MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 15

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Marina Bešker **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0193-2012

1. Riješiti jednadžbu: $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$ — 12

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. — 20 graf

3. Odrediti tok funkcije $f(x) = \dfrac{x^2+2}{x-8}$ i skicirati graf. — 20 graf

4. Zadana je funkcija $f(x) = \sqrt{8+8x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf. — 4+4+4+6

5. Gaussovom metodom riješiti sustav: — 15+3

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1}$. — 10+2

Ukupno: 30

4. $f(x) = \sqrt{8+8x}$

$$f'(x) = \frac{1}{2\sqrt{8+8x}} \cdot (8+8x)' = \frac{1}{2\sqrt{8+8x}} \cdot 8 = \frac{8}{2\sqrt{8+8x}}$$

$8 \neq 0$ NEMA STACIONARNIH TOČAKA

t... $y = (x - x_0) \cdot f'(x)$

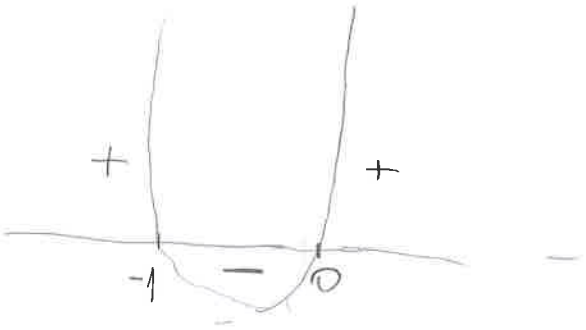
1. $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0 \qquad x+1 = 0$

$x = -1$



$x \in \langle -\infty, -1] \cup [0, +\infty \rangle$

0.93 > 0

2. $g(x) = \arctan(e^x)$ $\qquad \frac{1}{1+x^2}$

1. DOMENA

Df.. $x \in \mathbb{R}$

2 PARNOST NEPARNOST

$g(-x) = \arctan(e^{-x})$ funkcija nije niti parna niti neparna

3. ASIMPTOTE

$\lim_{x \to \infty}^{V.A} \arctan(e^x)$ V.A nema

H.A

$\lim_{x \to \pm\infty} \arctan(e^{-x}) = L'Hospital \lim_{x \to \pm\infty} \frac{1}{1+(e^{-x})^2} \cdot (e^{-x})'$

$\lim_{x \to \pm\infty} \frac{1}{1+(e^{-x})^2} \cdot e^{-x} \cdot (e^{-x})'$

Marenia Beseker

3. $f(x) = \frac{x^2+2}{x-8}$

1. DOMENA $\quad D_f \dots x \in \mathbb{R} \setminus \{8\}$

$x - 8 = 0$

$x = 8$

2. N.T

$x^2 + 2 = 0$

$x^2 = -2 \mid \sqrt{\ }$

$x = \pm\sqrt{-2}$

3. PARNOST NEPARNOST

$f(-x) = \frac{(-x)^2+2}{-x-8} = \frac{x^2+2}{-x-8} = -\frac{x^2-2}{x+8}$ funkcija je ne parn

4. ASIMPTOTE

V.A

$\lim_{x \to 8 \pm 0.1} \frac{x^2+2}{x-8} = +\infty$

V.A

$x = 8$

$y = ax + b$

$a = \lim \frac{f(x)}{x}$

$b = \lim (f(x) - a \cdot x)$

H.A

$\lim_{x \to \pm\infty} \frac{x^2+2 : x^2}{x-8 : x^2} = \lim_{x \to \pm\infty} \frac{\frac{x^2}{x^2} + \frac{2}{x^2}}{\frac{x}{x^2} - \frac{8}{x^2}} = \frac{1}{0} =$ nema H.A

K.A

$y = ax + b$

$a = \lim_{x \to \infty} \frac{f(x)}{x} = \frac{\frac{x^2+2}{x-8}}{\frac{x}{1}} = \frac{x^2+2}{x^2-8x} \begin{matrix} :x^2 \\ \end{matrix} = \frac{1 + \frac{2}{x^2}}{1 - \frac{8x}{x^2}} = \frac{1}{1}$

$\boxed{a = 1}$

$b = \lim_{x \to \infty} (f(x) - a \cdot x)$

$b = \lim_{x \to \infty} \frac{x^2+2}{x-8} - 1x = \lim_{x \to \infty} \frac{x^2+2-1x(x-8)}{x-8} =$

$\lim_{x \to \infty} \frac{x^2+2-x^2+8x}{x-8} = \lim_{x \to \infty} \frac{2+8x}{x-8} \begin{matrix} : x \\ \end{matrix} = \lim_{x \to \infty} \frac{\frac{2}{x} + \frac{8x}{x}}{\frac{x}{x} - \frac{8}{x}}$

$\boxed{b = 8}$

$y = ax + b$

$\boxed{y = 1x + 8}$

x	0	1
y	8	9

//

5.11. DERIVACIJA I STACIONARNE TOČKE Marina Bester

3 ZADATAK

$f(x) = \frac{x^2+2}{x-8}$

$\frac{f}{g} = \frac{(f)' \cdot (g) - (f) \cdot (g)'}{g^2}$

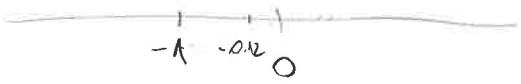
$f'(x) = \frac{(x^2+2)' \cdot (x-8) - (x^2+2) \cdot (x-8)'}{(x-8)^2}$

$f'(x) = \frac{2x(x-8) - (x^2+2)}{(x-8)^2}$

$f'(x) = \frac{2x(x-8) - x^2 - 2}{(x-8)^2}$

$f'(x) = \frac{2x^2 - 16x - x^2 - 2}{(x-8)^2}$

$f'(x) = \frac{x^2 - 16x - 2}{(x-8)^2}$ //



$x^2 - 16x - 2 = 0 \quad a = 1, \; b = -16, \; c = -2$

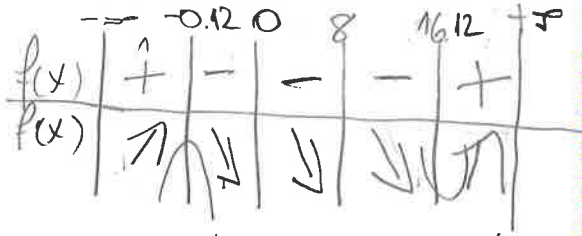
$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{16 \pm \sqrt{16^2 - 4 \cdot 1 \cdot (-2)}}{2} = \frac{16 \pm \sqrt{256+8}}{2} = \frac{16 \pm 2\sqrt{66}}{2}$

$x_1 = \frac{16 + 2\sqrt{66}}{2}, \quad x_2 = \frac{16 - 2\sqrt{66}}{2}$

$x_1 = 8 + \sqrt{66} \approx 16.12$ //

$x_2 = 8 - \sqrt{66} \approx -0.12$ //



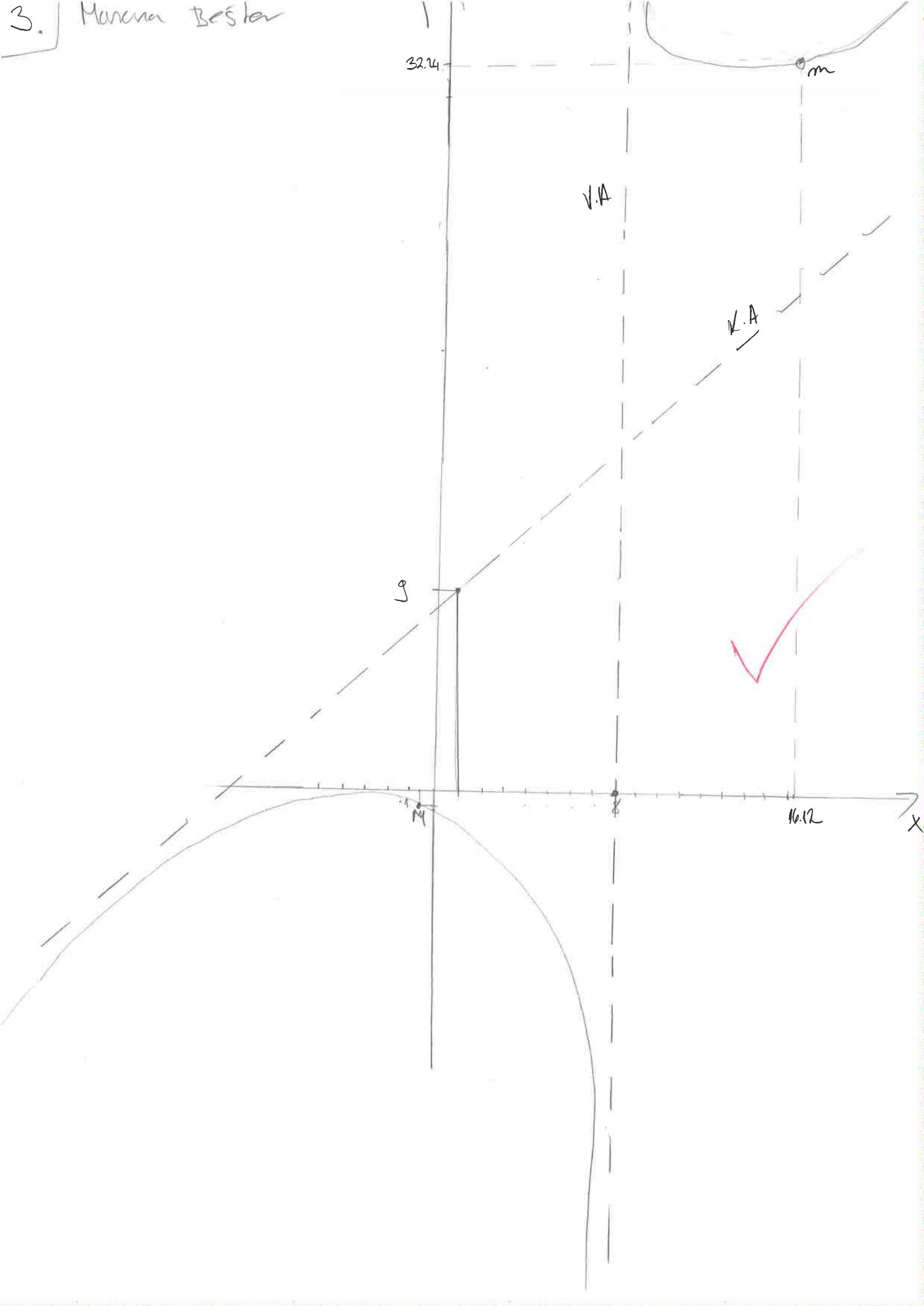
$y_1(-0.12) = \frac{-0.12^2 + 2}{-0.12 - 8} = -0.25$

$y_2(16.12) = \frac{16.12^2 + 2}{16.12 - 8} = 32.24$

M(-0.12, -0.25) m(16.12, 32,24)

(5)
$$\left[\begin{array}{cccc|c} 2 & -3 & -1 & 2 & 4 \\ 4 & -4 & -1 & 4 & 4 \\ 2 & -5 & -2 & 2 & 9 \\ 0 & 0 & 2 & 1 & -5 \end{array}\right] \approx \left[\begin{array}{c} \\ \\ \cdot(-1) \\ \\ \end{array}\right] + \sim \left[\right.$$

3. Marcha Bestar



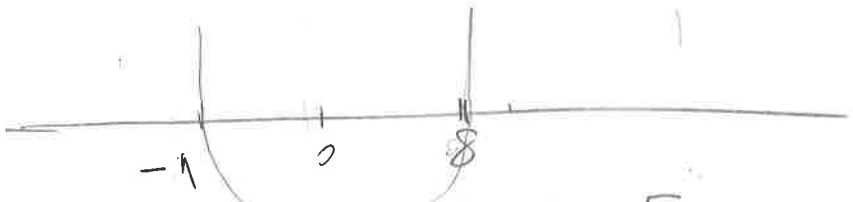
4. $f(x) = \sqrt{8+8x}$ Maruša Beškar

1. DOMENA

$8+8x \geq 0$

$8+8x = 0$

$8(1+x) = 0$



Df... $x \in \langle -\infty, -1] \cup [8, +\infty \rangle$

I. $8 = 0$ ✓ II. $1+x = 0$

$x = -1$ ✗

ASIMPTOTE

V.A

$\lim_{x \to 8} \sqrt{8+8x} = +\infty$

U.A

$x = 8$ desna stran

H.A

$\lim_{x \to \pm\infty} \sqrt{8+8x}$ Lhospital $\lim_{x \to \pm\infty} \frac{1}{2\sqrt{8+8x}} \cdot (8+8x)' = \lim_{x \to \pm\infty} \frac{1}{2\sqrt{8+8x}} \cdot 8$

$\lim_{x \to \pm\infty} \frac{8}{2\sqrt{8+8x}}$ NIMA H.A $\quad a = 0$

K.A $y = ax + b$

$a = \lim_{x \to \infty} \frac{f(x)}{x} = \frac{\sqrt{8+8x}}{x} = \frac{\sqrt{8+8x} : x}{x : x} = \lim_{x \to \infty} \frac{\sqrt{\frac{8}{x^2} + \frac{8x}{x^2}}}{1} = \frac{0}{1} = 0$

$b = \lim_{x \to \infty} (f(x) - a \cdot x) = \sqrt{8+8x} \cdot x : x = \lim_{x \to \infty} \sqrt{\frac{8}{x^2} + \frac{8x}{x^2}} \cdot 1$

$b = 0$

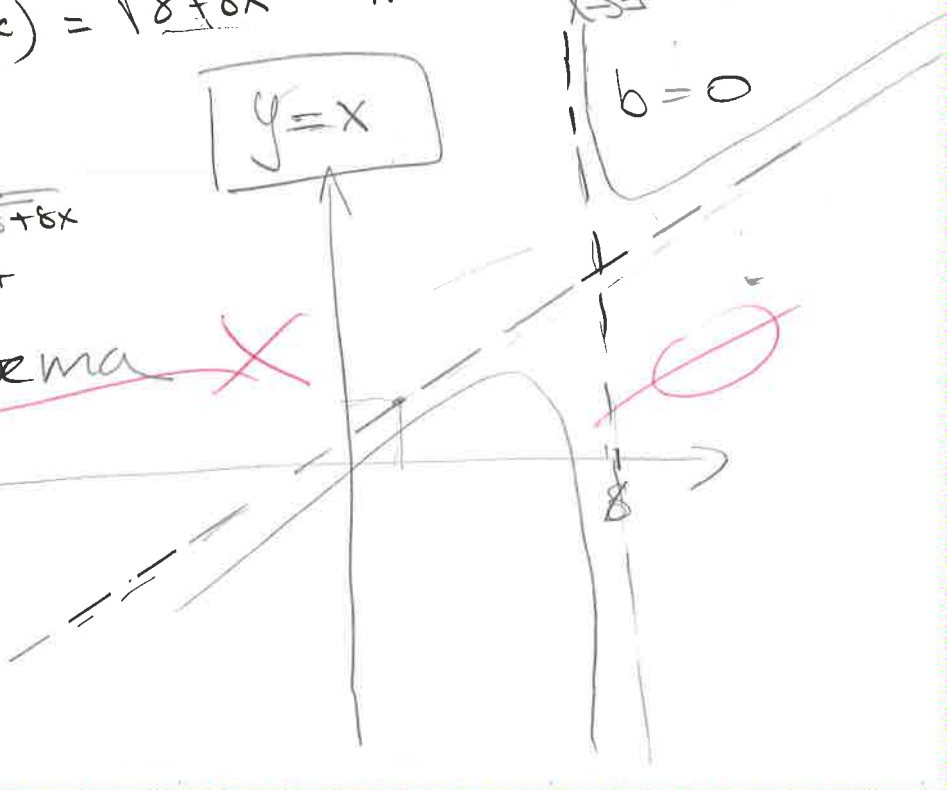
$y = x$

$f(x) = \sqrt{8+8x}$

$f'(x) = \frac{1}{2\sqrt{8+8x}} \cdot (8+8x)' = \frac{8}{2\sqrt{8+8x}}$

$8 = 0$ NIMA ST. TOČKA

nema lokalnih ekstrema ✗



6. $\lim_{x \to 1} \frac{\sqrt{2-x^2} - x}{x-1}$ $\left[\frac{0}{0}\right]$ L'Hospital

$$\lim_{x \to 1} \frac{(\sqrt{2-x^2})' - (x)'}{(x)' - (1)'} = \frac{\frac{1}{2\sqrt{2-x^2}} \cdot (2-x^2)' - 1}{1} =$$

$$\lim_{x \to 1} \frac{1}{2\sqrt{2-x^2}} \cdot (-2x) - 1 = \lim_{x \to 1} \frac{-2x-1}{2\sqrt{2-x^2}} \quad | : x$$

$$= \frac{\frac{-2x}{x} - \frac{1}{x}}{\frac{2}{x}\sqrt{\frac{2}{x^2} - \frac{x^2}{x^2}}} = \frac{-2}{1} = -2$$

$= -2$ ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 15

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: DINO PETEŠIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0314-2013.

1. Riješiti jednadžbu: $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$ — 12
2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. — 20 graf
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Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1}$. — 10+2

Ukupno: 22

1. $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$

$2 + \dfrac{2x+1}{2\sqrt{x(x+1)}} = 0$

$\dfrac{2\cdot 2\sqrt{x(x+1)} + 2x+1}{2\sqrt{x(x+1)}} = 0$

$4\sqrt{x(x+1)} + 2x+1 = 0$

$4\sqrt{x(x+1)} = -(2x+1)\,/^2$

$16x^2+16x = 4x^2+4x+1$

$12x^2+12x-1=0$

$x_{12} = \dfrac{-12 \pm \sqrt{144+48}}{24}$

$= \dfrac{-12\pm 8\sqrt{3}}{24}$

$x_1 = \dfrac{-3+2\sqrt{3}}{6}$ $\quad x_2 = \dfrac{-3-2\sqrt{3}}{6}$

PROVJERA ?

2. $f(x) = \arctan(e^x)$

① NULTOČKE

$F(x) = 0$

$\arctan(e^x) = 0$

$e^x = \tan(0) = 0$

VRIJEDI ZA $x = -\infty$

$$\frac{df(x)}{dx} = \frac{e^x}{e^{2x}+1}$$

$$\frac{df(x)}{dx} = 0 \rightarrow \text{NEMA LOKALNIH EKSTREMA}$$

$$\frac{d^2f(x)}{dx^2} = \frac{e^x(e^{2x}+1) - e^x(e^{2x})}{(e^{2x}+1)^2}$$

$e^{3x} + e^x - 2e^{3x} = 0$

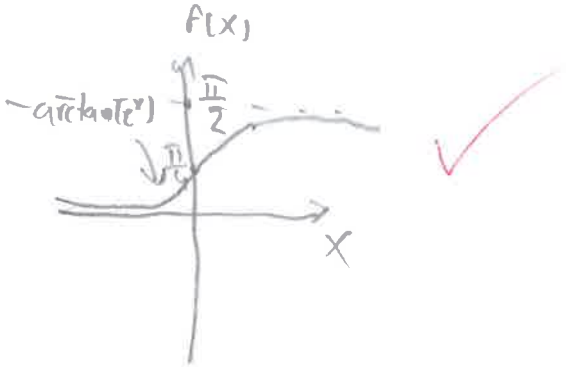
$-e^{3x} + e^x = 0$

$e^x(-e^{2x}+1) = 0$

$e^{2x} = 1$

$2x = \ln 1 = 0$

$x = 0$



$f(x=0) = \arctan(e^0) = \arctan(1) = \frac{\pi}{4}$

$T(x=0, x=\frac{\pi}{4})$ TOČKA INFLEKSIJE

4) $f(x)=\sqrt{8+8x}$

DOMENA $8+8x>0$

$8x>-8 /:8$

$x>-1$

LOKALNI EKSTREMI

$\frac{df(x)}{dx}=\frac{1}{2\sqrt{8+8x}}\cdot 8=0$

$\frac{4}{2\sqrt{n+x}}=0$ ✗

NULTOČKA $f(x)=0$ $x=-1$

$f(x=0)=\sqrt{8}=2\sqrt{2}$ ≠

$\frac{df(x)}{d(x)}=\frac{\sqrt{2}}{\sqrt{2+x}}=0$, NEMA LOKALNIH EKSTREMA, NEMA RJEŠENJA

TANGENTA U $x=2$

$\alpha=f'(x=2)=\frac{4}{\sqrt{8+n}}=\frac{\sqrt{6}}{3}$

$T(x=2, f(x)=2\sqrt{6})$

$y-2\sqrt{6}=\frac{\sqrt{6}}{3}(x-2)$

$y=\frac{\sqrt{6}}{3}x+\frac{4\sqrt{6}}{3}$

GLOBALNI EKSTREMI

najnegativnija točka domene je jedno : x koordinata globalnog minimuma

$f(x=-1)=0$ $T(-1,0)$ je globalni minimum ✓ a globalni maksimum se ne može jednoznačno odrediti ✗

DINO PEŠEŠIĆ 17-2-0314-2013

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 15

IME I PREZIME: FILIP MEDIĆ **VRIJEME POČETKA:** 09:49

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0395-2014

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$ — 12

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Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1}$. — 10+2

Ukupno: 20

(1) $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$

$\dfrac{2x+1}{2\sqrt{x^2+x}} = -2 \quad / \cdot 2\sqrt{x^2+x}$

$2x+1 = -4\sqrt{x^2+x} \quad /^2$

$(2x+1)^2 = 16 \cdot (x^2+x)$

$(2x+1)^2 = 16x^2 + x$

$4x^2 + 4x + 1 = 16x^2 + x$

$-16x^2 + 4x^2 + 4x - x + 1 = 0$

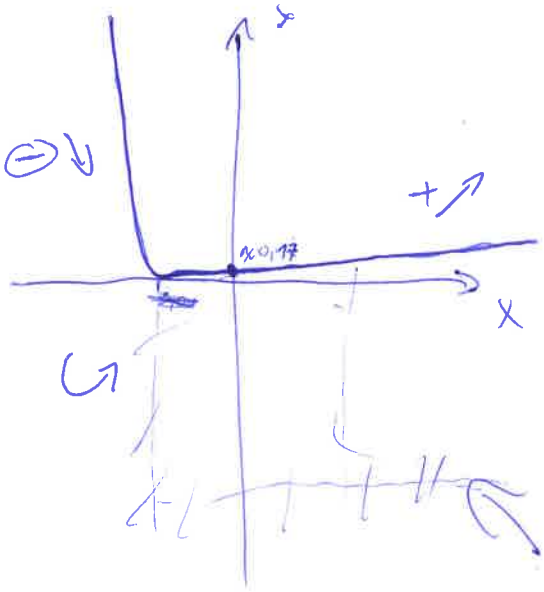
$-12x^2 + 3x + 1 = 0$

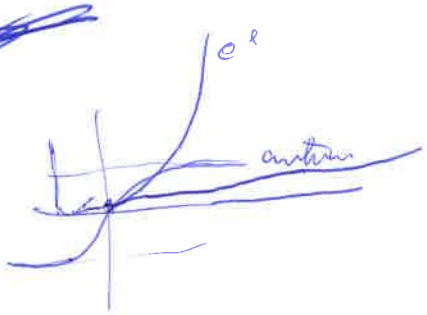
$x_{1,2} = \dfrac{-3 \pm \sqrt{9 - 4\cdot(-12)\cdot 1}}{-24}$

$x_{1,2} = \dfrac{-3 \pm \sqrt{9+48}}{-24} = \dfrac{-3 \pm \sqrt{57}}{-24}$

② $g(x) = \arctan(e^x)$

$D(\mathbb{R})$ $N(\mathbb{R}^+)$





$D(f) = \mathbb{R}$

$f(x) = \arctan(e^x)$

$f = \arctan$

$g = e^x$

$f' = \frac{1}{1+x^2}$

$g' = e^x$

$f'(g(x)) \cdot g' \Rightarrow \frac{1}{1+(e^x)^2} \cdot e^x$

$f' = \frac{1}{1+(e^x)^2} \cdot e^x$

$f'' = \frac{-2e^x \cdot e^x}{(e^x)^2} \cdot e^x + \frac{1}{1+(e^x)^2} \cdot e^x$

$= \frac{-2(e^x)^2}{(e^x)^2} \cdot e^x + \frac{1}{1+(e^x)^2} \cdot e^x$

$= -2e^x + \frac{e^x}{1+(e^x)^3}$

$f = 1 \quad f' = 0$

$f = x^2 \quad f' = 2x$

$g = e^x \quad g' = e^x$

$f'(g(x)) \cdot g' \quad 2(e^x) \cdot e^x$

$1 + (e^x)^2$

$f = e^x \quad g' = e^x$

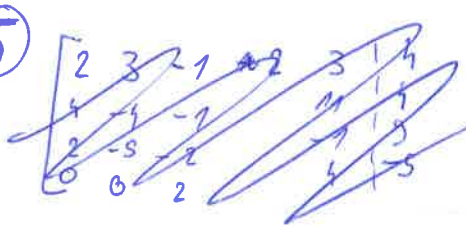
$\frac{f' \cdot g - f \cdot g'}{g^2} = \frac{0 \cdot e^x - 1 \cdot 2(e^x) \cdot e^x}{(e^x)^2}$

$= \frac{-2(e^x) \cdot (e)^x}{(e^x)^2}$

$\frac{-2(e^x) \cdot e^x}{(e^x)^2} \cdot e^x$

f' g'

(5)



$$\begin{array}{ccccc|c} x & y & z & w & v & \\ \hline 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \quad / R1 : 2$$

$$\begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & -1 & 0 & 4 & -5 \end{bmatrix} \begin{matrix} \\ R2-4R1 \\ R3-2R1 \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 0 & -3 & 0 & 5 & -4 \\ 0 & -2 & -1 & 2 & -4 & 5 \\ 0 & 2 & -1 & 0 & 4 & -5 \end{bmatrix} R2:2 \sim \begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & -\frac{3}{2} & 0 & \frac{5}{2} & -2 \\ 0 & -2 & -1 & 2 & -4 & 5 \\ 0 & 2 & -1 & 0 & 4 & 5 \end{bmatrix} \begin{matrix} R1+\frac{3}{2}R2 \\ \\ R3+2R2 \\ R4-2R2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -\frac{7}{4} & 1 & \\ 0 & 1 & -\frac{3}{2} & 0 & \frac{5}{2} \\ 0 & 0 & -2 & 2 & \\ 0 & 0 & -2 & 0 & \end{bmatrix} \quad \text{IZGLEDA NETOČNO}$$

(6) $\lim_{x \to 1} \dfrac{\sqrt{2-x^2}-x}{x-1} \quad \dfrac{\sqrt{1-1}}{0} : \dfrac{0}{0}$

?, ?, ?.

③ $f(x) = \frac{x^2+2}{x-8}$

$D(f) = \langle -\infty, 8 \rangle \langle 8, +\infty \rangle$

$x - 8 = 0$

$x = 8$

$-\infty \quad 6 \quad 8 \quad 10 \quad +\infty$

$\ominus \quad \oplus$

$\lim_{x\to\infty} \frac{x^2+2}{x-8} = \lim_{x\to\infty} \frac{x^2+2 \,/x^2}{x-8\,/x^2} = \lim_{x\to\infty} \frac{1+\frac{2}{x^2}}{\frac{1}{x}-\frac{8}{x^2}}$

$= \lim_{x\to\infty} \frac{1}{0} = 0$ D.H.A.

$\lim_{x\to-\infty} \frac{x^2+2}{x-8} \,|x\to -x| = \lim_{x\to\infty} \frac{x^2+2}{-x-8}\Big/{}_{x^2}$

$= \lim_{x\to\infty} \frac{1+\frac{2}{x^2}}{-\frac{1}{x}-\frac{8}{x^2}} = \frac{1}{0} = 0$ L.H.A.

V.A.

$\lim_{x\to 8^-} \frac{x^2+2}{x-8} = \lim_{x\to 8^-} \frac{+}{-0,1} = -$ (↘ 7,9)

$\lim_{x\to 8^+} \frac{x^2+2}{x-8} = \lim_{x\to 8^+} \frac{+}{+0,1} = +$ (↗ 8,1)

$f = x^2+2 \quad f' = 2x$

$f = \frac{x^2+2}{x-8} \quad g = x-8 \quad g' = 1$

$f'_{(x)} = \frac{f' \cdot g - f \cdot g'}{g^2} = \frac{2x(x-8) - 1(x^2+2)}{(x-8)^2}$

$= \frac{2x^2 - 16x - x^2 - 2}{(x-8)^2} = \frac{x^2-16x-2}{(x-8)^2}$

$f'(0) = -0,25$

$f(0) = \frac{x^2-16x-2}{(x-8)^2}$

$f(x) = 0$

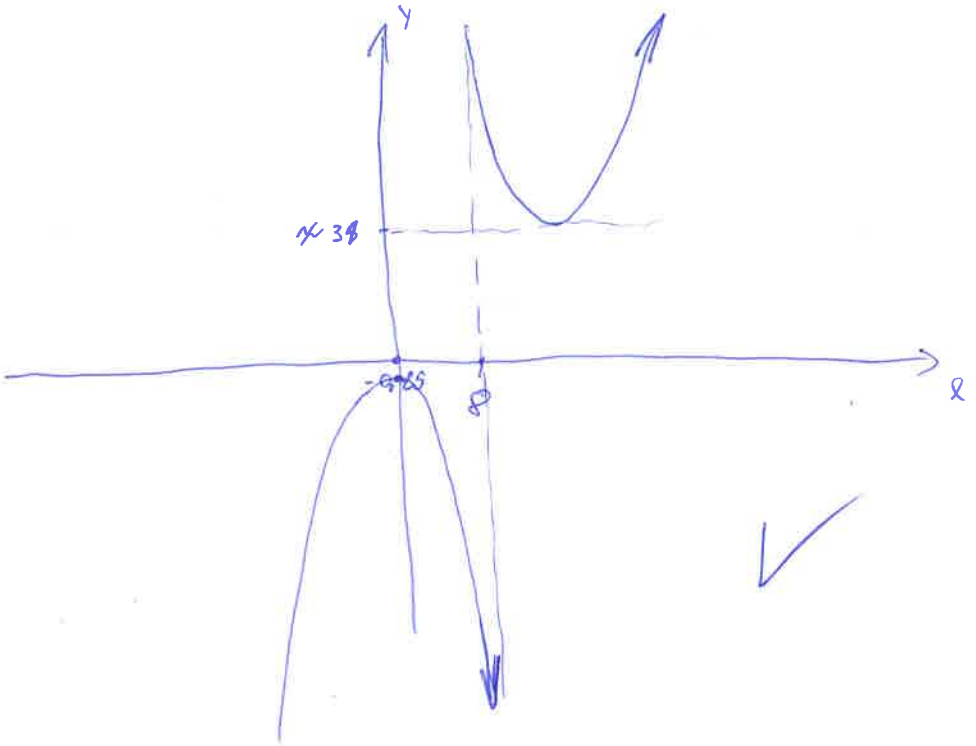
$\frac{x^2-16x-2}{(x-8)^2} = 0$

$x^2 - 16x - 2 = 0$

f' $\quad -\infty \quad 0 \quad 3 \quad 3 \quad +\infty$

f' $\quad \nearrow \quad \searrow \quad \searrow \quad \nearrow$

$y = -0,125 \quad x = 0$



$f'' = ?$

$f' = \frac{x^2 - 16x - 2}{(x-8)^2}$ — $f' = 2x - 16$, $f = 2x - 8$

$f \circ g = 2x - 8$

$f'(g(x)) \cdot g'$

$f = x^2$

$g = x - 8$

$f' = 2x$

$g' = 1$

$2(x-8) \cdot 1$

$2x - 8$

$f'' = \frac{2x-16}{2x-8}$

4) 1. DZ

$f(x) = \sqrt{8+8x}$ — $\sqrt{x}\left(-\frac{1}{2}\right)$, $8x+8$

tangenta en $x = 2$

$x_0 = -2$

$t = f'(x_0) + f(x_0)(x - x_0)$

$8 + 8x \geq 0$

$8x \geq 8 \mid :8$

$x \geq -1$

$D(f) = [-1, +\infty)$

$f \circ g = f'(g(x)) \cdot g'$

$f = \sqrt{x} = x^{-\frac{1}{2}}$

$g = 8x + 8$

$f' = -\frac{1}{2} x^{-\frac{3}{2}}$

$g' = 8$

x^2

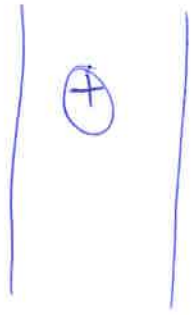
$2x$

$x^{-\frac{1}{2}}$

$-\frac{1}{2} x^{-\frac{3}{2}}$

$f' = -\frac{1}{2}(8x+8)^{-\frac{3}{2}} \cdot 8$

-1 +∞



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 15

IME I PREZIME: Šime Labus **VRIJEME POČETKA:** 9:30h

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
17-1-0291-2014
0269093263

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$ 12

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf

3. Odrediti tok funkcije $f(x) = \dfrac{x^2+2}{x-8}$ i skicirati graf. 20 graf

4. Zadana je funkcija $f(x) = \sqrt{8+8x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf. 4+4+4+6

5. Gaussovom metodom riješiti sustav: 15+3

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1}$. 10+2

Ukupno: 10

① $2 + \dfrac{2x+1}{2\sqrt{x^2+x}}$

$4\sqrt{x^2+x} + 2x + 1 = 0$

$4\sqrt{x^2+x} = -2x-1 \,/^2$

$16(x^2+x) = (-2x-1)^2$

$16x^2 + 16x = [-(2x+1)]^2 = (2x+1)^2 = 4x^2 + 4x + 1$

$12x^2 + 12x - 1 = 0$

$x_{1,2} = \dfrac{-12 \pm \sqrt{12^2 - 4\cdot 12\cdot(-1)}}{12\cdot 2}$

$x_1 = \dfrac{-3+2\sqrt{3}}{6}$ // $x_2 = -\dfrac{3+2\sqrt{3}}{6}$ //

PROVJERA ?

⑥ $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1} = \lim\limits_{x\to 1} \dfrac{\sqrt{2-1}-1}{1-1} = \left(\dfrac{0}{0}\right)$ H

$\lim\limits_{x\to 1} = \dfrac{\frac{1}{2\sqrt{2-x^2}}\cdot(-2x) - 1}{1} = \dfrac{\frac{-1}{\sqrt{2-1}} - 1}{1} = -2$ //

$x \to -1$

$\lim\limits_{x\to -1} \dfrac{\sqrt{2-(-1)^2}+1}{-1-1} =$

$\lim\limits_{x\to -1} \dfrac{2}{-2} = -1 \ldots$

⑤ $2x - 3y - z + 2w + 3v = 4$

$4x - 4y - z + 4w + 11v = 4$

$2x - 5y - 2z + 2w - v = 9$

$2y + z + 4v = -5$

$$\left[\begin{array}{ccccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \begin{array}{l} \\ \\ \\ \end{array} \begin{array}{l} \\ -2/-1 \\ \\ \end{array} \sim$$

$$\left[\begin{array}{ccccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 0 & 5 & -4 \\ 0 & -2 & -1 & 0 & -5 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \begin{array}{l} \\ 1/-1 \\ \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 0 & 5 & -4 \\ 0 & 0 & 0 & 0 & 0 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right]$$

S obzirom da je jedan red puna nula, a rješenje jednadžbe 1 zaključujemo da sustav nema rješenja.

$(0x = 1)$

$x \notin \mathbb{R}$

③ $f(x) = \frac{x^2+2}{x-8}$ $\quad D: x-8 \neq 0$, $x \neq 8$

$x^2 + 2 = 0$

$x^2 = 2$

$x \notin \mathbb{R} \rightarrow$ Nema nul točaka

$$\frac{df(x)}{dx} = \frac{2x(x-8) - (x^2+2)}{(x-8)^2} = \frac{2x^2 - 16x - x^2 - 2}{(x-8)^2} = \frac{x^2 - 16x + 2}{(x-8)^2}$$

$x^2 - 16x - 2 = 0$

$$x_{1,2} = \frac{16 \pm \sqrt{16^2 - 4 \cdot (-2)}}{2} \rightarrow \begin{array}{l} x_1 = 8 - \sqrt{66} \\ x_2 = 8 + \sqrt{66} \end{array}$$

SKICA?

④ $f(x) = \sqrt{8 + 8x}$

$$\frac{dF(x)}{dx} = \frac{8x}{2\sqrt{8+8x}} = \frac{4x}{\sqrt{8+8x}}$$

$4x = 0$

$x = 0 \rightarrow f(x) = 2\sqrt{2}$

Za $x = 0$ funkcija $f(x)$ postiže lokalni minimum $f(x) = 2\sqrt{2}$

$y = \frac{df(x)}{dx}_{x=2} \cdot (x - 2) =$

$y = \frac{2\sqrt{6}}{3} \quad x = \frac{4\sqrt{6}}{3}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 15

IME I PREZIME: FILIP GORŠEK **VRIJEME POČETKA:** 8:00 30.4.2015

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 3466 0 23782

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$ — 12

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. — 20 graf

3. Odrediti tok funkcije $f(x) = \dfrac{x^2+2}{x-8}$ i skicirati graf. — 20 graf 5

4. Zadana je funkcija $f(x) = \sqrt{8+8x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf. — 4+4+4+6

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Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1}$. — 10+2

Ukupno: 5

FILIP GORČEK

1. $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0 \;/ \cdot 2\sqrt{x^2+x}$

$4\sqrt{x^2+x} + 2x + 1 = 0 \;/^2$

$16(x^2+x) + 4x^2 + 2 \cdot 2x + 1 = 0$

$16x^2 + 16x + 4x^2 + 4x + 1 = 0$

$20x^2 + 20x + 1 = 0$

$x_{1/2} = \frac{-20 \pm \sqrt{20^2 - 4 \cdot 20 \cdot 1}}{40}$

$= \frac{-20 \pm 8\sqrt{5}}{40}$ $\quad x_1 = -0.05$ $\quad x_2 = -0.95$ ∅ PREVERI ?

4. $f(x) = \sqrt{8+8x}$ $\quad x_0 = 2$ $\quad f'(x) = \frac{1}{\sqrt{8+8x}}$ ×

$y - y_0 = f'(x_0) \cdot (x - x_0)$ $\quad y - y_0 = 0.20 \cdot (x-2)$

$y - y_0 = 0.20x - 0.40$

$f(x) = \sqrt{8+8x}$

$f'(x) = \frac{1}{2\sqrt{8+8x}}$ $\quad 2\sqrt{8+8x} = 2 \cdot (8+8x)^{1/2} = (16+16x)^{1/2}$

$f''(x) = \frac{-1 \cdot 1/2 \cdot (16+16x)^{-1/2} \cdot 16}{(2\sqrt{8+8x})^2} = \frac{-8 \cdot (16+16x)^{-1/2}}{(2\sqrt{8+8x})}$

$f'(x) = 0 \rightarrow \cancel{1 = 0}$ $\quad f''(x) = 0 \rightarrow -8 \cdot (16+16x)^{-1/2} = 0$

$-8 \cdot \frac{1}{\sqrt{16+16x}} =$

5.

$$\begin{array}{c} x\ \ y\ \ z\ \ w\ \ v \\ \left[\begin{array}{ccccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 1 & 1 & 0 & 4 & -5 \end{array}\right] \end{array} \cdot 1/2\ R1 \quad \left[\begin{array}{ccccc|c} \textcircled{1} & -3/2 & -1/2 & 1 & 3/2 & 2 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 1 & 1 & 0 & 4 & -5 \end{array}\right] \begin{array}{l} / \\ R2-4R1 \\ R3-2R1 \\ / \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -3/2 & -1/2 & 1 & 3/2 & 2 \\ 0 & 2 & 1 & 0 & 5 & -4 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 1 & 1 & 0 & 4 & -5 \end{array}\right] \cdot (1/2) \quad \left[\begin{array}{ccccc|c} 1 & -3/2 & -1/2 & 1 & 3/2 & 2 \\ 0 & \textcircled{1} & 1/2 & 0 & 5/2 & -2 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 1 & 1 & 0 & 4 & -5 \end{array}\right] \begin{array}{l} R1+3/2R2 \\ / \\ R3+2R2 \\ R4-R2 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1/4 & 1 & 21/4 & -1 \\ 0 & 1 & 1/2 & 0 & 5/2 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1/2 & 0 & 3/2 & -7 \end{array}\right] \quad \left[\begin{array}{ccccc|c} 1 & 0 & 1/4 & 1 & 21/4 & -1 \\ 0 & 1 & 1/2 & 0 & 5/2 & -2 \\ 0 & 0 & 1/2 & 0 & 3/2 & -7 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right] \cdot 2 \quad \left[\begin{array}{ccccc|c} 1 & 0 & 1/4 & 1 & 21/4 & -1 \\ 0 & 1 & 1/2 & 0 & 5/2 & -2 \\ 0 & 0 & \textcircled{1} & 0 & 3 & -14 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R1+1/4R3 \\ R2-1/2R3 \\ / \\ / \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 9/2 & 6/2 \\ 0 & 1 & 0 & 0 & 1 & 5 \\ 0 & 0 & 1 & 0 & 3 & -14 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

? RJEŠENJE ?

6. $\lim\limits_{x \to 1} \dfrac{\sqrt{2-x^2}-x}{x-1} = \dfrac{\sqrt{2-1}-1}{1-1} = \dfrac{1-1}{1-1} = 0$ ✗

FILIP GORSEK

3. $f(x) = \frac{x^2+2}{x-8}$

(1.) $D(f) = \mathbb{R} \setminus \{8\}$

$x - 8 \neq 0$

$x \neq 8$

(2.) V.A

$\lim_{x \to 8_-} \frac{x^2+2}{x-8} = \frac{66_-}{0_-} = -\infty$ V.A. = 8_

$\lim_{x \to 8_+} \frac{66_+}{0_+} = +\infty$ V.A. = 8

D.H.A.

$\lim_{x \to +\infty} \frac{x^2+2}{x-8} \overset{:x^2}{=} \left[\frac{\infty}{\infty}\right] = \frac{1+\frac{2}{x^2}}{\frac{x}{x^2} - \frac{8}{x^2}} = \frac{1}{0}$ NEMA D.H.A.

D.K.A.

$\lim_{x \to +\infty} \frac{\frac{x^2+2}{x-8}}{\frac{x}{1}} = \lim_{x \to +\infty} \frac{x^2+2}{x^2-8x} \overset{:x^2}{=} \left[\frac{\infty}{\infty}\right] = \frac{1+\frac{2}{x^2}}{1-\frac{8x}{x^2}} = \frac{1}{1} = a = 1$ ($\frac{2}{x^2} \to 0$, $\frac{8x}{x^2} \to 0$)

$\lim_{x \to +\infty} \frac{x^2+2}{x-8} - x = \lim_{x \to +\infty} \frac{x^2+2-x(x-8)}{x-8} = \lim_{x \to +\infty} \frac{x^2+2-x^2+8x}{x-8} \overset{:x}{=} \left[\frac{\infty}{\infty}\right] = \frac{\frac{2}{x}+8}{1-\frac{8}{x}} = 8 = b$

D.K.A $y = x + 8$

L.H.A. $\lim_{x \to -\infty} \frac{x^2+2}{x-8} = \left[\begin{matrix} x \to -x \\ -\infty \to +\infty \end{matrix}\right] = \lim_{x \to +\infty} \frac{x^2+2}{-x-8} \overset{:x^2}{=} \left[\frac{\infty}{\infty}\right] = \frac{1+\frac{2}{x^2}}{\frac{-x}{x^2} - \frac{8}{x^2}} = \frac{1}{0}$

L.K.A. $\lim_{x \to -\infty} \frac{\frac{x^2+2}{x-8}}{x} = \left[\begin{matrix} x \to -x \\ -\infty \to +\infty \end{matrix}\right] = \lim_{x \to +\infty} \frac{\frac{x^2+2}{-x-8}}{\frac{-x}{1}} = \lim_{x \to +\infty} \frac{x^2+2}{x^2+8x} \overset{:x^2}{=} \lim_{x \to +\infty} \frac{1+\frac{2}{x^2}}{1+\frac{8x}{x^2}} = a = 1$

$\lim_{x \to -\infty} \frac{x^2+2}{x-8} - x = \left[\begin{matrix} x \to -x \\ -\infty \to +\infty \end{matrix}\right] = \lim_{x \to +\infty} \frac{x^2+2}{-x-8} + x = \lim_{x \to +\infty} \frac{x^2+2+x(-x-8)}{-x-8} = \lim_{x \to +\infty} \frac{x^2+2-x^2-8x}{-x-8}$

$\lim_{x \to +\infty} \frac{2-8x}{-x-8} \overset{:x}{=} \left[\frac{\infty}{\infty}\right] = \frac{\frac{2}{x}-8}{-1-\frac{8}{x}} = \frac{0-8}{-1-0} = 8$ b = 8

GRAF?

K.A. $y = x + 8$

$f(x)=0 \Rightarrow x^2+2=0$

$x^2=-2 \quad /\sqrt{}$

$x=\sqrt{-2}$

$\downarrow$

NEMA NT

$f(0)=\frac{0^2+2}{0-8}=-\frac{1}{4}$ $\quad S(0,-\frac{1}{4})$

$$f'(x)=\frac{2x\cdot(x-8)-(x^2+2)\cdot 1}{(x-8)^2}=\frac{2x^2-16x-x^2-2}{(x-8)^2}=\frac{x^2-16x-2}{(x-8)^2}$$

$$f''(x)=\frac{(2x-16)\cdot(x-8)^2-(x^2-16x-2)\cdot 2(x-8)\cdot 1}{(x-8)^4}=\frac{(x-8)[2x^2-16x-16x+128-2x^2+32x+4]}{(x-8)^{*3}}$$

$$f''(x)=\frac{132}{(x-8)^3}$$

$f'(x)=0 \rightarrow x^2-16x-2=0$

$x_{1/2}=\frac{16\pm\sqrt{(-16)^2-4\cdot 1\cdot(-2)}}{2}$

$x_{1/2}=\frac{16\pm\sqrt{264+0}}{2}$

$x_1=16.12$ > STAC. TOČKE

$x_2=-0.12$

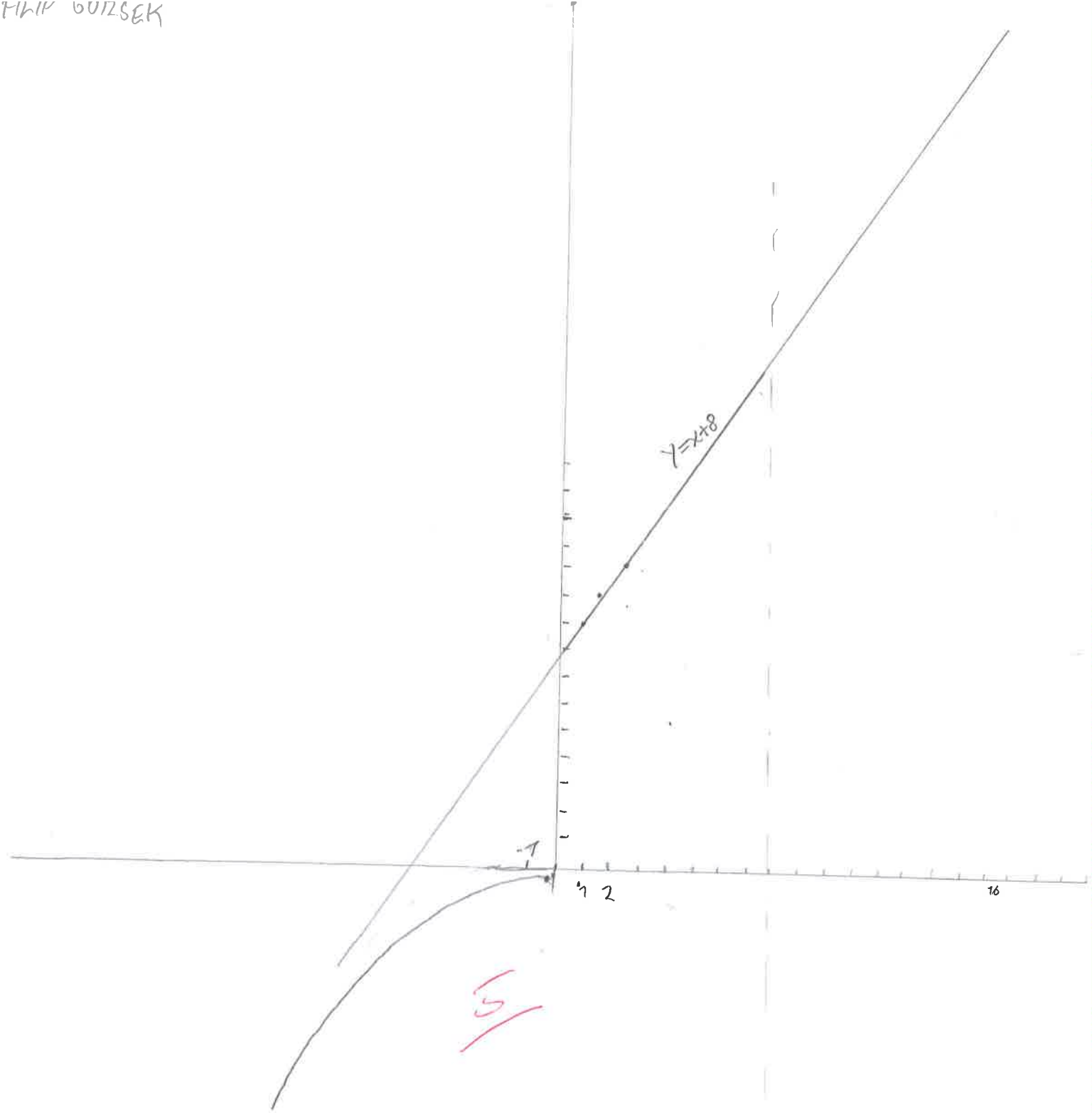
$f''(x)=0 \rightarrow 132\neq 0$ NEMA TOČKE INFLEKSIJE

$f(16.12)=32.26$

$f(-0.12)=-0.26$

	$-\infty$	-1	-0.26	0	16.12	$+\infty$
$f'(x)$		+		−		+
$f''(x)$		−		−		+
$f(x)$		⌒		⌣		⌒

FILIP GUZSEK



5

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 15

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: MATEO PEDIŠIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17 - 1 - 0306 - 2014

1. Riješiti jednadžbu: $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$ — 12
2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. — 20 graf
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6. Ispitati i na neki način provjeriti $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1}$. — 10+2

Ukupno:

3) $f(x) = \dfrac{x^2+2}{x-8}$

$x - 8 \neq 0$

$x = 8$

$x^2 + 2 = 0$

$x^2 = -2$

$x = \sqrt{-2}$

NEMA NUL TOČAKA

$D_f = \langle -\infty, 8 \rangle \cup \langle 8, +\infty \rangle$

$$f'(x) = \frac{(x^2+2)' \cdot (x-8) - (x^2+2) \cdot (x-8)'}{(x-8)^2}$$

$$f'(x) = \frac{2x \cdot (x-8) - (x^2+2) \cdot 1}{x^2 - 16x + 64}$$

$$f'(x) = \frac{2x^2 - 16x - x^2 - 2}{x^2 - 16x + 64}$$

$$f'(x) = \frac{x^2 - 16x - 2}{x^2 - 16x + 64}$$

$$f'(x) = \frac{(x^2-16x-2)' \cdot (x^2-16x+64) - (x^2-16x-2)\cdot(x^2-16x+64)'}{(x^2-16x+64)^2}$$

$$f''(x) = \frac{2x-16\cdot(x^2-16x+64) - (x^2-16x-2)\cdot(2x-16)}{(x^2-16x+64)^2}$$

$$f''(x) = \frac{2x^3-16x^2-32x^2+256x+128x-1024-(2x^3-16x^2-32x^2+256x-4x+32)}{(x^2-16x+64)^2}$$

$$f''(x) = \frac{2x^3-16x^2-32x^2+256x+128x-1024-2x^3+16x^2+32x^2-256x+4x-32}{(x^2-16x+64)^2}$$

$$f'(x) = \frac{132x-1056}{(x^2-16x+64)^2}$$

$132x - 1056 = 0$

$132x = 1056 \quad /:132$

$x = 8$

$$\lim_{x\to\infty} \frac{x^2+2}{x-8} \;/:x^2 = \frac{1+\frac{2}{x^2}}{\frac{x}{x^2}-\frac{8}{x^2}} = \frac{1+0}{0-0} = \frac{1}{0} = +\infty$$

NEMA H. A

FUNKCIJA RASTA

$$\lim_{x\to 8} \frac{x^2+2}{x-8} = \frac{8^2+2}{8-8} = \frac{66}{0} = +\infty$$

V. A JE 8

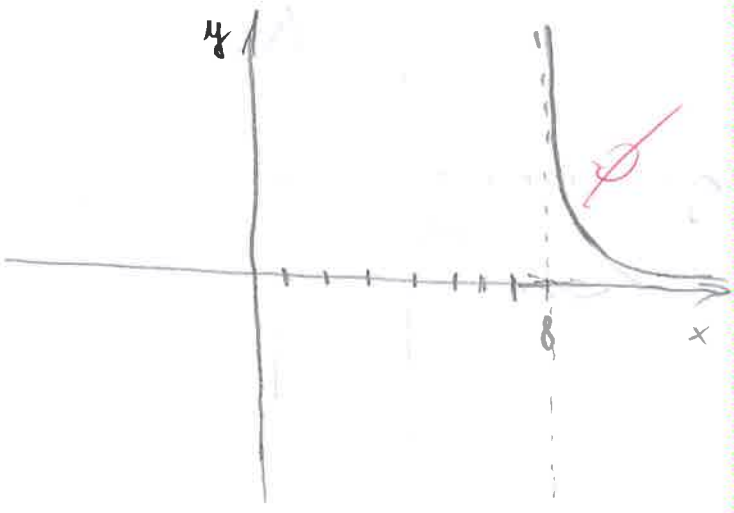
$x^2-16x-2=0$

$x_{12} = \frac{-b\cdot\sqrt{4ac}}{2a}$

$x_{12} = \frac{16\cdot\sqrt{4\cdot1\cdot-2}}{2\cdot1}$

$x_{12} = \frac{16\cdot\sqrt{-6}}{2}$

NEMA EKSTREMA



MATEO PEDIŠIĆ
17-1-0306-2014

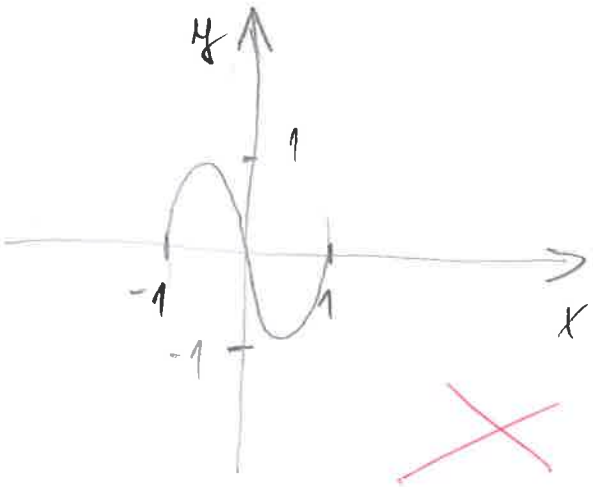
6.) $\lim_{x \to 1} \frac{\sqrt{2-x^2} - x}{x - 1} = \frac{\sqrt{2-1^2} - 1}{1 - 1} = \frac{0}{0}$

$\lim_{x \to 0{,}99999} \frac{\sqrt{2-(0{,}99999)^2} - 0{,}99999}{0{,}99999 - 1} = \frac{0{,}00002}{-0{,}00001} = -2$

$\lim_{x \to 1{,}00001} \frac{\sqrt{2-(1{,}00001)^2} - 1{,}00001}{1{,}00001 - 1} = \frac{-0{,}00002}{0{,}00001} = -2$

$\lim_{x \to 1} \frac{\sqrt{2-x^2} - x}{x-1} = -2$

2.) $g(x) = \arctan(e^x)$



$-1 \leq e^x \leq 1$

$-1 \leq e^x$ $\quad e^x \leq 1$

$-1 \leq e^x \,/\ln$ $\quad e^x \leq 1 \,/\ln$

$-1 \leq x$ $\quad x \leq 1$

5.)

$$\left[\begin{array}{ccccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] /:2 \left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \begin{matrix} \\ (-4) \\ (-2) \\ \\ \end{matrix}$$

$$\left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 2 & 1 & 0 & 5 & 4 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] :2 \left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 2 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \begin{matrix} \\ \left(\frac{3}{2}\right) \\ (2) \\ (-2) \end{matrix}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & 5 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 2 \\ 0 & 0 & 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 & -1 & -9 \end{array}\right]$$

OVA MATRICA NEMA RJEŠENJA.

$\emptyset$

1.)

$$\frac{2x+1}{2\sqrt{x^2+x}} \cdot \frac{2\cdot(2x+1)}{2\sqrt{x^2+x}} = \frac{(2x+1)\cdot(4x+2)}{2x^2+x} = \frac{8x+4x+4x+2}{2x^2+x} = \frac{16x+2}{2x^2+x}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 15

IME I PREZIME: JURE ŠUŠIĆ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-1-0259-2014

0269087848

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$ 12

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf

3. Odrediti tok funkcije $f(x) = \dfrac{x^2+2}{x-8}$ i skicirati graf. 20 graf 10

4. Zadana je funkcija $f(x) = \sqrt{8+8x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf. 4+4+4+6

5. Gaussovom metodom riješiti sustav: 15+3

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1}$. 10+2

Ukupno: 10

6.) $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x \,/:x^2}{x-1 \,/:x^2} = \lim\limits_{x\to 1} \dfrac{\sqrt{\frac{2}{x^2}-1} - \frac{x}{x^2}}{\frac{1}{x^2} - \frac{1}{x^2}}$ (2^0, 1^0; 0, 0)

$= \lim\limits_{x\to 1} \dfrac{\sqrt{-1}}{0} = \dfrac{1}{0} = \pm\infty$ ×

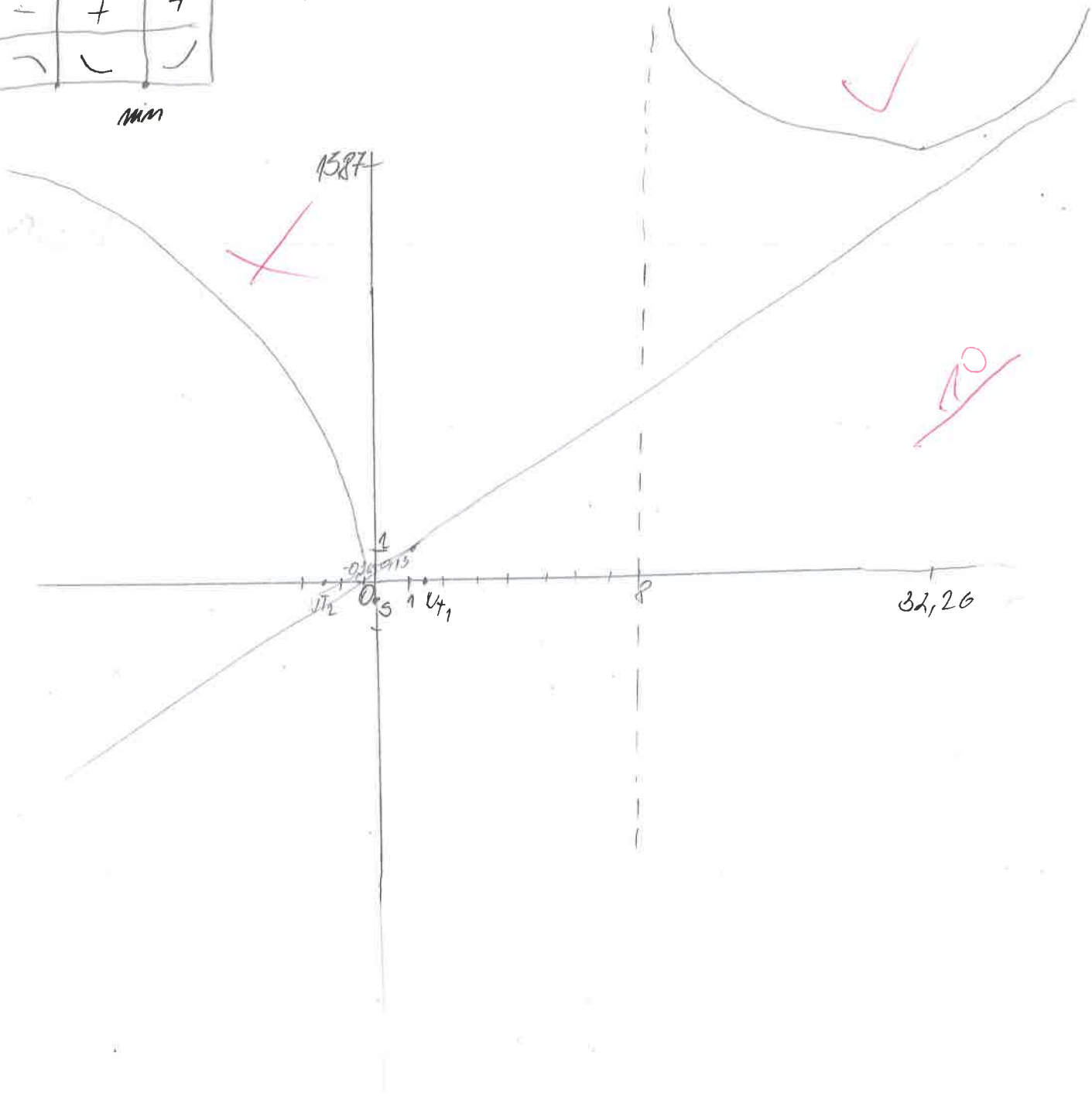
$\lim\limits_{x\to 1} \dfrac{\sqrt{2-1^2}-1}{1-1} = \lim\limits_{x\to 1} \dfrac{\sqrt{1}-1}{0} = \lim\limits_{x\to 1} = \dfrac{0}{0} = 0$

3.) VASTAVIK

	$-\infty$	0,13	15,87	∞
$f'(x)$	−	−	+	
$f''(x)$	−	+	+	
$f(x)$	⌒	⌣	⌣	
			min	

$f(0,13) = -0,26 \Rightarrow (-0,26; 0,13)$

$f(15,87) = 32,26 \Rightarrow \min (32,26; 15,87)$



KA...

$y = kx + l$

$k = \lim_{x \to \infty} \left(\frac{\frac{x^2+2}{x-8}}{x} \right)$

$\frac{x^2+2 \,/: x^2}{x^2-8x \,/: x^2} = \frac{1+0}{1-0} = 1$

$l = \lim_{x \to \infty} \frac{x^2+2 \,/: x^2}{x-8 \,/: x^2} - 1 = \frac{1+0}{0} - 1$

3.) $f(x) = \frac{x^2+2}{x-8}$

1° $x-8 \neq 0$

$x \neq 8 \quad D(f) = \mathbb{R} \setminus \{8\}$

2°

$VA = \lim_{x \to 8^-} \lim_{x \to 8} \frac{8^2+2}{8-8} = \lim_{x \to 8} \frac{66}{0^-} = -\infty$

OVA ... $x = 8$

$VA = \lim_{x \to 8^+} \lim_{x \to 8} \frac{8^2+2}{8^+-8} = \lim_{x \to 8} \frac{66}{0^+} = +\infty$

HA

$\lim_{x \to \pm\infty} \frac{x^2+2 \,/:x^2}{x-8 \,/:x^2} = \lim_{x \to \pm\infty} \frac{1+0}{0-0} = \frac{1}{0} = \infty$

KA

$y = kx + l \quad k = \lim_{x \to \infty} \frac{f(x)}{x} \quad l = \lim_{x \to \infty} (f(x) - kx)$

3° $f(x) = 0 \quad x^2+2 = 0$

$x^2 = -2 \,/\sqrt{}$ $\quad N_{T_1}(\sqrt{2}, 0)$

$x = \sqrt{-2}$ $\quad N_{T_2}(-\sqrt{2}, 0)$

$f(0) = 0 \quad \frac{0^2+2}{0-8} = \frac{2}{-8} = -\frac{1}{4} = -0{,}25 \quad S(0, -0{,}25)$

4° $f(-x) = \frac{-x^2+2}{-x-8} \Rightarrow$ NI PARNA NI NEPARNA, NIDE PERIODIČNA, NIDE OMEJENA

5° $f'(x) = \frac{2x \cdot (x-8) - (x^2+2) \cdot 1}{(x-8)^2} = \frac{2x^2 - 16x - x^2 - 2}{(x-8)^2} = \frac{x^2 - 16x - 2}{(x-8)^2}$

$f''(x) = \frac{2x - 16 \cdot (x-8)^2 - (x^2 - 16x - 2) \cdot 2 \cdot 1}{(x-8)^4} = \frac{(x-8) \cdot [2x - 16 \cdot (x-8) - (x^2 - 16x - 2) \cdot 2]}{(x-8)[(x-8)^3]}$

$= \frac{2x^2 - 16x - 16x + 128 - 2x^2 + 32x + 4}{(x-8)^3} = \frac{132}{(x-8)^3}$

6° $f'(x) = 0 \quad x^2 - 16x - 2 = 0$

$x_{1,2} = \frac{16 \pm \sqrt{256 - 8}}{2}$

$x_1 = 15{,}87$

$x_2 = 0{,}13$ → STACIONARNE TOČKE

$f''(x) = 0 \quad 132 = 0$

JORE ŠUŠIĆ

5.)

$$\begin{array}{ccccc|c} x & y & z & w & v & \\ 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \cdot \frac{1}{2}$$

$$\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \cdot(-4) \cdot(-2)$$

$$\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 2 & 1 & 0 & 5 & -4 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \cdot \frac{1}{2}$$

$$\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \cdot \frac{3}{2} \cdot 2 \cdot(-2)$$

$$\begin{array}{ccccc|c} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{array}$$

MATRICA IMA BESKONAČNO RJEŠENJA ZAŠTO?

KOJA?

PROVJERA!

9

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 15

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: Josip Mihoć **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0026908763

1. Riješiti jednadžbu: $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$ — 12
2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. — 20 graf
3. Odrediti tok funkcije $f(x) = \dfrac{x^2+2}{x-8}$ i skicirati graf. — 20 graf
4. Zadana je funkcija $f(x) = \sqrt{8+8x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf. — 4+4+4+6
5. Gaussovom metodom riješiti sustav: — 15+3

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1}$. — 10+2

Ukupno: 0

4.) $f(x) = \sqrt{8+8x}$

1. DOMENA

$8+8x \geq 0$

$8+8x = 0$

$x_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 8 \cdot 8}}{0}$

$x_{1,2} = \frac{-8}{0}$ ✗

EKSTREMI

$f(x) = \sqrt{8+8x}$

$f'(x) = \frac{1}{2\sqrt{8+8x}} \cdot (8+8x)'$

$f'(x) = \frac{8}{2\sqrt{8+8x}} = \frac{2 \cdot (2,2)}{2\sqrt{8+8x}} = \frac{4}{\sqrt{8+8x}} \cdot \frac{\sqrt{8+8x}}{\sqrt{8+8x}} = \frac{4 \cdot \sqrt{8+8x}}{8+8x}$ //

$f'(x) = 0$

$\frac{4 \cdot \sqrt{8+8x}}{8+8x} = 0 \quad / : 8+8x$

$4 \cdot \sqrt{8+8x} = 0$

$4 \neq 0$

$\sqrt{8+8x} = 0 \quad /^2$

$8+8x = 0$

$8x = -8 \quad / :8$

$x = -1$

H.A

$\lim_{x \to \infty} \frac{\sqrt{8+8x}}{1} \cdot \frac{\sqrt{8+8x}}{\sqrt{8+8x}} = \frac{\sqrt{8+8x}}{\sqrt{8+8x}} = \frac{8+8x \quad /:x}{\sqrt{8+8x} \quad /:x} = \frac{8}{0}$ //

V.A

$\lim_{x \to -1} = \sqrt{8+8 \cdot -1} = \sqrt{8-8} = 0$

$k = \lim_{x \to \infty} \frac{\sqrt{8+8x}}{x} \cdot \frac{\sqrt{8+8x}}{\sqrt{8+8x}} = \frac{8+8x \quad /:x}{x \cdot \sqrt{8+8x} \quad /:x} = \frac{0+8}{1 \cdot \sqrt{\infty}} = \frac{8}{\infty}$ nema

D.KA

L.KA

$k = \lim_{x \to -\infty} \frac{\sqrt{8+8x}}{x} = \frac{8+8x}{x \cdot \sqrt{8+8x}} = \frac{-8}{-1 \cdot \sqrt{0}} = \frac{-8}{0}$ nema

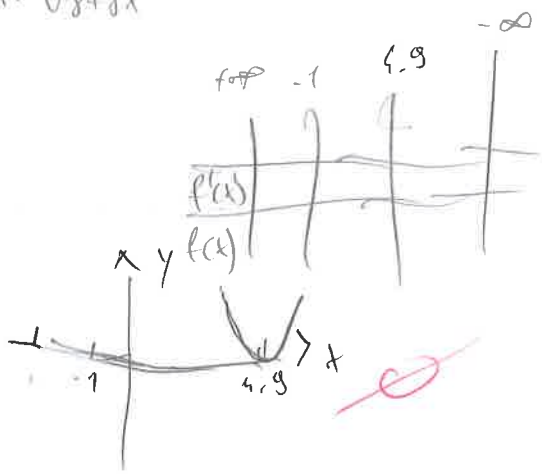
tangenta

$f(2) = \sqrt{8+8 \cdot 2}$

$f(2) = \sqrt{8+16}$

$f(2) = \sqrt{24}$

$f(2) = 4.8$ //



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6 $\lim_{x \to 1} \frac{\sqrt{2-2x^2} - x}{x-1} = \frac{0}{0} \cdot \frac{\sqrt{2-2x^2} - x}{x+1}$

JOSIP MIHOĆ

$2x - 3y - z + 2w + 3u = 4$
$4x - 4y - z + 4w + 11u = 4$
$2x - 5y - 2z + 2w - u = 9$
$2y + z + 4u = -5$

$$\sim \left[\begin{array}{ccccc|c} 2 & -3 & -1 & +2 & +3 & 4 \\ 4 & -4 & -1 & +4 & +11 & 4 \\ 2 & -5 & -2 & +2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right]$$

$1R : 2$

$$\sim \left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right]$$

$1R \cdot (-4) + 2R$
$1R \cdot (-2) + 3R$

$$\sim \left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 2 & 1 & 0 & 5 & -4 \\ 0 & -2 & -1 & 0 & -4 & 17 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \sim$$

$2R : 2$

$$\sim \left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & -2 & -1 & 0 & -4 & 17 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{4} & 1 & -1 \\ 0 & 1 & \frac{1}{2} & 0 & -2 \\ 0 & 0 & 0 & 0 & 13 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

$2R \cdot \frac{3}{2} + 1R$
$2R \cdot 2 + 3R$
$2R \cdot (-2) + 4R$

6

---

1. Rješiti jednadžbu $2x + \frac{2x+1}{2\sqrt{x^2+x}} = 0 \;/ \cdot 2\sqrt{x^2+x}$

$4\sqrt{x^2+x} + 2x + 1 = 0$

$4\sqrt{x^2+x} = -2x - 1 \;/^2$

$4^2\sqrt{x^2+x}^2 = (-2x-1)^2$

$16 \cdot (x^2+x) = (-2x-1)^2$

$16 \cdot (x^2+x) = (-2x)^2 + 2 \cdot (-2x) \cdot 1 + 1^2$

$16 \cdot (x^2+x) = 4x^2 - 4x + 1$

$16x^2 + 16x = 4x^2 - 4x + 1$

$16x^2 - 4x^2 + 16x + 4x - 1 = 0$

$12x^2 + 20x - 1 = 0$

PROVJERA ?

$x_1 = 0.048$
$x_2 = -1.71$

0

JOSIP MATIĆ

$f(x) = \frac{x^2+2}{x-8}$

Df $R \setminus \{8\}$

$y = kx + l$

$l = f(t) - kx$

1. DOMENA

$x - 8 \neq 0$

$x \neq 8$

2.

V.A

$\lim_{x \to 8} \frac{8^2+2}{8-8} = \frac{66}{0} = \checkmark$

V.A = 8 $\quad x = 8$

H.A

$\lim_{x \to \infty} \frac{x^2+2 \,/ :x^2}{x-8 \,/ :x^2} = \frac{1+0}{0+0} = \frac{1}{0} = +\infty$

nema H.A

$\lim_{x \to -\infty} \frac{x^2+2 \,/ :x^2}{x-8 \,/ :x^2} = \frac{-1+0}{0-0} = -\frac{1}{0} = -\infty$

DKA.

$k = \lim_{x \to \infty} \frac{\frac{x^2+2}{x-8}}{\frac{x}{1}} = \frac{x^2+2 \,/ :x^2}{x^2-8x \,/ :x^2} = \frac{1+0}{1-0} = 1$ $\quad k = 1$ //

$l = \lim_{x \to \infty} \frac{x^2+2}{x-8} \cdot \frac{(x^2-2)-1x}{(x+8)-1x} = \frac{(x^2-2)^2-1x}{x-8-1x} = \frac{(x^2)^2 - 2 \cdot x^2 \cdot (-2) + (-2)^2 - 1x}{x-8-1x} = \frac{x^4 - 4x^2 + 4 - 1x}{}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 15

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: MARIO MIKOŠ **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0170-2013

1. Riješiti jednadžbu: $2 + \dfrac{2x+1}{2\sqrt{x^2+x}} = 0$ 12

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf

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Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim\limits_{x\to 1} \dfrac{\sqrt{2-x^2}-x}{x-1}$. 10+2

Ukupno: 0

(5)

$$\left[\begin{array}{cccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \Big/ \cdot\tfrac{1}{2} \sim \left[\begin{array}{cccc|c} 1 & -\tfrac{3}{2} & -\tfrac{1}{2} & 1 & -\tfrac{3}{2} & 2 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \Big/ \cdot(-4) \Big/ \cdot(-2)$$

$$2 \left[\begin{array}{cccc|c} 1 & -\tfrac{3}{2} & -\tfrac{1}{2} & 1 & -\tfrac{3}{2} & 2 \\ 0 & 2 & 1 & 0 & 17 & -4 \\ 0 & -2 & -1 & 0 & 2 & 15 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \Big/ \cdot\tfrac{1}{2} \sim \left[\begin{array}{cccc|c} 1 & -\tfrac{3}{2} & -\tfrac{1}{2} & 1 & -\tfrac{3}{2} & 2 \\ 0 & 1 & \tfrac{1}{2} & 0 & \tfrac{17}{2} & -2 \\ 0 & -2 & -1 & 0 & 2 & 15 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array}\right] \Big/ \cdot\tfrac{3}{2} \Big/ \cdot 2 \Big/ \cdot(-2)$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & \tfrac{1}{4} & 1 & \tfrac{45}{4} & -1 \\ 0 & 1 & \tfrac{1}{2} & 0 & \tfrac{17}{2} & -2 \\ 0 & 0 & 0 & 0 & 19 & 1 \\ 0 & 0 & 0 & 0 & -13 & 9 \end{array}\right] \Big/ \cdot\tfrac{1}{19}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & \tfrac{1}{4} & 1 & \tfrac{45}{4} & -1 \\ 0 & 1 & \tfrac{1}{2} & 0 & \tfrac{17}{2} & -2 \\ 0 & 0 & 0 & 0 & 1 & \tfrac{1}{19} \\ 0 & 0 & 0 & 0 & 1 & -\tfrac{9}{13} \end{array}\right] \Big/ \cdot\left(-\tfrac{45}{4}\right) \Big/ \cdot\left(-\tfrac{17}{2}\right) \Big/ \cdot(-13)$$

0