MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 16

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: DANIEL ŠOŠA **VRIJEME POČETKA:** 09:20

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
17-2-0366-2014

1. Na temelju ispitivanja toka skicirati graf funkcije $f(x) = x - \sqrt{x^2 - 4}$. 20 graf
2. Odrediti cjelokupan tijek funkcije $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 1$ i skicirati graf. 20 graf
3. Koji su globalni i lokalni ekstremi funkcije $g(x) = \sqrt{2 - x^2}$? Posebno komentirati (ne)ograničenost. 6+6+3
4. Riješiti jednadžbu: $(1 - i)^6 - z^3 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 15+3
5. Razviti funkciju $f(x) = e^{2x}$ u Taylorov red po potencijama od x. Izračunati barem prva 4 člana. 15
6. Riješiti: $\cos x = 0.5$ 12

Ukupno: 82

1) $f(x) = x - \sqrt{x^2 - 4}$

1. $D_f \; \langle -\infty, -2] \cup [2, +\infty\rangle$

$x^2 - 4 \geq 0$
$x^2 - 4 = 0$
$x^2 = 4$
$x_1 = -2$
$x_2 = 2$

2. NULTOČKE

$x - \sqrt{x^2 - 4} = 0$

$\sqrt{x^2 - 4} = -x \;/^2$

$x^2 - 4 = x^2$

$\cancel{x^2} - x^2 = 4$

$0 = 4$ NEMA NULTOČAKA

3. ASIMPTOTE

$\lim_{x \to +\infty} x - \sqrt{x^2 - 4} = \infty - \infty = \lim_{x \to +\infty} x - \sqrt{x^2 - 4} \cdot \frac{-x - \sqrt{x^2 - 4}}{-x - \sqrt{x^2 - 4}} = \frac{\cancel{-x^2} + x^2 - 4}{-x - \sqrt{x^2 - 4}} = \frac{-4}{-x - \sqrt{x^2 - 4}} = \frac{-4}{-\infty} = 0$

$\boxed{\text{D.H.A } y = 0}$

$\lim_{x \to -\infty} x - \sqrt{x^2 - 4} = \lim_{x \to +\infty} -x - \sqrt{x^2 - 4} \cdot \frac{x - \sqrt{x^2 - 4}}{x - \sqrt{x^2 - 4}} = \frac{\cancel{-x^2} + x^2 - 4}{x - \sqrt{x^2 - 4}} = \frac{-4}{0} = -\infty$ NEMA H.A

$x - \sqrt{x^2 - 4} \to 0$

TRAŽIM KOSU

$\lim_{x \to -\infty} \frac{x - \sqrt{x^2 - 4}}{x} = \frac{-x - \sqrt{x^2 - 4}}{-x} \; \frac{:x}{:x} = \frac{-1 - 1}{-1} = \boxed{2 = a}$

$\boxed{\text{L.K.A: } y = 2x}$

$\lim_{x \to -\infty} x - \sqrt{x^2 - 4} - 2x \overset{\lim}{\underset{x \to \infty}{=}} -x - \sqrt{x^2 - 4} = \lim_{x \to +\infty} x - \sqrt{x^2 - 4} \cdot \frac{-x - \sqrt{x^2 - 4}}{-x - \sqrt{x^2 - 4}} = \frac{\cancel{-x^2} + x^2 - 4}{-x - \sqrt{x^2 - 4}} = \frac{-4}{-\infty} = 0$

V.A

$$\lim_{x\to -2} x-\sqrt{x^2-4} = -2-\sqrt{0} = -2$$ NEMA V.A

$$\lim_{x\to 2} x-\sqrt{x^2-4} = 2-\sqrt{0} = 2$$

STACIONARNE TOČKE

$$f'(x) = x-\sqrt{x^2-4} = 1-\frac{1}{2\sqrt{x^2-4}}\cdot 2x = 1-\frac{2x}{2\sqrt{x^2-4}} = \frac{2\sqrt{x^2-4}-2x}{2\sqrt{x^2-4}}$$

$$2\sqrt{x^2-4}-2x=0$$

$$2\sqrt{x^2-4}=2x \;/^2$$

$$4(x^2-4)=4x^2$$

$$\cancel{4x^2}-4=\cancel{4x^2}$$

$$-4=0$$

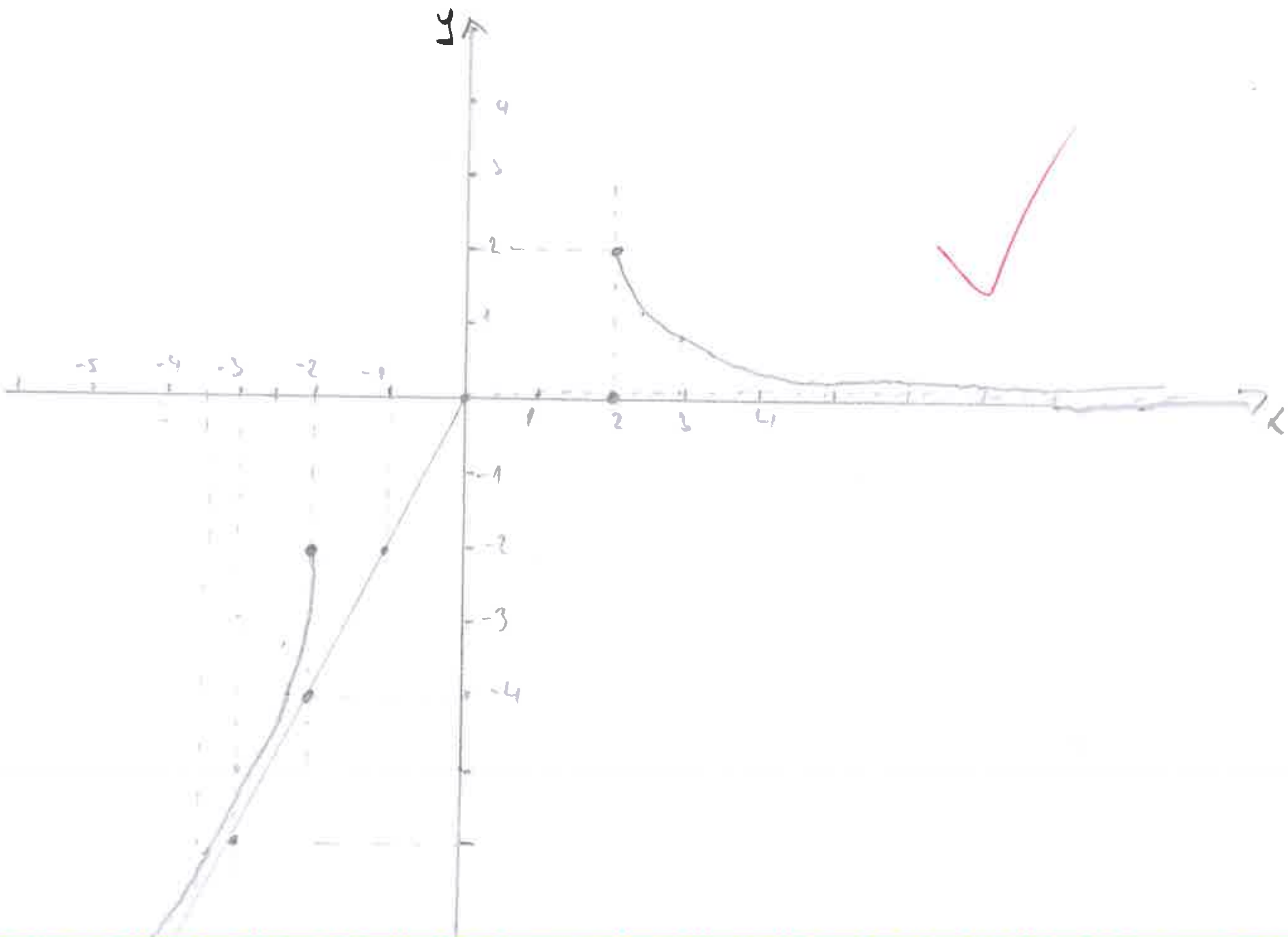
RAST I PAD FUNKCIJE

$-\infty$ -3 -2 0 2 3 $+\infty$

$f'(x)$	+	N/D	−
$f(x)$	↗	N/D	↘

$$f''(x) = \frac{2\sqrt{x^2-4}-2x}{2\sqrt{x^2-4}} = \frac{(2\sqrt{x^2-4}-2x)'\cdot 2\sqrt{x^2-4} - 2\sqrt{x^2-4}-2x\cdot(2\sqrt{x^2-4})'}{2\sqrt{x^2-4}}$$

$$f(x) = x-\sqrt{x^2-4}$$



(2) $f(x)=\frac{1}{3}x^3-x^2-3x+1$

$Df\langle -\infty, +\infty\rangle$

NULTOČKE

$\frac{1}{3}x^3-x^2-3x+1=0$ NE ZNAM RIJEŠITI

ASIMPTOTE

$\lim_{x\to+\infty} \frac{1}{3}x^3-x^2-3x+1=+\infty$

NEMA H.A

$\lim_{x\to-\infty} \frac{1}{3}x^3-x^2-3x+1=\lim_{x\to+\infty} -\frac{1}{3}x^3-x^2+3x+1=-\infty$

KOSE A.

$\lim_{x\to+\infty} \frac{\frac{1}{3}x^3-x^2-3x+1}{x} \begin{matrix} :x^3 \\ :x^3 \end{matrix} = \frac{\frac{1}{3}}{0}=+\infty$ NEMA KOSE

$\lim_{x\to-\infty} \frac{-\frac{1}{3}x^3-x^2+3x+1}{-x} \begin{matrix} :x^3 \\ :x^3 \end{matrix} = \frac{-\frac{1}{3}}{0}=-\infty$

$f'(x)=\frac{1}{3}x^3-x^2-3x+1= \quad x^2-2x-3=0$

$x^2-2x-3=0$

$\frac{2\pm\sqrt{4-4\cdot1\cdot(-3)}}{2}=\frac{2\pm\sqrt{16}}{2}=\frac{2\pm4}{2}= \begin{matrix} x_1=-1 \\ x_2=3 \end{matrix}$

STACIONARNE TOČKE

$T_1(-1, 2.66)$

$T_2(3,-8)$

$f''(x)=x^2-2x-3=\underline{2x-2}$

$2x-2=0$

$2x=2$

$\boxed{x=1}$

TOČKA INFLEKSIJE

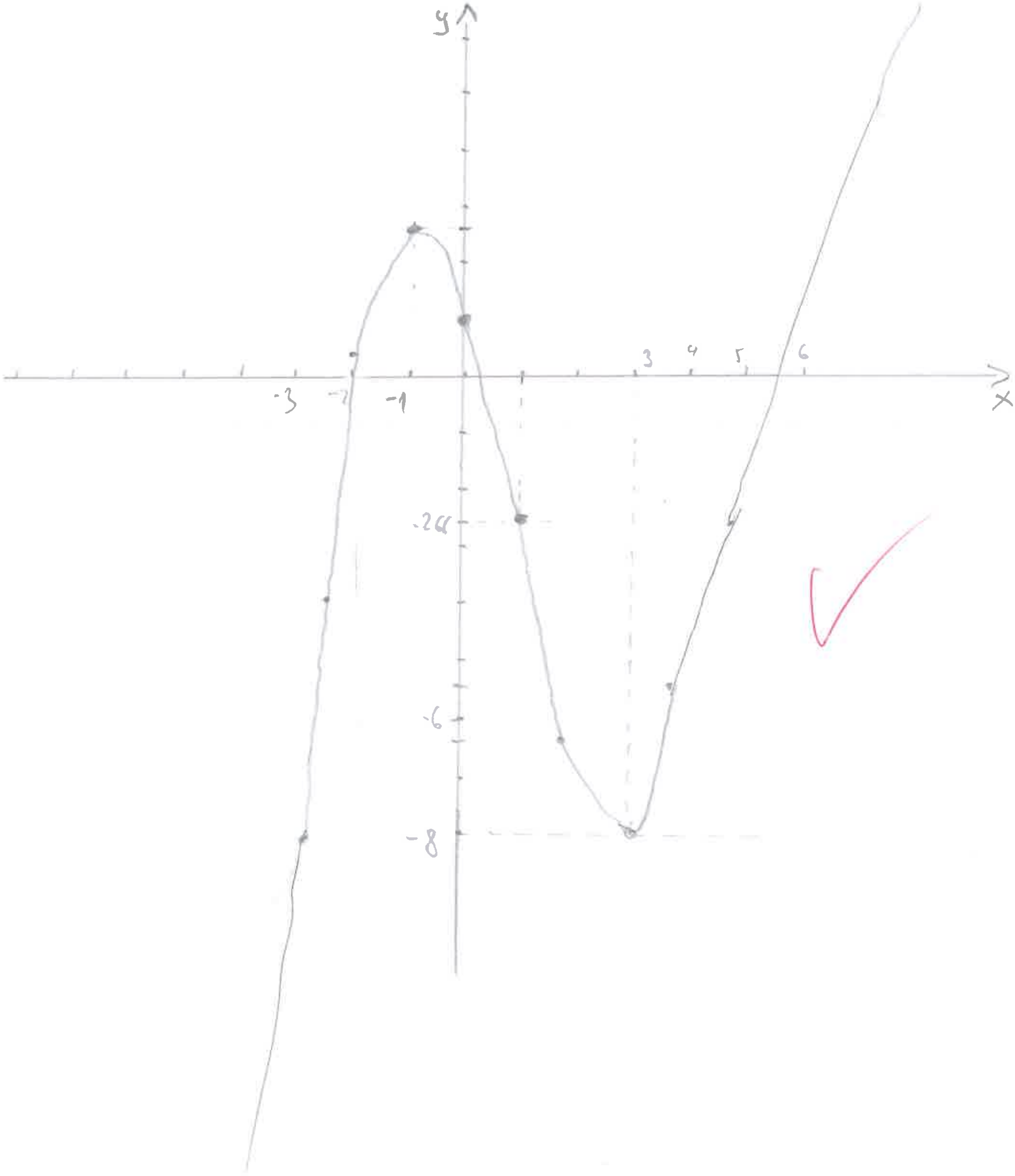
$T(1,-2.66)$

	$-\infty$	-2, -1	0, 3	4, $+\infty$
$f'(x)$		+	−	+
$f(x)$		↗	↘	↗

	$-\infty$, -2	-1, 0	1, 2	3, 4	$+\infty$
$f''(x)$	−	−	+	+	
$f(x)$	∩	∩	∪	∪	

DANIEL JOSA

$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 1$



3. $\sqrt{2-x^2}$

$Df\langle -1.41, 1.41\rangle$

$2-x^2 \geq 0$
$2-x^2=0$
$-x^2=-2$
$x^2=2$
$x=\pm\sqrt{2}$
$x_1=-1.41$
$x_2=1.41$

NULTOČKE

$\sqrt{2-x^2}=0 /^2$
$2-x^2=0$
$-x^2=-2$
$x^2=2$
$x_1=-1.41$
$x_2=1.41$

ASIMPTOTE

$$\lim_{x\to -1.41}\sqrt{2-x^2}=\sqrt{0}=0$$
$$\lim_{x\to 1.41}\sqrt{2-x^2}=\sqrt{0}=0$$
NEMA V.A

$$f'(x)=\sqrt{2-x^2}=\frac{1}{2\sqrt{2-x^2}}\cdot -2x=\frac{-2x}{2\sqrt{2-x^2}}$$

	-1.41	-1 0 1	1.41
$f'(x)$	+	-	
$f(x)$	↗	↘	

$-2x=0$
$\boxed{x=0}$ $T(0, 1.41) \Rightarrow$ STACIONARNA TOČKA

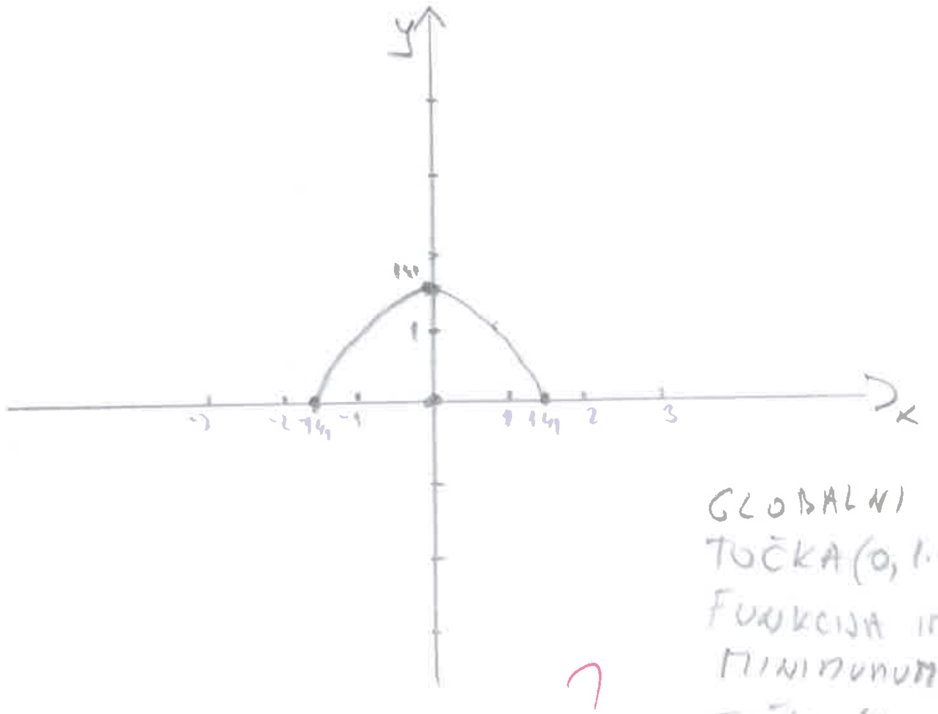
$$f''(x)=\frac{-2x}{2\sqrt{2-x^2}}=\frac{(-2x)'\cdot 2\sqrt{2-x^2}-(-2x)\cdot\left(2\sqrt{2-x^2}\right)'}{\left(2\sqrt{2-x^2}\right)^2}=\frac{-4\sqrt{2-x^2}+2x\cdot\frac{-4x}{2\sqrt{2-x^2}}}{\left(2\sqrt{2-x^2}\right)^2}$$

$$\left(2\sqrt{2-x^2}\right)'=\frac{2}{2\sqrt{2-x^2}}\cdot -2x=\frac{-4x}{2\sqrt{2-x^2}}$$

$$=\frac{-4\sqrt{2-x^2}-\frac{8x^2}{2\sqrt{2-x^2}}}{\left(2\sqrt{2-x^2}\right)^2}=0 \quad 2A \qquad -4\sqrt{2-x^2}-\frac{8x^2}{2\sqrt{2-x^2}}=\frac{-4\sqrt{2-x^2}-8x^2}{2\sqrt{2-x^2}}$$

DANIEL SOJA

$f(x) = \sqrt{2-x^2}$



LOKALNI EKSTREMI ?

GLOBALNI MAKSIMUM JE TOČKA $(0, 1.41)$. ✓

FUNKCIJA IMA DVA GLOBALNA MINIMUMA TOČKU $(-1.41, 0)$ ✓ TOČKU $(1.41, 0)$

FUNKCIJA JE OGRANIČENA ODOZGO TOČKOM $(0, 1.41)$, I TOČKAMA $(-1.41, 0)$ I $(-1.41, 0)$ ODOZDO. ✓ ✓

4 $(1-i)^6 - 2^3 = 0$

$(1-i)^6 = \binom{6}{0} \cdot 1^6 \cdot (-i)^0 + \binom{6}{1} \cdot 1^5 \cdot (-i)^1 + \binom{6}{2} \cdot 1^4 \cdot (-i)^2 + \binom{6}{3} \cdot 1^3 \cdot (-i)^3 + \binom{6}{4} \cdot 1^2 \cdot (-i)^4 + \binom{6}{5} \cdot 1^1 \cdot (-i)^5 + \binom{6}{6} \cdot 1^0 \cdot (-i)^6 = 1 + (-6i) - 15 + 20 \cdot 1 \cdot i + 15 \cdot 1 \cdot 1 + 6 \cdot 1 \cdot -i = -6i - 15 + 20i + 15 - 6i$

$= 8i$

PROVJERA

$(1-i)(1-i)(1-i)(1-i)(1-i)(1-i) =$

$(1-i)(1-i) = 1 - i - i + i^2 = 1 - 2i - 1 = -2i$

$-2i(1-i) = -2i + 2i^2 = -2i - 2$

$(-2i-2)(1-i) = -2i + 2i^2 - 2 + 2i = -2i - 2 - 2 + 2i = -4$

$-4(1-i) = -4 + 4i$

$(-4+4i)(1-i) = -4 + 4i + 4i - 4i^2 = -4 + 8i + 4 = 8i$ ✓✓

DRUGA STRANA

→

DANIEL ŚWOSA

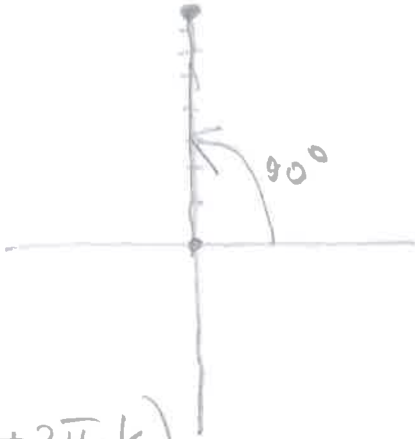
(4) $8i - z^3 = 0$

$-z^3 = -8i$

$z^3 = 8i$

$r = \sqrt{x^2+y^2} = \sqrt{Im^2+Re^2} = \sqrt{0+8^2} = 8$

$\varphi = \frac{\pi}{2} \approx 90^\circ$

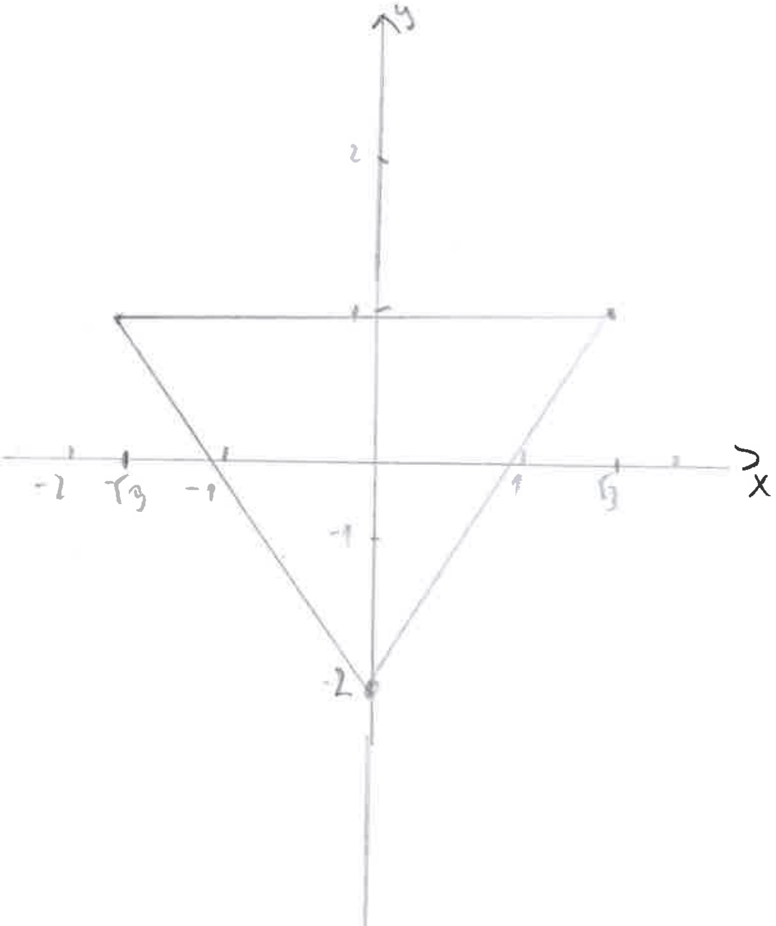


$$W = \sqrt[n]{r}\left(\cos\frac{\varphi+2\pi\cdot k}{n} + i\sin\frac{\varphi+2\pi\cdot k}{n}\right)$$

$$W_1 = \sqrt[3]{8}\left(\cos\frac{\frac{\pi}{2}+2\pi\cdot 0}{3} + i\sin\frac{\frac{\pi}{2}+2\pi\cdot 0}{3}\right) = 2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = \sqrt{3}+i$$

$$W_2 = \sqrt[3]{8}\left(\cos\frac{\frac{\pi}{2}+2\pi\cdot 1}{3} + i\sin\frac{\frac{\pi}{2}+2\pi\cdot 1}{3}\right) = 2\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = -\sqrt{3}+i$$

$$W_3 = \sqrt[3]{8}\left(\cos\frac{\frac{\pi}{2}+2\pi\cdot 2}{3} + i\sin\frac{\frac{\pi}{2}+2\pi\cdot 2}{3}\right) = 2(0+(-1)) = -2i$$



⑤ e^{2x}

$f(x_0)+f'(x_0)(x-x_0)+f''(x_0)\frac{(x-x_0)^2}{2!}+f'''(x_0)\frac{(x-x_0)^3}{3!}$

$x_0=0$ $f(x_0)=1$

$f'(x)=e^{2x}\cdot 2=2\cdot e^{2x}$

$f''(x)=2'\cdot e^{2x}+2\cdot(e^{2x})'=0+4e^{2x}$

$f'''(x)=4e^{2x}=4'\cdot e^{2x}+4\cdot(e^{2x})'=8e^{2x}$

$f''''(x)=8e^{2x}=8'\cdot e^{2x}+8\cdot(e^{2x})'=16e^{2x}$

$1+2x+\frac{4x^2}{2}+\frac{8x^3}{6}+\frac{16x^4}{24}$

$1+2(x-0)+4\cdot\frac{(x-0)^2}{2}+8\cdot\frac{(x-0)^3}{6}+16\cdot\frac{(x-0)^4}{24}=1+2x+4\cdot\frac{x^2}{2}+8\frac{x^3}{6}+16\frac{x^4}{24}$

✓

PROVJERA
$x=1$
$e^{2x}=e^2=7.389$

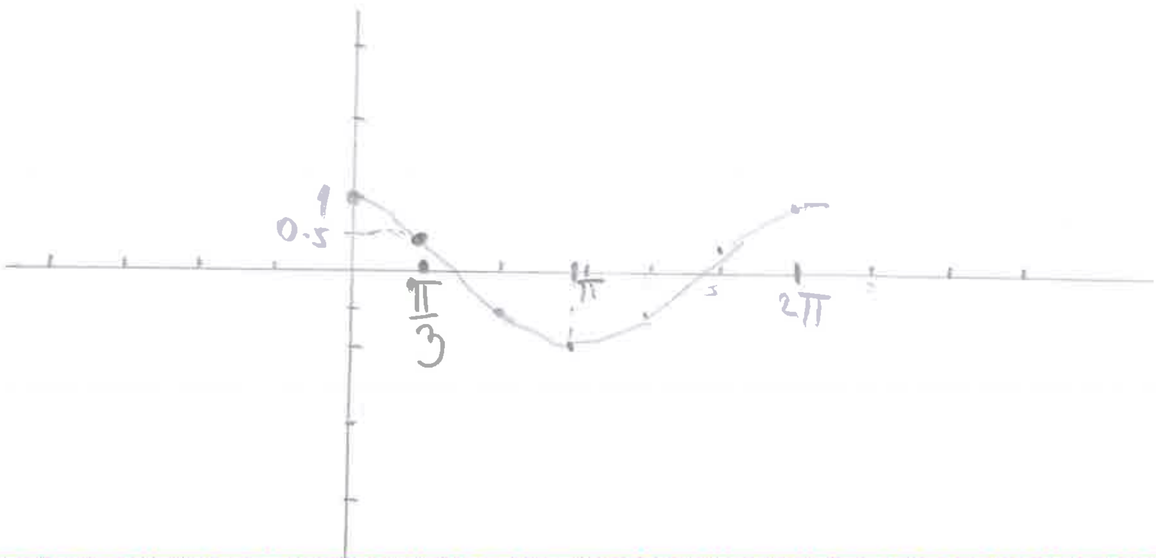
TAYLOROV RED
$x=1$
$1+2\cdot 1+4\cdot\frac{1}{2}+8\cdot\frac{1}{6}+16\cdot\frac{1}{24}=7$

⑥

$\cos x=0.5$ /arccos
$x=\frac{\pi}{3}$

PROVJERA
$\cos\left(\frac{\pi}{3}\right)=0.5$

Još?



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IME I PREZIME: KRISTIAN DUŠEVIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
0269081229

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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5. Razviti funkciju $f(x) = e^{2x}$ u Taylorov red po potencijama od x. Izračunati barem prva 4 člana. ~~15~~
6. Riješiti: $\cos x = 0.5$ ~~12~~

Ukupno:

61

4. ASIMPTOTE

NEMA V.A

$\lim_{x\to\infty} \frac{1}{3}x^3 - x^2 - 3x + 1 = [+\infty - \infty] \Rightarrow$ NEMA D.H.A

$\lim_{x\to-\infty} \frac{1}{3}x^3 - x^2 - 3x + 1 = \left[\begin{matrix} x \to -x \\ -\infty \to \infty \end{matrix}\right] = \lim_{x\to\infty} \frac{1}{3}(-x)^3 - x^2 + 3x + 1 \; [\infty - \infty]$

$\Rightarrow$ NEMA LH.A

$\lim_{x\to\infty} \frac{\frac{1}{3}x^3 - x^2 - 3x + 1}{x} = \lim_{x\to\infty} \frac{1}{3}x^2 - x^2 - 3 + \frac{1}{x} = [\infty - \infty] \Rightarrow$ NEMA DKA

$\lim_{x\to\infty} \frac{\frac{1}{3}x^3 - x^2 - 3x + 1}{x} \left[\begin{matrix} x \to -x \\ \infty \to \infty \end{matrix}\right] = \lim_{x\to\infty} \frac{1}{3}x^2 + x^2 + 3 - \frac{1}{x} \; [\infty + \infty]$

$\Rightarrow$ NEMA LK.A

5. ZAKRIVLJENOST / TAČKE INFLEKSIJE

$f''(x) = 2x - 2$

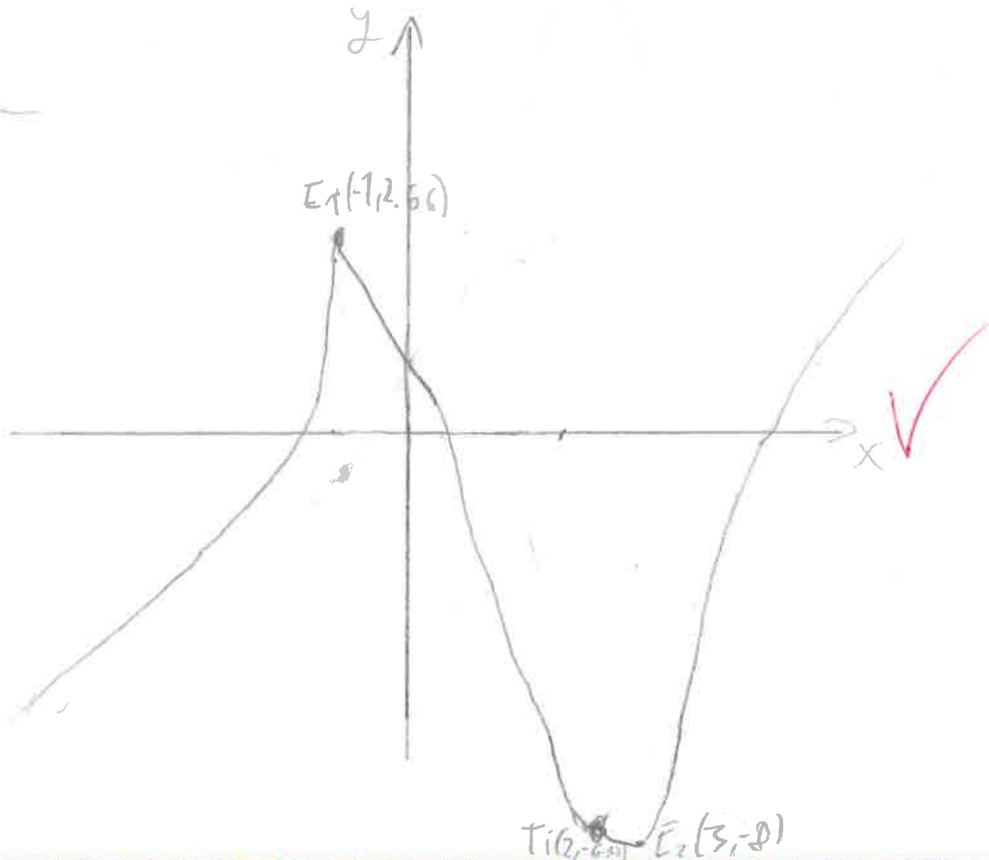
$2x - 2 = 0$

$2x = 2$ $\qquad T_i(2, -6.33)$

$x_i = 2$

$y = -6.33$

	$-\infty$	2	$+\infty$
	$-$	$+$	
$f''(x)$	$\cap$	$\cup$	



KRISTIAN DUJČEVIĆ

5. $f(x) = e^{2x}$

$$f(x) = e^{2x_0} + \frac{e^{x_0} \cdot 2}{1}(x-x_0) + \frac{e^{x_0} \cdot 2}{2}(x-x_0) + \frac{e^{x_0} \cdot 2}{6}(x-x_0)$$

2. $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 1$

1 DOMENA

$D_f < R >$

2. NULTOČKE

$$\frac{1}{3}x^3 - x^2 - 3x + 1 = 0$$

$$x\left(\frac{1}{3}x^2 - x - 3\right) = -1$$

$$x = 1$$

$$\frac{1}{3}x^2 - x - 3 = -1$$

$$\frac{1}{3}x^2 - x - 2 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+2.66}}{\frac{2}{3}}$$

$x_1 = 4.37$

$x_2 = -1.37$

3. EKSTREMI RAST/PAD

$f'(x) = x^2 - 2x - 3$

$$x^2 - 2x - 3 = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{16}}{2}$$

$$x_1 = \frac{2+4}{2} = 3$$

$x_2 = -1$

$y_1 = -8$

$y_2 = 2.666$

x	$-\infty$ ... -1	-1 ... 3	3 ... $+\infty$
$f'(x)$	+ ↗	− ↘	+ ↗

$E_1(3, -8) \Rightarrow$ LOKALNI MINIMUM

$E_2(-1, 2.66)$

1. $f(x) = x - \sqrt{x^2-4}$ KRISTIAN DUŠEVIĆ

1. DOMENA

$x^2 - 4 \geq 0$

$x^2 \geq 4$

$x \geq -2$

$x \geq 2$

$D_f \langle -\infty, -2] \cup [2, +\infty\rangle$

2. NULTOČKE

$x - \sqrt{x^2-4} = 0$

$x = \sqrt{x^2-4} \quad |^2$

$\cancel{x^2} = \cancel{x^2} - 4 \Rightarrow$ NEMA NULTOČAKI

3. EKSTREMI I RAST PAD

$f'(x) = 1 - \frac{2x}{2\sqrt{x^2-4}}$

$f'(x) = 1 - \frac{x}{\sqrt{x^2-4}}$

x	$-\infty$		-2		2		$+\infty$
		$+$				$-$	
$f'(x)$		$\nearrow$				$\searrow$	

$1 - \frac{x}{\sqrt{x^2-4}} = 0 \quad |\cdot\sqrt{x^2-4}$

$\sqrt{x^2-4} - x = 0$

$\sqrt{x^2-4} = x \quad |^2$

$\cancel{x^2} - 4 = \cancel{x^2} \Rightarrow$ NEMA EKSTREMA

3. ASIMPTOTE

$\lim_{x\to -2^-} x - \sqrt{x^2-4} \approx -2.2 \Rightarrow$ NEMA $\boxed{L.V.A}$

$\lim_{x\to -2^+} x - \sqrt{x^2-4} = -\infty$

$\lim_{x\to 2^+} x - \sqrt{x^2-4} \approx 1.8 \Rightarrow$ NEMA $\boxed{D.V.A}$

$\lim_{x\to 2^-} x - \sqrt{x^2-4} = -\infty$

$$\lim_{x\to\infty} x-\sqrt{x^2-4} = [\infty-\infty] = \lim_{x\to\infty} x-\sqrt{x^2-4} \cdot \frac{x+\sqrt{x^2-4}}{x+\sqrt{x^2-4}}$$

$$\lim_{x\to\infty} \frac{x^2-x^2+4 \,/:x}{x+\sqrt{x^2-4}} = \lim_{x\to\infty} \frac{4 \,/:x}{x+\sqrt{x^2-4}\,/:x} = \frac{0}{1} \Rightarrow \boxed{0 \text{ je } D.H.A}$$

$$\lim_{x\to-\infty} x-\sqrt{x^2-4} = \begin{bmatrix} x\to -x \\ -\infty \to \infty \end{bmatrix} = \lim_{x\to\infty} -x-\sqrt{x^2-4} \cdot \frac{(-x+\sqrt{x^2-4})}{-x+\sqrt{x^2-4}}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^2-x^2}+4 \,/:x}{-x+\sqrt{x^2-4}\,/:x} = \frac{0}{0} \Rightarrow \boxed{\text{NEMA L.H.A}}$$

$$\lim_{x\to-\infty} x-\sqrt{x^2-4} = \begin{bmatrix} x\to -x \\ -\infty \to \infty \end{bmatrix} = \lim_{x\to\infty} \frac{-x-\sqrt{x^2-4} \,/:x}{x \,/:x} = \frac{-1-1}{1} = -2 = k$$

$$\lim_{x\to\infty} -x-\sqrt{x^2-4} -2x = \lim_{x\to\infty} -3x-\sqrt{x^2-4} \cdot \frac{-3x+\sqrt{x^2-4}}{-3x+\sqrt{x^2-4}}$$

$$\lim_{x\to\infty} \frac{9x^2-x^2+4}{-3x+\sqrt{x^2-4}} \Rightarrow \boxed{\text{NEMA L.K.A}}$$

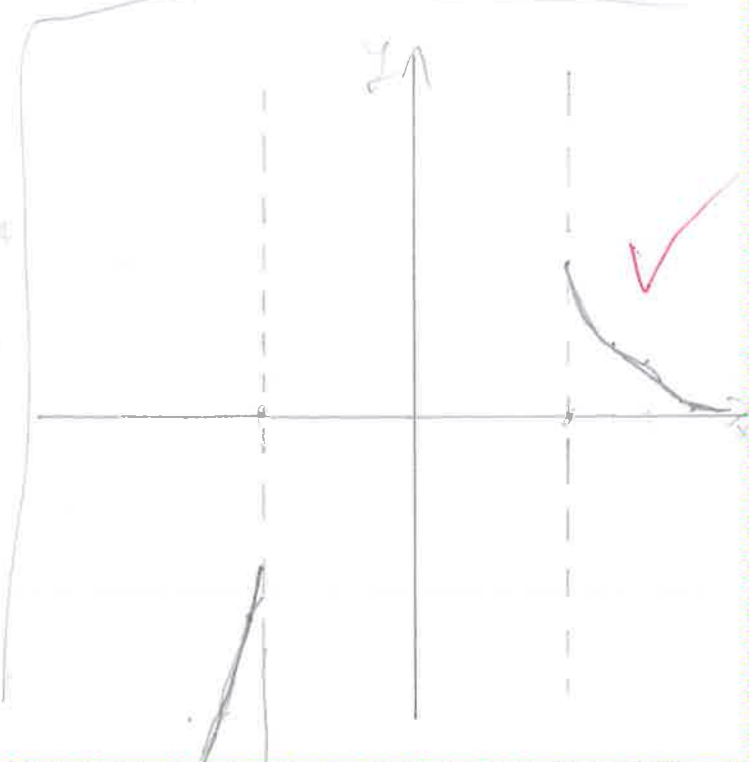
5. ZAKRIVLJENOST / TOČKE INFLEKSIJE

$$f''(x) = -\frac{\sqrt{x^2-4}-\left(\frac{x^2}{\sqrt{x^2-4}}\right)}{x^2-4}$$

$$-\left(\sqrt{x^2-4}-\frac{x^2}{\sqrt{x^2-4}}\right) = 0 \;/\cdot \sqrt{x^2-4}$$

$x^2-4 \neq x^2 \Rightarrow$ NEMA T.I.

	$-\infty$	-2	2	$+\infty$
	$-$		$-$	
$f''(x)$	$\cap$		$\cap$	



KRISTIAN DUŠEVIĆ

6.

$\cos x = 0.5 \,/\, \arccos$

$x = \arccos 0.5$

$x = 60^\circ = \frac{\pi}{3}$

~~$\emptyset$~~

JOŠ ?

4.

$(1-i)^6 - z^3 = 0$

$z^3 = (1-i)^6$

$z^3 = ((1-i)^2)^3$

$z^3 = (1-2i+i^2)^3$

$z^3 = (-2i)^3$

$z^3 = 8i$ ✓

$\operatorname{tg}\varphi = \frac{8}{0}$ $\qquad r = \sqrt{0^2+8^2}$

$\varphi = 90^\circ = \frac{\pi}{2}$ $\qquad r = 8$

$k=0$

$z_1 = \sqrt[3]{r} \cdot \left(\cos\frac{\varphi+2k\pi}{3} + i\sin\frac{\varphi+2k\pi}{3}\right)$

$z_1 = 2\left(\cos\frac{\frac{\pi}{2}}{\frac{3}{1}} + i\sin\frac{\frac{\pi}{2}}{\frac{3}{1}}\right)$

$z_1 = 1.73 + i$

$k=1$

$z_2 = 2\left(\cos\frac{\frac{\pi}{2}+2\pi}{3} + i\sin\frac{\frac{\pi}{2}+2\pi}{3}\right)$

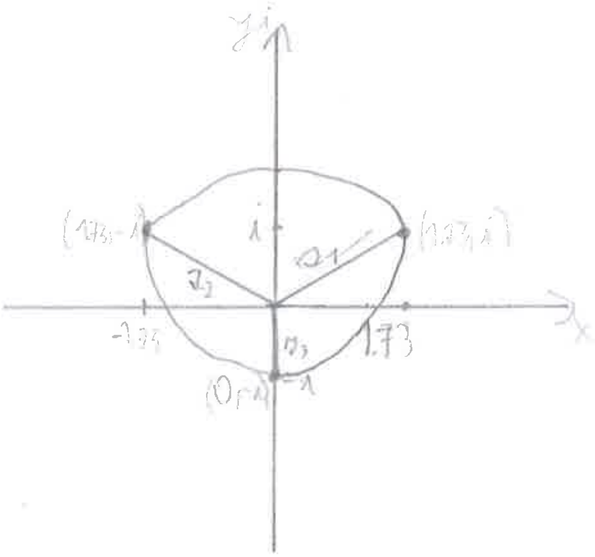
$z_2 = -1.73 + i$

$k = 2$

$$z_3 = 2 \cdot \left(\cos \frac{\varphi + 2 \cdot k \pi}{3} + i \sin \frac{\varphi + 2 \cdot k \cdot \pi}{3} \right)$$

$$z_3 = 2 \cdot \left(\cos \frac{\frac{\pi}{2} + 4\pi}{3} + i \sin \frac{\frac{\pi}{2} + 4\pi}{3} \right)$$

$$z_3 = -i$$



3.

KRISTIAN DUJEVIĆ

$f(x) = \sqrt{2-x^2}$

$f'(x) = \frac{1}{2\sqrt{2-x^2}} \cdot -2x$

$f'(x) = \frac{\cancel{-2}x}{\cancel{2}\sqrt{2-x^2}}$

$f'(x) = \frac{x}{\sqrt{2-x^2}}$

$2-x^2 \geq 0$

$2 \geq x^2$

$\pm\sqrt{2} \geq x$

$D_f\ [-\sqrt{2}, +\sqrt{2}]$

$E(0, 1.41)$ je ~~global~~ maksimum ✓

$x = 0$

$y = 1.41$

FUNKCIJA JE OGRANIČENA OD $-\sqrt{2}$ DO $+\sqrt{2}$ ✗

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 16

IME I PREZIME: MARKO MARIJANOVIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0437-2014 0171261712

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Na temelju ispitivanja toka skicirati graf funkcije $f(x) = x - \sqrt{x^2 - 4}$.	20 graf ?
2. Odrediti cjelokupan tijek funkcije $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 1$ i skicirati graf.	20 graf
3. Koji su globalni i lokalni ekstremi funkcije $g(x) = \sqrt{2 - x^2}$? Posebno komentirati (ne)ograničenost.	6+6+3
4. Riješiti jednadžbu: $(1 - i)^6 - z^3 = 0$. *Prikaži rješenja u kompleksnoj ravnini!*	~~15+3~~
5. Razviti funkciju $f(x) = e^{2x}$ u Taylorov red po potencijama od x. Izračunati barem prva 4 člana.	15
6. Riješiti: $\cos x = 0.5$	12

Ukupno: 26

1. GRAF

$f(x) = x - \sqrt{x^2-4}$

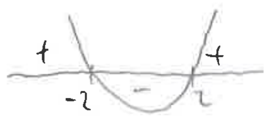
I DOMENA

$x^2 - 4 \geq 0$

$a=1$, $b=0$, $c=-4$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0 \pm \sqrt{0^2 - 4\cdot 1\cdot(-4)}}{2} =$

$x_1 = 2$ $\quad x_2 = -2$



$x \in \langle -\infty, -2] \cup [2, +\infty\rangle$ //

II ASIMPTOTE

V.A. - NEMA (JER SU RUBOVI DOMENE UKLJUČENI)

D.H.A.

$\lim_{x\to\infty} x - \sqrt{x^2-4} = \infty - \infty \quad \lim_{x\to\infty} x - \sqrt{x^2-4} \cdot \frac{x+\sqrt{x^2-4}}{x+\sqrt{x^2-4}} = \frac{x^2 - (\sqrt{x^2-4})^2}{x+\sqrt{x^2-4}} = \frac{-4}{\infty} = 0$ //

L.H.A.

$\lim_{x\to-\infty} x - \sqrt{x^2-4} = \infty + \infty = \infty$ NEMA L.H.A.

L.K.A. $y = k\cdot x + l$

$k = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x - \sqrt{x^2-4}}{x} \begin{matrix} /:x \\ /:x \end{matrix} = \lim_{x\to-\infty} \frac{-1 - \sqrt{1-0}}{1} = \frac{-2}{-1} = 2$ // $\quad k = 2$

$l = \lim_{x\to-\infty} [f(x) - kx] = \lim_{x\to\infty} x - \sqrt{x^2-4} - 2x = \lim_{x\to-\infty} -x - \sqrt{x^2-4} \cdot \frac{-x+\sqrt{x^2-4}}{-x+\sqrt{x^2-4}} = \frac{x^2 - (\sqrt{x^2-4})^2}{-x+\sqrt{x^2-4}} = \frac{-4}{\infty}$

$l = 0$ //

OSTALO NA STR 3 →

STR 1

SJ1(2)

III N.T.

$f(x)=0$ $\quad x-\sqrt{x^2-4}=0$

$\sqrt{x^2-4}=-x \quad |^2$

$\cancel{x^2}-4=\cancel{x^2}$

$-4\neq 0$ NE SIJE OS X

$f(0)=\infty$ NE SIJE OSY

IV DERIVACIJA

$$f'(x)=\left(x-\sqrt{x^2-4}\right)'=1-\frac{1}{2\sqrt{x^2-4}}\cdot 2x=1-\frac{2x}{2\sqrt{x^2-4}}=\frac{2\sqrt{x^2-4}-2x}{2\sqrt{x^2-4}}$$ //

$2\sqrt{x^2-4}-2x=0$ NEMA EKST.

$2\sqrt{x^2-4}=2x \quad |^2$

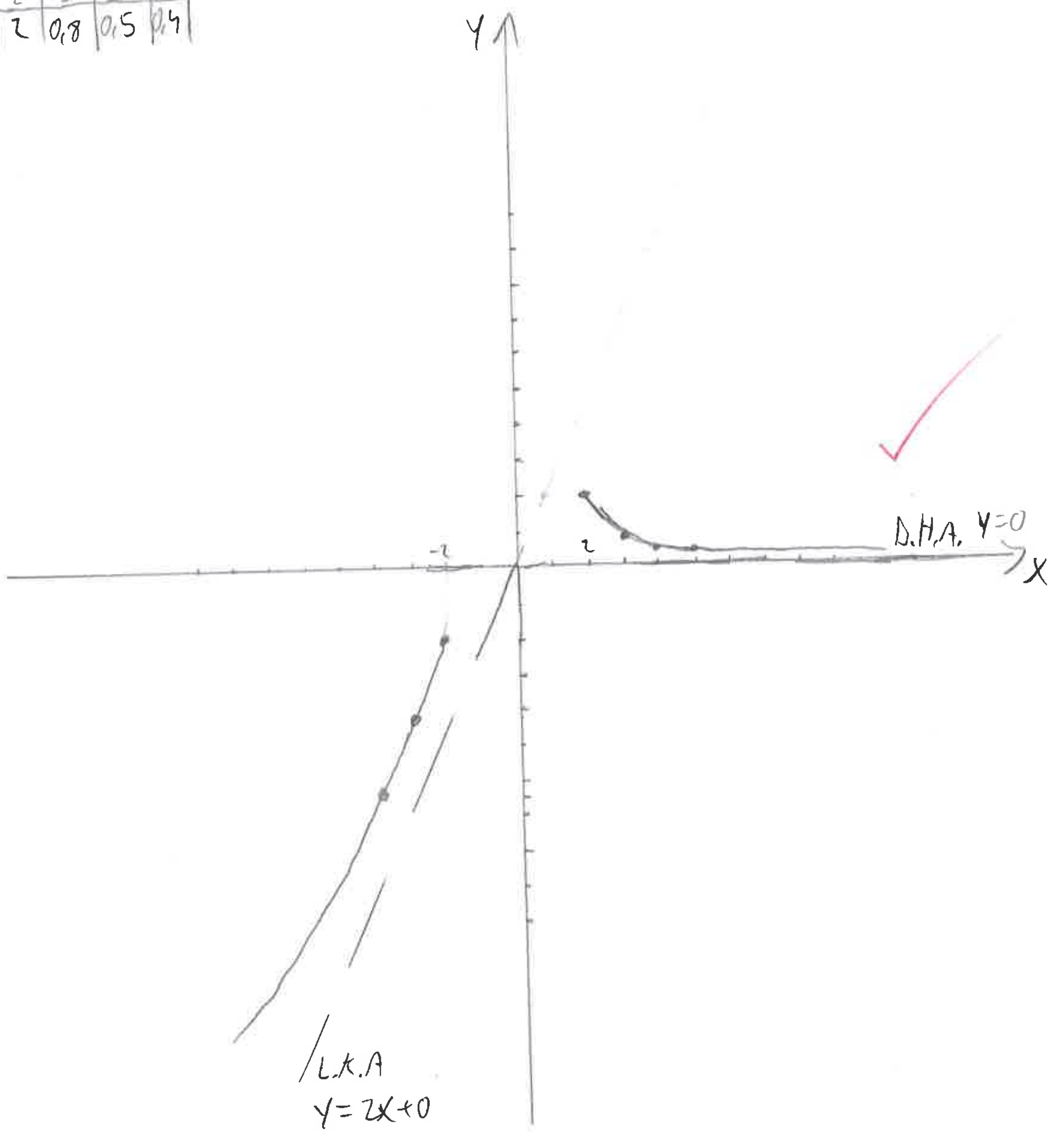
$4(x^2-4)=4x^2$

$\cancel{4x^2}-4=\cancel{4x^2}$

$-4\neq 0$

x	y
1	2
2	4
3	6

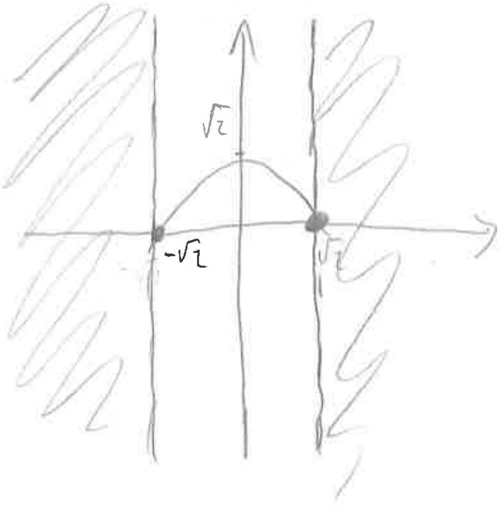
X	-4	-3	-2	2	3	4	5
f(x)	-7,5	-5,2	-2	2	0,8	0,5	0,4



③ LOKALNI I GLOBALNI EKSTREMI $g(x)=\sqrt{2-x^2}$

I. DOMENA

$2-x^2 \geq 0$ $\quad a=-1,\ b=0,\ c=2$ $\quad x_{1,2}=\frac{-0\pm\sqrt{0^2-4\cdot(-1)\cdot 2}}{-2}=\pm\sqrt{2}$



$x \in [-\sqrt{2}, \sqrt{2}]$

$g(\sqrt{2})=0$

$g(-\sqrt{2})=0$

II ASIMPTOTE

V.A. − NEMA $\quad$ K.A − NEMA

H.A. − NEMA

III N.T.

$f(x)=0 \quad \sqrt{2-x^2} \neq 0 \qquad f(0)=\sqrt{2}$ → TU SIJEČE OS y

IV DERIVACIJA

$g'(x)=\left(\sqrt{2-x^2}\right)'$

$g'(x)=\frac{1}{2\sqrt{2-x^2}}\cdot(-2x)=\frac{-2x}{2\sqrt{2-x^2}}$ // $\qquad -2x=0$

$g''(x)=\left(\frac{-2x}{2\sqrt{2-x^2}}\right)'=\frac{(-2x)'\cdot(2\sqrt{2-x^2})-(2\sqrt{2-x^2})'\cdot(-2x)}{(2\sqrt{2-x^2})^2}=$

$\cancel{g''(x)=\frac{-2\cdot(2\sqrt{2-x^2})-\frac{1}{4\sqrt{2-x^2}}\cdot(-2x)\cdot 2x}{8-4x^2}}$

FUNKCIJA PADA, LOKALNI EKSTREM JE $T_{max}(0,\sqrt{2})$ − LOKALNI MAKSIMUM, A VALJDA JE TO I GLOBALNI MAX. OTVORENA JE PUT DOLI (KONKAVNA)

GLOB. I LOKALNI MIN ?

(NE) OGRANIČENOST ?

(4) PRIKAŽI RJEŠENJA U KOMPLEKSNOJ RAVNINI

$z^3=(1-i)^6$

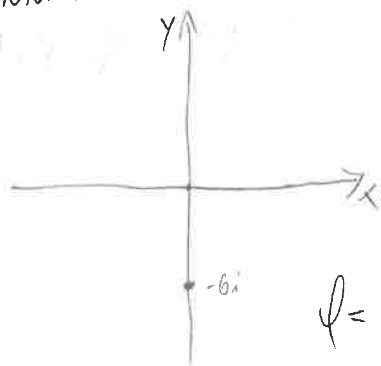
$r=\sqrt{x^2+y^2}$

$z^3=-6i$

$r=\sqrt{0^2+(-6)^2}$

$z=\sqrt[3]{-6}$

$r=6$

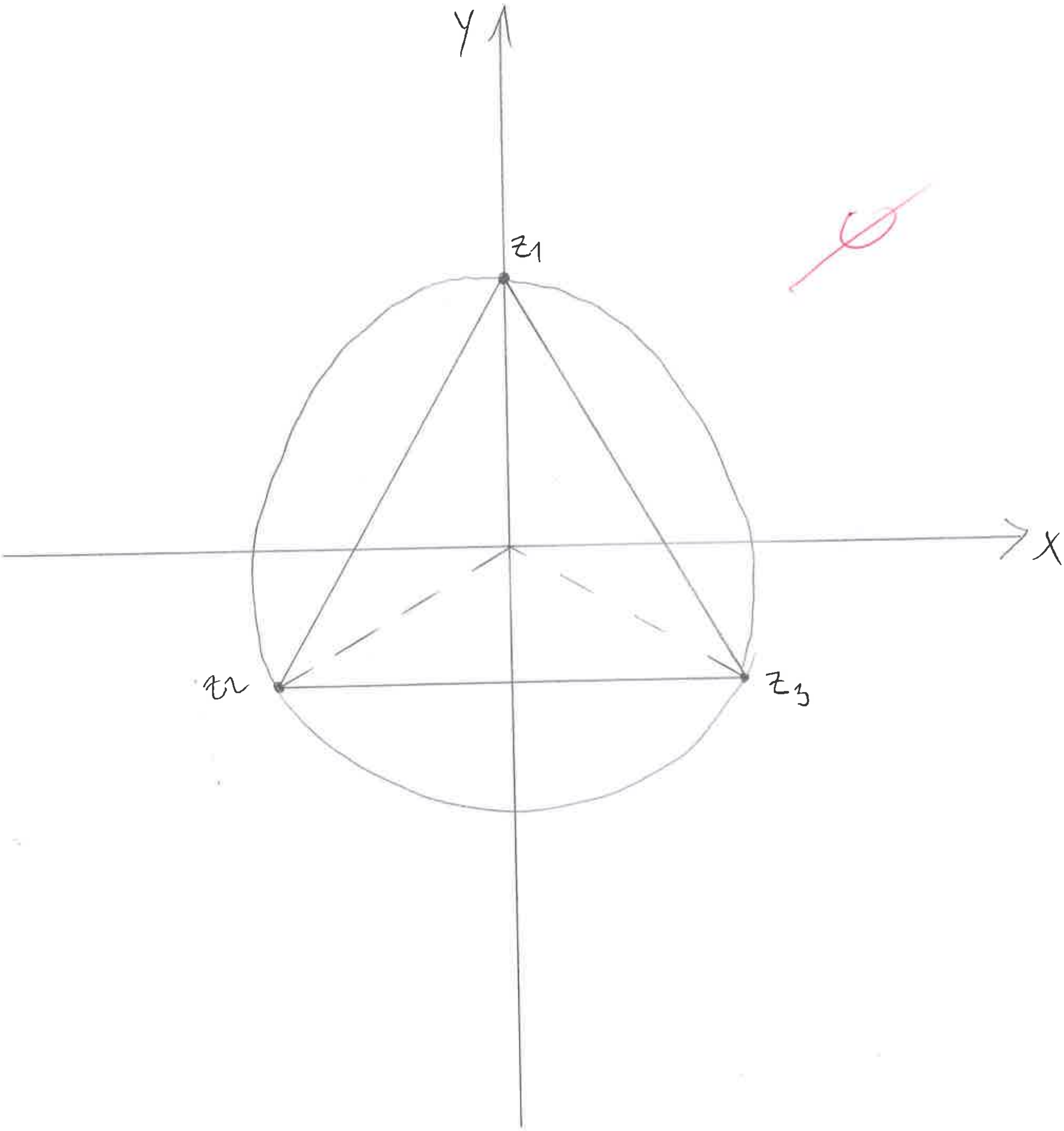


$\varphi=270°$

$\frac{1}{180}\cdot 270=$

$\varphi_0=\frac{270°}{3}=90°$ $\quad \Delta\varphi=120°$

$$\sqrt[3]{-6}=\sqrt[3]{6}\left(\cos\frac{\frac{3\pi}{2}+k\cdot 2\pi}{3}+i\sin\frac{\frac{3\pi}{2}+k\cdot 2\pi}{3}\right) \quad k=0,1,2$$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ 16	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: Mateo Bilaver **Vrijeme početka:**	
Matični broj studenta (iznad slike u indeksu): 17-2-0417-2014	

1. Na temelju ispitivanja toka skicirati graf funkcije $f(x) = x - \sqrt{x^2 - 4}$. — ~~20 graf~~
2. Odrediti cjelokupan tijek funkcije $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 1$ i skicirati graf. — 20 graf
3. Koji su globalni i lokalni ekstremi funkcije $g(x) = \sqrt{2 - x^2}$? Posebno komentirati (ne)ograničenost. — 6+6+3
4. Riješiti jednadžbu: $(1 - i)^6 - z^3 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* — ~~15+3~~
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6. Riješiti: $\cos x = 0.5$ — ~~12~~

Ukupno: Ø

1) $f(x) = x - \sqrt{x^2 - 4}$

	$-\infty$	-2	$+2$	$+\infty$
$x - \sqrt{x^2-4}$	+	• −	• +	

① Domena

$x^2 - 4 \geq 0$

$x^2 - 4 \geq 0$

$x^2 \geq 4 \quad /\sqrt{\ }$

$x = \pm\sqrt{2}$

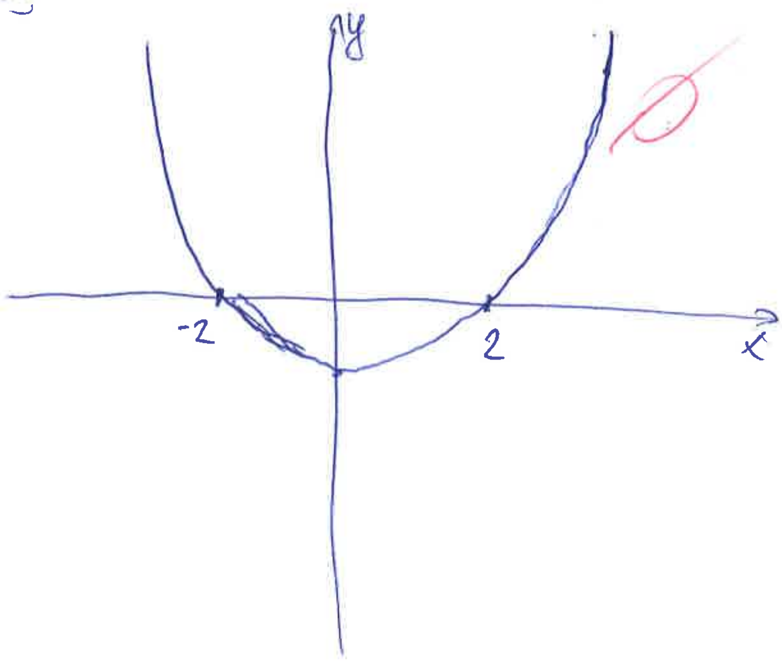
$x \in \langle -\infty, -2] \cup [2, +\infty\rangle$

② Asimptote

D.H.A $\lim\limits_{x \to \infty}$ $x - \sqrt{x^2 - 4} = \dfrac{x - \sqrt{x^2-4}}{\cancel{x}} \;/\!\cdot x = +\infty$

L.H.A $\lim\limits_{x \to -\infty}$ $x - \sqrt{x^2 - 4} = \left[\dfrac{x - \sqrt{x^2-4}}{\frac{x}{1}}\right] = +\infty$

Nema V.A



Ø

Derivacija

$x \geq \sqrt{x^2 + 4}$

$1 + 2x + (x^2 - 4) = 0$

$-2x - x^2 = -4 + 1$

4.) $(1-i)^6 - z^3 = 0$

$z^3 = (1-i)^6$

$$\sqrt[n]{2} = \sqrt[n]{r}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right.$$

$$z^n = r^n\left(\cos(n\cdot\varphi) + i\sin(n\cdot\varphi)\right)$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(1-i)^6 = 1^6 - 6\cdot 1^2\cdot(-i) + 6\cdot 1\cdot(-i)^2 - (-i)^6$$

$$= 1 + \cancel{6i} - \cancel{6i} + 1$$

$$= 2$$

$\cancel{\sqrt[6]{z} = \sqrt[6]{r}((}$

$r = \sqrt{x^2 + y^2}$

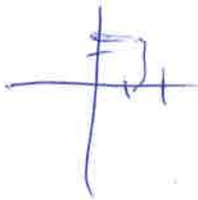
$r = \sqrt{1^2 + 2^2}$

$r = 2.24$

$\operatorname{tg}\varphi = \frac{y}{x}$

$\varphi = \frac{2}{1}$

$\varphi = 1.107$



$$\sqrt[6]{z} = \sqrt[6]{2.24}\left(\cos\frac{1.07 + 2k\pi}{6} + i\sin\frac{1.107 + 2k\pi}{6}\right)$$

$$= 1.14\,(0.9830) + (0.18345)$$

$z_0 = k_0 = 1.12 + 0.20i$

$z_1 = k_1 = 7.40 + 6.49i$

$z_2 = k_2 = 13.68 + 12.77i$

$f(x) = e^{2x}$

$x_0 = 0$

$f(x) = e^{2x} = x_0 = 0$

$f(x)' = 2 \cdot e^0 = 2$

$f'(x) = 2 \cdot 2 \cdot e^{2} = \cancel{}\ 4$

$f''(x) = 4 \cdot 2 \cdot 2 \cdot e^{2}$

$f''(x_0) = \cancel{}\ \cancel{54 \cdot 14}\ 16$

$f'''(x)\ \cancel{}\ 4 \cdot 2 \cdot 2 \cdot e \cdot 2$

$f(x_0) = 32$

$f^{IV}(x) = 512$

6.) $\cos x = 0.5$

$x = \frac{1}{3}\pi$

$\varnothing$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. PIŠITE DVOSTRANO! Obavezno popuniti sva polja ispod⇊ 16	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Ante Papić VRIJEME POČETKA:	
MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0211-2012	

1. Na temelju ispitivanja toka skicirati graf funkcije $f(x) = x - \sqrt{x^2 - 4}$. — 20 graf
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Ukupno: Ø

3) $g(x) = \sqrt{2-x^2}$

$g'(x) = \frac{2x}{2\sqrt{x^2-2}} = \frac{x}{\sqrt{x^2-2}}$

$g'(x) = 0 \rightarrow x = 0$

$g(0) = \sqrt{2} \rightarrow$ min $(0, \sqrt{2})$

$x^2 - 2 \geq 0$

$x^2 \geq 2$

$D(g) = \mathbb{R}$

Ø

~~4) $(1-i)^6 - z^3 = 0$~~

~~$z^3 = 6 - 6i - 1$~~

~~$z^3 = 5 - 6i$~~

~~$\varphi = \frac{6}{5}$~~

~~$r = \sqrt{5^2 + 6^2} = 7.8$~~

~~$t_g = \frac{1}{r}$~~