MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. PIŠITE DVOSTRANO! Obavezno popuniti sva polja ispod⇓

$\mathcal{D}1$

POPUNJAVA NASTAVNIK
Broj ↓
bodova

IME I PREZIME: JURE GENDA

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0326-2013

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 3x - 3}$ i skicirati graf.

20 graf

2. Riješiti: $\dfrac{z+i}{z-i} = \overline{2+3i}$. Prikaži rješenje u kompleksnoj ravnini!

12+3

3. Odrediti konvergenciju reda $\sum_n \dfrac{n^2}{2^n}$.

15

4. Odrediti tok funkcije $f(x) = \dfrac{x^2+3}{x^2-3}$ i skicirati graf.

20 graf

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost.

6+6+3

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2-3x}$, u točki gdje je $x = 0$.

15

Ukupno:

35

6.) $f(x) = e^{x^2-3x}$ 　　　 $x_1 = 0$

$y_0 = f(x_0) = 1$

$f'(x) = (2x-3)e^{x^2-3x}$

$f'(x_0) = 3$ 　✗　 $\boxed{f'(0) = (2\cdot 0 - 3)e^{0^2-3\cdot0} = -3}$

$t \ldots y - y_0 = f'(x_0)(x - x_0)$

$t \ldots y - 1 = (-3)(x - 0)$

$t \ldots y = 3x + 1$ ✗

3.) $\sum_n \dfrac{n^2}{2^n}$

$q = \lim\limits_{n\to\infty} \dfrac{Q_{n+1}}{Q_n} = \lim\limits_{n\to\infty} \dfrac{\frac{(n+1)^2}{2^{n+1}}}{\frac{n^2}{2^n}} = \lim\limits_{n\to\infty} \dfrac{\frac{n^2+2n+1}{2\cdot2^n}}{\frac{n^2}{2^n}} = \lim\limits_{n\to\infty} \dfrac{n^2+2n+1}{n^2} = \dfrac{1}{2} < 1$ ✓

PROMATRANI RED KONVERGIRA!

④ $f(x) = \dfrac{x^2+3}{x^2-3}$

$x^2-3 \neq 0$

$x \neq \pm\sqrt{3}$

$D = \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\} \rightarrow$ DOMENA

NEMA NULTOČAKA !

ASIMPTOTE :

→VERTIKALNE : $\left.\begin{array}{l} x = \sqrt{3} \\ x = -\sqrt{3} \end{array}\right\}$ VERTIKALNE ASIMPTOTE

→HORIZONTALNE : $\displaystyle\lim_{x\to\infty} \dfrac{x^2+5}{x^2-3} = 1$
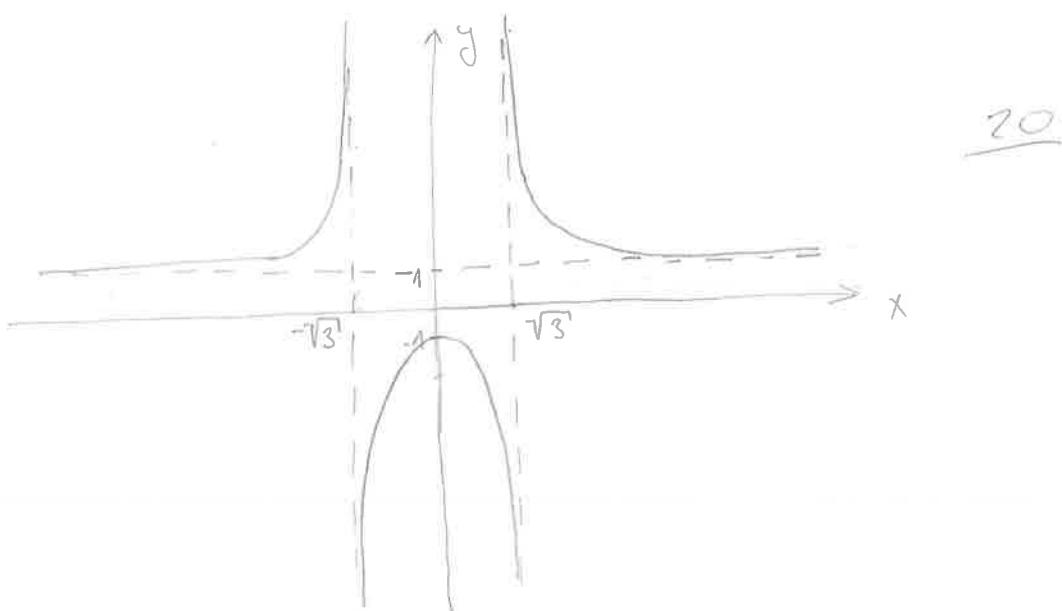
$y = 1 \rightarrow$ HORIZONTALNA ASIMPTOTA

$f'(x) = \dfrac{2x(x^2-3) - (x^2+3)\cdot 2x}{(x^2-3)^2} : 0 \Rightarrow x = 0$

ZA $x \in \langle-\infty, 0\rangle$ $f'(x) > 0$ pa je funkcija strogo rastuća

ZA $x \in \langle 0, +\infty\rangle$ $f'(x) < 0$ pa je funkcija strogo padajuća

STOGA FUNKCIJA IMA MAKSIMUM U TOČKI $M(0, -1)$

<u>20</u>

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{D}1$

IME I PREZIME: TONI GRBIĆ   VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-1-0288-2014

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 3x - 3}$ i skicirati graf.   ~~20 graf~~

2. Riješiti: $\dfrac{z + i}{z - i} = \overline{2 + 3i}$. *Prikaži rješenje u kompleksnoj ravnini!*   ~~12+3~~

3. Odrediti konvergenciju reda $\sum_n \dfrac{n^2}{2^n}$.   15

4. Odrediti tok funkcije $f(x) = \dfrac{x^2 + 3}{x^2 - 3}$ i skicirati graf.   ~~20 graf~~

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x + 2} + \sqrt{4 - x}$. Posebno komentirati (ne)ograničenost.   ~~6+6+3~~

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 3x}$, u točki gdje je $x = 0$.   ~~15~~

Ukupno:

~~∅~~

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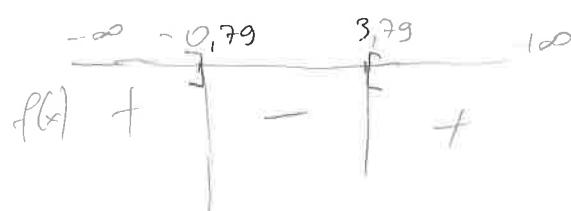
① $f(x) = x + \sqrt{x^2 - 3x - 3}$

$x^2 - 3x - 3 \geqslant 0$

$x^2 - 3x - 3 = 0$
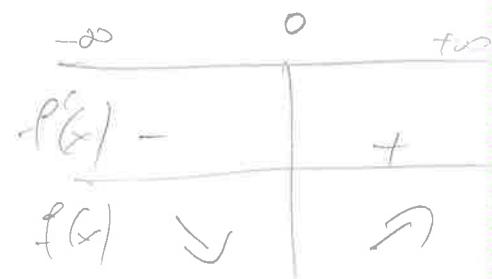
$x_{1,2} = \dfrac{3 +- \sqrt{9 - 4 \cdot (-3)}}{2} = \dfrac{3 +- \sqrt{21}}{2}$ $x_1 = 3,79$ $x_2 = -0,79$

$$f'(x) = 1 + \frac{1}{2\sqrt{x^2 - 3x - 3}} \cdot 2x - 1$$

$$= 1 + \frac{2x - 1}{2\sqrt{x^2 - 3x - 3}} = \frac{x}{\sqrt{x^2 - 3x - 3}}$$

N.T $x = 0$

$f(x)$	$-\infty$	$+$	$-0,79$	$-$	$3,79$	$+$	$+\infty$

	$-\infty$		0		$+\infty$
$f'(x)$		$-$		$+$	
$f(x)$		$\searrow$		$\nearrow$	

$Df: \langle -\infty, -0,79] \cup [3,79, +\infty\rangle$
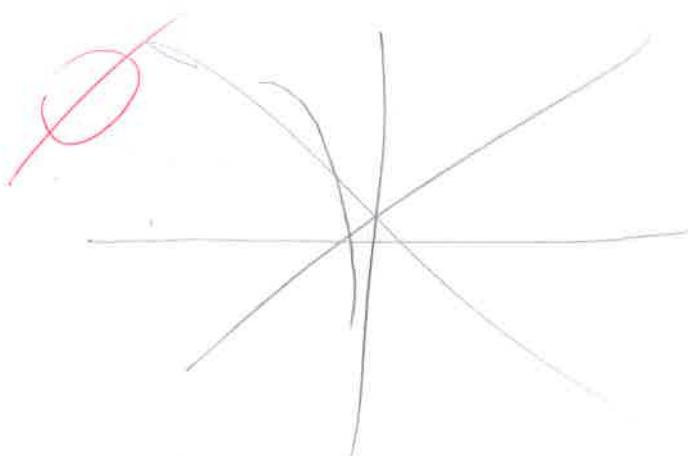
N.T. $x + \sqrt{x^2 - 3x - 3} = 0 \Big/^2$

$x^2 + x^2 - 3x - 3 = 0$

$2x^2 - 3x - 3 = 0$

② $\dfrac{2+i}{2-i} = \overline{2+3i}$

$\dfrac{x+yi+i}{x+yi-i} = 2-3i \quad / \cdot (x+yi-i)$

$x+yi+i = 2x + 2yi - 2i - 3xi - 3yi^2 + 3i^2$

$x+yi+i = 2x + 2yi - 2i - 3xi + 3y - 3$

$yi + i = 2yi - 2i + 3y - 3$

$yi = 2yi - 2i + 3y - 3 - i$

$yi = 2yi - 3i + 3y - 3 \quad / \cdot i$

$y = \dfrac{2yi - 3i + 3y - 3}{i}$

∅

⑤ $f(x) = \sqrt{x+2} + \sqrt{4-x}$

$f'(x) = \dfrac{1}{2\sqrt{x+2}} - \dfrac{1}{2\sqrt{4-x}}$

$f'(x)$

Ime i prezime: TONI GRBIC **Matični broj u indeksu:**

4) $f(x) = \dfrac{x^2+3}{x^2-3}$

$DP = x^2 - 3 \neq 0$

$x^2 \neq 3 /\sqrt{}$

$x \neq \pm\sqrt{3}$

$\mathcal{D}f, \langle-\infty, -\sqrt{3}\rangle \cup \langle-\sqrt{3}, \sqrt{3}\rangle \cup \langle\sqrt{3}, +\infty\rangle$

N.T./
NEMA

$f'(x) = \dfrac{2x(x^2+3) - 2x(x^2-3)}{(x^2-3)^2} =$

$f'(x) = \dfrac{2x^3 + 6x - 2x^3 + 6x}{(x^2-3)^2} =$

V.A.

$\lim\limits_{x\to-\sqrt{3}} \dfrac{x^2+3}{x^2-3} = 0$

NEMA . V.A.

$= \dfrac{12x}{(x^2-3)^2}$

$\lim\limits_{x\to\sqrt{3}} \dfrac{x^2+3}{x^2-3} =$
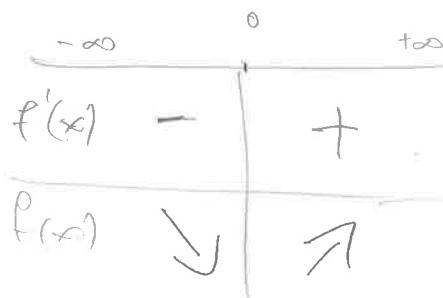
$12x = 0$

$x = 0$
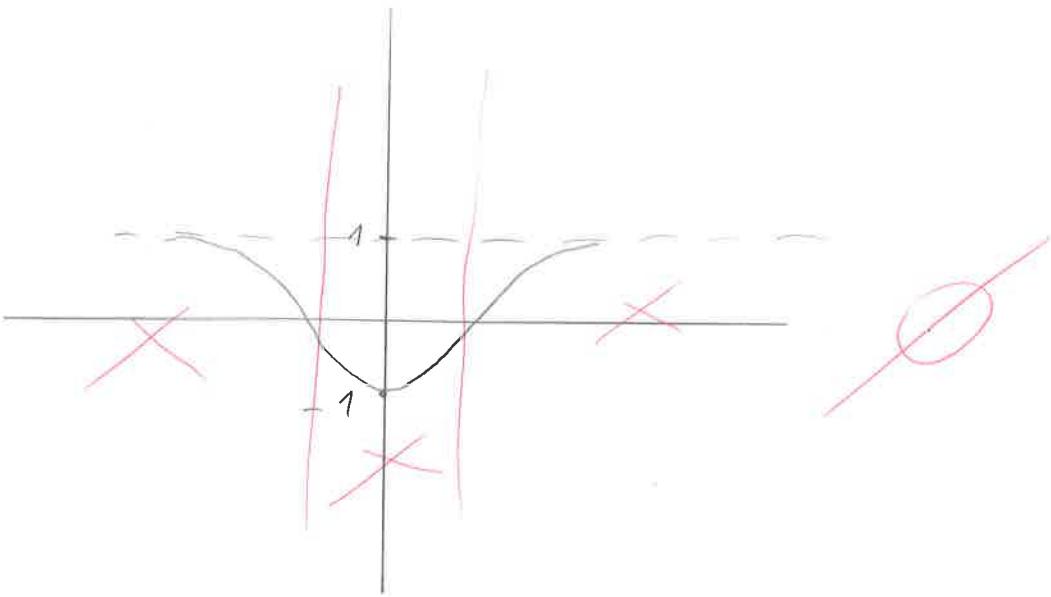
H.A. $\lim\limits_{x\to-\infty} \dfrac{x^2+3/:x^2}{x^2-3/:x^2} \quad \dfrac{\infty}{\infty}$

$\lim\limits_{x\to-\infty} \dfrac{\dfrac{x^2}{x^2}+\dfrac{3}{x^2}}{\dfrac{x^2}{x^2}-\dfrac{3}{x^2}} = \dfrac{1+0}{1-0} = 1$

$\lim\limits_{x\to\infty} = \dfrac{\dfrac{x^2}{x^2}+\dfrac{3}{x^2}}{\dfrac{x^2}{x^2}-\dfrac{3}{x^2}} = \dfrac{1+0}{1+0} = 1$

	$-\infty$	0	$+\infty$
$f'(x)$	$-$		$+$
$f(x)$	$\searrow$		$\nearrow$

$Tmin(0, -1)$

6)

$$y - f(x_0) = f'(x_0)(x - x_0) \quad , \quad x = 0$$

$$f(x_0) = e^0 = 1$$

$$y - 1 = 1(x - 0)$$

$$y - 1 = x$$

$$y = x + 1$$

$$f'(x) = e^{x^2 - 3x} \cdot 1$$

$$f'(x) = e^{x^2 - 3x}$$

$$f'(x_0) =$$

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 3x - 3}$ i skicirati graf.   20 graf

2. Riješiti: $\dfrac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!*   12+3

3. Odrediti konvergenciju reda $\sum\limits_{n} \dfrac{n^2}{2^n}$.   15

4. Odrediti tok funkcije $f(x) = \dfrac{x^2+3}{x^2-3}$ i skicirati graf.   20 graf

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost.   6+6+3

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 3x}$, u točki gdje je $x = 0$.   15

Ukupno:

$2.$ $\dfrac{z+i}{z-i} = \overline{2+3i}$

$\dfrac{z+i}{z-i} = 2 - 3i \; / \cdot (z-i)$

$z+i = 2z - 2i - 3zi + 3i^2$

$z+i = 2z - 2i - 3zi - 3$

$-z + 3zi = -3 - 3i$ $z = ?$

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 3x - 3}$ i skicirati graf.

    20 graf

2. Riješiti: $\dfrac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!*

    12+3

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    15

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    6+6+3

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    15

Ukupno:

$\mathcal{O}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{D}1$

IME I PREZIME: ANTE SKOBLAR   VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 17-2-0132-11

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 3x - 3}$ i skicirati graf.   20 graf

2. Riješiti: $\dfrac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!*   12+3

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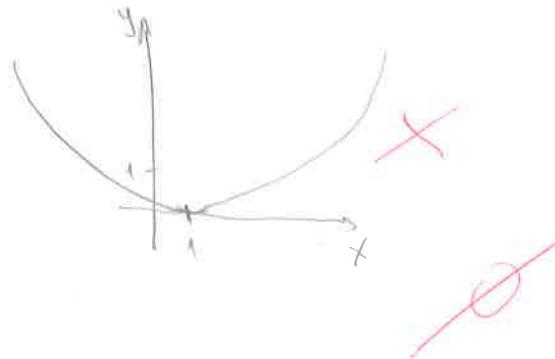
6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 3x}$, u točki gdje je $x = 0$.   15

Ukupno:

6. $f(x) = e^{x^2 - 3x}$ $f_{(0)} = e^0$

   $x = 0$ $f_{(0)} = 1$

1. $f(x) = x + \sqrt{x^2 - 3x - 3}$

$a = 1$
$b = -3$
$c = -3$

$x_{1,2} = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \dfrac{-3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$
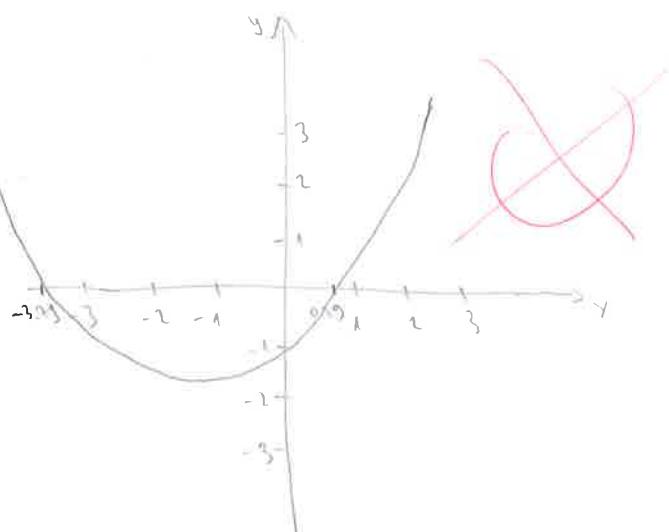
$x_{1,2} = \dfrac{-3 \pm \sqrt{9 + 12}}{2}$

$x_{1,2} = \dfrac{-3 \pm \sqrt{21}}{2}$
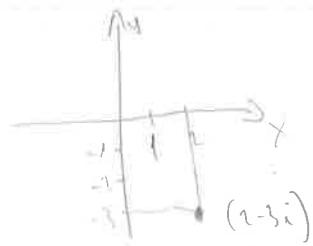
$x_{1,2} = \dfrac{-3 \pm 4.58}{2}$

$x_1 = 0.79$

$x_2 = -3.79$

2. $\dfrac{z+1}{z-1} = \overline{2+3i}$

$\dfrac{z+1}{z-1} = 2-3i \;\Big|\cdot z-1$

$z+1 = z-1(2-3i)$

$z = ?$

4. $f(x) = \dfrac{x^2+3}{x^2-3}$

$x^2-3 \le 0$

$x^2-3 = 0$

$x^2 - 3$

$x = \sqrt{3}$

$x = 1.73$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

$\mathcal{D}1$

IME I PREZIME: Josip Mihoć   VRIJEME POČETKA: 27

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0363088763

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 3x - 3}$ i skicirati graf.

    20 graf

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    15

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    ~~20 graf~~

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x + 2} + \sqrt{4 - x}$. Posebno komentirati (ne)ograničenost.

    6+6+3

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 3x}$, u točki gdje je $x = 0$.

    15

$f(x) = e^{x^2 - 3x}$

Ukupno:

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓

D1

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Josipa Jozević   VRIJEME POČETKA: 08:30

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0265 078354

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 3x - 3}$ i skicirati graf.   20 graf

2. Riješiti: $\dfrac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!*   12+3

3. Odrediti konvergenciju reda $\sum\limits_n \dfrac{n^2}{2^n}$.   15

4. Odrediti tok funkcije $f(x) = \dfrac{x^2+3}{x^2-3}$ i skicirati graf.   20 graf 10

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost.   6+6+3

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 3x}$, u točki gdje je $x = 0$.   15

Ukupno:

10

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① $f(x) = x + \sqrt{x^2 - 3x - 3}$

1) DOMENA

$x^2 - 3x - 3 \geq 0$

$x^2 - 3x \geq 3$

$x(x - 3x) \geq 3$

$\boxed{x \geq 3}$ ✗

$x - 3 \geq 3$

$x \geq 3$

	$+\infty$	3	$+\infty$
x	$+$	$+$	
$x-3$	$-$	$+$	
		$+$	⊕

$D(x): x \in \mathbb{R} \setminus [3, +\infty)$ ✗
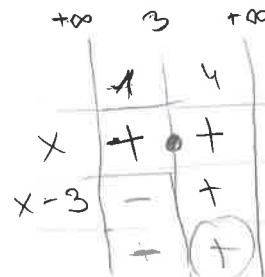
② NULTOČKE

$x + \sqrt{x^2 - 3x - 3} = 0 \quad /^2$

$x^2 + x^2 - 3x - 3 = 0$

$2x^2 - 3x - 3 = 0$

$x_{1,2} = \dfrac{3 \pm \sqrt{9 - 4 \cdot 2 \cdot (-3)}}{4} = \dfrac{3 \pm \sqrt{33}}{4}$

$x_1 = \dfrac{3 + \sqrt{33}}{4} = 2,2$

$x_2 = \dfrac{3 - \sqrt{33}}{4} = -0,7$

$N_1 (2,2 , 0)$

$N_2 (-0,7 , 0)$

③ VA

$\lim\limits_{x \to 3} x + \sqrt{x^2 - 3x - 3} = \infty - \infty \quad / \cdot \dfrac{x - \sqrt{x^2 - 3x - 3}}{x - \sqrt{x^2 - 3x - 3}}$

$\lim\limits_{x \to 3} \dfrac{x^2 - (x^2 - 3x - 3)}{x - \sqrt{x^2 - 3x - 3}} = \lim\limits_{x \to 3} \dfrac{3x + 3}{x - \sqrt{x^2 + 3x - 3}} = \dfrac{12}{3 - \sqrt{9 - 5 - 3}} = \dfrac{12}{3 - \sqrt{-3}}$

nema V.A.

H.A.

$$\lim_{x \to \pm\infty} \frac{3x+3 \mid :x}{x - \sqrt{x^2-3x-3} \mid :x} = \lim_{x \to \pm\infty} \frac{3 + \frac{3}{x}}{\frac{x}{x} - \sqrt{\frac{x^2}{x} - \frac{3x}{x} - \frac{3}{x}}}$$

JOSIPA JOZELIĆ

② $\dfrac{2+i}{z-i} = \overline{2+3i} \Big/ \cdot (z-i)$

$z^2 = -1$

$2+i = 2-3i \cdot (z-i)$
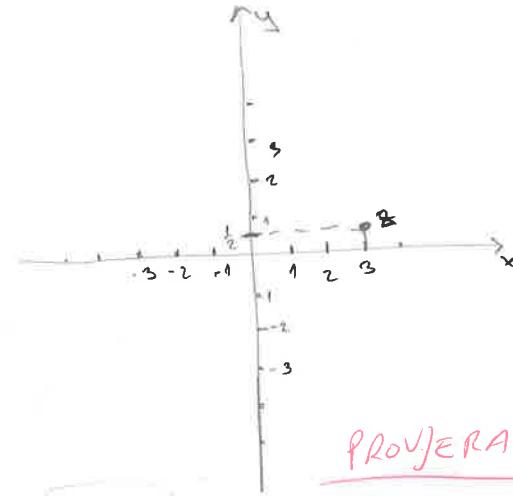
$x+yi+i = (2-3i)(x+yi-i)$

$x+yi+i = 2x+2yi-2i-3xi-3yi^2+3i^2$

$\underline{x}+\underline{yi}+\underline{i} = \underline{2x}+\underline{2yi}-\underline{2i}-\underline{3xi}+\underline{3y}-\underline{3}$

$x = 2x-3 \Rightarrow -x = -3/(-1) \Rightarrow \boxed{x=3}$

$y = 2y-2+3y \quad y-5y = -2 \Rightarrow -4y = -2/(-4) \Rightarrow \boxed{y=\dfrac{1}{2}}$

$\boxed{z = 3 + \dfrac{1}{2}i}$

④ $f(x) = \dfrac{x^2+3}{x^2-3}$

1) $x^2 - 3 \neq 0$

$x^2 \neq 3 \Big/ \sqrt{}$

$x \neq \pm\sqrt{3}$

$Df \; x \in \mathbb{R} \backslash \{\pm\sqrt{3}\}$

$\dfrac{x^2+3}{x^2-3} = 0 \Big/ (x^2-3)^2$

$(x^2+3)(x^2-3) = 0$

$x^4 + 3x^2 + 3x^2 - 9 = 0$

$x^4 = 9 \Big/ \sqrt{}$

$x^2 = 3 \Big/ \sqrt{}$

$\boxed{x = \pm\sqrt{3}}$

$N_1(+\sqrt{3}, 0)$
$N_2(-\sqrt{3}, 0)$

V.A
$\lim\limits_{x\to\sqrt{3}^+} \dfrac{x^2+3}{x^2-3} = \dfrac{(\sqrt{3}^+)^2+3}{(\sqrt{3}^+)^2-3} = \dfrac{6}{0^+} \Rightarrow +\infty$

$\lim\limits_{x\to\sqrt{3}^-} \dfrac{x^2+3}{x^2-3} = \dfrac{(\sqrt{3}^-)^2+3}{(\sqrt{3}^-)^2-3} = \dfrac{6}{0^+} \Rightarrow +\infty$

$\lim\limits_{x\to-\sqrt{3}^+} \dfrac{x^2+3}{x^2-3} = \dfrac{(-\sqrt{3}^-)^2+3}{(-\sqrt{3}^-)^2-3} = \dfrac{6}{0^+} \Rightarrow +\infty$

$\lim\limits_{x\to-\sqrt{3}^-} \dfrac{x^2+3}{x^2-3} = \dfrac{(-\sqrt{3}^+)^2+3}{(-\sqrt{3}^+)^2-3} = \dfrac{6}{0^+} \Rightarrow +\infty$

H.A
$\lim\limits_{x\to\infty} \dfrac{x^2+3 \;:x^2}{x^2-3 \;:x^2} = \lim \dfrac{1+\left(\dfrac{3}{x^2}\right)^{=0}}{1-\dfrac{3}{x^2}} = \boxed{1} \qquad \boxed{y=1 \; je \; H.A.}$

$$f(x) = \frac{x^2+3}{(x^2-3)}$$

$$f'(x) = \frac{(x^2+3)'\cdot(x^2-3) - (x^2+3)\cdot(x^2-3)'}{(x^2-3)^2} = \frac{2x(x^2-3) - (x^2+3)2x}{(x^2-3)^2} = \frac{2x^3-6x-2x^3-6x}{(x^2-3)^2} =$$

$$f'(x) = \frac{-12x}{(x^2-3)^2}$$
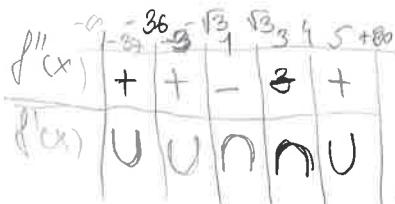
$$f'(x) = \frac{-12x}{(x^2-3)^2}$$

$$f''(x) = \frac{(-12x)'\cdot((x^2-3)^2) - (-12x)\cdot((x^2-3)^2)'}{(x^2-3)^4} =$$

$$f''(x) = \frac{-12((x^2-3)^2) + 12x\cdot 2(x^2-3)\cdot(x^2-3)'}{(x^2-3)^4}$$

$$= \frac{-12(x^4-6x^2+9) + 12x\cdot(2x^2-6)\cdot 2x}{(x^2-3)^4} =$$

$$= \frac{-12x^4+72x^2-108 + (24x^3-72x^2)\cdot 2x}{(x^2-3)^4} =$$

$$= \frac{-12x^4+72x^2-108 + 48x^4-144x^3}{(x^2-3)^4} = \frac{36x^4-144x^3+72x^2-108}{(x^2-3)^4}$$

$-12x = 0 \;/:-12$

$\boxed{x=0}$

max(0,0)

$f''(x) = 0$

$$f'(x) = \frac{-12x}{(x^2-3)^2}$$

$$f''(x) = -12$$

$36x^4-144x^3+72x^2 = 108$

$36x^2(x^2-4x+36) = 108$

$36x^2 = 108 \;/:36$

$x^2 = 3 \;|\sqrt{}$

$x = \pm\sqrt{3}$

$x^2-4x+36 = 0$

$x(x-4) = -36$

$\boxed{x = -36}$

$x-4 = 0$

$\boxed{x = 4}$

10

V. KA.

1. ZADATAK
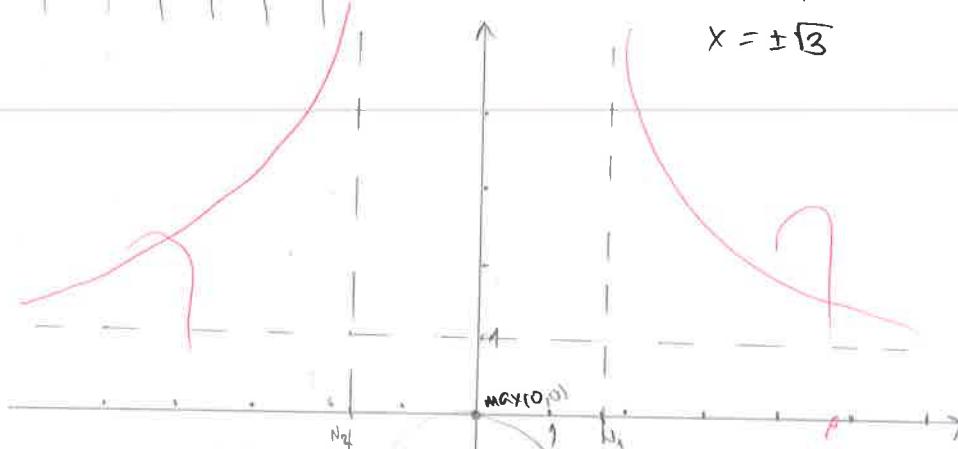
$$\lim_{x \to -\infty} x + \sqrt{x^2-3x-3} = [\infty, \infty] = \frac{x+\sqrt{x^2-3x-3}}{x} \cdot \frac{x-\sqrt{x^2-3x-3}}{x-\sqrt{x^2-3x-3}} = \frac{x-x^2-3x-3}{x^2-\sqrt{x^2-3x-3}} = \frac{-x^2-2x-3}{x^2-\sqrt{x^2-3x-3}}$$

$$L'H \lim_{x \to -\infty} \frac{-2x-2}{\left(x^2-\sqrt{x^2-3x-3}\right)^2} \Rightarrow \frac{-2 \cdot 3{,}79-2}{\left((-3{,}79)^2-\sqrt{(3{,}79)^2 \cdot 3 \cdot (3{,}79)-3}\right)^2} = \frac{-9{,}58}{\left(14{,}36-\sqrt{14{,}36-11{,}383}\right)^2}$$

$$= \frac{-9{,}58}{\left(14{,}36-\sqrt{0}\right)^2} = \frac{-9{,}58}{206{,}2} = 0{,}046$$

VIDI BURALO.

MAJA ŠOGORIĆ   D1

1. $f(x) = x + \sqrt{x^2 - 3x - 3}$
   $\underset{\geqslant 0}{}$

   $x^2 - 3x - 3 \geqslant 0$

   $x^2 - 3x - 3 = 0$

   $x_{1,2} = \dfrac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \dfrac{3 \pm \sqrt{9 + 12}}{2} = \dfrac{3 \pm \sqrt{21}}{2}$

   $x_1 = \dfrac{3 + \sqrt{21}}{2} = \dfrac{3 + 4,58}{2} = 3,79$

   $x_2 = \dfrac{3 - \sqrt{21}}{2} = \dfrac{3 - 4,58}{2} = -1.58$

   $$D(f) = \langle -\infty, -1.58 \rangle \cup \langle -1.58, 3,79 \rangle \cup \langle 3,79, +\infty \rangle$$

① V.A.
$\displaystyle\lim_{x \to -1.58^-} x + \sqrt{x^2 - 3x - 3} = \lim_{x \to -1.58^-} (-1.58) + \sqrt{(-1.58)^2 - 3 \cdot (-1.58) - 3} = \lim_{x \to -1.58^-} -1.58 + \sqrt{2,49 + 4,74} - 3 = \lim_{x \to -1.58^-} -1.58 + \sqrt{4,23}$

$\displaystyle\lim_{x \to -1.58^-} -1.58 + 2,05 = \lim_{x \to -1.58^-} 0.47$ NEMA V.A.

$\displaystyle\lim_{x \to 3,79^-} 3,79 + \sqrt{\underset{14,36 - 11,37 - 3}{3,79^2 - 3 \cdot 3,79 - 3}} = \lim_{x \to 3,79^-} 3,79 + (-0.01) = \lim_{x \to 3,79} 3,79$ nema V.A.

② H.A.
$\displaystyle\lim_{x \to +\infty} x + \sqrt{x^2 - 3x - 3} = \dfrac{x + \sqrt{x^2 - 3x - 3}}{1} \cdot \dfrac{x - \sqrt{x^2 - 3x - 3}}{x - \sqrt{x^2 - 3x - 3}} = \dfrac{x - x^2 - 3x - 3}{x - \sqrt{x^2 - 3x - 3}} = \dfrac{-x^2 - 2x - 3}{x - \sqrt{x^2 - 3x - 3}} = \left[\dfrac{-\infty}{\infty}\right]$

$\displaystyle\lim_{x \to \infty} \dfrac{-x^2 - 2x - 3}{x - \sqrt{x^2 - 3x - 3}} \Big|{:x^2} \;{:x^2} = \lim_{x \to \infty} \dfrac{-1 - \frac{2}{x}^0 - \frac{3}{x}^0}{\frac{1}{x} - \sqrt{1 - \frac{3}{x} - \frac{3}{x^2}}} = \dfrac{-1}{\sqrt{1}} = -1 \quad \boxed{y = -1}$

$\displaystyle\lim_{x \to -\infty} x + \sqrt{x^2 - 3x - 3} = \left\{ \begin{array}{c} x \to -x \\ -\infty \to +\infty \end{array} \right\} \lim_{x \to +\infty} -x + \sqrt{(-x)^2 - 3(-x) - 3} = \lim_{x \to +\infty} \dfrac{-x + \sqrt{x^2 + 3x - 3}}{1} \cdot \dfrac{x - \sqrt{x^2 + 3x - 3}}{x - \sqrt{x^2 + 3x - 3}}$

$\displaystyle\lim_{x \to +\infty} \dfrac{-x - x^2 - 3x - 3}{x - \sqrt{x^2 - 3x - 3}} = \dfrac{-x^2 - 4x - 3}{x - \sqrt{x^2 - 3x - 3}} \left[\dfrac{\infty}{\infty}\right] \begin{array}{c} :x^2 \\ = \\ :x^2 \end{array} \dfrac{-\frac{x^2}{x^2} - \frac{4x}{x^2} - \frac{3}{x^2}}{\frac{x}{x^2} - \sqrt{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}}} = \dfrac{-1 - 0 - 0}{0 - \sqrt{1 - \infty}} = \dfrac{-1}{\sqrt{1}} = 1 \quad \boxed{y = 1}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. PIŠITE DVOSTRANO! Obavezno popuniti sva polja ispod⇊ $\mathcal{D}1$

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: *LOVRE BUBALO*   VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):
17-2-0389-2014

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 3x - 3}$ i skicirati graf. — 20 graf *10*

2. Riješiti: $\dfrac{z+i}{z-i} = \overline{2+3i}$. Prikaži rješenje u kompleksnoj ravnini! — 12+3

3. Odrediti konvergenciju reda $\sum_n \dfrac{n^2}{2^n}$. — 15

4. Odrediti tok funkcije $f(x) = \dfrac{x^2 + 3}{x^2 - 3}$ i skicirati graf. — 20 graf *4*
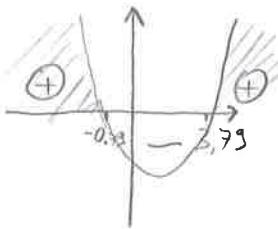
5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost. — 6+6+3

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 3x}$, u točki gdje je $x = 0$. — 15

Ukupno: 29

① $f(x) = x + \sqrt{x^2 - 3x - 3}$

DOMENA:

$x^2 - 3x - 3 \geq 0$

$x_{1,2} = \dfrac{3 \pm \sqrt{9 + 12}}{2} = \dfrac{3 \pm \sqrt{21}}{2}$

$D_f : \langle -\infty, -0.79] \cup [3.79, +\infty\rangle$

$x_1 = 3.79$
$x_2 = -0.79$

V.A

$\lim_{x \to 3.79} \left(x + \sqrt{x^2 - 3x - 3}\right) \cdot \dfrac{x - \sqrt{x^2 - 3x - 3}}{x - \sqrt{x^2 - 3x - 3}} = \lim_{x \to 3.79} \dfrac{x^2 - x^2 + 3x + 3}{x - \sqrt{x^2 - 3x - 3}} = \lim_{x \to 3.79} \dfrac{3x + 3 \,/ \cdot x}{x - \sqrt{x^2 - 3x - 3} \,/ \cdot x} = \dfrac{3 \cdot 1 + 0}{1 - 1} =$

$= \dfrac{3}{0} = +\infty$ V.A → $x = 3.79$

$\lim_{x \to -0.79} \dfrac{3x + 3}{x - \sqrt{x^2 - 3x - 3}} = \lim_{x \to -0.79} \dfrac{-3x + 3 / \cdot x}{-x - \sqrt{x^2 + 3x - 3} / \cdot x} = \dfrac{-3 + 0}{-1 - 1} = \dfrac{3}{2}$ ⟸ NIJE V.A.

H.A

- D.HA,

$\lim_{x \to \infty} \left(x + \sqrt{x^2 - 3x - 3}\right) \cdot \dfrac{x - \sqrt{x^2 - 3x - 3}}{x - \sqrt{x^2 - 3x - 3}} = \lim_{x \to \infty} \dfrac{x^2 - x^2 + 3x + 3}{x - \sqrt{x^2 - 3x - 3}} = \lim_{x \to \infty} \dfrac{3x + 3 / \cdot x}{x - \sqrt{x^2 - 3x - 3} / \cdot x} = \dfrac{3}{1 - 1} = \dfrac{3}{0}$

NEMA D.HA

- L.HA

$\lim_{x \to -\infty} \dfrac{3x + 3}{x - \sqrt{x^2 - 3x - 3}} = \lim_{x \to -\infty} \dfrac{-3x + 3 / \cdot x}{-x - \sqrt{x^2 + 3x - 3} / \cdot x} = \dfrac{-3}{-1 - 1} = \dfrac{3}{2}$ $y = \dfrac{3}{2}$ → L.HA ✓

K.A

$y = kx + l$

$k = \lim_{x \to \infty} \dfrac{f(x)}{x}$

$l = \lim_{x \to \infty} [f(x) - kx]$ $y = 2x - \dfrac{3}{2}$

y	x
$-\frac{3}{2}$	0
$-\frac{1}{2}$	1
2.5	2
4.5	3

D.K.A

$k = \lim_{x \to \infty} \dfrac{x + \sqrt{x^2 - 3x - 3}\;/:x}{x\;/:x} = \dfrac{1+1}{1} = 2$ $\boxed{k = 2}$

$l = \lim_{x \to \infty}\left[x + \sqrt{x^2 - 3x - 3} - 2x\right] = \lim_{x \to \infty}\left(\sqrt{x^2 - 3x - 3} - x\right) \cdot \dfrac{\sqrt{x^2 - 3x - 3} + x}{\sqrt{x^2 - 3x - 3} + x} = \lim_{x \to \infty} \dfrac{x^2 - 3x - 3 - x^2}{\sqrt{x^2 - 3x - 3} + x}$

$= \lim_{x \to \infty} \dfrac{-3x - 3\;/:x}{\sqrt{x^2 - 3x - 3} + x\;/:x} = \dfrac{-3}{1+1} = -\dfrac{3}{2}$ $\boxed{l = -\dfrac{3}{2}}$

$\boxed{y = 2x - \dfrac{3}{2}} \rightarrow D.K.A$

L.K.A

$k = \lim_{x \to -\infty} \dfrac{x + \sqrt{x^2 - 3x - 3}}{x} = \lim_{x \to -\infty} \dfrac{-x + \sqrt{x^2 + 3x - 3}\;/:x}{-x\;/:x} = \dfrac{-1+1}{-1} = \dfrac{0}{-1}$ NEMA L.K.A

PRVA I DRUGA DERIVACIJA

$f(x) = x + \sqrt{x^2 - 3x - 3}$

$f'(x) = 1 + \dfrac{1}{2\sqrt{x^2 - 3x - 3}} \cdot 2x - 3$

$f'(x) = \dfrac{2x - 3 + 2\sqrt{x^2 - 3x - 3}}{2\sqrt{x^2 - 3x - 3}}$

$f''(x) = 0 + \dfrac{2 \cdot 2\sqrt{x^2 - 3x - 3} - (2x - 3) \cdot \frac{1}{4\sqrt{x^2 - 3x - 3}} \cdot (2x - 3)}{\left(2\sqrt{x^2 - 3x - 3}\right)^2}$
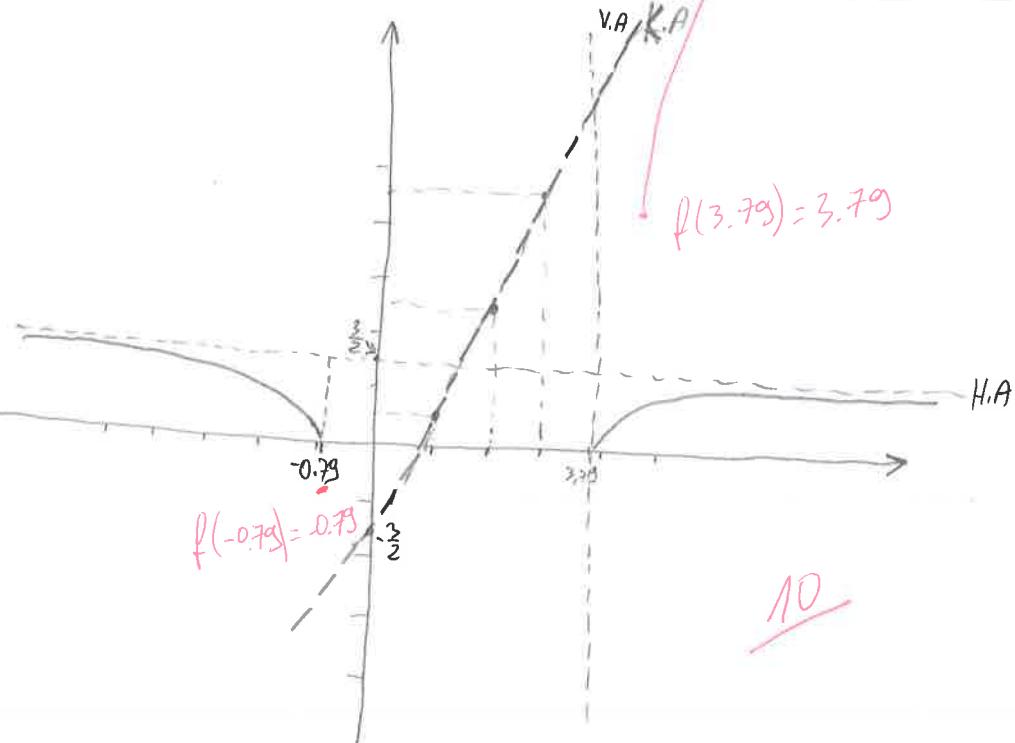
$f''(x) = \dfrac{4\sqrt{x^2 - 3x - 3} - \frac{(2x - 3)^2}{4\sqrt{x^2 - 3x - 3}}}{4x^2 - 12x - 12}$

$f'(x) = 0$

$2x - 3 + 2\sqrt{x^2 - 3x - 3} = 0$

$2\sqrt{x^2 - 3x - 3} = 3 - 2x\;/^2$

$4x^2 - 12x - 12 = 9 - 12x + 4x^2$

$-12 = 9$

$\hookrightarrow$ NEMA EKSTREMA

	$-\infty$	-0.79	3.79	$+\infty$
$f'(x)$	$-$			$+$
$f(x)$	$\searrow$			$\nearrow$

V.A K.A

$f(3.79) = 3.79$

-0.79

$f(-0.79) = 0.79$ $-\frac{3}{2}$

3.79

H.A

10

Ime i prezime: LOVRE BUBALO   **Matični broj u indeksu:** 17-2-0389-2014

②

$$\frac{z+i}{z-i} = \overline{2+3i}$$

$$z = x+yi \qquad \bar{z} = x - yi$$

$$\frac{x+yi+i}{x+yi-i} = 2-3i \quad / \cdot (x+yi-i)$$

$$\boxed{z = -\frac{33}{5} + \frac{6}{5}i}$$

$$x+yi+i = (2-3i)(x+yi-i)$$

$$x+yi+i = 2x + 2yi - 2i - 3xi + 3y + 3$$

Re $\quad x = 2x + 3y - 3$

Im $\quad y+1 = 2y - 2 - 3x$

$$x + 3y = 3 \ /\cdot 3 \ \Rightarrow \ x + 3 \cdot \frac{6}{5} = -3 \quad \text{✗}$$

$$y - 3x = 3$$

$$3x + 9y = 9$$

$$-3x + y = 3 \ / +$$

$$x = -3 - 3 \cdot \frac{6}{5}$$

$$\boxed{x = -\frac{33}{5}} \quad \text{✗}$$
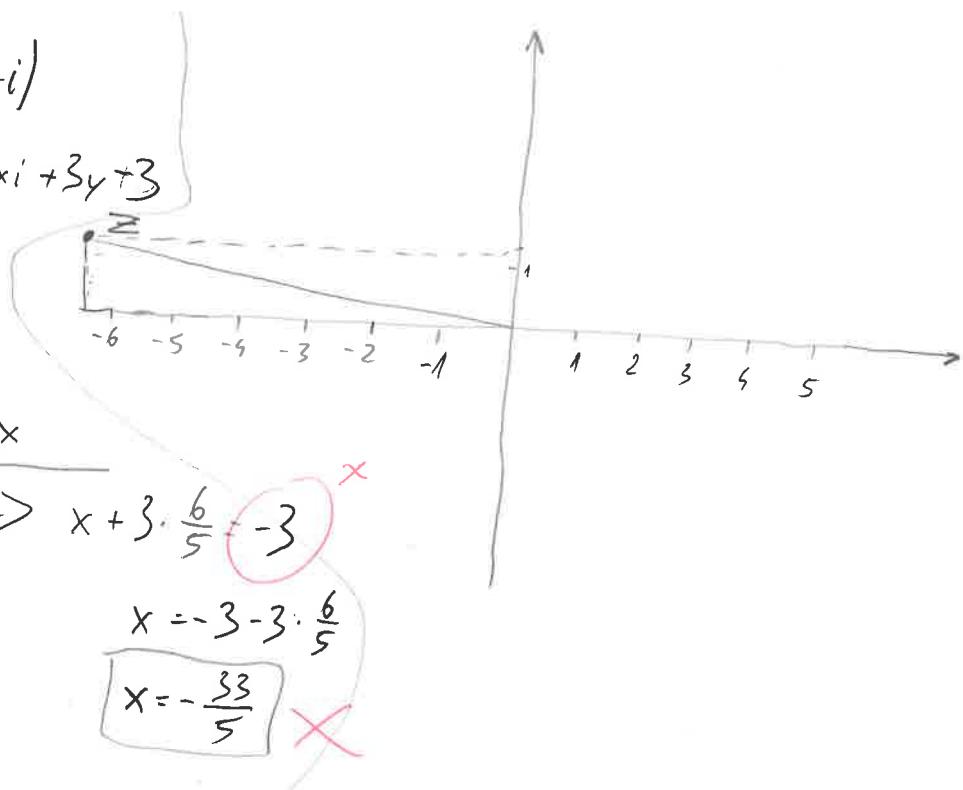
$$10y = 12$$

$$y = \frac{12}{10}$$

$$\boxed{y = \frac{6}{5}}$$

PROVJERA

VIDI JOZELJIĆ

(4) $f(x) = \dfrac{x^2+3}{x^2-3}$

$x^2+3=0$

x^2-3 NEMA

DOMENA:

$Df: \mathbb{R} \setminus \{-3,3\}$ ✗

$x^2-3 \neq 0$

$x^2 \neq 3$

$x \neq \pm 3$ ✗

V.A

$\lim\limits_{x \to 3} \dfrac{9+3}{9-3} = \dfrac{12}{6} = 2 \quad \to$ NEMA

$\lim\limits_{x \to -3} \dfrac{9+3}{9-3} = 2 \quad \to$ NEMA

H.A

D.H.A

$\lim\limits_{x \to \infty} \dfrac{x^2+3 \;/:x^2}{x^2-3 \;/:x^2} = \dfrac{1+0}{1-0} = 1 \qquad \boxed{Y=1} \to$ D.HA

L.H.A

$\lim\limits_{x \to -\infty} \dfrac{x^2+3 \;/:x}{x^2-3 \;/:x} = 1$

K.A

D.K.A $\qquad Y=kx+l \qquad k=\lim\limits_{x \to \infty} \dfrac{f(x)}{x} \qquad l=\lim\limits_{x \to \infty}\left[f(x)-kx\right]$

$k=\lim\limits_{x \to \infty} \dfrac{\frac{x^2+3}{x^2-3}}{x} = \lim\limits_{x \to \infty} \dfrac{x^2+3 \;/:x^3}{x^3-3 \;/:x^3} = \dfrac{0}{1} = 0 \qquad \boxed{k=0} \to$ NEMA D.K.A

L.K.A

$k=\lim\limits_{x \to \infty} \dfrac{\frac{x^2+3}{x^2-3}}{x} = \dfrac{x^2+3 \;/:x^3}{x^3-3 \;/:x^3} = \dfrac{0}{-1} \qquad$ NEMA L.K.A

$f'(x) = \dfrac{2x(x^2-3)-(x^2+3)\cdot 2x}{(x^2-3)^2} = \dfrac{2x\left[x^2-3-x^2-3\right]}{(x^2-3)^2} = \dfrac{-12x}{(x^2-3)^2}$

$f''(x) = \dfrac{-12\cdot(x^2-3)^2 + 12x\cdot 2(x^2-3)\cdot 2x}{(x^2-3)^4} = \dfrac{-12\cdot(x^2-3)\left[(x^2-3)-4x^2\right]}{(x^2-3)^4} = \dfrac{-12\cdot(-3x^2-3)}{(x^2-3)^3} = \dfrac{36x^2+36}{(x^2-3)^3}$

④→ NASTAVAK

$f'(x)=0$ $f(0)=\dfrac{0+3}{0-3}=-1$ $f''(x)=0$
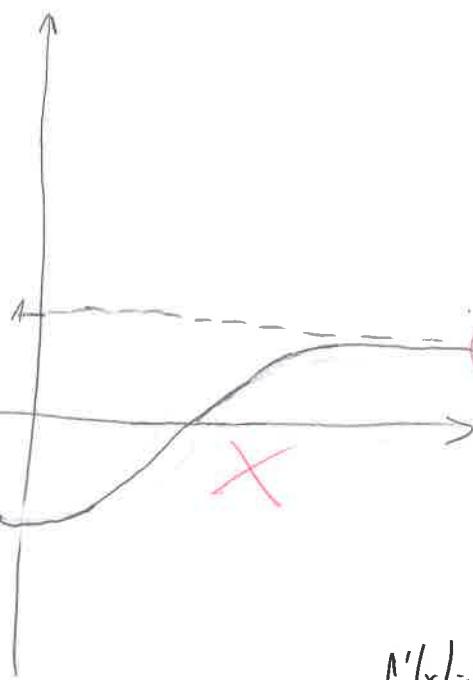
$-12x=0$ $36x^2+36=0/:36$

$x=0$ $\boxed{MIN(0,-1)}$ $x^2+1=0$

$\cancel{x^2=-1}$ NEMA TOČKI INFLEKSIJE

	$-\infty$	0	$+\infty$
$f'(x)$	$+$	$\vdots$	$-$
$f(x)$	↗	$\vdots$	↘

$\underline{4}$

⑤

$f(x)=\sqrt{x+2}+\sqrt{4-x}$

$f'(x)=\dfrac{1}{2\sqrt{x+2}}\cdot 1+\dfrac{1}{2\sqrt{4-x}}\cdot(-1)$

$f'(x)=\dfrac{1}{2\sqrt{x+2}}\cdot\dfrac{2\sqrt{x+2}}{2\sqrt{x+2}}-\dfrac{1}{2\sqrt{4-x}}\cdot\dfrac{2\sqrt{4-x}}{2\sqrt{4-x}}$

$f'(x)=\dfrac{2\sqrt{x+2}}{4x+4}-\dfrac{2\sqrt{4-x}}{16-4x}$

$f'(x)=\dfrac{2\sqrt{x+2}\,(4-x)-2\sqrt{4-x}\cdot(x+1)}{4(x+1)(4-x)}$

$f'(x)=0$

$2\sqrt{x+2}\cdot(4-x)-2\cdot\sqrt{4-x}\cdot(x+1)=0$

$2\sqrt{x+2}\cdot(4-x)=2\sqrt{4-x}\,(x+1)/^2$

$(4x+8)\cdot(16-8x+x^2)=(16-4x)(x^2+2x+1)$

$64x-32x^2+4x^3+128-64x+8x^2=16x^2+32x+16$
$\underline{-4x^3-8x^2-4x}$

$8x^3+16x^2+28x+112=0$ ⊘

⑥ $f(x) = e^{x^2 - 3x}$

$\begin{array}{|c|}\hline x_0 = 0 \\ \hline y_0 = 1 \\ \hline \end{array}$

$t: \quad y - y_0 = f'(x_0)(x - x_0)$

$f(0) = e^{0^2 - 3 \cdot 0} = e^0 = 1$

$y - 1 = 3(x - 0)$

$\boxed{y = -3x + 1}$ ✓

$f'(x) = e^{x^2 - 3x} \cdot (2x - 3)$

$f'(x_0) = e^{0 - 3 \cdot 0} \cdot (2 \cdot 0 - 3) = -3$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓

$\mathcal{D}1$

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: MAJA ŠOGORIĆ   VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

0269040783

1. Odrediti tok funkcije $f(x) = x + \sqrt{x^2 - 3x - 3}$ i skicirati graf.

    20 graf

2. Riješiti: $\dfrac{z+i}{z-i} = \overline{2+3i}$. *Prikaži rješenje u kompleksnoj ravnini!*

    12+3

3. Odrediti konvergenciju reda $\displaystyle\sum_n \frac{n^2}{2^n}$.

    15

4. Odrediti tok funkcije $f(x) = \dfrac{x^2 + 3}{x^2 - 3}$ i skicirati graf.

    20 graf

5. Navesti posebno lokalne, a posebno globalne ekstreme funkcije $f(x) = \sqrt{x+2} + \sqrt{4-x}$. Posebno komentirati (ne)ograničenost.

    6+6+3

6. Pronaći tangentu na graf funkcije $f(x) = e^{x^2 - 3x}$, u točki gdje je $x = 0$.

    15

    Ukupno:

    Ø

5. $f(x) = \sqrt{x+2} + \sqrt{4-x}$

$x+2 \geq 0$

$x+2 = 0$

$x \pm 2 \quad Df \{-2, 2\}$

$\sqrt{-3+2} + \sqrt{4-(-3}$

$\sqrt{-1} + \sqrt{7}$

$-\infty \quad -2 \quad 2 \quad +\infty$

$\sqrt{3+2} + \sqrt{4-3} = \sqrt{5} + \sqrt{1}$

DkA

$\lim\limits_{x \to \infty^+} \dfrac{\sqrt{x+2} + \sqrt{4-x} + x}{x}$

$\lim\limits_{x \to 2^-} \sqrt{2+2} + \sqrt{4-2} = \sqrt{4} + \sqrt{2} = 2 + \sqrt{2}$ mena V.A.

$\lim\limits_{x \to -2} \sqrt{2+2} + \sqrt{4-(-2} = \sqrt{0} + \sqrt{6} =$

$= \lim\limits_{x \to \infty}$

$\lim\limits_{x \to -\infty} \sqrt{x+2} + \sqrt{4-\infty} =$

EKSTREMI ?

$\lim\limits_{x \to 2^+} \left. \sqrt{x+2} + \sqrt{4-x} \right|^2 = (2+2) + (4-2) = 4+2$ mena V.A $= 6$ mena V.A

$\lim\limits_{x \to -2} \left. \sqrt{x+2} + \sqrt{4-x} \right|^2 = (-2+2)(4-(-2) = 0 + 6 = 6$ mena V.A.

$\lim\limits_{x \to +\infty} \sqrt{x+2} + \sqrt{4-x} = [\infty, \infty]_{1:x}^{1:x} \quad \sqrt{\tfrac{x}{x} + \tfrac{2}{x}} + \sqrt{\tfrac{4}{x} - \tfrac{x}{x}} = \sqrt{1} + \sqrt{1} = 2$

$1 \quad 0 \quad 0 \quad 1 \quad y = 2$

$\lim\limits_{x \to -\infty} \begin{Bmatrix} x \to -x \\ -\infty \to +\infty \end{Bmatrix} \sqrt{-x-2} + \sqrt{4+x} = \lim\limits_{x \to +\infty}^{\lim\limits_{x \to +\infty}} \sqrt{(-x-2) + \sqrt{(4+x)}} \begin{matrix} 1:x \\ 1:x \end{matrix} = 2$

$\lim\limits_{x \to +\infty} \sqrt{\tfrac{-x}{x} - \tfrac{2}{x}} + \sqrt{\tfrac{4}{x} + \tfrac{x}{x}} = -\sqrt{1} + \sqrt{1} = 0 \quad y = 0$

$-1 \quad 0 \quad 0 \quad 1$

$\lim\limits_{x \to 2} \sqrt{x+2} + \sqrt{4-x} \overset{L'H}{=} (\sqrt{x+2})' + (\sqrt{4-x})' = \left((x+2)^{\frac{1}{2}}\right)' + \left((4-x)^{\frac{1}{2}}\right)' = \frac{1}{2}(1+0) + \frac{1}{2}(0-1)$

$= \frac{1}{2} - \frac{1}{2} = 0$ mena LKA
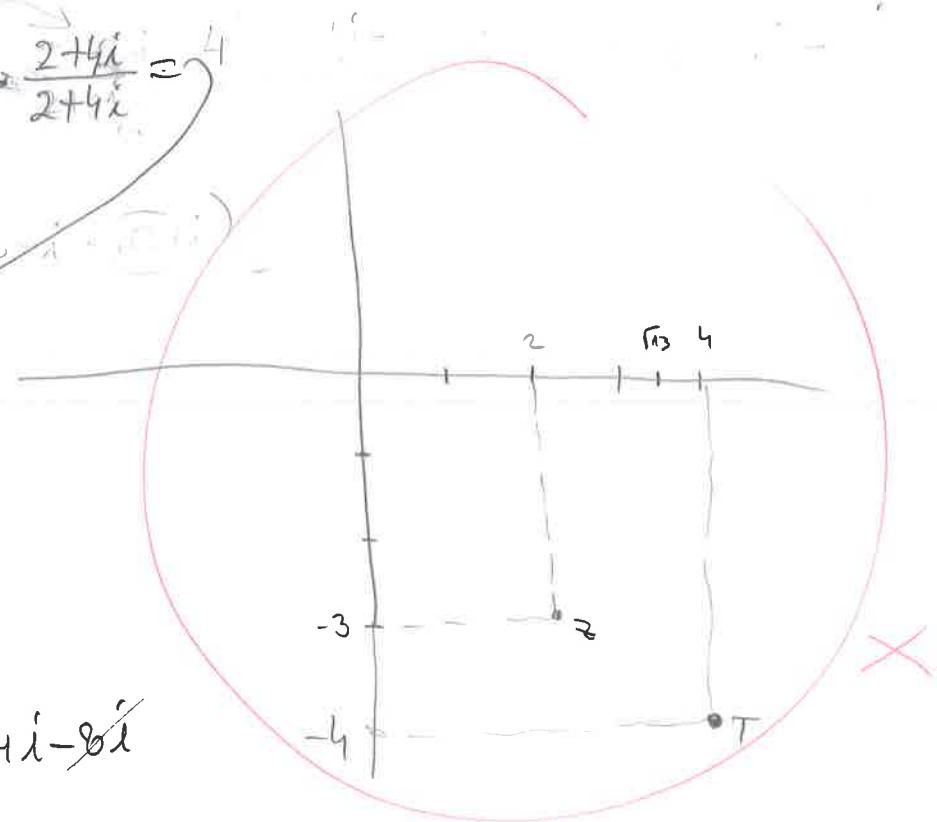
2. $\dfrac{z+i}{z-i} = \overline{2+3i}$

$z = 2-3i$ $\begin{cases} x = 2 \\ y = -3 \end{cases}$ $z = \sqrt{x^2+y^2} = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

$\dfrac{2-3i+i}{2-3i-i} = \dfrac{2-2i}{2-4i} \cdot \dfrac{2+4i}{2+4i} = $

$\dfrac{\sqrt{13}+i}{\sqrt{13}-i} = \dfrac{\sqrt{13}+i}{\sqrt{13}+i} = $

$= (2-2i)(2+4i) = 4 + 8i - 4i - 8i$

$= 4 - 4i \quad (T)$

$x = 4, \quad y = -4$

VIDI ŠOGORIĆ

$$f'(x) = \frac{x^2+3}{x^2-3} = \frac{(x^2+3)'\cdot(x^2-3)-(x^2+3)(x^2-3)'}{(x^2-3)^2} = \frac{2x(x^2-3)-(x^2+3)\cdot 2x}{(x^2-3)^2}$$

4. zadatak

$$= \frac{2x^3-6x-2x^3-6x}{(x^2-3)^2} = \frac{-12x}{(x^2-3)^2}$$

$$f''(x) = \frac{(-12x)'(x^2-3)^2-(-12x)\left((x^2-3)^2\right)'}{(x^2-3)^4} = \frac{-12x(x^2-3)^2+(-12x)(2(x^2-3)}{(x^2-3)^4}$$

$$= \frac{-12x(x^2-3)^2+12x\cdot 2(x^2-3)}{(x^2-3)^4} \overset{(2x^2-6)\cdot 12x}{=} \frac{-12x(x^2-3)^2+24x^3-72x}{(x^2-3)^4{}^2} = \frac{-12x+24x^3-72x}{(x^2-3)^2}$$

$$= \frac{24x^3-84x}{(x^2-3)^2}$$

Stacion. točke
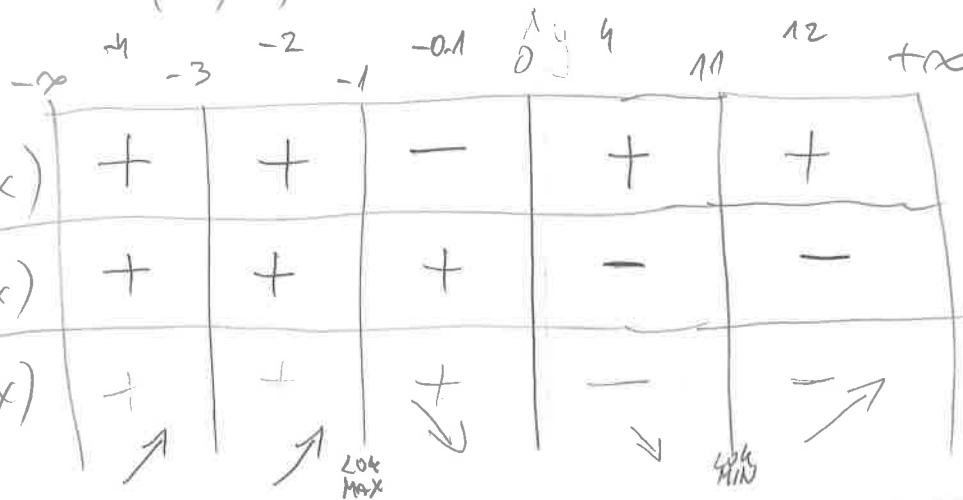
$$f'(x) = \frac{-12x}{(x^2-3)^2} = \frac{-12\cdot 3}{(3^2-3)^2} = \frac{-36}{(6)^2} = \frac{-36}{36} = -1 \quad T_1(-1,0)$$

$$f''(x) = \frac{24\cdot 3^3-84\cdot 3}{(3^2-3)^2} = \frac{648-252}{36} = \frac{396}{36} = 11 \quad T_2(0,11)$$

-1530

parnost/neparnost

$$f(-x) = \frac{(-x)^2+3}{(-x^2-3)} = \frac{x^2+3}{x^2-3} \implies = f(x) \text{ parna, Nije periodična (memo trig.tablice)}$$

$$f''(x) \frac{24\cdot(-4)^3-84\cdot(-4)}{(-4^2-3)^2} = \frac{1866}{169} > +$$

$$f''(-2) \frac{24\cdot(-2)^3-84\cdot(-2)}{(-2^2-3)^2} = +$$

$$f''(-0.1) \frac{24\cdot(-0.1)^3-84\cdot(-0.1)}{(-0.1^2-3)^2} = +$$

$$f''(4) \frac{24\cdot 4-84\cdot 4}{(4^2-3)^2} = \frac{96-336}{169} = -$$

$$f(12) \frac{24\cdot 12-84\cdot 12}{(12^2-3)^2} = -$$

	$-\infty$	-4	-3	-2	-1	-0.1	0	4	11	12	$+\infty$
$f(x)$		$+$		$+$		$-$		$+$		$+$	
$f'(x)$		$+$		$+$		$+$		$-$		$-$	
$f''(x)$		$+$		$\perp$		$+$		$-$		$\nearrow$	

↗   ↗ LOK MAX   ↘   ↘   LOK MIN ↗

$$f(x) \Rightarrow -4 \Rightarrow \frac{(-4)^2+3}{(-4)^2-3} = \frac{16+3}{16-3} = \frac{19}{13} = + \bigg/ f(-2)\frac{(-2)^2+3}{(-2)^2-3} = \frac{7}{1} = + \bigg/ f(0)\frac{-0.1^2+3}{-0.1^2-3} = - \bigg/ f(4)\frac{4^2+3}{4^2-3} = \frac{19}{13} = +$$

$$f'(x) = \frac{-12\cdot(-4)}{(-4^2-3)^2} = \frac{48}{169} = + \bigg/ \frac{-12\cdot(-2)}{(-2^2-3)^2} = \frac{24}{1} = + \bigg/ \frac{-12\cdot(-0.1)}{(-0.1^2-3)^2} = \frac{12.1}{8.9} = + \bigg/ \frac{-12\cdot 4}{(4^2-3)^2} = \frac{-48}{169} = - \bigg/ \frac{-12\cdot 12}{(12^2-3)^2} = \frac{-144}{19881} = -$$

4. $f(x) = \dfrac{x^2+3}{x^2-3}$

$x^2 - 3 \geqslant 0$

$x^2 - 3 = 0$

~~$x^2 = \pm 3$~~ ✗

~~$D(f) = \mathbb{R}$~~ ✗

V.A.

$\lim\limits_{x \to 3^+} \dfrac{x^2+3}{x^2-3} = \dfrac{3^2+3}{3^2-3} = \dfrac{12}{6} = 2$ nije V.A.

$\lim\limits_{x \to -3^+} \dfrac{(-3)^2+3}{(-3)^2-3} = \dfrac{12}{6} = 2$ nije V.A.

H.A.

$\lim\limits_{x \to +\infty} \dfrac{x^2+3}{x^2-3} = \left[\dfrac{\infty}{\infty}\right] \begin{array}{c} |:x^2 \\ |:x^2 \end{array} = \lim\limits_{x \to +\infty} \dfrac{1+\frac{3}{x^2}}{1-\frac{3}{x^2}} = \dfrac{1^+}{1_+} = 1 \Rightarrow y = 1$

$\lim\limits_{x \to -\infty} \dfrac{x^2+3}{x^2-3} \left\{\dfrac{\infty}{\infty}\right\} \begin{Bmatrix} x \to -x \\ -\infty \to +\infty \end{Bmatrix} \lim\limits_{x \to +\infty} \dfrac{(-x)^2+3}{(-x)^2-3} = \dfrac{x^2+3}{x^2-3} \begin{array}{c} |:x^2 \\ |:x^2 \end{array} = \dfrac{1+\frac{3}{x^2}}{1-\frac{3}{x^2}} = \dfrac{1}{1} = 1 \;\boxed{y=1}$

LVA

$\lim\limits_{x \to \infty} \dfrac{x^2+3}{x^2-3} = \left[\dfrac{\infty}{\infty}\right] \overset{L'H}{=} \lim\limits_{x \to \infty} \dfrac{(x^2+3)'\cdot(x^2-3)-(x^2+3)(x^2-3)'}{(x^2-3)^2} = \dfrac{2x(x^2-3)-(x^2+3)(2x)}{(x^2-3)^2}$

$= \dfrac{2x^3-6x-2x^3-6x}{(x^2-3)^2} \overset{L'H}{=} \dfrac{-12x}{(x^2-3)^2} \overset{L'H}{=} \dfrac{-12(3)}{(3^2-3)^2} = \dfrac{-36}{(6)^2} = \dfrac{-36}{36} = -1 \Rightarrow a = -1$

$\lim\limits_{x \to -\infty} \dfrac{x^2+3}{x^2-3} + (-1) = \dfrac{(x^2+3)-1(x^2-3)}{x^2-3} = \lim\limits_{x \to \infty} \dfrac{x^2+3-x^2+3}{(x^2-3)} = \dfrac{6}{(x^2-3)} = \dfrac{6}{(3^2-3)} = \dfrac{6}{6} = 1$

$y = -x + 1$

x	0	1
$y = -x+1$	1	0

sjecište s osi $x \Rightarrow f(x) = x^2+3 > 0$ nema

$x^2 + 3 = 0$

sjecište s osi $y \Rightarrow f(0) = \dfrac{0^2+3}{0^2-3} = \dfrac{3}{-3} = -1 \quad S(0,-1)$