

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: ANORIJA VAMBER

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0448-1014

- Riješiti jednadžbu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$ 12
- Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. ~~20 graf~~ 15
- Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf. ~~20 graf~~ 6
- Zadana je funkcija $f(x) = \sqrt{7+5x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf. ~~4+4+4+6~~
- Gaussovom metodom riješiti sustav: ~~15+3~~

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

Provjeri uvrštavanjem!

- Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$. ~~10+2~~

Ukupno:

35

5.
$$\left[\begin{array}{cccc|ccc} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{array} \right] \begin{array}{l} | :2 \\ \\ \\ \sim \end{array}$$

~~0 0 2 1 4 -5~~

$2R = 2I$

$$\left[\begin{array}{cccc|ccc} 0 & 0 & 2 & 1 & 4 & -5 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 4 & -5 \end{array} \right]$$

$$\left[\begin{array}{cccc|ccc} 1 & \frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{array} \right] \sim \left[\begin{array}{cccc|ccc} 1 & \frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & -10 & -3 & 0 & 5 & -4 \\ 0 & -8 & -1 & 0 & -4 & 5 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{array} \right] | :(-1)$$

$1R \cdot (-4) + 2R$

$1R \cdot (-2) + 3R$

$$\left[\begin{array}{cccc|ccc} 1 & \frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & -10 & -3 & 0 & 5 & -4 \\ 0 & 8 & 1 & 0 & 4 & -5 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{array} \right]$$

$3R \cdot \frac{1}{2} + 1R \quad 3R \cdot 3 + 2R \quad 2R \cdot (-2) + 4R$

$$\sim \left[\begin{array}{ccccc|c} 1 & \frac{11}{2} & 0 & 1 & \frac{7}{2} & -\frac{1}{2} \\ 0 & 14 & 0 & 0 & 17 & -19 \\ 0 & 8 & 1 & 0 & 4 & -5 \\ 0 & -16 & 0 & 0 & 4 & 5 \end{array} \right] \quad | : 14$$

$$\sim \left[\begin{array}{ccccc|c} 1 & \frac{11}{2} & 0 & 1 & \frac{7}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & \frac{17}{14} & -\frac{19}{14} \\ 0 & 8 & 1 & 0 & 4 & -5 \\ 0 & -16 & 0 & 0 & 4 & 5 \end{array} \right]$$

$2R \cdot (-\frac{11}{2}) + 1R \quad 2R \cdot (-8) + 3R \quad 2R \cdot 14 + 4R$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & \frac{91}{28} & \frac{5}{28} \\ 0 & 1 & 0 & 0 & \frac{17}{14} & -\frac{19}{14} \\ 0 & 0 & 1 & 0 & -\frac{40}{7} & \frac{41}{7} \\ 0 & 0 & 0 & 0 & 21 & -14 \end{array} \right] \quad | : 21$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & \frac{81}{28} & \frac{5}{28} \\ 0 & 1 & 0 & 0 & \frac{17}{14} & -\frac{19}{14} \\ 0 & 0 & 1 & 0 & -\frac{40}{7} & \frac{41}{7} \\ 0 & 0 & 0 & 0 & 1 & -\frac{14}{21} \end{array} \right]$$

$$4R \cdot (-\frac{81}{28}) + 1R$$

$$4R \cdot (-\frac{17}{14}) + 2R$$

$$4R \cdot \frac{40}{7} + 3R$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 0 & \frac{59}{28} \\ 0 & 1 & 0 & 0 & 0 & -\frac{331}{84} \\ 0 & 0 & 1 & 0 & 0 & \frac{43}{21} \\ 0 & 0 & 0 & 0 & 1 & -\frac{14}{21} \end{array} \right]$$

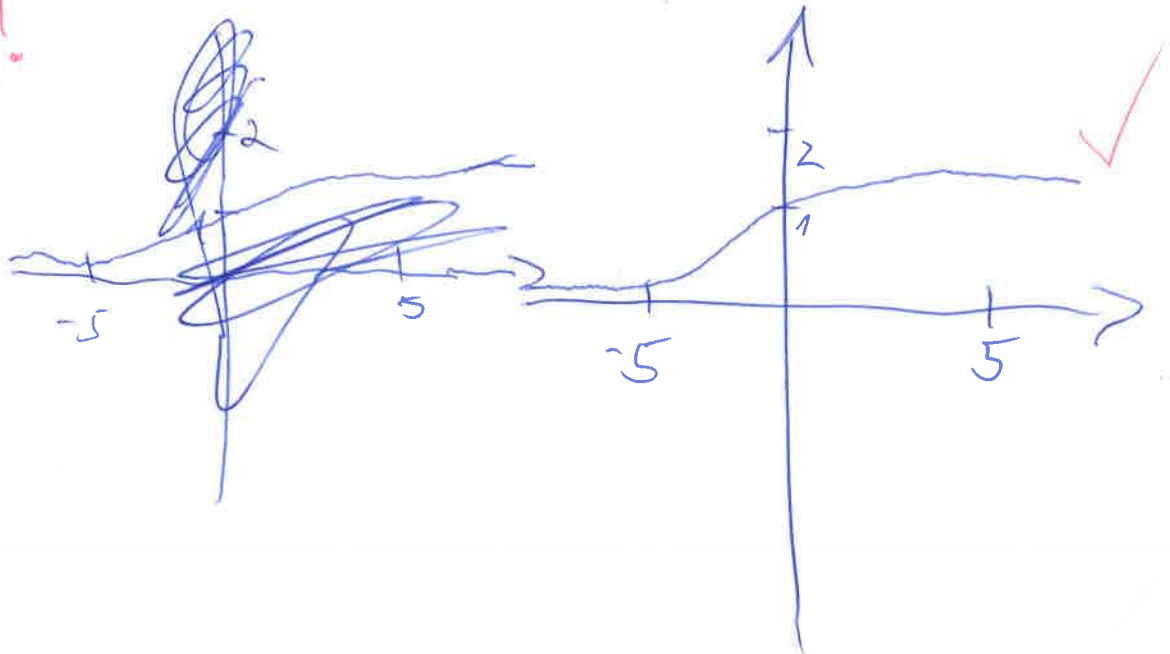
$$2y + z$$

KOJA SU
RJEŠENJA 0

2. $\lim_{x \rightarrow +\infty} \arctg(e^x) = \arctg(\infty) = \frac{\pi}{2} \quad \checkmark$

DERIVACIJE!

$\lim_{x \rightarrow -\infty} \arctg(e^x) = \arctg(\frac{1}{\infty}) = 0 \quad \checkmark \quad \underline{15}$



~~get distance~~ $y = \sqrt{7+5x}$

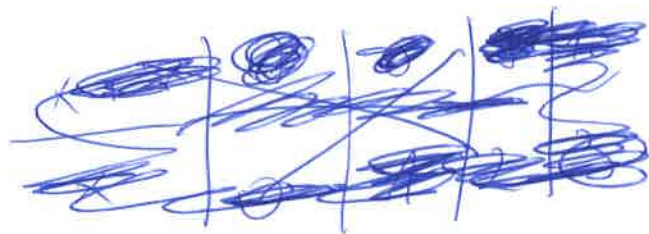


$$4. f(x) = \sqrt{7+5x}$$

$$f'(x) = \frac{1}{2\sqrt{7+5x}} \cdot (7+5x)'$$

$$f'(x) = \frac{1}{2\sqrt{7+5x}} \cdot 5$$

$$f'(x) = \frac{5}{2\sqrt{7+5x}}$$



Matematika 1

Ime i prezime: ANORIJA KUMBER

Matični broj u indeksu: 17-02-0448-2014

$$6. \frac{\sqrt{2-01^2} - 1}{1-1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} L'H$$

~~$$\frac{\sqrt{2-x^2}-x}{x-1} \cdot \frac{x}{x} = \frac{\sqrt{\frac{2-x^2}{x^2}} - \frac{x}{x}}{\frac{x}{x} - \frac{1}{x}} = \frac{\sqrt{\frac{2}{x^2} - 1} - 1}{1 - \frac{1}{x}}$$~~

$$3. f(x) = \frac{x^2+8}{x-7}$$

DOMENA $x-7 \neq 0$ $D_f: \mathbb{R} \setminus \{7\}$

~~$x=7$~~ NT: $x^2+8=0$

V.A $\lim_{x \rightarrow 7^+} \frac{x^2+8}{x-7} = +\infty$

$x \cdot (x+8) = 0$

$x_1 = 0$

$\lim_{x \rightarrow 7^-} \frac{x^2+8}{x-7} = -\infty$

$x_2 = -8$

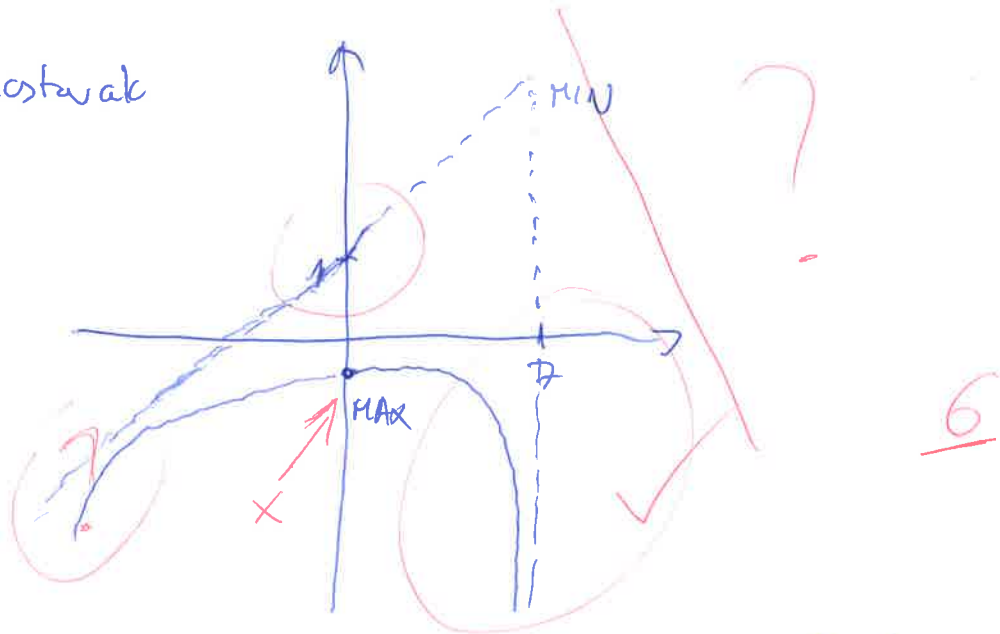
~~$$\frac{x^2+8}{x-7} = \frac{x^2-7x+7x+8}{x-7} = \frac{x^2-7x}{x-7} + \frac{7x+8}{x-7}$$~~

H.A. $\lim_{x \rightarrow +\infty} \frac{x^2+8}{x-7} = \frac{x^2+8}{x-7} = \frac{\frac{x^2}{x^2} + \frac{8}{x^2}}{\frac{x}{x^2} - \frac{7}{x^2}} = \frac{1+0}{0-0} = \frac{1}{0} = \infty$

$\lim_{x \rightarrow -\infty} \frac{x^2+8}{x-7} = x \quad (x \rightarrow -x, -\infty \rightarrow +\infty)$

~~$$\frac{x^2+8}{x-7} = \frac{x^2+8-x(x-7)}{x-7} = \frac{x^2+8-x^2+7x}{x-7} = \frac{7x+8}{x-7}$$~~

3. zadatok mostarac



$x+7$	7	8	9	10
x	0	1	2	3

$$f'(x) = \frac{x^2+8}{x^2-7}$$

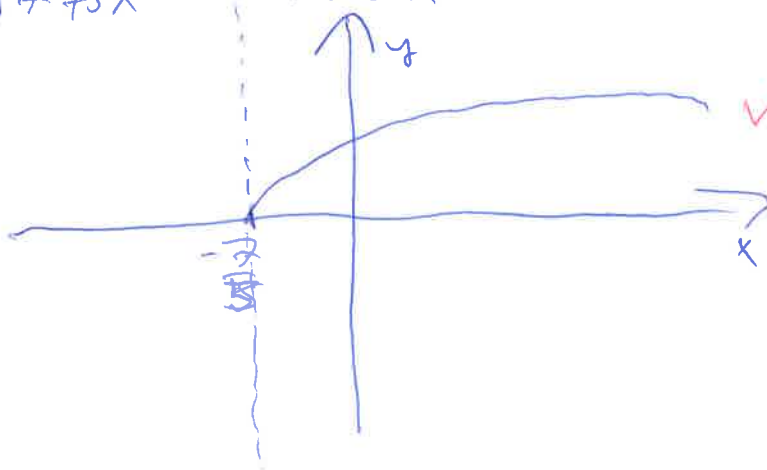
$$f'(x) = \frac{(x^2+8)' \cdot (x^2-7) - (x^2+8) \cdot (x^2-7)'}{(x^2-7)^2}$$

$$f'(x) = \frac{2x \cdot (x^2-7) - (x^2+8) \cdot 2x}{(x^2-7)^2} = \frac{-7+8}{(x^2-7)^2} = \frac{1}{(x^2-7)^2}$$

4. DOMENA $7+5x \geq 0$
 $x \geq -7,5$

NEGMA EKSTREMA X

$$\lim_{x \rightarrow +\infty} \sqrt{7+5x} \cdot \frac{\sqrt{7+5x}}{\sqrt{7+5x}} = \frac{7+5x}{\sqrt{7+5x}} = +\infty$$



$$6. \frac{\sqrt{2-x^2} - x}{x-1}$$

$$x \rightarrow 1 \quad x-1$$

$$\frac{2-x^2-x^2}{(x-1) \cdot (\sqrt{2-x^2} + x)}$$

$$(x-1) \cdot (\sqrt{2-x^2} + x)$$

$$= \frac{2-2x^2}{(x-1)(\sqrt{2-x^2} + x)}$$

$$= \frac{2 \cdot (1-x^2)}{(x-1) \cdot (\sqrt{2-x^2} + x)}$$

$$= \frac{2 \cdot (x-1)(x+1)}{(x-1)(\sqrt{2-x^2} + x)}$$

$$= \frac{2 \cdot \cancel{(x-1)} (x+1)}{\cancel{(x-1)} (\sqrt{2-x^2} + x)}$$

$$= \frac{-2(1+1)}{(\sqrt{2-1} + 1)}$$

$$= \frac{-2-2}{1+1} = \frac{-4}{2} = -2$$

$$= -2$$

$$= \frac{-2-2}{1+1} = \frac{-4}{2} = -2$$

PROVJERA...

VIDI FRIZOP

3. zadatak nastavak

$$\frac{x^2+8-x^2+x}{x-7} : x$$

$$\frac{x+8}{x-7} = 7$$

$$y = kx + l$$

$$y = x + 7$$

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

ε1

NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **JURE FRIO**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0264-2013

1. Riješiti jednačbu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

12

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije.

20 graf

3. Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf.

20 graf

18

4. Zadana je funkcija $f(x) = \sqrt{7+5x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x=2$ i skicirati je uz graf.

4+4+4+6

5. Gaussovom metodom riješiti sustav:

15+3

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

Ukupno:

30

~~1.1)~~

1.) $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

$\frac{2x+1}{2\sqrt{x^2+x}} = -2 \quad | \cdot 2\sqrt{x^2+x}$

$2x+1 = -2\sqrt{x^2+x} \quad |^2$
 $(2x+1)^2 = -16(x^2+x)$

$4x^2+4x+1 = -16x^2-16x$

$4x^2+16x^2+4x+16x+1=0$

$20x^2+20x+1=0 \quad | :$

$\Delta = \sqrt{b^2-4ac}$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
 $x_{1,2} = \frac{-20 \pm \sqrt{20^2-4 \cdot 20 \cdot 1}}{2 \cdot 20}$

$x_{1,2} = \frac{-10 \pm \sqrt{400-80}}{40}$

$x_{1,2} = \frac{-20 \pm \sqrt{320}}{40}$

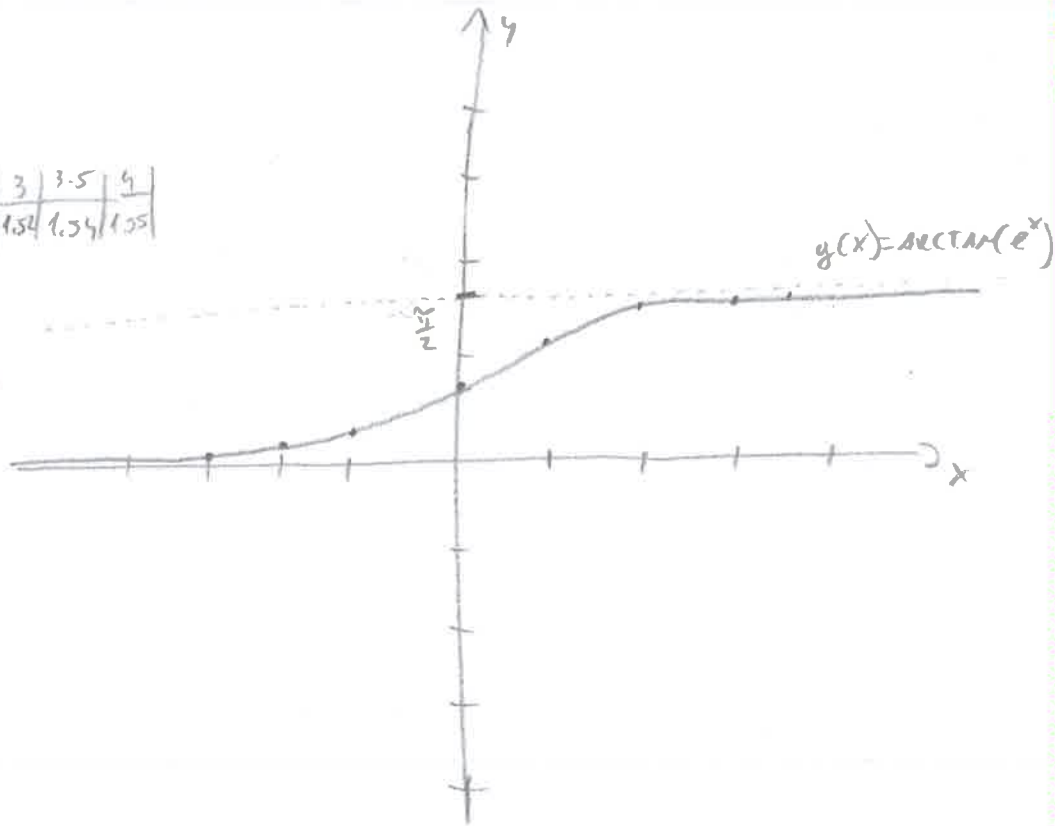
$x_1 = -0.05$
 $x_2 = -0.95$

PROVJERA?

2. $g(x) = \text{ARCTAN}(e^x)$

x	-2	-1	0	1	2	3	3.5	4
y	0.13	0.50	0.79	1.11	1.70	2.0	2.54	2.55

FUNKCIJSKI TIJEK?



3. $f(x) = \frac{x^2+8}{x-7}$

DOMENA $x-7 \neq 0$ ili $(-\infty, 7) \cup (7, +\infty)$

V.A. $\lim_{x \rightarrow 7^-} \frac{x^2+8}{x-7} = -\infty$

$\lim_{x \rightarrow 7^+} \frac{x^2+8}{x-7} = +\infty$

H.A. $\lim_{x \rightarrow +\infty} \frac{x^2+8}{x-7} \stackrel{1/\infty}{=} \frac{1+0}{0-0} = \frac{1}{0} = +\infty$

$\lim_{x \rightarrow -\infty} \frac{x^2+8}{x-7} \stackrel{1/\infty}{=} \lim_{x \rightarrow +\infty} \frac{x^2-x}{-x-7} \stackrel{1/\infty}{=} \frac{1-0}{-0-0} = -\infty$

D. KOSA

$k = \lim_{x \rightarrow +\infty} \frac{f(x) - \lim_{x \rightarrow +\infty} \frac{x^2+8}{x-7}}{\frac{x}{1}} = \lim_{x \rightarrow +\infty} \frac{x^2+8}{x^2-7x} \stackrel{1/\infty}{=} \frac{1+0}{1-0} = \frac{1}{1} = 1$

$l = f(x) - kx = \lim_{x \rightarrow +\infty} \frac{x^2+8}{x-7} - x = \lim_{x \rightarrow +\infty} \frac{x^2+8-x^2+7x}{x-7} = \lim_{x \rightarrow +\infty} \frac{7x+8}{x-7} \stackrel{1/\infty}{=} \frac{7+0}{1-0} = \frac{7}{1} = 7$

$y = kx + l$

$y = x + 7$

L. KOSA $k = \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \frac{x^2+8}{x(x-7)} = \lim_{x \rightarrow +\infty} \frac{x^2+8}{x^2-7x} \stackrel{1/\infty}{=} \lim_{x \rightarrow +\infty} \frac{x^2+8}{x^2+7x} \stackrel{1/\infty}{=} \frac{1}{1} = 1$

$l = f(x) - kx = \lim_{x \rightarrow +\infty} \frac{x^2+8}{x-7} - x = \lim_{x \rightarrow +\infty} \frac{x^2+8-x^2+7x}{x-7} = \lim_{x \rightarrow +\infty} \frac{7x+8}{x-7} \stackrel{1/\infty}{=} \lim_{x \rightarrow +\infty} \frac{-7x+8}{-x+7} \stackrel{1/\infty}{=} \frac{-7}{-1} = 7$

x	1	2	3	4
y	8	9	10	11



Matematika 1

Ime i prezime:

JURE FRŽOP

Matični broj u indeksu:

17-2-0269-2013

NASTAVNIK GRAFA:

SJ. SA FOR ODMA

$$\frac{x^2+8}{x-7} = 0 / (x-7)$$

$$x^2+8=0$$

$$x^2 = -8$$

LEMA
14

$$f'(x) = \frac{(x^2+8)' \cdot (x-7) - (x^2+8) \cdot (x-7)'}{(x-7)^2}$$

$$f'(x) = \frac{2x(x-7) - (x^2+8) \cdot 1}{(x-7)^2} = \frac{2x^2 - 14x - x^2 - 8}{(x-7)^2}$$

$$= \frac{x^2 - 14x - 8}{(x-7)^2}$$

$$f''(x) = \frac{(x^2 - 14x - 8)' \cdot (x-7)^2 - (x^2 - 14x - 8) \cdot ((x-7)^2)'}{(x-7)^4}$$

$$f''(x) = \frac{(2x - 14) \cdot (x^2 - 14x + 49) - (x^2 - 14x - 8) \cdot (2x - 14)}{(x-7)^3}$$

$$f''(x) = \frac{(2x^3 - 28x^2 + 98x - 14x^2 + 196x - 686) - (3x^2 - 14x^2 - 28x^2 + 196x - 16x + 112)}{(x-7)^3}$$

$$f''(x) = \frac{2x^3 - 28x^2 + 98x - 14x^2 + 196x - 686 - 3x^2 + 14x^2 + 28x^2 - 196x + 16x - 112}{(x-7)^3}$$

$$f''(x) = \frac{114x - 798}{(x-7)^3}$$

KR. TOČKE

MONOTONOST

-∞ -0.55 7 14.5 +∞

+	-	-	+
↗	↘	↘	↗

LOK. MAX
 $y_1 = 29.1$ $f(14.5, 29.1)$

LOK. MIN
 $y_2 = -1.1$ $f(-0.55, -1.1)$

INFLEKSIJA

$$\frac{114x - 798}{(x-7)^3} = 0 / (x-7)^3$$

$$114x - 798 = 0$$

$$114x = 798 / :114$$

$$x = 7$$

-∞ -0.55 7 14.5 +∞

-	-	+	+
∩	∩	∪	∪

GRAFA



$$\frac{x^2 - 14x - 8}{(x-7)^2} = 0 / (x-7)^2$$

$$x^2 - 14x - 8 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{14 \pm \sqrt{(14)^2 - 4 \cdot 1 \cdot (-8)}}{2}$$

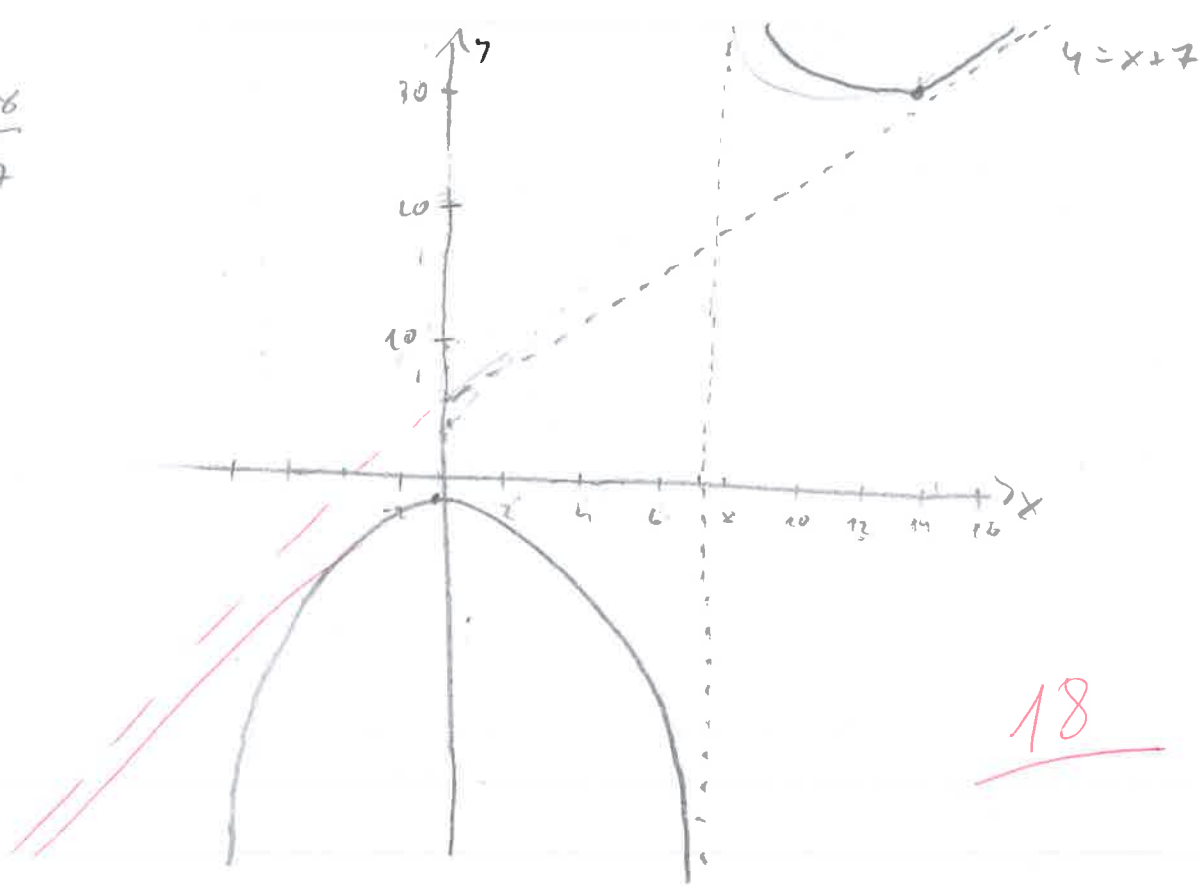
$$x_{1,2} = \frac{14 \pm \sqrt{196 + 32}}{2}$$

$$x_{1,2} = \frac{14 \pm \sqrt{228}}{2}$$

$$x_1 = 14.5$$

$$x_2 = -0.55$$

$$\text{GRAE} = \frac{2x+6}{x-7}$$



$$4. f(x) = \sqrt{7+5x}$$

LOK EK? GL. EK? GRAE TANGENIA x=2 GRAE

$$f'(x) = \frac{1}{2\sqrt{7+5x}} \cdot 5 = \frac{5}{2\sqrt{7+5x}}$$

$-\infty$	-1.4	5	$+\infty$
	↘	+	+
		↗	↗

x	10	12	20
y	-6.8	-5.6	-0.7

$$\frac{5}{2\sqrt{7+5x}} = 0 / (2\sqrt{7+5x})$$

$$5 = 0$$

GLOBAL MINIMUM = -1.4 ✓
 LOCAL MINIMUM = -1.4 ✓
 GLOBAL MAXIMUM = +∞ MUDA
 LOCAL MAXIMUM = 0

$$\text{DI } [-1.4, +\infty)$$

$$y = k \cdot (x - x_0) + y_0$$

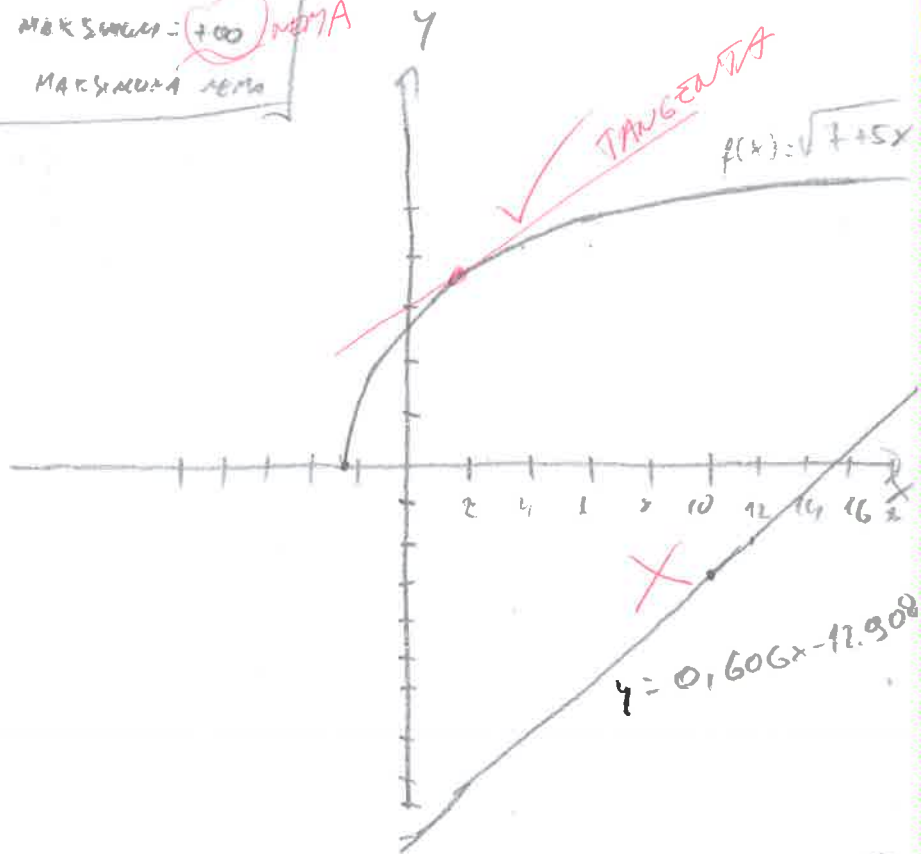
$$y_0 = f(2) = \sqrt{7+5 \cdot 2} = \sqrt{17} = 4.12$$

$$k = f'(x_0) = 0.606$$

$$y = 0.606 \cdot (x - 2) + 4.12$$

$$y = 0.606x - 1.212 + 4.12$$

$$y = 0.606x - 12.908$$



$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

$\begin{matrix} x \\ y \\ z \\ w \\ v \end{matrix}$

$$\left[\begin{array}{ccccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \xrightarrow{1:2} \left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \begin{matrix} (1-f(1)) / (1-f(2)) \\ C_2 \end{matrix}$$

$$\left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 2 & 1 & 0 & 5 & -4 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \xrightarrow{1:2} \left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \begin{matrix} (1+\frac{3}{2}) / (1-2) / (-2) \\ C_4 \end{matrix}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{array} \right] \xrightarrow{1:(4)} \left[\begin{array}{ccccc|c} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{array} \right]$$

MATRICA IMA
BESKONAČNO RIJEŠENJA

A KOJA SU TO RIJEŠENJA?

$$\left[\begin{array}{ccccc|c} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

6. $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2-x}}{x-1}$

$\lim_{x \rightarrow 1^-} \frac{\sqrt{2-x^2-x}}{x-1} = -1.9$

$\lim_{x \rightarrow 1^+} \frac{\sqrt{2-x^2-x}}{x-1} = -2$

NEMA POSTUPKA?

x	$\frac{\sqrt{2-x^2-x}}{x-1}$
-1	-1 TEŽI U $\boxed{-1}$
-1.5	MEMO
-1.1	-0.94
2	MEMO
0	-1.41

$\lim_{x \rightarrow +\infty} \frac{\sqrt{2-x^2-x}}{x-1} \stackrel{1/x^2}{=} \sqrt{\frac{2}{x^2} - \frac{x^2}{x^2} - \frac{x}{x}} = \sqrt{0-1-1} = \boxed{-2}$

$\lim_{x \rightarrow -\infty} \frac{\sqrt{2-x^2-x}}{x-1} = -\lim_{x \rightarrow +\infty} \frac{\sqrt{2+x^2+x}}{x-1} = -\lim_{x \rightarrow +\infty} \sqrt{\frac{2}{x^2} + \frac{x^2}{x^2} + \frac{x}{x}} = -\sqrt{0+1+1} = \boxed{0}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

MAJA ŠIKIĆ

VRIJEME POČETKA:

08:00

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

- 10:00

1. Riješiti jednačinu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

12

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije.

20 graf

3. Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf.

20 graf

4. Zadana je funkcija $f(x) = \sqrt{7+5x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf.

4+4+4+6

5. Gaussovom metodom riješiti sustav:

15+3

$$2x - 3y - z + 2w + 3v = 4$$

$$4x - 4y - z + 4w + 11v = 4$$

$$2x - 5y - 2z + 2w - v = 9$$

$$2y + z + 4v = -5$$

Provjeri vrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

Ukupno:

10

Matematika 1

Ime i prezime: MAJA ŠIKIĆ

Matični broj u indeksu: 17-1-0101-2011

$$3. f(x) = \frac{x^2 + 8}{x - 7}$$

1) DOMENA

$$f(x) = \frac{x^2 + 8}{x - 7}$$

$$D_f(x) = \{x \neq 7\}$$

$$x - 7 \neq 0$$

$$x \neq 7$$

2) PARNOST I NEPARNOST

$$f(-x) = \frac{(-x^2) + 8}{-x - 7} = \frac{x^2 + 8}{-x - 7}$$

Funkcija je neparna!

3) PERIODIČNOST

Funkcija nije periodična jer u sebi ne sadrži
 $\sin, \cos, \tan, \cot, \dots$

4) ASIMPTOTE

a) HORIZONTALNA

$$f(x) = \lim_{x \rightarrow +\infty} \frac{x^2 + 8}{x - 7} = \lim_{x \rightarrow +\infty} \frac{\frac{x^2}{x^2} + \frac{8}{x^2}}{\frac{x}{x^2} - \frac{7}{x^2}} = \frac{1}{0} = \infty = \infty$$

$$f(x) = \lim_{x \rightarrow -\infty} \frac{x^2 + 8}{x - 7} = \lim_{x \rightarrow -\infty} \frac{\frac{x^2}{x^2} + \frac{8}{x^2}}{\frac{x}{x^2} - \frac{7}{x^2}} = \frac{1}{0} = \infty = \infty$$

b) VERTIKALNA ASIMPTOTA

$$f(x) = \lim_{x \rightarrow 7^+} \frac{x^2 + 8}{x - 7} = \lim_{x \rightarrow 7} \frac{7^2 + 8}{7 - 7} = \frac{57}{0} = \infty$$

c) KOSA ASIMPTOTA

$$y = kx + l$$

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x}$$

$$k = \lim_{x \rightarrow \infty} \frac{x^2 + 8}{x - 7} = \lim_{x \rightarrow \infty} \frac{1 \cdot (x^2 + 8)}{x(x - 7)} = \lim_{x \rightarrow \infty} \frac{x^2 + 8 \quad | : x^2}{x^2 - 7x \quad | : x^2} = \frac{1 + \frac{8}{x^2}}{1 - \frac{7}{x}}$$

$$\frac{\frac{x^2}{x^2} + \frac{8}{x^2}}{\frac{x^2}{x^2} - \frac{7x}{x^2}} = \frac{1 + 0}{1 - 0} = \frac{1}{1} = 1 \quad k = 1$$

$$l = \lim_{x \rightarrow \infty} \left(f(x) - k \cdot x \right) = \lim_{x \rightarrow \infty} \left(\frac{x^2 + 8}{x - 7} - 1 \cdot x \right) = \lim_{x \rightarrow \infty} \left(\frac{x^2 + 8 \quad | : x^2}{x - 7 \quad | : x} - x \right)$$

$$= \lim_{x \rightarrow \infty} \left(\frac{\frac{x^2}{x^2} + \frac{8}{x^2}}{\frac{x}{x} - \frac{7}{x}} - \frac{x}{x} \right) = \frac{1 + 0}{0 - 0} - 0 = \frac{1}{0} = 0$$

$$y = 1x + 0$$

$$\frac{x^2 + 8}{x - 7} - x = \frac{x^2 + 8 - x^2 + 7x}{x - 7}$$

$$= \frac{7x + 8}{x - 7}$$

$$= \frac{7 + \frac{8}{x}}{1 - \frac{7}{x}} = 7$$

5. NUL TOČKE

$$f(x) = \frac{x^2 + 8}{x - 7}$$

MAJA ŠUKIĆ

17-1-0101-2011

$$\frac{x^2 + 19 - 8}{(x-7)}$$

$$f(0) = \frac{0^2 + 8}{0 - 7} = \frac{8}{-7} = -1.2 \quad \text{NT}(1, 2, 0)$$

6. STACIONARNE TOČKE

$$f'(x) = \frac{x^2 + 8}{x - 7} = \frac{x^2 + 19 - 8}{(x - 7)}$$

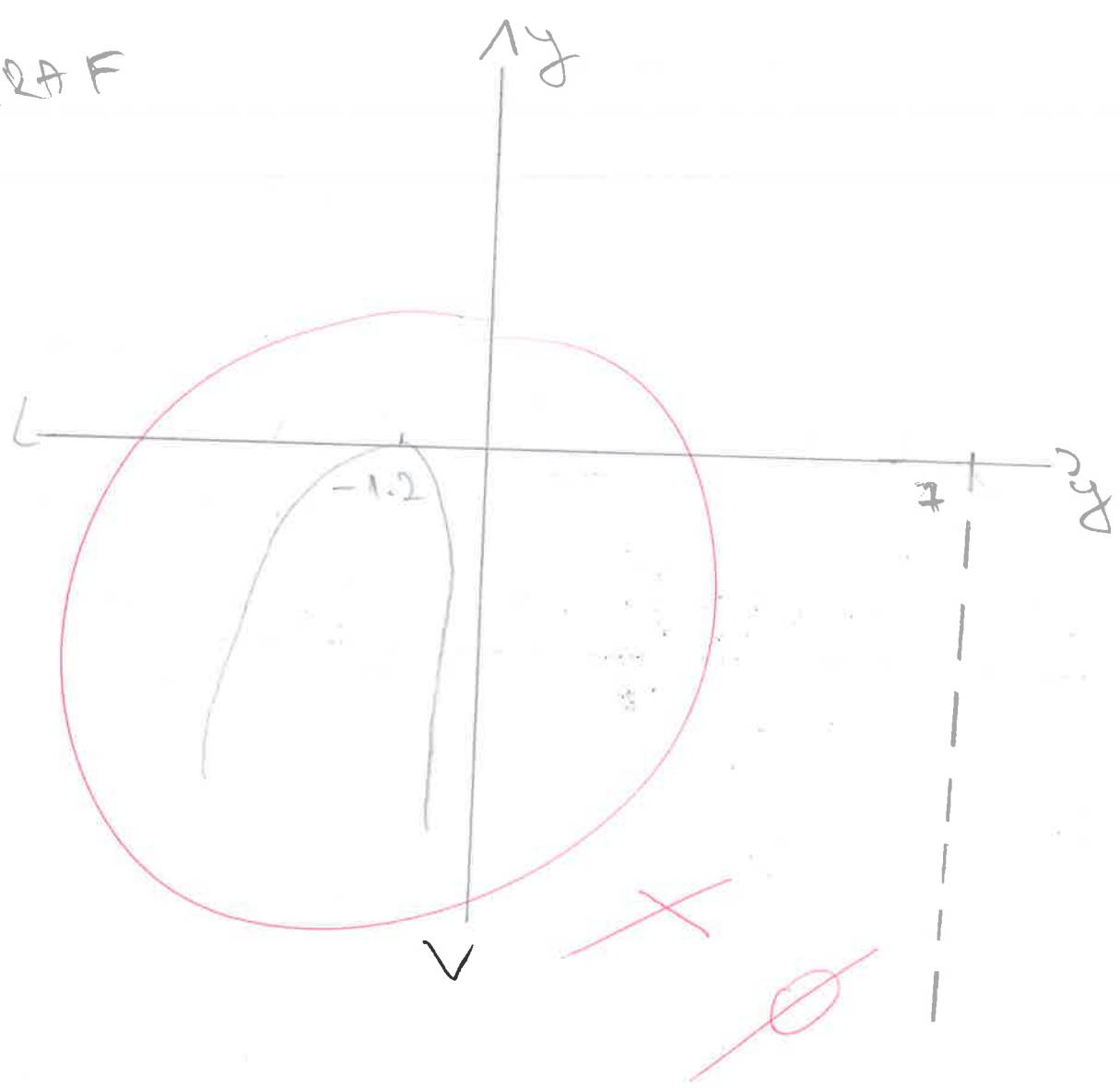
	$-\infty$	1.2	0	$+\infty$
$f(x)$	-	-	-	-
$f'(x)$	↗	↘	↘	↘

$$f'(x) = \frac{x^2 + 8}{x - 7} = \frac{(x-7)' \cdot (x^2 + 8) + (x-7) \cdot (x^2 + 8)'}{(x-7)^2}$$

$$= \frac{1(x^2 + 8) + \cancel{(x-7)} \cdot 2x}{(x-7)^2} = \frac{(x^2 + 8) + 2x}{(x-7)^2}$$

$$\frac{2x^2 + 8}{x-7}$$

I. GRAF



$$\begin{aligned}
 6. \lim_{x \rightarrow 1} \frac{\sqrt{2-x^2-x}}{x-1} \cdot \frac{\sqrt{2-x^2+x}}{\sqrt{2-x^2+x}} &= \lim_{x \rightarrow 1} \frac{2-x^2+x}{(x-1)(\sqrt{2-x^2+x})} \\
 &= \lim_{x \rightarrow 1} \frac{2(1-x^2)}{-(1-x)(\sqrt{2-x^2+x})} = \lim_{x \rightarrow 1} \frac{2(1-x)(1+x)}{-1(1-x)(\sqrt{2-x^2+x})} \\
 \lim_{x \rightarrow 1} \frac{2(1+x)}{-\sqrt{2+1+1}} &= \frac{4}{-2} = -2 \quad \checkmark \quad \text{PROVJERA...}
 \end{aligned}$$

1:

$$\begin{aligned}
 2 + \frac{2x+1}{2\sqrt{x^2+x}} &= 0 \quad \rightarrow \quad \begin{aligned} 2x+1 &\leq 0 \\ 2x &\leq -1 \\ x &\leq -1/2 \end{aligned} \\
 \frac{2x+1}{2\sqrt{x^2+x}} &= -2 \quad \text{POGREŠNO I KRAČIJE} \\
 \frac{2x+1}{x^2+x} &= -2 \\
 \frac{2x+1}{x^2+x} &= 1 \\
 \frac{4x^2+4x+1}{x^2+x} &= 0 \quad \rightarrow \\
 \frac{4x^2+4x+1-x}{x^2+x} &= 0 \\
 \frac{3x^2+3x-1}{x^2+x} &= 0
 \end{aligned}$$

$$2x+1 \leq 0$$

$$2x \leq -1$$

$$x \leq -1/2$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **GORDAN J AČAN**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **0269 0865 92**

1. Riješiti jednadžbu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

12

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije.

20 graf

3. Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf.

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4+4+4+6

5. Gaussovom metodom riješiti sustav:

15+3

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

6. $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1} = \frac{\sqrt{2-1}-1}{1-1} = \frac{0}{0} = 0$ ✗

Ukupno:

7

1. $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

$$\frac{2 \cdot (2\sqrt{x^2+x}) + 2x+1}{2\sqrt{x^2+x}} = 0 \quad | \cdot 2\sqrt{x^2+x}$$

$$4\sqrt{x^2+x} + 2x+1 = 0 \quad | ^2$$

$$16x^2 + 16x + 2x+1 = 0 \quad \text{✗}$$

$$16x^2 + 18x + 1 = 0$$

$$x_{1,2} = \frac{-18 \pm \sqrt{324 - 64}}{32}$$

$$x_1 = \frac{-9 + \sqrt{65}}{16} = -0.0586$$

$$x_2 = \frac{-9 - \sqrt{65}}{16} = -1.066 \quad \checkmark$$

$$\begin{aligned} x^2 + x &\geq 0 \\ x \cdot (x+1) &\geq 0 \\ x_1 &\geq 0 \quad x+1 \geq 0 \\ x_2 &\geq -1 \end{aligned}$$



$$(3) f(x) = \frac{x^2 + 8}{x - 7}$$

$$x - 7 \neq 0 \quad Df = \mathbb{R} \setminus \{7\}$$

$$x \neq 7$$

ASIMPTOTE

L.V.A $\lim_{x \rightarrow 7^-} \frac{x^2 + 8}{x - 7} = \frac{(6,9999)^2 + 8}{6,9999 - 7} = \frac{48,9986 + 8}{0^-} = \frac{56,9986}{0^-} = -\infty$

D.V.A $\lim_{x \rightarrow 7^+} \frac{x^2 + 8}{x - 7} = \frac{(7,0001)^2 + 8}{7,0001 - 7} = \frac{57,0014}{0^+} = +\infty$

H.A. $\lim_{x \rightarrow +\infty} \frac{x^2 + 8}{x - 7} \stackrel{/:x^2}{=} \lim_{x \rightarrow +\infty} \frac{1 + \frac{8}{x^2}}{\frac{1}{x} - \frac{7}{x^2}} = \frac{1}{0^-} = -\infty$

$\lim_{x \rightarrow -\infty} \frac{x^2 + 8}{x - 7} \stackrel{/:x^2}{=} \lim_{x \rightarrow -\infty} \frac{1 + \frac{8}{x^2}}{\frac{1}{x} - \frac{7}{x^2}} = \frac{1}{0^-} = -\infty$

K.A. $y = kx + l$

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x} \quad l = \lim_{x \rightarrow \infty} [f(x) - kx]$$

$$y = x + 7$$

$$k = \lim_{x \rightarrow \infty} \frac{\frac{x^2 + 8}{x - 7}}{\frac{x}{1}} = \lim_{x \rightarrow \infty} \frac{x^2 + 8}{x^2 - 7x} \stackrel{/:x^2}{=} \lim_{x \rightarrow \infty} \frac{1 + \frac{8}{x^2}}{1 - \frac{7}{x}} = 1$$

$$l = \lim_{x \rightarrow \infty} \frac{x^2 + 8}{x - 7} - x = \lim_{x \rightarrow \infty} \frac{x^2 + 8 - (x^2 - 7x)}{x - 7} = \lim_{x \rightarrow \infty} \frac{8 + 7x}{x - 7} \stackrel{/:x}{=} \lim_{x \rightarrow \infty} \frac{\frac{8}{x} + 7}{1 - \frac{7}{x}} = 7$$

NULTOCKE

$$\frac{x^2 + 8}{x - 7} = 0 \quad / \cdot (x - 7)$$

$$x^2 + 8 = 0$$

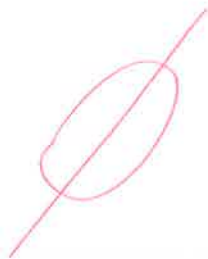
$$x^2 = -8$$

$$x_{1,2} = \pm \sqrt{-8}$$

$$x_1 = 2\sqrt{2} = 2,828$$

$$x_2 = -2\sqrt{2} = -2,828$$

GRAF?



3. (NE) PARNOST

GORDAN JACAN
0269086592

$$f(-x) = \frac{(-x)^2 + 8}{-x - 7} = \frac{x + 8}{-x - 7} = \frac{x + 8}{-(x + 7)} = - \frac{x + 8}{x + 7}$$

ni parna ni
neparna

DERIVACIJA

$$f(x) = \frac{x^2 + 8}{x - 7}$$

$$f'(x) = \frac{(x^2 + 8)' \cdot (x - 7) - (x^2 + 8) \cdot (x - 7)'}{(x - 7)^2} = \frac{2x \cdot (x - 7) - (x^2 + 8) \cdot 1}{(x - 7)^2} = \frac{2x^2 - 14x - x^2 - 8}{(x - 7)^2}$$

$$f'(x) = \frac{x^2 - 14x - 8}{(x - 7)^2}$$

KRITIČNE TOČKE

$$f'(x) = 0$$

$$\frac{x^2 - 14x - 8}{(x - 7)^2} = 0 \quad / \cdot (x - 7)^2$$

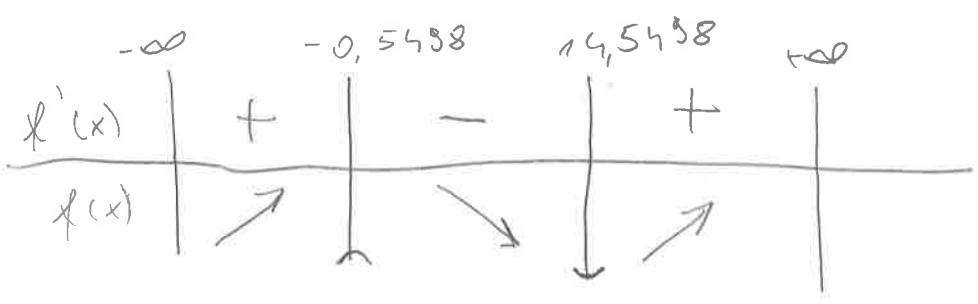
$$x^2 - 14x - 8 = 0$$

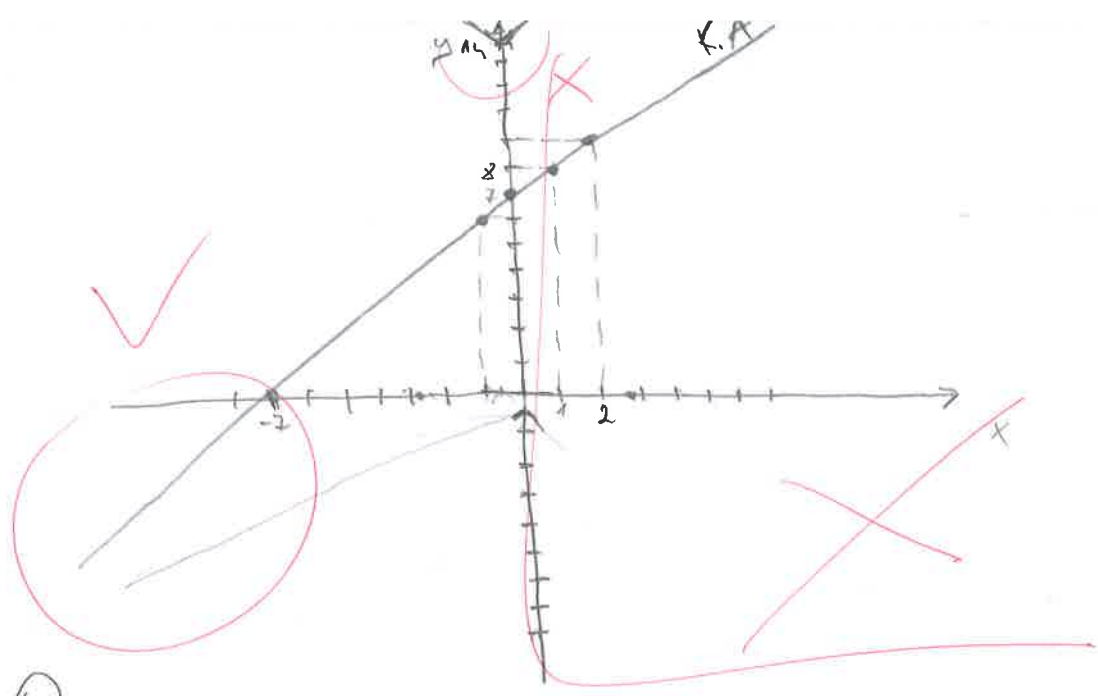
$$x_{1,2} = \frac{14 \pm \sqrt{196 + 32}}{2}$$

$$x_1 = 7 + \sqrt{57} = 14,5498$$

$$x_2 = 7 - \sqrt{57} = -0,5498$$

MONOTONOST





x	1	2	0	-1
y	8	9	7	6

4) $f(x) = \sqrt{7+5x}$

$$f'(x) = \frac{1}{2\sqrt{7+5x}} \cdot 5 = \frac{5}{2\sqrt{7+5x}}$$

KRITISCHE WERTE

$$f'(x) = 0$$

$$\frac{5}{2\sqrt{7+5x}} = 0 \quad | \cdot 2\sqrt{7+5x}$$

$$5 \neq 0$$

KEINE GLOBALE EKSTREMA X

x	0	1	2	-1
y	2,645	3,6	4,12	1,9

$$x_0 = 2 \quad y_0 = \sqrt{17}$$

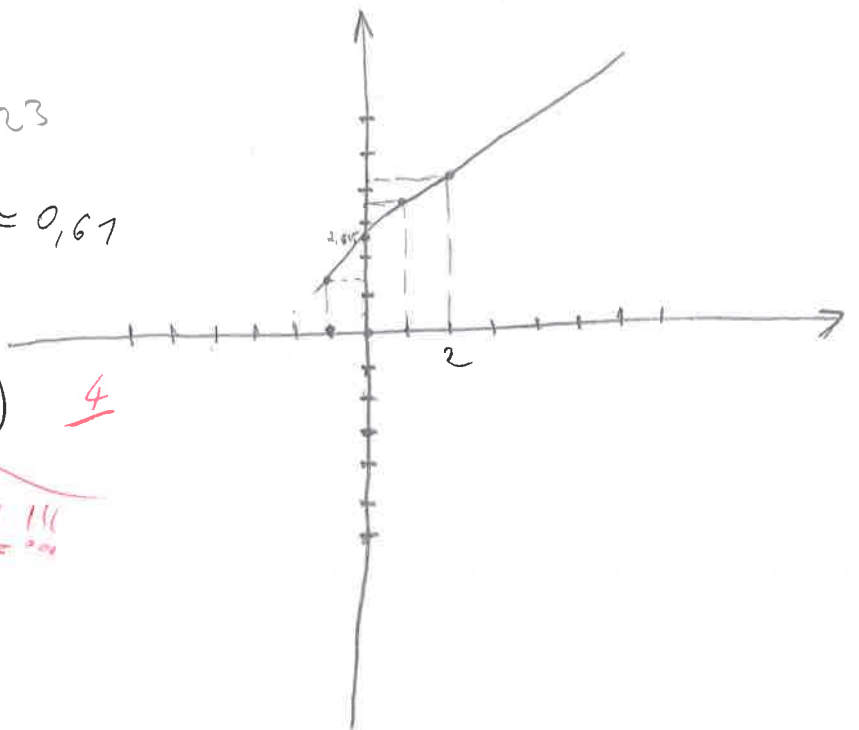
$$y - y_0 = f'(x_0) (x - x_0)$$

$$f(2) = \sqrt{7+5 \cdot 2} = \sqrt{17} = 4,123$$

$$f'(2) = \frac{5}{2\sqrt{17}} = \frac{5\sqrt{17}}{34} = 0,67$$

$$y - \sqrt{17} = \frac{5\sqrt{17}}{34} \cdot (x - 2) \quad \underline{4}$$

NACHTAT III



Matematika 1

Ime i prezime: GORDAN JACAN

Matični broj u indeksu: 0269 0865 92

5.

$$\left[\begin{array}{ccccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \cdot 1/2 \sim \left[\begin{array}{ccccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 1 & -\frac{5}{2} & -1 & 1 & -\frac{1}{2} & \frac{9}{2} \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \sim$$

$$\sim \left[\begin{array}{ccccc|c} 1 & -\frac{5}{2} & -1 & 1 & -\frac{1}{2} & \frac{9}{2} \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -3 & -1 & 2 & 3 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \cdot (-4) \cdot (-2) \sim \left[\begin{array}{ccccc|c} 1 & -\frac{5}{2} & -1 & 1 & -\frac{1}{2} & \frac{9}{2} \\ 0 & 6 & 3 & 0 & 13 & -14 \\ 0 & 2 & 1 & 0 & 4 & -5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc|c} 1 & -\frac{5}{2} & -1 & 1 & -\frac{1}{2} & \frac{9}{2} \\ 0 & 1 & \frac{1}{2} & 0 & \frac{13}{6} & -\frac{15}{6} \\ 0 & 2 & 1 & 0 & 4 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \cdot (-2) \sim$$

II : 6
IV - III

BESKONAČNO
MNOGO RJEŠENJA

$x_5 = \tau$ ✓ $\tau, \rho \in \mathbb{R}$
 $x_4 = \rho$ ✓

$2x_2 + x_3 + 4\tau = -5$
 $x_3 = -5 - 4\tau - 2x_2$

$x_2 + \frac{1}{2}(-5 - 4\tau - 2x_2) + \frac{13}{6}\tau = -\frac{15}{6}$
 $x_2 - \frac{5}{2} - 2\tau - x_2 + \frac{13}{6}\tau = -\frac{15}{6}$

$\frac{1}{6}\tau = \frac{1}{6} \quad / \cdot 6$
 ~~$\tau = 1$~~ ✗

KOJA?

O.K.

$x_2 = ?$
 $x_1 = ?$

$$x_1 - \frac{5}{2}x_2 - 1 \cdot (-5 - 4\pi - 2x_2) + 0 - \frac{1}{2}\pi = \frac{9}{2}$$

$$x_1 - \frac{5}{2}x_2 + 5 + 4\pi + 2x_2 + 0 - \frac{1}{2}\pi = \frac{9}{2}$$

$$0 = -\frac{1}{2}x_2 - \frac{1}{2} - \frac{7}{2}\pi - x_1$$

NA KRAJU UOPĆE NIJE JASNO

KOJE JE RJEŠENJE OVIH JEDNAŽBI.



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

POPUNJAVA

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod

ε1

NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **JURE ŠUŠIĆ**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **17-1-0259-2014**

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Provjeri uvrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

Ukupno:

10

3.) $f(x) = \frac{x^2+8}{x-7}$ $1^\circ x-7 \neq 0$ $D(f) = \mathbb{R} \setminus \{7\}$
 $x \neq 7$

2° ASIMPTOTE

VA. $\lim_{x \rightarrow 7^-} \frac{x^2+8}{x-7} = \lim_{x \rightarrow 7^-} \frac{7^2+8}{7-7} = \frac{57}{0} = -\infty$ OVA... $x=7$

$\lim_{x \rightarrow 7^+} \frac{7^2+8}{7-7} = \frac{57}{0} = +\infty$

HA. $\lim_{x \rightarrow \pm\infty} \frac{x^2+8}{x-7} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \rightarrow \pm\infty} \frac{1 + \frac{8}{x^2}}{\frac{1}{x} - \frac{7}{x^2}} = \frac{1}{0} = \infty$ NEMA HA.

KA. **KOSE ASIMPTOTE?**

3.) $f(x) = 0$ $x^2+8=0$ NEMA NULTOČKI
 $x^2 = -8$
 $x = \pm\sqrt{-8}$

$f(0) = \frac{0^2+8}{0-7} = -\frac{8}{7} = -1,14$ S $(0, -1,14)$

4. $f(-x) = \frac{(-x)^2 + 8}{(-x) - 7} = \frac{x^2 + 8}{-x - 7}$ NI PARNA NI NEPARNA, NIJE OMEĐENA, NIJE PERIODIČNA

5. $f'(x) = \frac{2x \cdot (x-7) - (x^2+8) \cdot 1}{(x-7)^2} = \frac{2x^2 - 14x - x^2 - 8}{(x-7)^2} = \frac{x^2 - 14x - 8}{(x-7)^2}$

$f''(x) = \frac{2x \cdot (-14)(x-7)^2 - (x^2 - 14x - 8) \cdot 2(x-7) \cdot 1}{(x-7)^4}$
 $= \frac{(x-7) [2x \cdot (-14)(x-7) - (x^2 - 14x - 8) \cdot 2]}{(x-7)^4}$
 $= \frac{-28x^2 + 196x - 2x^2 + 28x + 16}{(x-7)^3} = \frac{-30x^2 + 224x + 16}{(x-7)^3}$

6. $f'(x) = 0 \quad x^2 - 14x - 8 = 0$
 $x_{1,2} = \frac{14 \pm \sqrt{196 - 32}}{2}$
 $x_{1,2} = \frac{14 \pm 2\sqrt{41}}{2}$

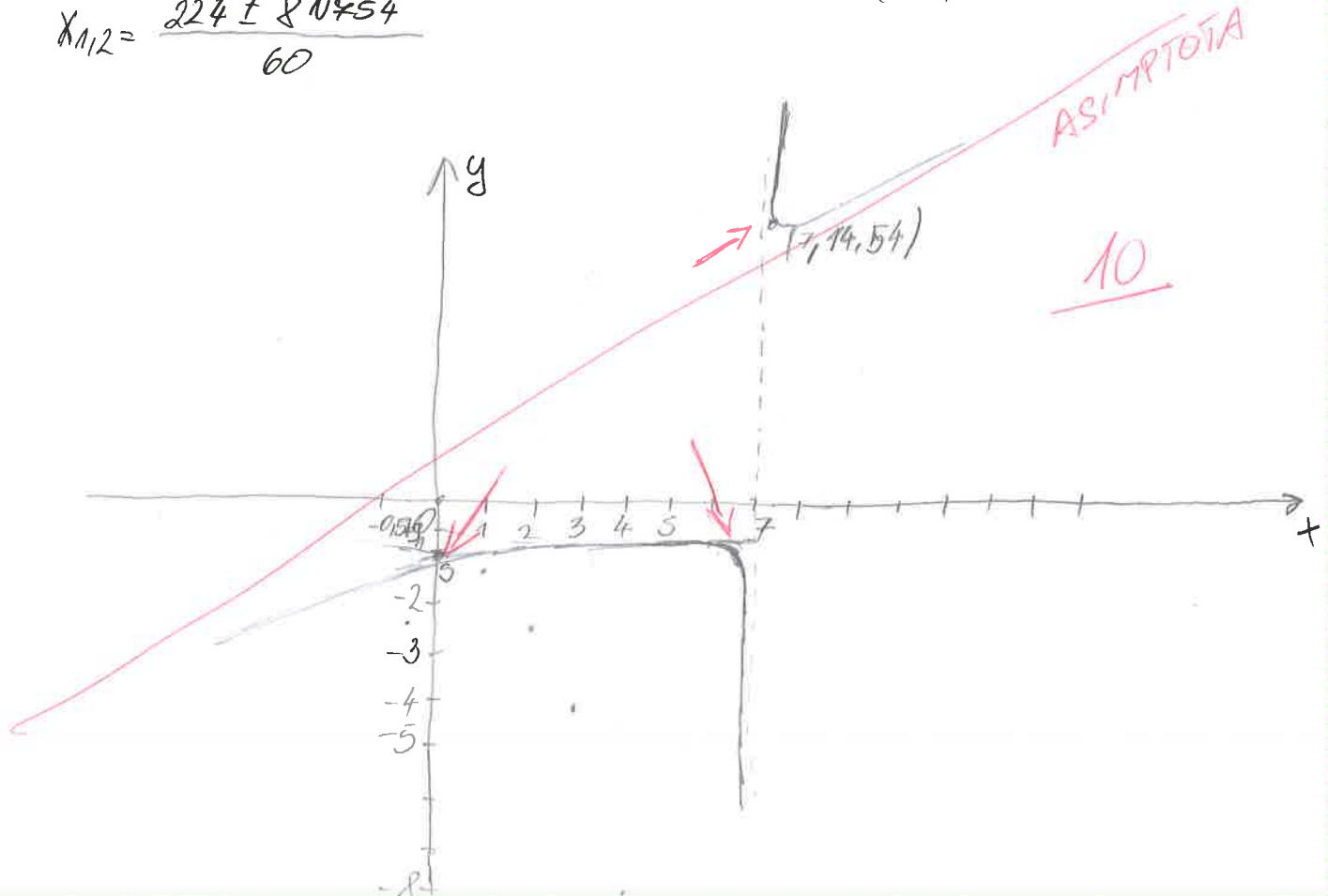
$x_1 = 13,4$
 $x_2 = 0,16$

OVOG NEMA NA GRAFU!

$f''(x) = 0 \quad -30x^2 + 224x + 16 = 0 \quad | \cdot (-1)$
 $30x^2 - 224x - 16 = 0$
 $x_{1,2} = \frac{224 \pm \sqrt{50176 - 1920}}{60}$
 $x_{1,2} = \frac{224 \pm 8\sqrt{754}}{60}$
 $x_1 = 7,39$
 $x_2 = 0,07$

	$-\infty$	$0,07$	$0,16$	7	$7,39$	$13,4$	$+\infty$
$f'(x)$	-	-	-	-	-	+	
$f''(x)$	-	-	-	+	-	-	
$f(x)$	\nearrow	\nearrow	\nearrow	\searrow	\searrow	\searrow	

$f(7,39) =$



Matematika 1

Ime i prezime: JURE ŠUŠIĆ

Matični broj u indeksu: 17-1-0259-2014

5.)

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 2 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \xrightarrow{\cdot \frac{1}{2}} \left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 4 & -4 & -1 & 4 & 11 & 4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \begin{array}{l} \cdot (-4) \cdot (-2) \\ \leftarrow + \right. \\ \left. \leftarrow + \right. \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 2 & 1 & 0 & 5 & -4 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \xrightarrow{\cdot \frac{1}{2}} \left[\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \end{array} \right] \begin{array}{l} \cdot (\frac{3}{2}) \cdot (2) \cdot (-2) \\ \leftarrow + \right. \\ \leftarrow + \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{array} \right] \xrightarrow{\cdot 2} \left[\begin{array}{ccccc|c} 1 & 0 & \frac{1}{2} & 2 & \frac{21}{2} & -2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{array} \right] \begin{array}{l} \leftarrow + \right. \\ \leftarrow + \right. \\ \cdot (-\frac{1}{2}) \cdot (-\frac{1}{4}) \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & \frac{15}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 0 & 6 & -3 \\ 0 & 0 & 0 & 1 & \frac{11}{4} & -\frac{5}{4} \end{array} \right] \xrightarrow{\cdot (-1)} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & \frac{15}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 0 & 6 & -3 \\ 0 & 0 & 0 & 1 & \frac{11}{4} & -\frac{5}{4} \end{array} \right] \begin{array}{l} \leftarrow + \right. \\ \leftarrow + \right. \\ \leftarrow + \right. \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & \frac{15}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 0 & 6 & -3 \\ 0 & 0 & 0 & 1 & \frac{11}{4} & -\frac{5}{4} \end{array} \right] \xrightarrow{\cdot (-1)} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & \frac{15}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 0 & 6 & -3 \\ 0 & 0 & 0 & 1 & \frac{11}{4} & -\frac{5}{4} \end{array} \right]$$

$r(M) = 4$

MATRICA IMA BESKONAČNO RJEŠENJA

KOJA SU TO RJEŠENJA ?

$$6.) \lim_{x \rightarrow 1} \frac{\sqrt{2-x^2} - x}{x-1} = \frac{L'H}{x \rightarrow 1} \lim_{x \rightarrow 1} \frac{-\frac{1}{2\sqrt{2-x^2}} \cdot (-2x) - 1}{1} = \lim_{x \rightarrow 1} \frac{x}{\sqrt{2-x^2}} - 1$$

$$= \lim_{x \rightarrow 1} \frac{1}{\sqrt{2-1}} - 1 = \frac{1}{\sqrt{1}} - 1 = 1 - 1 = 0$$

PROVJERA!

$$4.) f(x) = \sqrt{7+5x}$$

$$7+5x \geq 0 \quad D(f) = \left[-\frac{7}{5}, +\infty\right)$$

$$5x \geq -7$$

$$x \geq -\frac{7}{5}$$

VA

$$HA \lim_{x \rightarrow +\infty} \sqrt{7+5x} / x = \lim_{x \rightarrow +\infty} \sqrt{\frac{7}{x} + 5} = \sqrt{5} = 2,24$$

$$f(x) = 0 \quad \sqrt{7+5x} = 0 / ^2$$

$$7+5x = 0$$

$$x = -\frac{7}{5} = -1,4$$

NT(-1,4, 0)

$$f(0) = \sqrt{7+0} = \sqrt{7} = 2,65 \quad S(0, 2,65)$$

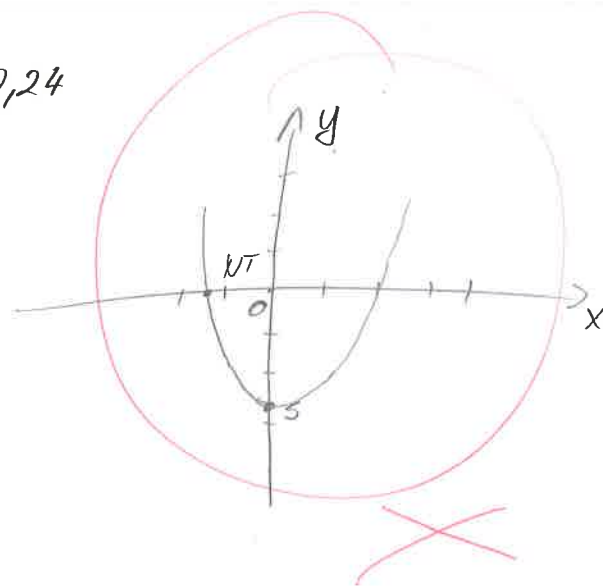
$$f'(x) = \frac{1}{2\sqrt{7+5x}} \cdot 5 = \frac{5}{2\sqrt{7+5x}}$$

$$f''(x) = \frac{-5 \cdot \frac{1}{4\sqrt{7+5x}} \cdot 5}{(2\sqrt{7+5x})^2} = \frac{-25}{4\sqrt{7+5x}}$$

$$f'(x) = 0 \quad 5 \neq 0$$

$$f''(x) = \frac{-25}{4\sqrt{7+5x}} = 0 / \cdot (4\sqrt{7+5x})$$

$$= -25 \neq 0$$



$$1.) \quad 2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0 \quad | \cdot (2\sqrt{x^2+x})$$

$$4\sqrt{x^2+x} + (2x+1) = 0 \quad |^2$$

$$16(x^2+x) + 4x^2 + 4x + 1 = 0$$

$$16x^2 + 16x + 4x^2 + 4x + 1 = 0$$

$$16x^2 + 4x + 17 = 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 1088}}{32}$$

$$x_{1,2} = \frac{-4 \pm \sqrt{-1072}}{32}$$

$$16x^2 + 16x + 4x^2 + 4x + 1 = 0$$

$$20x^2 + 20x + 1 = 0$$

$$x_{1,2} = \frac{-20 \pm \sqrt{400 - 80}}{40} = \frac{-20 \pm 8\sqrt{5}}{40}$$

$$x_1 = -0,05 \quad x_2 = -0,95$$

$$20x^2 + 16x + 1 = 0$$

$$x_{1,2} = \frac{-16 \pm \sqrt{256 - 80}}{40}$$

$$x_{1,2} = \frac{-16 \pm 4\sqrt{11}}{40}$$

$$x_1 = -0,43 \quad x_2 = -0,07$$

$$2.) \quad g(x) = \arctan(e^x)$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **KARLO KOLJAJA**

VRIJEME POČETKA: **9:15**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **0269087607**

ε1

1. Riješiti jednadžbu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

~~12~~

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije.

~~20 graf~~

3. Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf.

~~20 graf~~

4. Zadana je funkcija $f(x) = \sqrt{7+5x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf.

~~4+4+4+6~~

5. Gaussovom metodom riješiti sustav:

~~15+3~~

$$2x - 3y - z + 2w + 3v = 4$$

$$4x - 4y - z + 4w + 11v = 4$$

$$2x - 5y - 2z + 2w - v = 9$$

$$2y + z + 4v = -5$$

Provjeri vrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

Ukupno:


KARLO KOLAJA

5. $\begin{pmatrix} 1 & -3 & -1 & 2 & 3 & | & 4 \\ 4 & -4 & -1 & 4 & 11 & | & 4 \\ 2 & -5 & -2 & 2 & -1 & | & 9 \\ 0 & 2 & 1 & 0 & 4 & | & -5 \end{pmatrix} \xrightarrow{1:2} \begin{pmatrix} 0 & 2 & 1 & 0 & 4 & | & -5 \\ 4 & -4 & -1 & 4 & 11 & | & 4 \\ 2 & -5 & -2 & 2 & -1 & | & 9 \\ 2 & -3 & -1 & 2 & 3 & | & 4 \end{pmatrix}$

$\begin{pmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & | & 2 \\ 4 & -4 & -1 & 4 & 11 & | & 4 \\ 2 & -5 & -2 & 2 & -1 & | & 9 \\ 0 & 2 & 1 & 0 & 4 & | & -5 \end{pmatrix} \xrightarrow{1:(-4), 1:(-2)} \begin{pmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & | & 2 \\ 0 & 2 & 1 & 0 & 5 & | & -4 \\ 0 & -2 & -1 & 0 & -4 & | & 5 \\ 0 & 2 & 1 & 0 & 4 & | & -5 \end{pmatrix}$

$\begin{pmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & | & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & | & -2 \\ 0 & -2 & -1 & 0 & -4 & | & 5 \\ 0 & 2 & 1 & 0 & 4 & | & -5 \end{pmatrix} \xrightarrow{1:2, 1:(-2), 1:\frac{3}{2}} \begin{pmatrix} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & | & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & | & -2 \\ 0 & 0 & 0 & 0 & 1 & | & 1 \\ 0 & 0 & 0 & 0 & -1 & | & 1 \end{pmatrix}$

KOJE JE RJEŠENJE?

$$1. \quad 2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0 \quad | \cdot 2\sqrt{x^2+x}$$

$$= 4\sqrt{x^2+x} + 2x+1 = 0$$

$$4x + 4\sqrt{x} + 2x+1 = 0 \quad \times$$

$$6x + 4\sqrt{x} + 1 = 0 \quad \times$$

$$36x^2 + 16x + 1 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-256 \pm \sqrt{256 - 4 \cdot 36 \cdot 1}}{72}$$

$$\frac{-256 \pm 12}{72}$$

$$x_1 = -3,39 \quad \times$$

$$x_2 = -3,72 \quad \times$$

PROVJERA...

$$3. \quad f(x) = \frac{x^2+8}{x-7}$$

$$f'(x) = \frac{2 \cdot x^{2-1} + 8}{x-7} = \frac{2x+8}{x-7} \quad | \cdot x-7 = (2x+8)(x-7)$$

$$= 2x^2 + 8x + (-14x) - 56$$

$$= 2x^2 - 6x - 56$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$$

$$= \frac{36 \pm \sqrt{36 - 4 \cdot 2 \cdot (-56)}}{4} = \frac{36 \pm 22}{4}$$

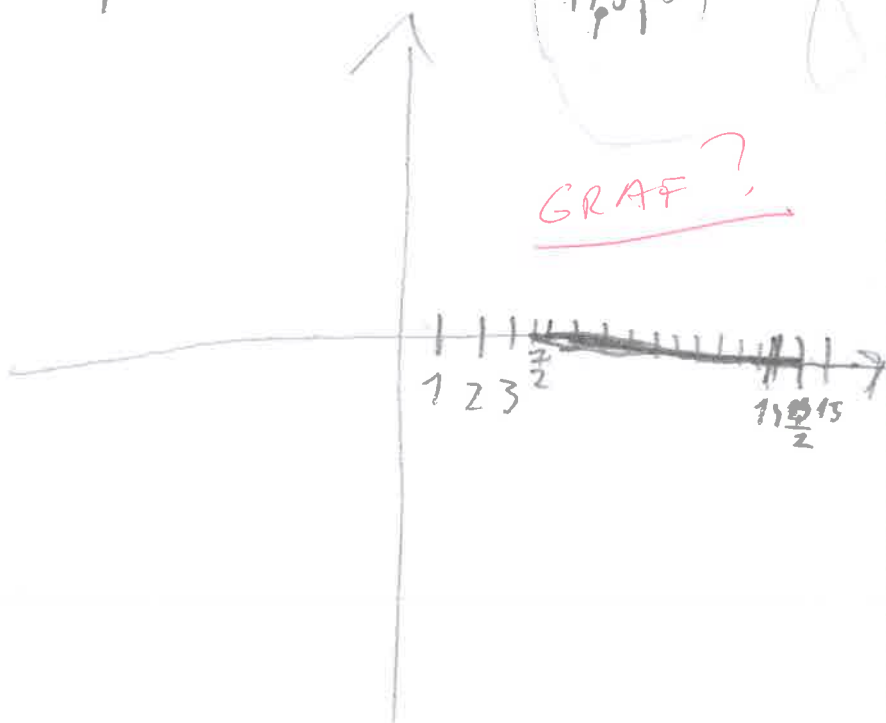
$$x_1 = \frac{29}{2} \quad x_2 = \frac{7}{2}$$

$$x \in \left\langle \frac{7}{2}, \frac{29}{2} \right\rangle$$

$$(35, 0)$$

$$(17,5, 0)$$

GRAF?



KARLO KOČAJA

$$7. f(x) = \sqrt{7+5x} \quad /^2$$

$$f(x) = 7+5x$$

lokalni ekstremi \emptyset
globalni \exists



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
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IME I PREZIME: **JOSIP MATEŠIĆ**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

0269075368

- Riješiti jednadžbu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$ 12
- Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf
- Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf. 20 graf
- Zadana je funkcija $f(x) = \sqrt{7+5x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf. 4+4+4+6
- Gaussovom metodom riješiti sustav: 15+3

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

Provjeri vrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

⑥

$$\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1} = \lim_{x \rightarrow 1} \frac{\sqrt{2-1^2}-1}{1-1} = \lim_{x \rightarrow 1} \frac{1-1}{1-1} = +\infty$$

Ukupno:

~~0~~

③ $f(x) = \frac{x^2+8}{x-7}$

$x-7 \neq 0 \quad D = X \in \mathbb{R} / \{7\}$

$x-7 = 0$

$x = 7$

$\frac{x^2+8}{x-7} = 0 / x-7$

$x^2+8 = 0$

$x^2 = -8 / (-1)$

$-x^2 = 8 / \sqrt{\quad}$

$x = \pm 2\sqrt{2}$

D.T. $(-2\sqrt{2}, 0)$

$(2\sqrt{2}, 0)$

$(0, -\frac{8}{7})$

V.A.

$$\lim_{x \rightarrow 7} \frac{x^2 + 8}{x - 7} \rightarrow \lim_{x \rightarrow 7} \frac{7^2 + 8}{7 - 7} = \lim_{x \rightarrow 7} \frac{57}{0} = +\infty$$

$$x = 7$$

H.A.

$$\lim_{x \rightarrow \infty} \frac{x^2 + 8}{x - 7} \cdot \frac{x^2}{x^2} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2} + \frac{8}{x^2}}{\frac{x}{x^2} - \frac{7}{x^2}} = \lim_{x \rightarrow \infty} \frac{1 + 0}{1 - 0} = \lim_{x \rightarrow \infty} \frac{1}{1} = 1$$

$$y = 1$$

Kosa asimptota ne postoji!

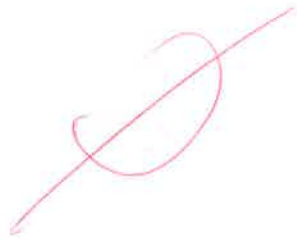
$$f(x) = \frac{x^2 + 8}{x - 7}$$

$$f(x)' = \frac{x^2 + 8 + (x - 7) \cdot 2x}{(x - 7)^2}$$

$$f(x)' = \frac{x^2 + 8 - 2x^2 + 14x}{(x - 7)^2}$$

$$f(x)' = \frac{-x^2 + 14x + 8}{(x - 7)^2}$$

SKACA 7
GRAFA



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

POPUNJAVA

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod

£1

NASTAVNIK

IME I PREZIME:

Sejla Šakanović

VRIJEME POČETKA:

Broj ↓
bodova

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0386-2014

1. Riješiti jednačinu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

12

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije.

20 graf

3. Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf.

20 graf

4. Zadana je funkcija $f(x) = \sqrt{7+5x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x=2$ i skicirati je uz graf.

4+4+4+6/4

5. Gaussovom metodom riješiti sustav:

15+3

12

$$2x - 3y - z + 2w + 3v = 4$$

$$4x - 4y - z + 4w + 11v = 4$$

$$2x - 5y - 2z + 2w - v = 9$$

$$2y + z + 4v = -5$$

Provjeri vrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

Ukupno:

4

1. $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

$$\frac{2x+1}{2\sqrt{x^2+x}} = -2 \quad | \cdot 2\sqrt{x^2+x}$$

$$2x+1 = -2 \cdot 2\sqrt{x^2+x}$$

$$2x+1 = -4\sqrt{x^2+x} \quad | \cdot 2$$

$$2x+4\sqrt{x^2+x} = -1 \quad | \cdot 2$$

$$4x^2 + 16(x^2+x) = -1 \quad | \cdot 2$$

$$4x^2 + 16x^2 + 16x = -1$$

$$20x^2 + 16x + 1 = 0$$

$$x_{1,2} = \frac{-16 \pm \sqrt{16^2 - 4 \cdot 20 \cdot 1}}{40} = \frac{-16 \pm 4\sqrt{11}}{40}$$

PROVJERA?

2. $g(x) = \arctan(e^x)$

$$Df = R$$

$$f'(x) = \frac{1}{1+(e^x)^2} \cdot (e^x)' = \frac{e^x}{1+e^{2x}}$$

$$③ f(x) = \frac{x^2 + 8}{x - 7}$$

1°

$$x - 7 \neq 0$$

$$x \neq 7$$

$$Df = \mathbb{R} \setminus \{7\}$$

2° V. asymptota

$$\lim_{x \rightarrow 7^+} \frac{x^2 + 8}{x - 7} = +\infty$$

$$\lim_{x \rightarrow 7^-} \frac{x^2 + 8}{x - 7} = -\infty$$

$$O.V.A. \quad x = 7$$

Horizontalna a. 14

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 + 8}{x - 7} \Big| : x^2 = \lim_{x \rightarrow \pm\infty} \frac{1 + \frac{8}{x^2}}{\frac{1}{x} - \frac{7}{x^2}} = \frac{1 + 0}{0 - 0} = \frac{1}{0} = 0$$

$$O.H.A. \quad y = 0$$

3° $f(x) = 0$

$$x^2 + 8 = 0$$

$$x^2 = 8$$

$$x_1 = \sqrt{8}$$

$$x_2 = -\sqrt{8}$$

$$NT(\sqrt{8}, 0)$$

$$(-\sqrt{8}, 0)$$

$$f(0) = -1,14$$

$$ST(0, -1,14)$$

$$4° f'(x) = \frac{(x^2 + 8)' \cdot (x - 7) - (x^2 + 8) \cdot (x - 7)'}{(x - 7)^2} = \frac{2x \cdot (x - 7) - (x^2 + 8) \cdot 1}{(x - 7)^2}$$

$$= \frac{2x^2 - 14x - x^2 - 8}{(x - 7)^2} = \frac{x^2 - 14x - 8}{(x - 7)^2}$$

$$f'(x) = 0$$

$$x^2 - 14x - 8 = 0$$

$$x_{1,2} = \frac{14 \pm \sqrt{14^2 - 4 \cdot 1 \cdot (-8)}}{2} = \frac{14 \pm \sqrt{228}}{2}$$

$$x_1 = 14,55 \quad (14,55, 29,09) \text{ max}$$

$$x_2 = -0,55 \quad (-0,55, -1,02) \text{ min}$$

5.
$$\begin{bmatrix} 2 & -3 & -1 & 2 & 3 & | & 4 \\ 4 & -4 & -1 & 4 & 11 & | & 4 \\ 2 & -5 & -2 & 2 & -1 & | & 9 \\ 0 & 0 & 2 & 1 & 4 & | & -5 \end{bmatrix} \xrightarrow{1 \cdot (-2)}$$

$$\begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & | & 2 \\ 4 & -4 & -1 & 4 & 11 & | & 4 \\ 2 & -5 & -2 & 2 & -1 & | & 9 \\ 0 & 0 & 2 & 1 & 4 & | & -5 \end{bmatrix} \xrightarrow{\begin{matrix} \cdot (-4) / \cdot (-2) \\ \leftarrow + \end{matrix}}$$

$$\begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & | & 2 \\ 0 & 2 & 1 & 0 & 5 & | & -4 \\ 0 & -2 & -1 & 0 & -4 & | & 5 \\ 0 & 0 & 2 & 1 & 4 & | & -5 \end{bmatrix} \xrightarrow{1 \cdot (2)}$$

$$\begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & | & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & | & -2 \\ 0 & -2 & -1 & 0 & -4 & | & 5 \\ 0 & 0 & 2 & 1 & 4 & | & -5 \end{bmatrix} \xrightarrow{\begin{matrix} \cdot \frac{3}{2} / \cdot 2 \\ \leftarrow \end{matrix}}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & | & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & | & -2 \\ 0 & 0 & 0 & 0 & 1 & | & 9 \\ 0 & 0 & 2 & 1 & 4 & | & -5 \end{bmatrix} \xrightarrow{1 \cdot (-2)}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & | & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & | & -2 \\ 0 & 0 & 1 & \frac{1}{2} & 3 & | & \frac{13}{2} \\ 0 & 0 & 0 & 0 & 1 & | & 9 \end{bmatrix} \xrightarrow{\begin{matrix} \leftarrow + \\ \leftarrow + \\ \cdot \frac{1}{2} / \cdot \left(-\frac{1}{4}\right) \end{matrix}}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & | & 11 \\ 1 & 0 & 0 & \frac{7}{8} & \frac{9}{2} & | & \frac{11}{8} \\ 0 & 1 & 0 & -\frac{1}{4} & 1 & | & \frac{19}{8} \\ 0 & 0 & 1 & \frac{1}{4} & 3 & | & \frac{13}{4} \\ 0 & 0 & 0 & 0 & 1 & | & \frac{9}{2} \end{bmatrix}$$

$$\begin{aligned} x_4 &= t = 0 \\ t + 1x_5 &= 9 \implies x_5 = 9 - t \\ x_3 &= \frac{1}{7} + \frac{1}{2}t \\ 1x_2 - \frac{1}{4}t + 9 - t &= -\frac{19}{4} \\ x_2 &= \frac{19}{5} + \frac{5}{4}t \end{aligned}$$

$$\begin{aligned} x_1 + \frac{7}{8}t + \frac{9}{2}(9-t) &= -\frac{11}{8} \\ x_1 + \frac{7}{8}t + \frac{81}{2} - \frac{9}{2}t &= -\frac{11}{8} \\ x_1 &= -\frac{165}{8} + \frac{29}{8}t \end{aligned}$$

PROUJERA

$$2y + z + 4v = 2 \cdot \frac{19}{5} + \frac{5}{2}t + \frac{1}{7} + \frac{1}{2}t + 4 \cdot (9-t) = \frac{38}{5} + \frac{1}{7} + 36 + \left(\frac{5}{2} + \frac{1}{2} - 4\right)t \neq -5$$

6.
$$\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2} - x}{x-1} = \frac{\sqrt{2-1^2} - 1}{1-1} = \frac{0}{0} = +\infty$$

$$\lim_{x \rightarrow 1^-} \frac{\sqrt{2-x^2} - x}{x-1} = -\infty$$

$$\lim_{x \rightarrow 1^+} \frac{\sqrt{2-x^2} - x}{x-1} = +\infty$$

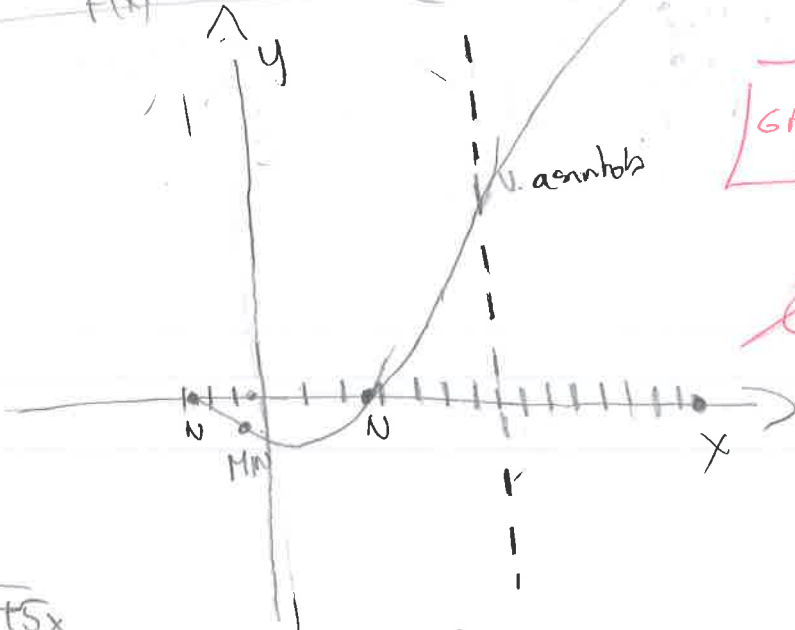
Matematika 1

Ime i prezime: Šejla Šakarović

Matični broj u indeksu: 17-2-0386-2014

③ nastavak skacio
verh

	$-\infty$	$-0,55$	2	$4,55$	$+\infty$
$f'(x)$		-	-	+	+
$f(x)$					



GRAF PRILJEŽE UZ ASIMPTOTE

④ $f(x) = \sqrt{7+5x}$

$$f'(x) = \frac{1}{2\sqrt{7+5x}} \cdot (7+5x)' = \frac{5}{2\sqrt{7+5x}}$$

$f'(x) = 0$ Nema
 $5 = 0$ // min
// max

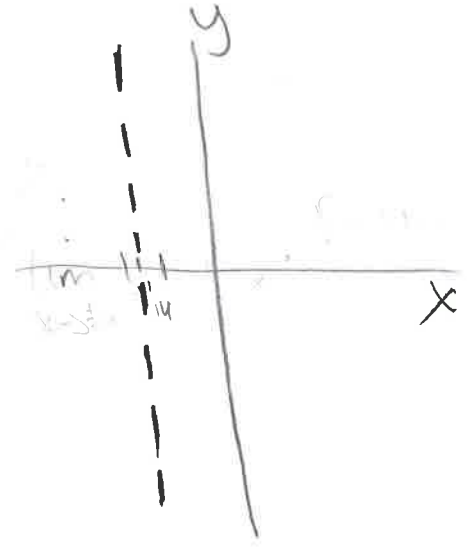
$7+5x \neq 0$
 $5x \neq -7$
 $x \neq -\frac{7}{5}$

$D_f = \mathbb{R} \setminus \left\{ -\frac{7}{5} \right\}$

V. asimptota

$$\lim_{x \rightarrow -\frac{7}{5}^+} \sqrt{7+5x} = +\infty$$

$$\lim_{x \rightarrow -\frac{7}{5}^-} \sqrt{7+5x} = -\infty$$



tangenta

$f(x) = \sqrt{7+5x}$ $x = 2$

$y = 4,12$

$k = f'(x_0)$

$f'(x) = \frac{5}{2\sqrt{7+5x}}$

$y - y_0 = f'(x_0)(x - x_0)$

$k = f'(2)$

$y = 0,61(x - 2) + 4,12$

$k = 0,61$

$y = 0,61x + 2,9$

SKICE?

4

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

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ε1

IME I PREZIME: **MARIO MATEO TORIĆ**

VRIJEME POČETKA: **9:15**

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

026 0308 0456

1. Riješiti jednačbu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$ 12
2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf
3. Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf. 20 graf
4. Zadana je funkcija $f(x) = \sqrt{7+5x}$. Koji su lokalni ekstremi? Koji su globalni ekstremi? Skicirati graf. Pronaći tangentu za $x = 2$ i skicirati je uz graf. 4+4+4+6
5. Gaussovom metodom riješiti sustav: 15+3

$$\begin{aligned}2x - 3y - z + 2w + 3v &= 4 \\4x - 4y - z + 4w + 11v &= 4 \\2x - 5y - 2z + 2w - v &= 9 \\2y + z + 4v &= -5\end{aligned}$$

Provjeri vrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

Ukupno:

1.) $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0 \quad / \cdot 2\sqrt{x^2+x}$

$$2\sqrt{x^2+x} + 2x+1 = 0 \quad / \cdot 2$$
$$4x^2 + 4x + 4x^2 + 1 = 0 \quad \times$$
$$8x^2 + 4x + 1 = 0$$
$$a = 8 \quad b = 4 \quad c = 1$$
$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 4 \cdot 8 \cdot 1}}{16}$$
$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 32}}{16}$$
$$x_{1,2} = \frac{-4 \pm \sqrt{-16}}{16} \Rightarrow \text{NEMA RIJEŠENJA}$$

3. tip $\frac{a}{b} = 0$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

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IME I PREZIME: *Ivan Štefanić*

VRIJEME POČETKA: *8:15*

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0268-2013

- Riješiti jednačbu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$ 12
- Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf
- Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf. 20 graf
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$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

Provjeri vrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2} - x}{x-1}$.

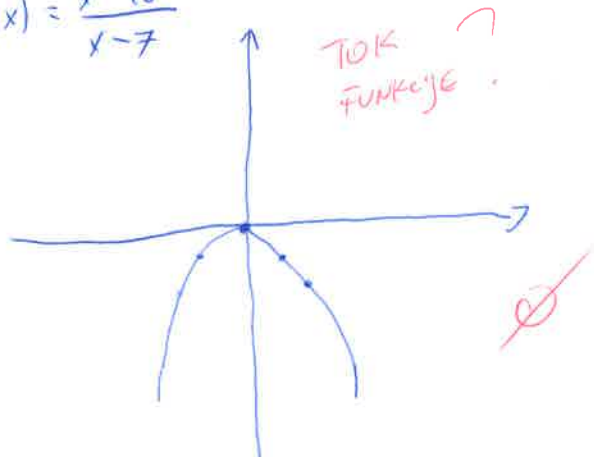
$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sqrt{2-x^2} - x}{x-1} &= \frac{0}{0} \\ \lim_{x \rightarrow 1} \frac{1}{0} &= 1 \end{aligned}$$

10+2

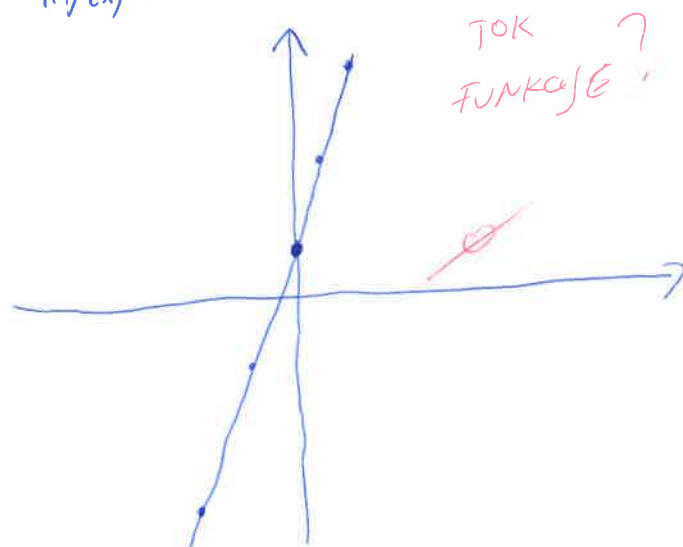
Ukupno:

~~0~~

3. $f(x) = \frac{x^2+8}{x-7}$



4. $f(x) = \sqrt{7+5x}$



1. $x = \frac{1}{2}$

$$2 + \frac{2 \cdot \frac{1}{2} + 1}{2 \sqrt{\left(\frac{1}{2}\right)^2 + \frac{1}{2}}} = -$$

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NASTAVNIK

IME I PREZIME: **BEUNO STIPANOVIĆ**

VRIJEME POČETKA:

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MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **17-1-0189-2013**

- Riješiti jednadžbu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$ 12
- Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije. 20 graf
- Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf. 20 graf
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$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

$$\frac{-9}{1} \frac{41}{8} = -\frac{369}{8} + \frac{7}{8} = \frac{376}{8}$$

Provjeri vrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

Ukupno:

⑤

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= 4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned} \quad \begin{bmatrix} 2 & -3 & -1 & +2 & +3 & | & 4 \\ 4 & -4 & -1 & 4 & 11 & | & 4 \\ 2 & -5 & -2 & 2 & -1 & | & 9 \\ 0 & 2 & 1 & 0 & 4 & | & 5 \end{bmatrix} \begin{matrix} :2 \\ \sim \\ \sim \\ \sim \end{matrix}$$

$$\begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & | & 2 \\ 0 & -2 & 1 & 0 & 5 & | & -4 \\ 0 & -2 & -3 & 0 & -4 & | & 5 \\ 0 & 2 & 1 & 0 & 4 & | & 5 \end{bmatrix} \begin{matrix} \cdot(-4), \cdot(-2) \\ :2 \\ \sim \end{matrix} \rightarrow \begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & | & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & | & -2 \\ 0 & -2 & -3 & 0 & -4 & | & 5 \\ 0 & 2 & 1 & 0 & 4 & | & 5 \end{bmatrix} \begin{matrix} \sim \\ \cdot(\frac{3}{2}), \cdot 2, \cdot(-2) \\ \sim \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & | & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & | & -2 \\ 0 & 0 & -2 & 0 & 1 & | & 1 \\ 0 & 0 & 0 & 0 & -1 & | & 9 \end{bmatrix} \begin{matrix} \sim \\ \cdot(-2) \\ \sim \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{4} & 1 & \frac{21}{4} & | & -1 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & | & -2 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & | & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & -1 & | & 9 \end{bmatrix} \begin{matrix} \sim \\ \cdot(\frac{1}{2}), \cdot(-\frac{1}{2}) \\ \sim \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & \frac{41}{8} & | & \frac{7}{8} \\ 0 & 1 & 0 & 0 & \frac{11}{4} & | & -\frac{7}{4} \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & | & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & | & -9 \end{bmatrix} \begin{matrix} \sim \\ \cdot(-1) \\ \sim \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & \frac{41}{8} & | & \frac{7}{8} \\ 0 & 1 & 0 & 0 & \frac{11}{4} & | & -\frac{7}{4} \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & | & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & | & -9 \end{bmatrix} \begin{matrix} \sim \\ \cdot(\frac{1}{2}), \cdot(\frac{11}{4}), \cdot(-\frac{41}{8}) \\ \sim \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & | & 47 \\ 0 & 1 & 0 & 0 & 0 & | & 82 \\ 0 & 0 & 1 & 0 & 0 & | & -5 \\ 0 & 0 & 0 & 0 & 1 & | & -9 \end{bmatrix}$$

$$x+w=47$$

$$y = \frac{82}{2}$$

$$z = -5$$

$$v = -9$$

PROUJERA:

$$\begin{aligned} 2y + z + 4v &= 2 \cdot \frac{82}{2} - 5 + 4 \cdot (-9) \\ &= 82 - 5 - 36 \neq -5 \end{aligned}$$

~~∅~~

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

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ε1

IME I PREZIME: Kristijan Kokić

VRIJEME POČETKA: 9⁰⁰

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

57652-2009

1. Riješiti jednačbu: $2 + \frac{2x+1}{2\sqrt{x^2+x}} = 0$

~~12~~

2. Za funkciju $g(x) = \arctan(e^x)$ temeljem ispitivanja funkcijskog tijeka napraviti skicu grafa funkcije.

20 graf

3. Odrediti tok funkcije $f(x) = \frac{x^2+8}{x-7}$ i skicirati graf.

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4+4+4+6

5. Gaussovom metodom riješiti sustav:

~~15+3~~

$$2x - 3y - z + 2w + 3v = 4$$

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Provjeri vrštavanjem!

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{\sqrt{2-x^2}-x}{x-1}$.

10+2

Ukupno:

5.

$$\left[\begin{array}{cccccc|c} 2 & -3 & -1 & 2 & 3 & 1 & 4 \\ 4 & -4 & -1 & 4 & 11 & & 4 \\ 2 & -5 & -2 & 2 & -1 & & 9 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \rightarrow \left[\begin{array}{cccccc|c} 2 & -3 & -1 & 2 & 3 & 1 & 4 \\ 2 & -5 & -2 & 2 & -1 & & 9 \\ 4 & -4 & -1 & 4 & 11 & & 4 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \leftarrow (-2) \cdot (-1)$$
$$\sim \left[\begin{array}{cccccc|c} 0 & 2 & 1 & 0 & 4 & & -5 \\ -4 & 10 & 4 & -4 & 2 & & -18 \\ 4 & -4 & -1 & 4 & 11 & & 4 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \cdot (-1) \rightarrow \left[\begin{array}{cccccc|c} 0 & 2 & 1 & -1 & 0 & & 10 \\ 0 & 6 & 3 & 0 & 13 & & -14 \\ 4 & -4 & -1 & 4 & 11 & & 4 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right]$$
$$\left[\begin{array}{cccccc|c} 4 & -4 & -1 & 4 & 11 & & 4 \\ 0 & 6 & 3 & 0 & 13 & & -14 \\ 0 & 2 & -1 & -1 & 0 & & 0 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \begin{array}{l} : (4) \\ : (6) \\ \leftarrow \downarrow + \cdot (-4) \end{array} \rightarrow \left[\begin{array}{cccccc|c} 1 & -1 & \frac{1}{4} & 1 & \frac{11}{4} & & 1 \\ 0 & 6 & 3 & 0 & 13 & & -14 \\ 0 & 0 & -2 & -1 & 0 & & 0 \\ 0 & -2 & 3 & 2 & 4 & & 0 \end{array} \right]$$

$$\textcircled{1} \quad 2f \quad \frac{2x+1}{2\sqrt{x^2+x}} = 0$$

$$\frac{\cancel{2}x+3}{\sqrt{x+x}} = \frac{x+3}{\sqrt{2x}}$$

