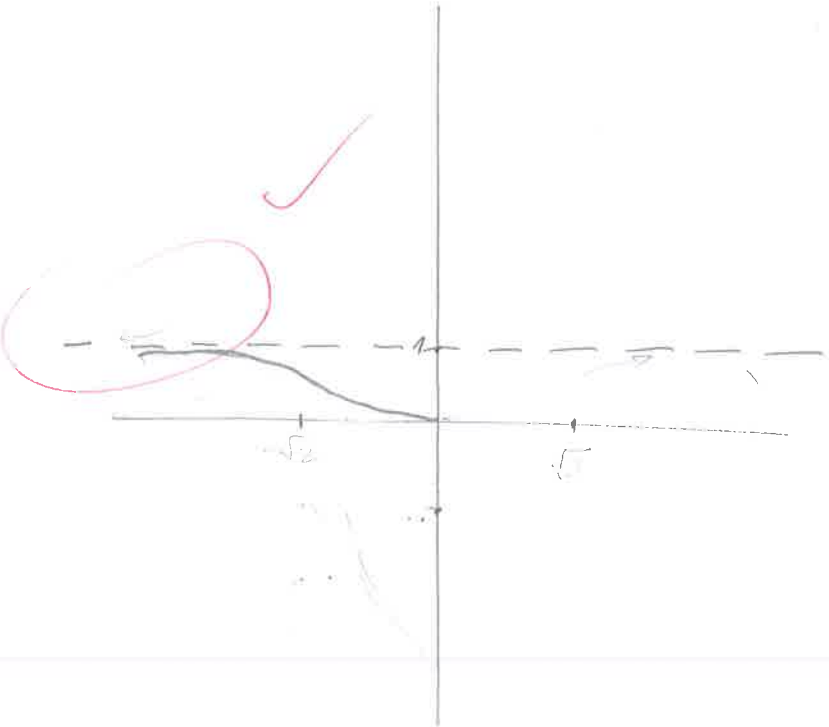
$f''(x) = \dfrac{2(x^2+3)^{\cancel{2}} - 2\cancel{(x^2+3)}\cdot 2x\cdot(2x)}{(x^2+3)^{\cancel{4}\,3}} = \dfrac{2x^2+6-8x^2}{(x^2+3)^3} = \dfrac{-6x^2+6}{(x^2+3)^3}$

$-6x^2+6=0$ $a=-6$ $b=0$ $c=6$

$x_{1,2} = \dfrac{0 \pm \sqrt{0-4\cdot(-6)\cdot 6}}{-12} = \dfrac{\pm 12}{-12}$ $x_1=-1$, $x_2=1$

	$-\sqrt{2}$	-1	1	$\sqrt{2}$	
$f''(x)$	$-$	$-$	$+$	$+$	$+$
	$\cap$	$\cap$	$\cup$	$\cup$	$\cup$

$f''(-1)=0$



6

$$\begin{array}{c}\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & -1 & -3 & 2 \\ 1 & -8 & -9 & -8 \\ 5 & 5 & 0 & 14 \end{array}\right]\end{array}\begin{array}{l} \\ II-2I \\ III-I \\ IV-5I \end{array} \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -5 & -5 & -6 \\ 0 & -10 & -10 & -12 \\ 0 & -5 & -5 & -6 \end{array}\right] \cdot\left(-\frac{1}{5}\right) \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 1 & 6/5 \\ 0 & -10 & -10 & -12 \\ 0 & -5 & -5 & -6 \end{array}\right]\begin{array}{l} I-2II \\ \\ III+10II \\ IV+5II \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 8/5 \\ 0 & 1 & 1 & 6/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 13

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: SANDRO VELIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0281-2013

1. Za funkciju $f(x) = \sqrt{x^2+8x+15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf

2. Odrediti tok funkcije $f(x) = \dfrac{x^2-2}{x^2+3}$ i skicirati graf. 20 graf

3. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = 2^{x^2-3}$. Posebno komentirati (ne)ograničenost. 7+7+6

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$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

Provjeri uvrštavanjem!

5. Odrediti kada je $\dfrac{x+1}{\sqrt{x^2-x}} + 1 > 0$. 15

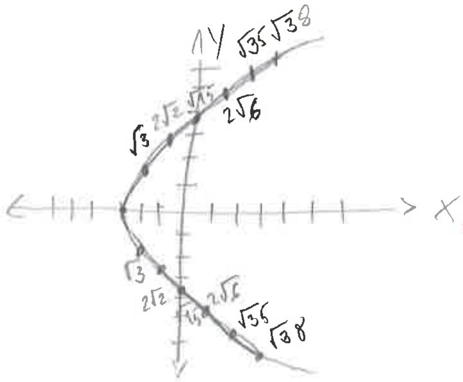
6. Razvojem u Taylorov red oko nule provjeriti binomnu formulu za $f(x) = (x-2)^4$. 15

Ukupno: ~~15~~ 3

Kor

1. $f(x) = \sqrt{x^2+8x+15}$

X	-3	-2	-1	0	1	2	3
Y	0	$\sqrt{3}$	$2\sqrt{2}$	$\sqrt{15}$	$2\sqrt{6}$	$\sqrt{35}$	$\sqrt{38}$



4.

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & -1 & -3 & 2 \\ 1 & -8 & -9 & -8 \\ 5 & 5 & 0 & 14 \end{array}\right] \begin{array}{l} II + 3I \\ III - 3II \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 5 & 5 & 0 & 14 \\ -5 & 5 & 0 & -14 \\ 5 & 5 & 0 & 14 \end{array}\right] \ldots \left[\begin{array}{ccc|c} 5 & 5 & 0 & 14 \\ 5 & 5 & 0 & 14 \\ -5 & 5 & 0 & -14 \\ 1 & 2 & 1 & 4 \end{array}\right] I - II \ldots \left[\begin{array}{ccc|c} 5 & 5 & 0 & 14 \\ -5 & 5 & 0 & -14 \\ 1 & 2 & 1 & 4 \end{array}\right] I + II$$

$$\left[\begin{array}{ccc|c} 0 & 5 & 0 & 0 \\ -5 & 5 & 0 & -14 \\ 1 & 2 & 1 & 4 \end{array}\right] \ldots \left[\begin{array}{ccc|c} -5 & 5 & 0 & -14 \\ 0 & 5 & 0 & 0 \\ 1 & 2 & 1 & 4 \end{array}\right] I - II \ldots \left[\begin{array}{ccc|c} -5 & 0 & 0 & -14 \\ 0 & 5 & 0 & 0 \\ 1 & 2 & 1 & 4 \end{array}\right] \begin{array}{l} I : (-5) \\ II : 5 \end{array} \ldots \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{14}{5} \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 4 \end{array}\right] III - 2II;\ III - I \ldots$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{14}{5} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{6}{5} \end{array}\right] \quad a = \frac{14}{5} \quad b = 0 \quad c = \frac{6}{5} \quad \begin{bmatrix} \frac{14}{5} \\ 0 \\ \frac{6}{5} \end{bmatrix}$$

P: $a + 2b + c = 4 \Rightarrow \frac{14}{5} + 2 \cdot 0 + \frac{6}{5} = 4$ ✓

$2a - b - 3c = 2 \Rightarrow 2\frac{14}{5} - 0 - 3 \cdot \frac{6}{5} = 2$ ✓

$a - 8b - 9c = -8 \Rightarrow \frac{14}{5} - 8 \cdot 0 - 9 \cdot \frac{6}{5} = -8$ ✓

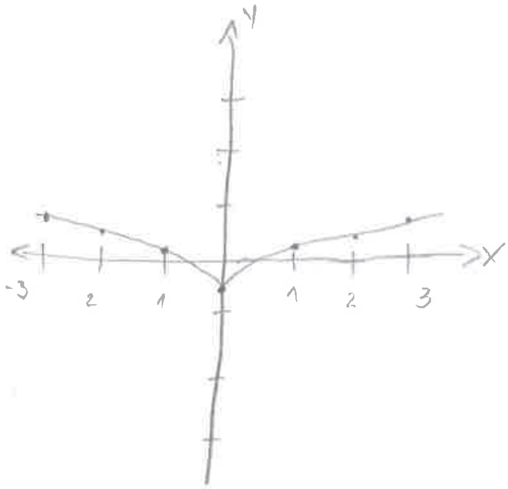
$5a - 5b = 14 \Rightarrow 5 \cdot \frac{14}{5} - 5 \cdot 0 = 14$ ✓

✓

SAMO JEDNO OD BESKONAČNO RJEŠENJA.

2. $f(x) = \frac{x^2 - 2}{x^2 + 3}$

x	-3	-2	-1	0	1	2	3
y	$\frac{7}{12}$	$\frac{2}{7}$	$-\frac{1}{4}$	$-\frac{2}{3}$	$-\frac{1}{4}$	$\frac{2}{7}$	$\frac{7}{12}$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 13

IME I PREZIME: GORDAN JAČAN **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17 - 1 - 0254 - 2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Za funkciju $f(x) = \sqrt{x^2 + 8x + 15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf
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5. Odrediti kada je $\dfrac{x+1}{\sqrt{x^2 - x}} + 1 > 0$. 15
6. Razvojem u Taylorov red oko nule provjeriti binomnu formulu za $f(x) = (x-2)^4$. 15

Ukupno: $\emptyset$

1. $f(x) = \sqrt{x^2 + 8x + 15}$

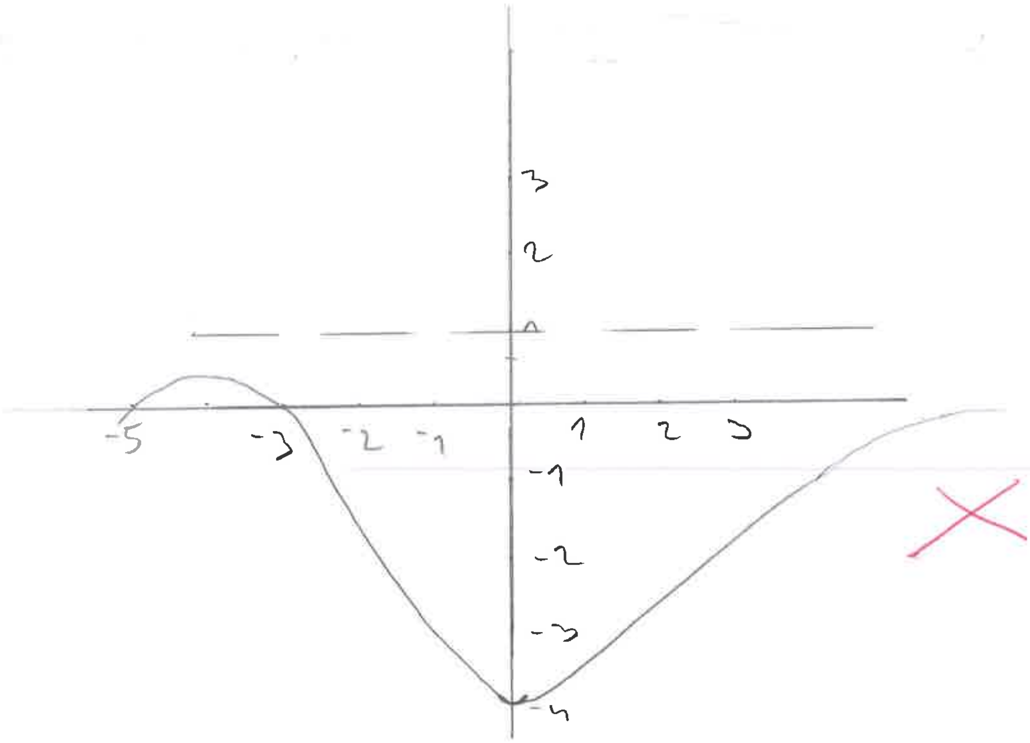
DOMENA

$x^2 + 8x + 15 \geq 0$

$x_{1,2} = \dfrac{-8 \pm \sqrt{64 - 60}}{2}$

$x_1 = \dfrac{-8 + 2}{2} = -3$ $\quad x_2 = \dfrac{-8 - 2}{2} = -5$

$D_f = \mathbb{R}$



4.

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & -1 & -3 & 2 \\ 1 & -8 & -9 & -8 \\ 5 & 5 & 0 & 14 \end{array}\right] \begin{array}{l} \cdot(-2) \cdot(-1) \cdot(-5) \\ + \\ + \\ + \end{array} \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -5 & -5 & -6 \\ 0 & -10 & -10 & -12 \\ 0 & -5 & -5 & -6 \end{array}\right] \begin{array}{l} \\ \cdot(-1) \\ \\ + \end{array} \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -5 & -5 & -6 \\ 0 & -10 & -10 & -12 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -5 & -5 & -6 \\ 0 & -10 & -10 & 12 \end{array}\right] \begin{array}{l} \\ \cdot(-2) \\ + \end{array} \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -5 & -5 & -6 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right] /:(-5) \sim \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 1 & \frac{6}{5} \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & \frac{8}{5} \\ 0 & 1 & 1 & \frac{6}{5} \end{array}\right]$$

$x_2 + x_3 = \frac{6}{5}$ $\qquad$ $x_3 = t \quad , \quad t \in \mathbb{R}$

$x_2 = \frac{6}{5} - t$

$t = 1$

$x_1 + t = \frac{8}{5}$ ✗

$x_1 = \frac{8}{5} - t$ ✗

$x_2 = \frac{6}{5} - 1$ $\qquad$ $x_1 = \frac{8}{5} - 1$

$x_2 = \frac{1}{5}$ $\qquad$ $x_1 = \frac{3}{5}$

PROVJERA

$x_1 = \frac{8}{5} + t$

0

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2. $f(x) = \frac{x^2-2}{x^2+3}$

DOMENA $\qquad D_f = \mathbb{R}$

$x^2+3 \neq 0$

$x^2 = -3$

$x = \pm\sqrt{-3}$ $\qquad$ DOMENA JE $\mathbb{R}$ PA NEMA V. A.

H. A.

$$\lim_{x\to\infty} \frac{x^2-2 \;/ : x^2}{x^2+3 \;/ : x^2} = \lim_{x\to 0} \frac{1-\frac{2}{x^2}}{1+\frac{3}{x^2}} = 1$$

KADA IMAMO H. A. ONDA NEMA K. A.

NULTOČKE

$f(x) = 0$

$\frac{x^2-2}{x^2+3} = 0 \quad / \cdot x^2+3$

$x^2-2=0$

$x^2 = 2$

$x_{1,2} = \pm\sqrt{2}$ $\qquad x_1 = 1{,}414$

$x_2 = -1{,}414$

(NE)PARNOST

$f(-x) = \frac{(-x)^2-2}{(-x)^2+3} = \frac{x^2-2}{x^2+3}$ $\qquad$ PARNA

DERIVACIJA

$f'(x) = \frac{(2x)\,(x^2+3) - (x^2-2)\cdot(2x)}{(x^2+3)^2}$

$= \frac{2x^3+6x-(2x^3-4x)}{(x^2+3)^2}$

$= \frac{\cancel{2x^3}+6x-\cancel{2x^3}+4x}{(x^2+3)^2}$

$f'(x) = \frac{10x}{(x^2+3)^2}$

KRITIČNA TOČKA

$f'(x) = 0$

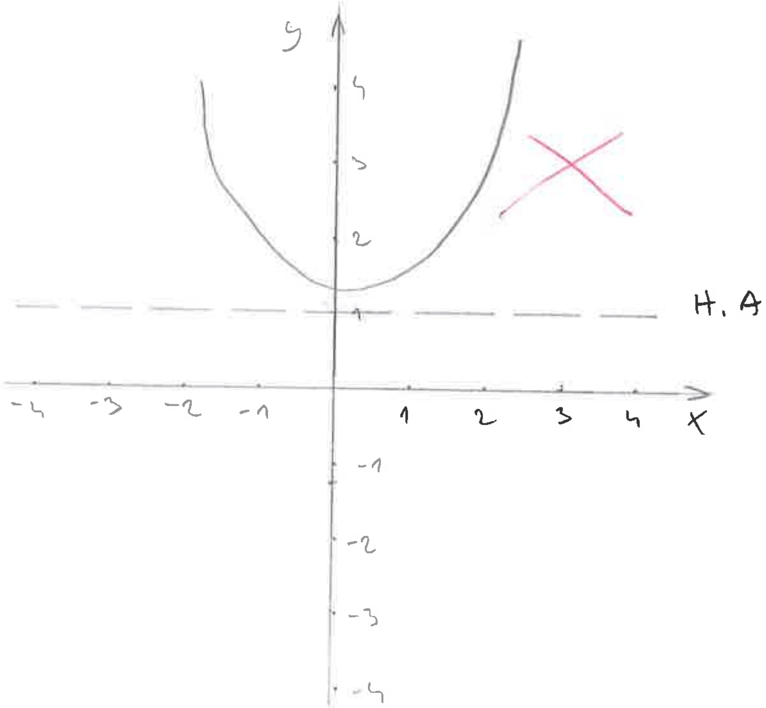
$\frac{10x}{(x^2+3)^2} = 0 \quad / \cdot (x^2+3)^2$

$10x = 0 \quad / : 10$

$x = 0$

MONOTONOST

		0	
$f'(x)$	$-$		$+$
$f(x)$	$\searrow$	minimum	$\nearrow$



GORDAN JAČAN 17 - 1 - 0254 - 2014

5.

$$\frac{x+1}{\sqrt{x^2-x}} + 1 > 0$$

$x^2 - x \neq 0$

$x \cdot (x-1) \neq 0$

$x \neq 0 \quad x-1 \neq 0$

$x \neq 1$

$$\frac{x+1+\sqrt{x^2-x}}{\sqrt{x^2-x}} > 0 \quad / \cdot \sqrt{x^2-x}$$

$$x+1+\sqrt{x^2-x} > 0 \quad /^2$$

$$x^2+1+x^2-x > 0$$

$$2x^2 - x + 1 > 0$$

$$2x^2 - x + 1 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1-8}}{4} \qquad D = \mathbb{R} \setminus \{0, 1\}$$

$\frac{x+1}{\sqrt{x^2-\cancel{x}}} + 1$ je veće od 0 kada je $x \in \langle 0, 1 \rangle \cup \langle 1, +\infty \rangle$

0

1. $f(x) = \sqrt{x^2+8x+15}$

$D_f = \mathbb{R}$ NEMA V. A.

H. A.

$\lim_{x\to\infty} \sqrt{x^2+8x+15} \; /:x = \lim_{x\to\infty} \sqrt{1+\frac{8}{x}+\frac{15}{x^2}} = 1$

IMA H. A. NEMA K. A

(NE) PARNOST

$f(-x) = \sqrt{(-x)^2+8\cdot(-x)+15}$

$= \sqrt{x-8x+15}$

$= \sqrt{-(-x+8x-15)}$

NI PARNA NI NEPARNA

NULTOČKE

$x^2+8x+15=0$

$x_{1,2} = \frac{-8\pm\sqrt{64-60}}{2}$

$x_1 = -3 \qquad x_2 = -5$

DERIVACIJA

$f'(x) = \frac{2x+8}{2\sqrt{x^2+8x+15}}$

KRITIČNE TOČKE

$f'(x) = 0$

$\frac{2x+8}{2\sqrt{x^2+8x+15}} = 0 \; /\cdot 2\sqrt{x^2+8x+15}$

$2x+8=0$

$2x=-8$

$x=-4$

		-4	
$f'x$	$-$		$+$
$f(x)$	↘	↓	↗

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 13	POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: Jobipa Jozeyić **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0269078354

1. Za funkciju $f(x) = \sqrt{x^2+8x+15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. — 20 graf

2. Odrediti tok funkcije $f(x) = \dfrac{x^2-2}{x^2+3}$ i skicirati graf. — 20 graf

3. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = 2^{x^2-3}$. Posebno komentirati (ne)ograničenost. — 7+7+6

4. Gaussovom metodom riješiti matrični sustav: — 12+3

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

Provjeri uvrštavanjem!

5. Odrediti kada je $\dfrac{x+1}{\sqrt{x^2-x}} + 1 > 0$. — 15

6. Razvojem u Taylorov red oko nule provjeriti binomnu formulu za $f(x) = (x-2)^4$. — 15

Ukupno: $\emptyset$

② $f(x) = \dfrac{x^2-2}{x^2+3}$

① DOMENA

$x^2+3 \neq 0$

$x^2 \neq -3 \;/\sqrt{\;}$

$x \neq \pm\sqrt{3}$ $D(f) \in \mathbb{R} \setminus \{\pm\sqrt{3}\}$

ASIMPTOTE

V.A.

$\lim\limits_{x\to\sqrt{3}^+} \dfrac{x^2-2}{x^2+3} = \dfrac{(\sqrt{3}^+)^2-2}{(\sqrt{3}^+)^2+3} = \dfrac{\sqrt{3}^+-2}{\sqrt{3}^++3} = \boxed{-\infty}$ $\sqrt{3} = 1{,}7$

$\lim\limits_{x\to\sqrt{3}^-} \dfrac{x^2-2}{x^2+3} = \dfrac{(\sqrt{3}^-)^2-2}{(\sqrt{3}^+)^2+3} = \dfrac{\sqrt{3}^+-2}{\sqrt{3}^++3} = \boxed{-\infty}$

$\lim\limits_{x\to-\sqrt{3}^+} \dfrac{x^2-2}{x^2+3} = \dfrac{(-\sqrt{3}^+)^2-2}{(-\sqrt{3}^+)^2+3} = \dfrac{\sqrt{3}^+-2}{\sqrt{3}^++3} = \dfrac{-}{+} = \boxed{-\infty}$ $\boxed{x = \pm\sqrt{3}}$

$\lim\limits_{x\to-\sqrt{3}^-} \dfrac{x^2-2}{x^2+3} = \dfrac{(-\sqrt{3}^-)^2-2}{(-\sqrt{3}^+)^2+3} = \dfrac{\sqrt{3}^+-2}{\sqrt{3}^++3} = \boxed{-\infty}$

H.A. $\lim\limits_{x\to+\infty} \dfrac{x^2-2 \;/:x^2}{x^2+3 \;/:x^2} = \lim \dfrac{\frac{x^2}{x^2} - \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{3}{x^2}} = \dfrac{1}{1} = \boxed{1} \to +\infty$

$\lim\limits_{x\to-\infty} \dfrac{x^2-2 \;/:x^2}{x^2+3 \;/:x^2} = \lim \dfrac{1 - \frac{2}{x^2}}{1 + \frac{3}{x^2}} = \dfrac{1}{1} = \boxed{1} \Rightarrow -\infty$ $\boxed{y = 1}$

⟶ nema kosu asimptotu

③ NULTOČKE

$\dfrac{x^2-2}{x^2+3} = 0 \;/\cdot x^2+3$

$x^2 - 2 = 0$

$x^2 = 2 \;/\sqrt{\;}$

$x = \pm\sqrt{2}$

$N_1(+\sqrt{2}, 0)$

$N_2(-\sqrt{2}, 0)$

④ Parnost / Neparnost

$f(x) = \dfrac{x^2-2}{x^2+3}$ $f(-x) = \dfrac{(-x^2)-2}{-x^2+3} =$

$= -\left(\dfrac{x^2+2}{x^2+3}\right) \Rightarrow$ funkcija nije parna ni neparna

(2) → nastavak

④ Derivacija

$$f(x) = \frac{x^2-2}{x^2+3} \Big/ '$$

$$\frac{f'g - fg'}{g^2}$$

$$f'(x) = \frac{(x^2-2)'\cdot(x^2+3) - (x^2-2)\cdot(x^2+3)'}{(x^2+3)^2} = \frac{2x\cdot(x^2+3) - (x^2-2)\cdot 2x}{(x^2+3)^2} = \frac{2x^3+6x-(2x^3-4x)}{(x^2+3)^2} =$$

$$\frac{\cancel{2x^3}+6x-\cancel{2x^3}+4x}{(x^2+3)^2} = \frac{10x}{(x^2+3)^2} \qquad f'(x) = \frac{10x}{(x^2+3)^2}$$

$$f'(x) = 0$$

$$\frac{10x}{(x^2+3)^2} = 0 \Big/ \cdot (x^2+3x)^3 \qquad \frac{10\cdot(-3)}{(-3)^2+3)^2} = -0{,}2$$

$$10x\cdot(x^2+3x) = 0 \qquad 10x+30=0 \qquad X_T(-3,-0{,}2)$$

$$10x^3+30x^2=0 \qquad 10x = -30 /10$$

$$x^2(10x+30) = 0 \qquad \boxed{x=-3} \text{ → kritična točka}$$

$$x^2 = 0 /\sqrt{\ }$$

	$-\infty$	-3	$-\sqrt{3}$	1	$+\sqrt{3}$	$+\infty$
	-4	-2	$-0{,}5$	$1{,}5$	4	
$f'(x) = \frac{10x}{(x^2+3)^2}$	$-$	$-$	$-$	$+$	$+$	
$f(x)$	↘	↘	↘ minimum	↗	↗	

$$I^2 \pm 2\cdot I\cdot II + II^2$$

$$f'(x) = \frac{10x}{(x^2+3)^2} \Big/ '$$

$$f''(x) = \frac{(10x)'\cdot(x^2+3)^2 - (10x)\cdot((x^2+3)^2)'}{(x^2+3)^4} = \frac{10(x^4+6x^2+9) - (10x)(2\cdot(x^2+3)\cdot(x^2+3)')}{(x^2+3)^4}$$

$$= \frac{(10x^4+60x^2+90)-(10x)\cdot(2x^2+6)\cdot 2x}{(x^2+3)^4} = \frac{(10x^4+60x^2+90)-(20x^3+60x)\cdot(2x)}{(x^2+3)^4}$$

$-40x^4$ $-120x^2$

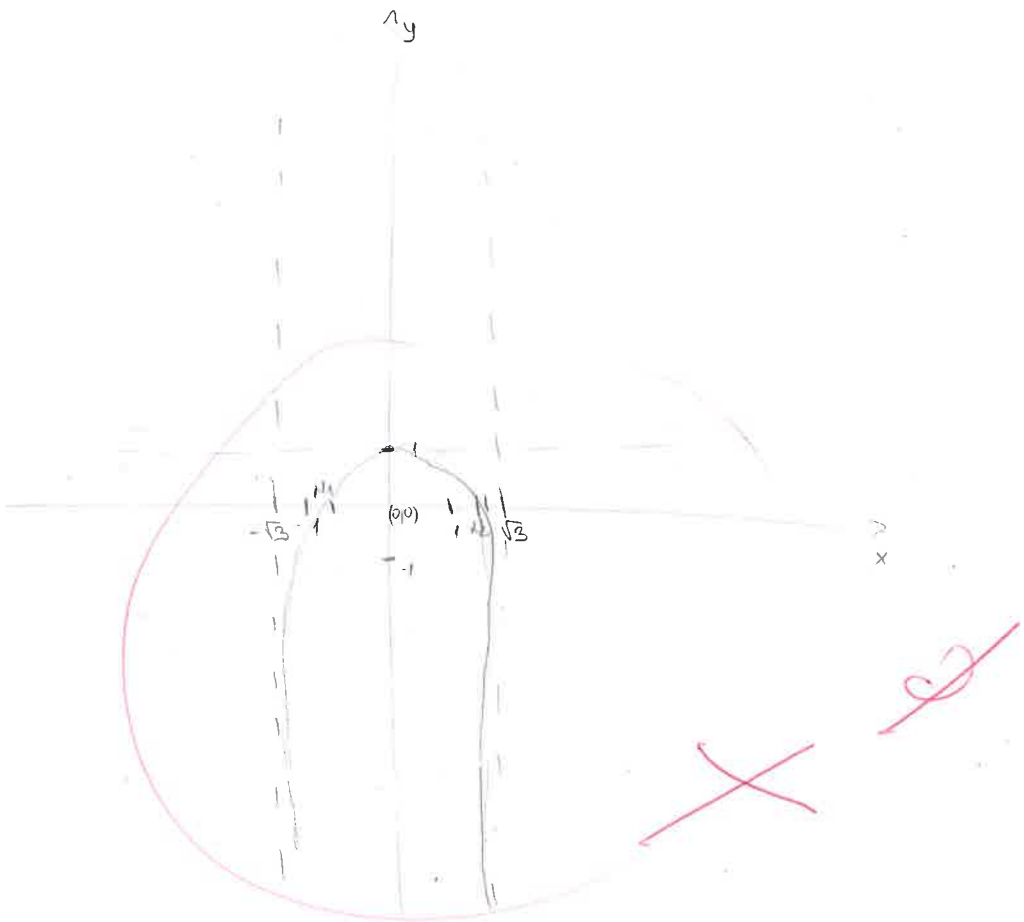
$$= \frac{-30x^4-60x^2+90}{(x^2+3)^4} = \frac{-30(x^4-2x^2+3)}{(x^2+3)^4}$$

	$-\infty$	-3	$-\sqrt{3}$	1	$+\sqrt{3}$	$+\infty$
	-4	-2	$-0{,}5$	$1{,}5$	4	
$\frac{-30(x^4-2x^2+3)}{(x^2+3)^4}$	$-$	$-$	$-$	$+$	$+$	
$f(x)$	∩	∩	∩	∪	∪	

~~NEMA SKICE~~

0

(2) GRAF



(4)

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & -1 & -3 & 2 \\ 1 & -8 & -5 & -8 \\ 5 & 5 & 0 & 14 \end{array}\right] \begin{array}{l} R2-2 \cdot R1 \\ R3-R1 \\ R4-5 \cdot R1 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -5 & -5 & -6 \\ 0 & -6 & -8 & -4 \\ 0 & -5 & -5 & -6 \end{array}\right] \begin{array}{l} R2-R3 \\ R3 : 2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 3 & -2 \\ 0 & -3 & -4 & -2 \\ 0 & -5 & -5 & -6 \end{array}\right] \begin{array}{l} R3+3 \cdot R2 \\ R4+5 \cdot R2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 5 & -8 \\ 0 & 0 & -10 & -16 \end{array}\right] R3 \leftrightarrow R4 \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 5 & -8 \\ 0 & 0 & 0 & 0 \end{array}\right] :5 \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -\frac{8}{5} \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R1-R3 \\ R2-3R3 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 28/5 \\ 0 & 1 & 0 & -2/5 \\ 0 & 0 & 1 & -8/5 \\ 0 & 0 & 0 & 0 \end{array}\right] R1-2R2 \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 24/5 \\ 0 & 1 & 0 & -2/5 \\ 0 & 0 & 1 & -8/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

RJEŠENJE ?

$\emptyset$

(5)

$$\frac{x+1}{\sqrt{x^2-x}} + 1 > 0 \quad / \cdot \sqrt{x^2-x}$$

$$x+1 + 1(\sqrt{x^2-x}) > 0$$

$$x+1 + \sqrt{x^2-x} > 0 \quad /^2$$

$$(x+1)^2 + x^2 - x > 0$$

$$x^2+2x+1+x^2-x > 0$$

$$2x^2+x+1 > 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1^2-4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{-1 \pm \sqrt{1-8}}{2}$$

$\emptyset$

(1) $f(x) = \sqrt{x^2 - 8x + 15}$

(1) $x^2 - 8x + 15 \geq 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{8 \pm \sqrt{64 - 4 \cdot 1 \cdot 15}}{2} = \frac{8 \pm \sqrt{64 - 60}}{2} = \frac{8 \pm \sqrt{4}}{2} = \frac{8 \pm 2}{2}$$

$$x_1 = \frac{8+2}{2} = \frac{10}{2} = 5 \qquad x_2 = \frac{8-2}{2} = \frac{6}{2} = 3$$

$$\boxed{\begin{matrix} x \geq 5 \\ x \geq 3 \end{matrix}}$$

$\frac{5}{0} \infty \quad \frac{0}{5} = 0$

ASIMPTOTE

V.A.

$$\lim_{x \to \infty} \sqrt{x^2 - 8x + 15} = \infty - \infty \Big/ \cdot \frac{\sqrt{x^2 - 8x + 15}}{\sqrt{x^2 - 8x + 15}}$$

$$\lim_{x \to \infty} \frac{x^2 - 8x + 15 \, /:}{\sqrt{x^2 - 8x + 15}} = \frac{\infty}{\infty}$$

$$\lim_{x \to 5^+} \frac{5^2 - 8 \cdot 5 + 15}{\sqrt{5^2 - 8 \cdot 5 + 15}} = \frac{0}{\sqrt{0}} \Rightarrow \text{nema V.A.}$$

$$\lim_{x \to 3^+} \frac{3^{+2} - 8 \cdot 3^+ + 15}{\sqrt{3^{+2} - 8 \cdot 3^+ + 15}} = \frac{15^+ - {}^+24 + 15}{\sqrt{15^+ - 24 + 15}} = \boxed{-\infty}$$

$$\lim_{x \to 3^-} = \frac{(3^-)^2 - 8 \cdot 3^- + 15}{\sqrt{(-3^-)^2 \, 21 + 15}} = \boxed{-\infty}$$

H.A.

$$\lim_{x \to +\infty} \frac{x^2 - 8x + 15 \, /:x^2}{\sqrt{x^2 - 8x + 15} \, /:x^2} = \lim_{x \to \infty} \frac{1 - \frac{8x}{x^2} + \frac{15}{x^2}}{\sqrt{x - 8 + \frac{15}{x^2}}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \, , 0{,}35$$

$$\lim_{x \to -\infty} \frac{x^2 - 8x + 15 \, /:x^2}{\sqrt{x^2 - 8x + 15} \, /:x^2} = \lim_{x \to \infty} \frac{1 - \frac{8}{x} + \frac{15}{x^2}}{\sqrt{\frac{1}{x} - 8 + \frac{15}{x}}} = -0{,}35$$

NULTOČKE

$N_1(5, 0) \qquad N_2(3, 0)$

$f(x) = \sqrt{x^2 - 8x + 15} \, /'$

$\sqrt[n]{x} = \frac{1}{n} x^{n-1}$

$$f'(x) = \frac{1}{2}(x^2 - 8x + 5)^{-1/2} \cdot (x^2 - 8x + 15)' = \frac{1}{2\sqrt{x^2 - 8x + 5}} \cdot 2x - 8 = \frac{2x - 8}{2(\sqrt{x^2 - 8x + 5})} =$$

$$= \frac{2(x-4)}{2\sqrt{x^2 - 8x + 15}} \Rightarrow f'(x) = \frac{x - 4}{\sqrt{x^2 - 8x + 15}} \Big/ '$$

$$f''(x) = \frac{(x-4)' \cdot (\sqrt{x^2 - 8x + 15}) - (x-4)(\sqrt{x^2 - 8x + 15})'}{(\sqrt{x^2 - 8x + 15})^2} = \frac{1(\sqrt{x^2 - 8x + 5}) - (x-4)\left(\frac{1}{2\sqrt{x^2 - 8x + 15}}\right)}{x^2 - 8x + 15}$$

$$\cdot (x^2 - 8x + 15)'$$

$$f'' \Rightarrow \frac{\sqrt{x^2-8x+15} - \frac{(x-4)}{2\sqrt{x^2-8x+15}} \cdot (2x-8)}{x^2-8x+15} =$$

$$\frac{\sqrt{x^2-8x+15} - \frac{2x^2-8x-8x+16}{2\sqrt{x^2-8x+15}}}{x^2-8x+15} = \frac{\sqrt{x^2-8x+15} - \frac{\cancel{2}(x^2-8x+8)}{\cancel{2}\sqrt{x^2-8x+15}}}{\frac{x^2-8x+15}{1}}$$

$$= \frac{\sqrt{x^2-8x+15} - x^2-8x+8}{(x^2-8x+15)\cdot(\sqrt{x^2-8x+15})}$$

NEMA SKICE

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 13

IME I PREZIME: COURE KAŠTROPIL **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0255-2014 0269086961

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Za funkciju $f(x) = \sqrt{x^2 + 8x + 15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf

~~2.~~ Odrediti tok funkcije $f(x) = \dfrac{x^2 - 2}{x^2 + 3}$ i skicirati graf. ~~20 graf~~

3. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = 2^{x^2-3}$. Posebno komentirati (ne)ograničenost. 7+7+6

4. Gaussovom metodom riješiti matrični sustav: 12+3

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

Provjeri uvrštavanjem!

~~5.~~ Odrediti kada je $\dfrac{x+1}{\sqrt{x^2 - x}} + 1 > 0$. ~~15~~

6. Razvojem u Taylorov red oko nule provjeriti binomnu formulu za $f(x) = (x-2)^4$. 15

Ukupno: ∅

(2) $f(x) = \dfrac{x^2-2}{x^2+3}$

1) DOMENA

$x+3=0$

$x=3$

$x \in \langle -\infty, 3\rangle \cup \langle 3, +\infty\rangle$

2) ASIMPTOTE

V.A. $x=a$

$\lim\limits_{x\to 3} \dfrac{x^2-2}{x^2+3}$

$\lim \dfrac{3^2-2}{3^2+3} = \dfrac{9-2}{9+3} = \dfrac{7}{11}$

H.A. $y=b$

$\lim\limits_{x\to\infty} \dfrac{x^2-2}{x^2+3} = \lim\limits_{x\to\infty} \dfrac{x^2-2 /:x^2}{x^2+3 /:x^2} = \dfrac{\frac{x^2}{x^2} - \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{3}{x^2}} = \dfrac{1-0}{1+0} = 1$

3) SJECIŠTA

za x | $f(x)=0$

$\dfrac{x^2-2}{x^2+3} = 0 \quad / \cdot x^2+3$

$x^2-2=0$

$x^2=2$

$x_1=\sqrt{2}$ $x_2=-\sqrt{2}$

za y | $f(0)=?$

$f(0)=\dfrac{x^2-2}{x^2+3}$

$\dfrac{0-2}{0+3} = -\dfrac{2}{3}$

$y=-\dfrac{2}{3}$

5) Derivacija

$$f(x)' = \frac{x^2-2}{x^2+3} = \frac{(x^2-2)' \cdot (x^2+3) - (x^2-2) \cdot (x^2+3)'}{(x^2+3)^2} = \cancel{(2 \cdot x^1 - 0)}$$

$$= \frac{(2 \cdot x^1 - 0) \cdot (x^2+3) - (x^2-2) \cdot (2 \cdot x^1 + 0)}{x^4+6x^2+9} = \frac{2x^3+6x-(2x^3-4x)}{x^4+6x^2+9}$$

$$= \frac{\cancel{2x^3}+6x-\cancel{2x^3}+4x}{x^4+6x^2+9} = \frac{10x}{x^4+6x^2+9} //$$

6) KRITIČNE TOČKE

$$f(x)' = 0 \rightarrow \frac{10x}{x^4+6x^2+9} = 0 / \cdot x^4+6x^2+9 \qquad 10x = 0$$

$$x = 0$$

Nema ih

7) MONOTONOST

$$f(x)' = \frac{10x}{x^4+6x^2+9}$$

$-\infty$ 1	3	5 $+\infty$
$\frac{10x}{x^4+6x^2+9}$		$\frac{10x}{x^4+6x^2+9}$
$\frac{10}{16}$ ↗		$\frac{25}{392}$ ↘

8) i 9) LOKALNIH I GLOBALNIH EKSTREMA NEMA

10) TOČKE PREGIBA GDJE JE 2. der. = 0

GRAF ?

$$f(x)'' = 0$$

$$f(x)'' = \frac{10x}{x^4+6x^2+9} = \frac{\cancel{10x'} \cdot (x^4+6x^2+9) - 10x \cdot (x^4+6x^2+9)'}{(x^4+6x^2+9)^2} =$$

$$= -\frac{(10x \cdot (4 \cdot x^3 + 6 \cdot 2 \cdot x^1 + 0))}{x^8+6x^4+18} = -\frac{(40x^4+120x^2)}{x^8+6x^4+18} = \frac{-40x^4-120x^2}{x^8+6x^4+18} //$$

5) $\frac{x+1}{\sqrt{x^2-x}}+1>0 \quad / \cdot \sqrt{x^2-x}$

$x=1$

Provjera: $\frac{-1+1}{\sqrt{-1^2+1}}+1=\frac{0}{\sqrt{2}}+1$

$=0+1=1$

$x+1+1>0$

$x>-2$ //

$\cancel{3)\ f(x)=2x^2-3}$

$\cancel{6)\ f(x)=(x-2)^4}$

4) Gauss

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

LOVRE KAŠTROPIL

0269086961

L. Kaštropil

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 13

IME I PREZIME: KREŠIMIR ANTOLOVIĆ **VRIJEME POČETKA:** 09:35

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0402-2014 0111117055

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Za funkciju $f(x) = \sqrt{x^2 + 8x + 15}$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf

2. Odrediti tok funkcije $f(x) = \dfrac{x^2 - 2}{x^2 + 3}$ i skicirati graf. 20 graf

3. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = 2^{x^2-3}$. Posebno komentirati (ne)ograničenost. 7+7+6

4. Gaussovom metodom riješiti matrični sustav: 12+3

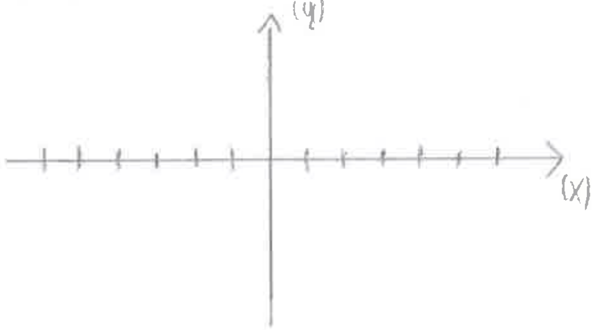
$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

Provjeri uvrštavanjem!

5. Odrediti kada je $\dfrac{x+1}{\sqrt{x^2 - x}} + 1 > 0$. 15

6. Razvojem u Taylorov red oko nule provjeriti binomnu formulu za $f(x) = (x-2)^4$. 15

Ukupno: $\emptyset$



1. $f(x) = \sqrt{x^2+8x+15}$

$\sqrt{x^2+8x+15} = 0 \;/^2$

$x^2+8x+15=0$

$a=1 \quad x_1 = -3$

$b=8 \quad x_2 = -5$

$c=15$

2. $f(x) = \dfrac{x^2-2}{x^2+3}$

$Df(x) = \setminus \mathbb{R}$

$x^2+3 \neq 0$

$x^2+3=0$

$x_1 = \sqrt{3}\,i$

$x_2 = -\sqrt{3}\,i$

3.

$f(x) = 2^{x^2-3}$

$2^{x^2-3} =$

4.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

$$\begin{array}{c} (-2) + \\ (-1)? \\ (-5) + \end{array} \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ 1 & -8 & -9 \\ 5 & 5 & 0 \end{bmatrix} = + \begin{bmatrix} 1 & 2 & 1 \\ 0 & -5 & -5 \\ 0 & -10 & -10 \\ 0 & -5 & -5 \end{bmatrix} = \Big[$$

5.

$$\frac{x+1}{\sqrt{x^2-x}} + 1 > 0$$

$$\frac{5+3\sqrt{5}}{5}$$

$5+3\sqrt{5} > 0$

$5 > 0$