

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

POPUNJAVA

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod

A1

NASTAVNIK

IME I PREZIME: Šime Labus

VRIJEME POČETKA: 8:30h

Broj ↓
bodova

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): 0269093263

17-1-0291-2014

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf.

15 graf

2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf.

15 graf

3. Navesti posebno sve lokalne, a posebno sve globalne ekstreme funkcije $f(x) = \sqrt{4x-x^2}$. Komentirati (ne)omeđenost.

8+8+4

4. Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$. Prikaži rješenja u kompleksnoj ravnini!

12+3

5. Ispitati konvergenciju reda: $\sum_{n=1}^{\infty} (\sqrt{n^2-4n}-n)$

15

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$.

12+3

Ukupno:

27

3) $f(x) = \sqrt{4x-x^2}$

$$\frac{df(x)}{dx} = \frac{1}{2\sqrt{4x-x^2}} \cdot (4-2x) = \frac{2-x}{\sqrt{4x-x^2}}$$

$$\frac{df(x)}{dx} = 0 \Rightarrow 2-x=0$$

$x=2 \rightarrow f(x)=2$

x	2
f(x)	2

Globalni minimum i lokalni maksimum jest $f(x)=2$, a postiže se za $x=2$.

6) $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} = \left(\frac{0}{0}\right)^{\frac{1}{3}}$ $\lim_{x \rightarrow 1} \frac{1}{\sqrt[3]{x^2}} = \lim_{x \rightarrow 1} 3\sqrt[3]{x^2} = 3 = 3$ ✓

PROVERA?

4) $\sqrt[3]{\frac{4+3i}{2-4i}} = \sqrt[3]{\frac{4+3i}{2-4i} \cdot \frac{2+4i}{2+4i}} = \sqrt[3]{\frac{(3+12) + (6-15)i}{4+16}} = \sqrt[3]{1 - \frac{1}{2}i} = \sqrt[3]{\frac{\sqrt{5}}{2} \cdot (\cos(-26,565^\circ) + i \sin(-26,565^\circ))}$

$i \sin(-26,565^\circ) = 1,03789$

$\operatorname{tg} \rho = \frac{-1}{2} \Rightarrow \rho = -26,565^\circ$

$r = \sqrt{1^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

$z = \frac{\sqrt{5}}{2} (\cos(-26,565^\circ) + i \sin(-26,565^\circ))$

$\left[\cos\left(\frac{-26,565^\circ}{3} + \frac{24\pi}{3}\right) + i \sin\left(\frac{-26,565^\circ}{3} + \frac{24\pi}{3}\right) \right]$

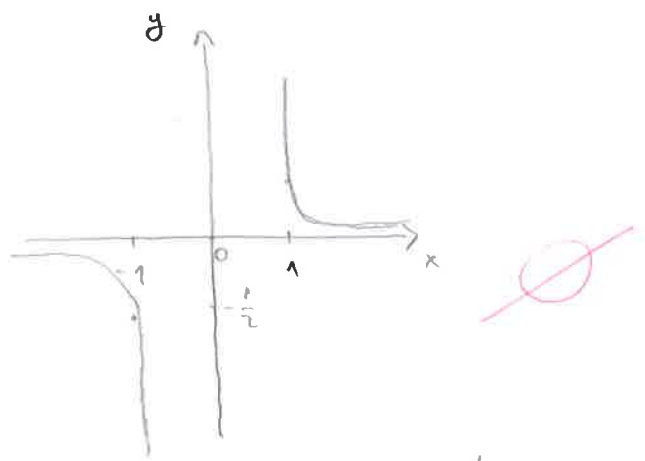
3 RJEŠENJA

② $f(x) = \frac{e^x}{x}$

$\frac{df(x)}{dx} = \frac{xe^x - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$

$\frac{df(x)}{dx} = 0 \Rightarrow e^x(x-1) = 0$

$e^x = 0$
 $x \in \mathbb{R}$ $x-1=0$
 $x=1$



$\lim_{x \rightarrow -\infty} \frac{e^x}{x} = \left(\frac{0}{\infty}\right)^{\text{L'H}} = \lim_{x \rightarrow -\infty} \frac{e^x}{1} = 0$

$\lim_{x \rightarrow +\infty} \frac{e^x}{x} = \left(\frac{\infty}{\infty}\right)^{\text{L'H}} = \lim_{x \rightarrow +\infty} e^x = \infty$

	$-\infty$		$+\infty$
x		1	
$\frac{df(x)}{dx}$		0	↗
$f(x)$		e	

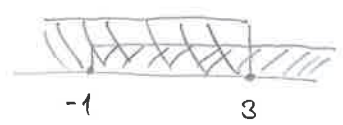
$\lim_{x \rightarrow \infty} \frac{e^x}{x} = +\infty$

$\lim_{x \rightarrow 0} \frac{e^x}{x} = -\infty$

① $f(x) = \sqrt{1+x} - \sqrt{3-x}$

$\frac{df(x)}{dx} = \frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{3-x}} \cdot (-1) = \frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{3-x}}$

$1+x \geq 0 \Rightarrow x \geq -1$
 $3-x \geq 0 \Rightarrow x \leq 3$



$DF = [-1, 3]$

$\sqrt{2-x} + \sqrt{1+x} = 0 \quad |^2$

$3-x + 2\sqrt{(3-x)(1+x)} + 1+x = 0$

$2\sqrt{(3-x)(1+x)} = -4$

$\sqrt{(3-x)(1+x)} = -2 \quad |^2$

$(3-x)(1+x) = 4$

$3+3x-x-x^2 = 4$

$-x^2+2x-1 = 0$

$x^2-2x+1 = 0$

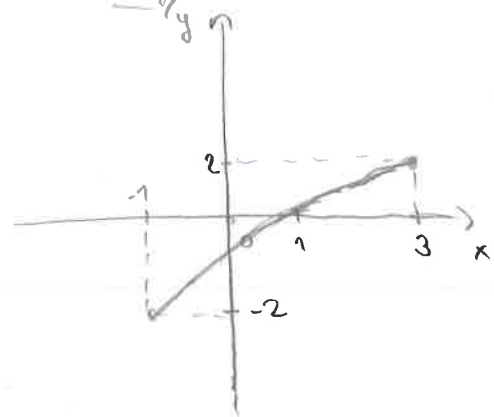
$x_{1,2} = \frac{2 \pm \sqrt{4-4}}{2} = 1$

$x = 1$

	-1	0		1		3
x						
$\frac{df(x)}{dx}$		+		0		+
$f(x)$		↗		0		↗

Wanna check!
 $\lim_{x \rightarrow 1} \sqrt{1+x} - \sqrt{3-x} = -2$

$\lim_{x \rightarrow 3} \sqrt{1+x} - \sqrt{3-x} = 2$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

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A1

IME I PREZIME: Ante Mikelic

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

0269078840

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf. 15 graf
2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf. 15 graf
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4. Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$. Prikaži rješenja u kompleksnoj ravnini! 12+3
5. Ispitati konvergenciju reda: $\sum_{n=1}^{\infty} (\sqrt{n^2-4n}-n)$ 15
6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$. 12+3

Ukupno:

27

3. $f(x) = \sqrt{4x-x^2}$

$f'(x) = \frac{1 \cdot (4-2x)}{2\sqrt{4x-x^2}}$

$f'(x) = 0$

$4-2x=0$

$3x^2=4$

$x^2 = \frac{4}{3}$

$x_1 = \frac{2\sqrt{3}}{3}$

$x_2 = \frac{2\sqrt{3}}{-3}$

→ nije u domeni

$x_1 = \frac{2\sqrt{3}}{3} \Rightarrow$ lokalni maksimum

$4x-x^2 \geq 0$

$x(4-x) \geq 0$

$x(2-x)(2+x) \geq 0$

$-x(x+2)(x-2) \geq 0$

$x(x+2)(x-2) \leq 0$



$DF = (-\infty, -2] \cup [0, 2]$



$DF: x \in (-\infty, -2] \cup [0, 2]$

$4-x=0$
 $x_2=4$

$DF: x \in [0, 4]$

2.

$$6. \lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} = \frac{0}{0} \quad \lim_{x \rightarrow 1} \frac{(\sqrt[3]{x})^3 - 1^3}{\sqrt[3]{x}-1} = \lim_{x \rightarrow 1} \frac{(\cancel{\sqrt[3]{x}-1})(\sqrt[3]{x^2} + \sqrt[3]{x} + 1)}{\cancel{\sqrt[3]{x}-1}}$$

$$\lim_{x \rightarrow 1} (\sqrt[3]{x^2} + \sqrt[3]{x} + 1) = 1 + 1 + 1 = 3 \quad \checkmark$$

PROYERA?

$$1. f(x) = \sqrt{1+x} - \sqrt{3-x} \quad \begin{array}{ll} 1+x \geq 0 & x \geq -1 \\ 3-x \geq 0 & x \leq 3 \end{array}$$

$$DF = [-1, 3]$$

$$\lim_{x \rightarrow -1^+} (\sqrt{1+x} - \sqrt{3-x}) = (0 - \sqrt{4}) = -2$$

$$\lim_{x \rightarrow 3^-} (\sqrt{1+x} - \sqrt{3-x}) = (\sqrt{4} - 0) = 2$$

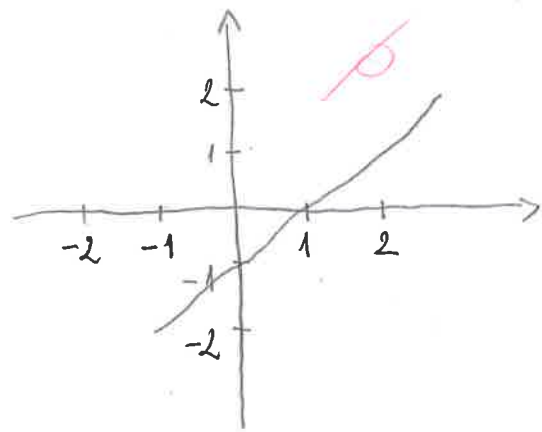
$$f'(x) = \frac{1}{2\sqrt{1+x}} - \frac{1 \cdot (-1)}{2\sqrt{3-x}}$$

$$f'(x) = \frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{3-x}}$$

$$f'(x) = 0 \quad \underbrace{\frac{1}{2\sqrt{1+x}}}_{>0} + \underbrace{\frac{1}{2\sqrt{3-x}}}_{>0} = \emptyset \rightarrow \text{tidak ada penyelesaian}$$

→ tidak ada stasioner titik

1. graf



$$f(0) = 1 - \sqrt{3}$$

$$f(1) = 0$$

4. $\sqrt[3]{\frac{4+3i}{2-4i}}$

$$z = \sqrt[3]{\frac{4+3i}{2-4i} \cdot \frac{2+4i}{2+4i}} = \sqrt[3]{\frac{(4+3i)(2+4i)}{2^2+4^2}} = \sqrt[3]{\frac{8+16i+6i-12}{20}}$$

$$= \sqrt[3]{\frac{-4+22i}{20}} = \sqrt[3]{-\frac{1}{3} + \frac{11}{10}i}$$

$$(r = \sqrt{\frac{1}{25} + \frac{121}{100}} = \sqrt{\frac{4+121}{100}} = \sqrt{\frac{125}{100}} = \frac{5\sqrt{5}}{10} = \frac{\sqrt{5}}{2})$$

$$= \sqrt[3]{\frac{\sqrt{5}}{2} \left(\frac{-2}{5\sqrt{5}} + \frac{11}{5\sqrt{5}}i \right)}$$

$$\frac{\sqrt{5}}{2} \cdot x = \frac{11}{20}$$

$$\sqrt{5} x = \frac{11}{5}$$

$$x = \frac{11}{5\sqrt{5}}$$

$$\cos \varphi = \frac{2}{5\sqrt{5}} \quad \sin \varphi = \frac{11}{5\sqrt{5}}$$

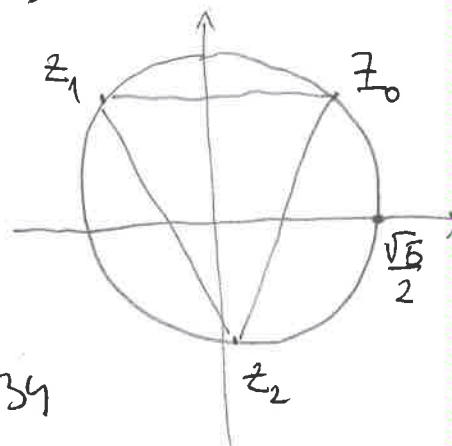
$$\varphi = 100 \cdot 3048463^\circ = 1,750649827 \quad \checkmark$$

$$z = \frac{\sqrt[6]{5}}{\sqrt[3]{2}} \operatorname{cis} \left(\frac{1,750649827 + 2k\pi}{3} \right) - k = 0,1,2$$

$$z_0 = \frac{\sqrt[6]{5}}{\sqrt[3]{2}} \operatorname{cis} 0,58355$$

$$z_1 = \frac{\sqrt[6]{5}}{\sqrt[3]{2}} \operatorname{cis} 2,6779$$

$$z_2 = \frac{\sqrt[6]{5}}{\sqrt[3]{2}} \operatorname{cis} 4,77234$$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

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IME I PREZIME: ANTE NEGRO

VRIJEME POČETKA: 0815H

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0399-2014

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf. 15 graf
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Ukupno:

22

$$1.) f(x) = \sqrt{1+x} - \sqrt{3-x}$$

$$f'(x) = \frac{1}{2(1+x)} \cdot (1+x)' - \frac{1}{2(3-x)} \cdot (3-x)'$$

$$f'(x) = \frac{1}{2+2x} \cdot 1 - \frac{1}{6-2x} \cdot (-1)$$

$$f'(x) = \frac{1}{2+2x} + \frac{1}{6-2x}$$

$$f'(x) = \frac{-6-2x+2+2x}{(2+2x)(6-2x)}$$

$$f'(x) = \frac{6-2x+2+2x}{12+12x-12x-4x^2}$$

$$f'(x) = \frac{4}{12+8x-4x^2}$$

$$f'(x) = \frac{4}{4(3+2x-2x^2)}$$

$$f'(x) = \frac{1}{3+2x-2x^2}$$

$$f'(x) = 0$$

$$\frac{1}{3+2x-2x^2} = 0$$

$$1 \neq 0$$

2)

$$f(x) = \frac{e^x}{x}$$

$$\text{DOMENA } D_f = \mathbb{R} \setminus \{0\}$$

$$f(x) = 0$$

$$\frac{e^x}{x} = 0 \quad | \cdot x$$

$$e^x \neq 0 \quad \text{НЕМА НУЛТОВЕ}$$

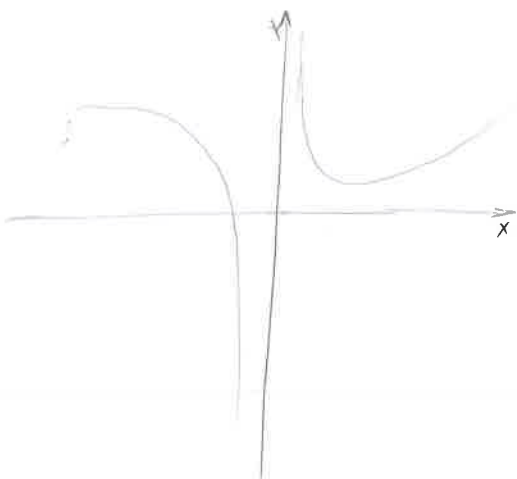
$$f'(x) = \frac{e^x \cdot x - e^x \cdot 1}{x^2} = e^x \cdot \frac{x-1}{x^2}$$

$$f'(x) = 0$$

$$e^x \cdot \frac{x-1}{x^2} = 0$$

$$x = 1 \quad \text{СТАЦИОНАРНА ТОЧКА}$$

	$-\infty$	0	1	$+\infty$
f	-	o	+	+
f'	-	-	o	+
f''				



АСИМПТОТЕ?

$$f''(x) = e^x \cdot \frac{x-1}{x^2} + e^x \cdot \frac{x^2 - (x-1) \cdot 2x}{x^4}$$

$$= e^x \cdot \frac{x^2 - 2x^2 + 2x}{x^4} =$$

$$= e^x \cdot \frac{-x^2 + 2x}{x^4} =$$

$$= e^x \left(\frac{x-1}{x^3} + \frac{-x^2+2x}{x^4} \right) =$$

$$= e^x \frac{x^3 - x^2 - x^2 + 2x}{x^4} =$$

$$= e^x \frac{x^3 - 2x^2 + 2x}{x^4} =$$

$$= e^x \frac{x(x^2 - 2x + 2)}{x^4}$$

$$= e^x \cdot \frac{x^2 - 2x + 2}{x^3}$$

$$f''(x) = 0$$

$$e^x \cdot \frac{x^2 - 2x + 2}{x^3} \neq 0$$

7

Matematika 1

Ime i prezime: ANTE NEGO

Matični broj u indeksu: 17-2-0399-2014

4.)

$$\sqrt[3]{\frac{4+3i}{2-4i} \cdot \frac{2+4i}{2+4i}} = \sqrt[3]{\frac{8+16+6i+12i^2}{4+16}} =$$

$$= \sqrt[3]{\frac{-4+22i}{20}} = \sqrt[3]{-\frac{1}{5} + \frac{11}{10}i}$$

$$r = \sqrt{\left(-\frac{1}{5}\right)^2 + \left(\frac{11}{10}\right)^2} = \frac{\sqrt{5}}{2}$$

$$\operatorname{tg} \varphi = \frac{\frac{11}{10}}{-\frac{1}{5}} = -\frac{11}{2}$$

$$\varphi = -0^\circ 5' 47'' + 360 = 359^\circ 54' 13''$$

$$\sqrt[3]{\frac{\sqrt{5}}{2} \left(\cos 359^\circ 54' 13'' + i \sin 359^\circ 54' 13'' \right)} =$$

$$= \frac{\sqrt[3]{5}}{\sqrt[3]{2}} \left(\frac{\cos 359^\circ 54' 13'' + 24i}{3} + i \frac{\sin 359^\circ 54' 13'' + 24i}{3} \right)$$

$k=0,1,2$

$$k_1 = \frac{\sqrt[3]{5}}{\sqrt[3]{2}} \left(\frac{1}{3} + i \frac{(-0.004)}{3} \right)$$

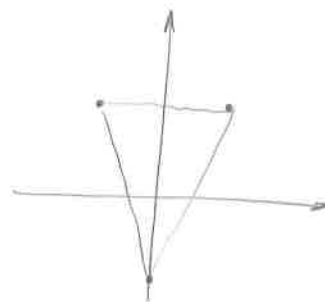
$$= 0.34 - i 0.0002$$

$$k_2 = \frac{\sqrt[3]{5}}{\sqrt[3]{2}} \left(\frac{0.98}{3} + i \frac{0.17}{3} \right)$$

$$= 0.33 + i 0.05$$

$$k_3 = \frac{\sqrt[3]{5}}{\sqrt[3]{2}} \left(\frac{0.49}{3} + i \frac{(-0.001)}{3} \right)$$

$$= 0.17 - i 0.0003$$



6.)

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt[3]{x^2} + \sqrt[3]{x} + 1}{\sqrt[3]{x^2} + \sqrt[3]{x} + 1} = \lim_{x \rightarrow 1} \frac{\cancel{(x-1)} (\sqrt[3]{x^2} + \sqrt[3]{x} + 1)}{\cancel{x-1}} = \lim_{x \rightarrow 1} \sqrt[3]{x^2} + \sqrt[3]{x} + 1 = 3 \quad \checkmark$$

PROVERA;

$$f(x) = \frac{x-1}{\sqrt[3]{x}-1}$$

$$f(2) = 3.86$$

$$f(1.5) = 3.45$$

$$f(1.3) = 3.28$$

$$f(1.1) = 3.09$$

$$f(0.9) = 2.89$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} = 3 \quad \checkmark$$

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IME I PREZIME: **DAJIEL ŠOŠA**

VRIJEME POČETKA:

A1

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **17-2-0366-2014**

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf. 15 graf
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Ukupno:

21

①
 $f(x) = \sqrt{1+x} - \sqrt{3-x}$ Df $\langle -1, 3 \rangle$

$$\begin{aligned} 1+x &\geq 0 & 3-x &\geq 0 \\ 1+x &= 0 & 3-x &= 0 \\ x &= -1 & -x &= -3 \\ & & x &= 3 \end{aligned}$$

NUKTOČKE
 $\sqrt{1+x} - \sqrt{3-x} = 0 \quad | \cdot 2$
 $\sqrt{1+x} = \sqrt{3-x} \quad | ^2$
 $1+x = 3-x$
 $1+x = 3-x$
 $2x = 3-1$
 $x = 1$ JE NUKTOČKA

ASIMPTOTE

$$\lim_{x \rightarrow -1} \sqrt{1+x} - \sqrt{3-x} = \sqrt{0} - \sqrt{2} = 0 - \sqrt{2} = -\sqrt{2} = -\infty \quad \boxed{\text{VA } x = -1}$$

$$\lim_{x \rightarrow 3} \sqrt{1+x} - \sqrt{3-x} = 2 - \sqrt{0} = 2 - (-\infty) = 2 + \infty = +\infty \quad \boxed{x = 3 \text{ V.A.}}$$

RAST. I PADO

$$f'(x) = \sqrt{1+x} \cdot \sqrt{3-x} = \frac{1}{2\sqrt{1+x}} \cdot (1+x)' = \frac{1}{2\sqrt{3-x}} \cdot (3-x)' = \frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{3-x}} = \frac{2\sqrt{3-x} + 2\sqrt{1+x}}{2\sqrt{1+x} \cdot 2\sqrt{3-x}}$$

$$2\sqrt{3-x} + 2\sqrt{1+x} = 0 \quad | ^2$$

$$2\sqrt{3-x} = -2\sqrt{1+x} \quad | ^2$$

$$4(3-x) = 4(1+x)$$

$$12-4x = 4+4x$$

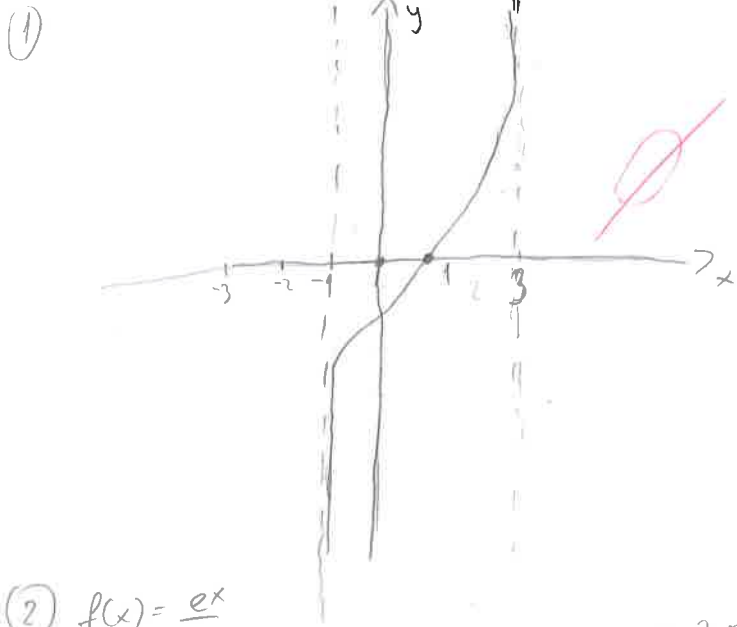
$$-8x = 4-12$$

$$-8x = -8$$

$$\boxed{x = 1}$$

$$T(1,0)$$

	$-\infty$	0	1	2	$+\infty$
$f'(x)$	+		+		
$f(x)$	\nearrow		\nearrow		



② $f(x) = \frac{e^x}{x}$
 $D_f = (-\infty, 0) \cup (0, +\infty)$
 $x \neq 0$
 $x = 0$

NULTORDE
 $e^x = 0$ NIKAD

	$-\infty$	-1	0	1	$+\infty$
$f'(x)$	-	-	+		
$f(x)$	\searrow	\searrow	\nearrow		

	$-\infty$	-1	0	1	$+\infty$
$f'(x)$	-	+	+		
$f(x)$	\cap	\cup	\cup		

ASIMPTOTE

$\lim_{x \rightarrow +\infty} \frac{e^x}{x} = \frac{\infty}{\infty} = \lim_{x \rightarrow +\infty} \frac{e^x}{x} = \frac{\infty}{1} = \frac{\infty}{1} = \infty$ KENAL D.H.A

TRAJIN K.S.V
 $\lim_{x \rightarrow +\infty} \frac{e^x}{x} = \frac{e^x \cdot (x^2)'}{x^2 \cdot 1} = \frac{e^x \cdot 2x}{2x} = \frac{e^x}{1} = \frac{\infty}{1} = \infty$

$\lim_{x \rightarrow -\infty} \frac{e^x}{x} = \frac{e^{-x}}{-x} = \frac{0}{\infty} = 0$ y=0 L.H.A

$\lim_{x \rightarrow +\infty} \frac{e^x}{x} - x = \frac{e^x - x^2}{x \cdot x^2} = \frac{\infty}{\infty} = \frac{\infty}{\infty} = \infty$

D.K.A y=x

$\lim_{x \rightarrow 0^-} \frac{e^x}{x} = \frac{e^0}{0} = \frac{1}{0} = -\infty$ V.A x=0

$\lim_{x \rightarrow 0^-} \frac{e^x}{x} = \frac{e^{-0}}{-0} = \frac{1}{-0} = -\infty$

$\lim_{x \rightarrow 0^+} \frac{e^x}{x} = \frac{e^{0^+}}{0^+} = \frac{1}{0^+} = +\infty$

RAST IPAD

$f'(x) = \frac{e^x}{x} = \frac{(e^x)' \cdot x - e^x \cdot (x)'}{x^2} = \frac{e^x \cdot x - e^x \cdot 1}{x^2} = \frac{e^x(x-1)}{x^2}$

$f'(x) = 0$ ZA $e^x(x-1) = 0$

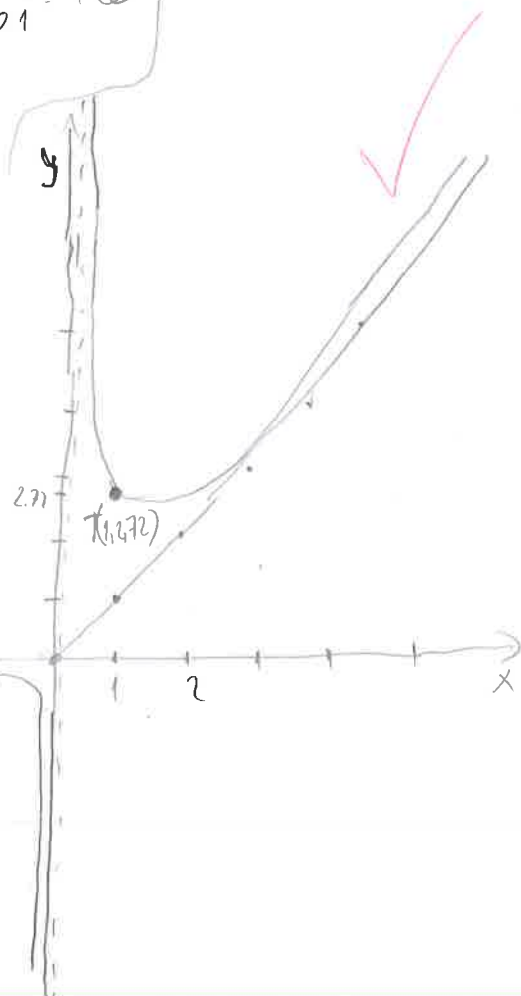
$e^x = 0$ NIKAD
 $x-1 = 0$
x=1

$T_1(1, 2.72)$ MINIMUM

ZAKRIVJENOST

$f''(x) = \frac{e^x(x-1)}{x^2} = \frac{(e^x(x-1))' \cdot x^2 - e^x(x-1) \cdot (x^2)'}{x^4}$
 $= \frac{e^x(x-1) + e^x \cdot x^2 - e^x(x-1) \cdot 2x}{x^4}$
 $= \frac{e^x \cdot 2x^3}{x^4}$

$e^x(x-1)' = (e^x)' \cdot (x-1) + e^x \cdot (x-1)'$
 $= e^x \cdot (x-1) + e^x \cdot 1 = e^x(x-1) + e^x$



Matematika 1

Ime i prezime: DANIEL JOŠA

Matični broj u indeksu: 17-2-0366-2014

3

$$\sqrt{4x-x^2}$$

$$f'(x) = \frac{1}{2\sqrt{4x-x^2}} \cdot (4x-x^2)' = \frac{4-2x}{2\sqrt{4x-x^2}} = \frac{2-x}{\sqrt{4x-x^2}}$$

$$D_f \subset (0, 4)$$

$$4x-x^2 \geq 0$$

$$4x-x^2 = 0$$

$$x(4-x) = 0$$

$$x=0 \quad 4-x=0$$

$$x=4$$

$$2-x=0$$

$$-x=-2$$

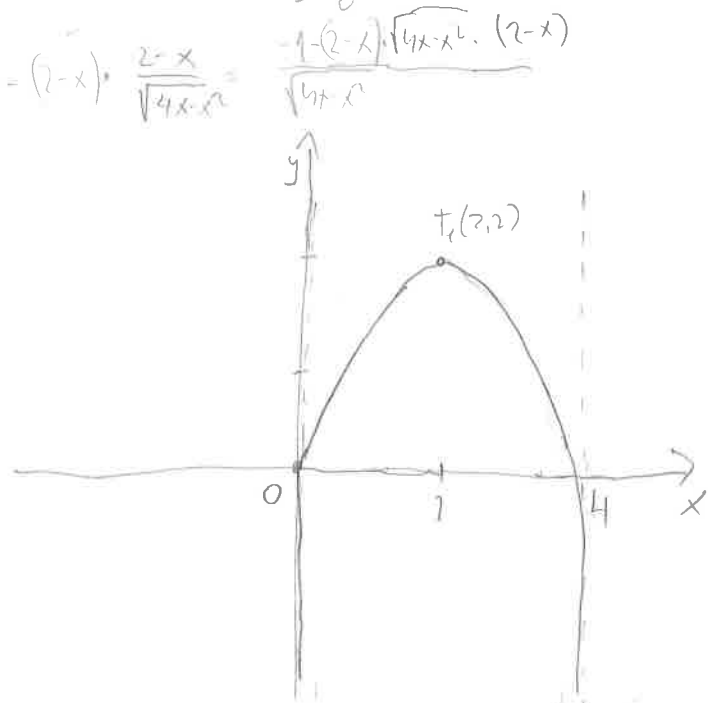
$$x=2$$

$T_1(2, 2)$ STACIONARNA TOČKA

$$f''(x) = \frac{2-x}{\sqrt{4x-x^2}} = \frac{(2-x)' \cdot \sqrt{4x-x^2} - (2-x) \cdot (\sqrt{4x-x^2})'}{(\sqrt{4x-x^2})^2} = \frac{-1 \cdot \sqrt{4x-x^2} - (2-x) \cdot \frac{2-x}{\sqrt{4x-x^2}}}{4x-x^2} = \frac{-1 \cdot \sqrt{4x-x^2} - \frac{(2-x)^2}{\sqrt{4x-x^2}}}{4x-x^2}$$

	0	2	3	4
$f'(x)$		-	-	
$f(x)$		∩	∩	

	0	1	2	3	4
$f'(x)$		+		-	
$f(x)$		↗		↘	



$$\lim_{x \rightarrow 0^+} \sqrt{4x-x^2} = \sqrt{0} = 0 \quad \text{V.A. } x=0$$

$$\lim_{x \rightarrow 4^-} \sqrt{4x-x^2} = \sqrt{16-16} = \sqrt{0} = 0 \quad \text{V.A. } x=4$$

$\lim_{x \rightarrow 0^-}$ NIJE U DOMENU
 $\lim_{x \rightarrow 4^+}$

$T(2, 2)$ GLOBALNI MAKSIMUM ✓
 f je $\sqrt{4x-x^2}$ JE NEOMEĐENA OD 0 DO 2
 - II - JE OMEĐENA OD 0 DO 4 ✓

6

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} = \frac{1-1}{\sqrt[3]{1}-1} = \frac{0}{0} = \lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} : x = \frac{1}{\frac{x}{\sqrt[3]{x}} - \frac{1}{\sqrt[3]{x}}} = \frac{1}{\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}} = \frac{1}{0} = +\infty$$

$$(4) \sqrt[3]{\frac{4+3i}{2-4i}} = \frac{4+3i}{2-4i} \cdot \frac{2+4i}{2+4i} = \frac{(4+3i)(2+4i)}{(2-4i)(2+4i)} = \frac{8+16i+6i+12i^2}{4+8i-8i-16i^2} = \frac{22i+8-12}{20} = \frac{22i-4}{20}$$

$$W_1 = \sqrt[3]{1.12} \left(\cos \frac{2.96+2.0\pi}{3} + i \sin \frac{2.96+2.0\pi}{3} \right)$$

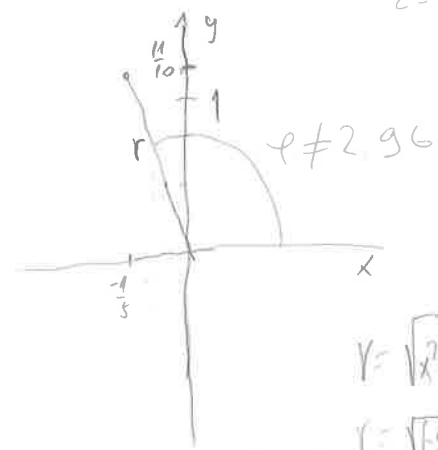
$$= 1.04(0.55 + 0.83i) = 0.57 + 0.86i$$

$$W_2 = \sqrt[3]{1.12} \left(\cos \frac{2.96+2.1\pi}{3} + i \sin \frac{2.96+2.1\pi}{3} \right)$$

$$= 1.04(-0.99 + 0.06i) = -1.03 + 0.06i$$

$$W_3 = \sqrt[3]{1.12} \left(\cos \frac{2.96+2.2\pi}{3} + i \sin \frac{2.96+2.2\pi}{3} \right)$$

$$= 1.04(0.45 - 0.89i) = 0.47 - 0.93i$$



$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{11}{10}\right)^2}$$

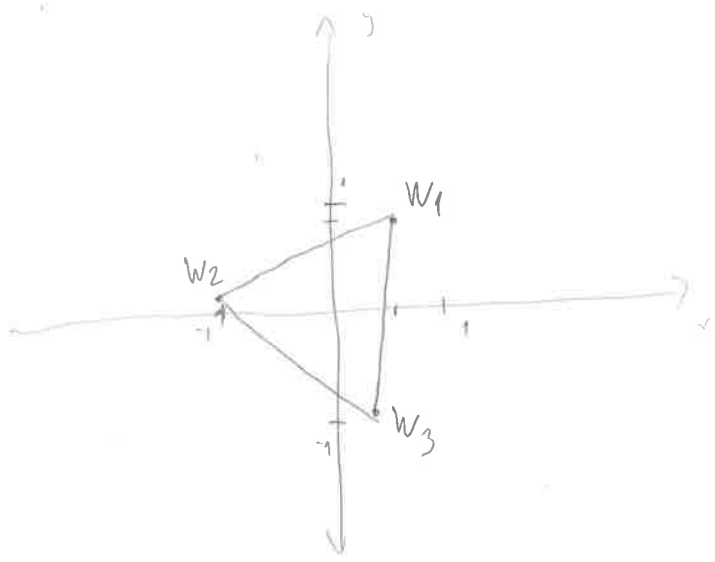
$$r = \sqrt{0.04 + 1.21}$$

$$r = 1.12$$

$$\phi = \arctan \frac{y}{x} + \pi$$

$$= \arctan \frac{11/10}{-1/5} + \pi = -0.17 + \pi = 2.96$$

$\phi = 2.96$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: *Antonio Jurić*

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): *0269076958*

A1

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf.

15 graf

2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf.

15 graf

3. Navesti posebno sve lokalne, a posebno sve globalne ekstreme funkcije $f(x) = \sqrt{4x-x^2}$. Komentirati (ne)omeđenost.

8+8+4

4. Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$. Prikaži rješenja u kompleksnoj ravnini!

12+3

5. Ispitati konvergenciju reda: $\sum_{n=1}^{\infty} (\sqrt{n^2-4n}-n)$

15

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$.

12+3

Ukupno:

11

6. $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$

PROVJERA:

$\lim_{x \rightarrow 1^+} \frac{x-1}{\sqrt[3]{x}-1} = 3$

ZASTO?

$\lim_{x \rightarrow 1^-} \frac{x-1}{\sqrt[3]{x}-1} = 3$

x	lim	x	lim
0.98	2,97	1,001	3,00
0.95	2,94	1,01	3,00
0.90	2,89	1,1	3,09

1. $f(x) = \sqrt{1+x} - \sqrt{3-x}$

1. $1+x \geq 0 \quad 3-x \geq 0$

$x \geq -1 \quad \Rightarrow x \geq -3$

$x \leq 3$

$x \in [-1, 3]$



2.

$\sqrt{1+x} - \sqrt{3-x} = 0$

$\sqrt{1+x} = \sqrt{3-x} \quad |^2$

$1+x = 3-x$

$2x = 2$

$x = 1$

$$f(x) = \sqrt{1+x} - \sqrt{3-x}$$

NASTAVAK 1. ZADATKA

$$f'(x) = \frac{1}{2\sqrt{1+x}} \cdot 1 - \frac{1}{2\sqrt{3-x}} \cdot (-1)$$

$$f'(x) = \frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{3-x}}$$

re. točke

$$\frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{3-x}} = 0$$

$$\frac{1}{2\sqrt{1+x}} = -\frac{1}{2\sqrt{3-x}} \quad |^2$$

$$\frac{1}{4(1+x)} = \frac{1}{4(3-x)}$$

$$\frac{1}{4+4x} = \frac{1}{12-4x} \quad |^{-1}$$

~~XXXXXXXXXX~~

$$4+4x = 12-4x$$

$$8x = 8$$

$$x = 1$$

$$y = 0$$

$$2. f(x) = \frac{e^x}{x} \quad x \neq 0$$

Df: $\mathbb{R} \setminus \{0\}$

GRAF:

$$\frac{e^x}{x} = 0$$

$$e^x = 0 \quad x = -\infty$$

$$f'(x) = \frac{e^x \cdot x - e^x \cdot 1}{x^2}$$

$$f'(x) = \frac{xe^x - e^x}{x^2} = \frac{x(e^x - 1)}{x^2} = \frac{e^x - 1}{x}$$

$$\frac{e^x - 1}{x} = 0 \quad x = 0$$

$$y = \frac{e^1}{1} = 2.71$$

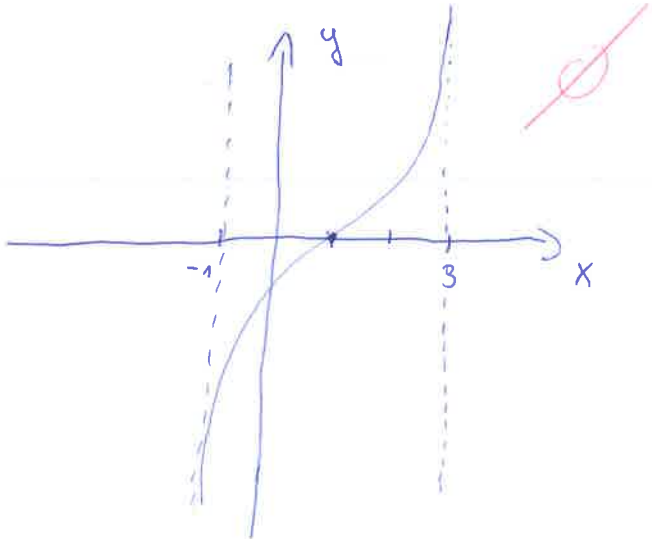
MONOTONOST



	$-\infty$	-1	3	$+\infty$	
x	N.D.		+	N.D.	
$f'(x)$			↗		

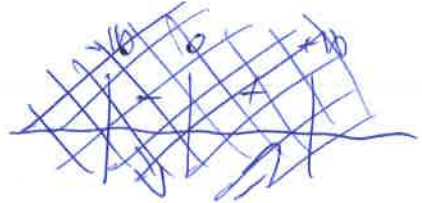
b.m.a.

GRAF:



x	2	-1	1	2	$-\frac{1}{2}$
$f(x)$	0.06	-0.7	2.72	3.7	-1.21

GRAF NIJE
 TEMELJEN
 NA ISPITIVANJU
 FUNKCIJSKOG
 TIJEKA



Matematika 1

Ime i prezime: Andrija Jović

Matični broj u indeksu: 0269076958

③

$$f(x) = \sqrt{4x - x^2}$$

$$4x - x^2 \geq 0$$

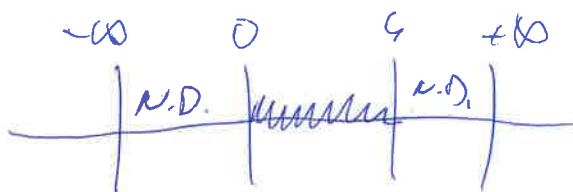
~~$$x(4-x) \geq 0$$~~

$$x(4-x) \geq 0$$

$$x \geq 0$$

$$4-x \geq 0$$

$$x \leq 4$$



$$Df(x) \in [0, 4]$$

$$\sqrt{4x - x^2} = 0 \quad |^2$$

$$4x - x^2 = 0$$

$$x(4-x) = 0$$

$$\boxed{\begin{matrix} x=0 \\ x=4 \end{matrix}}$$

$$f'(x) = \frac{1}{2\sqrt{4x-x^2}} \cdot (4-2x)$$

~~$$f'(x) = \frac{4-2x}{2\sqrt{4x-x^2}}$$~~

~~$$f'(x) = \frac{4-2x}{2\sqrt{4x-x^2}}$$~~

~~$$f'(x) = \frac{4-2x}{2\sqrt{4x-x^2}}$$~~

$$f'(x) = \frac{4-2x}{2\sqrt{4x-x^2}} = \frac{2(2-x)}{2\sqrt{4x-x^2}} = \frac{2-x}{\sqrt{4x-x^2}}$$

~~$$f'(x) = \frac{4-2x}{2\sqrt{4x-x^2}}$$~~

~~$$f'(x) = \frac{4-2x}{2\sqrt{4x-x^2}}$$~~

~~$$f'(x) = \frac{4-2x}{2\sqrt{4x-x^2}}$$~~

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~~$$f'(x) = \frac{4-2x}{2\sqrt{4x-x^2}}$$~~

x	0	1	2	2.5	2.5
y	0	1.41	2	1.96	1.93

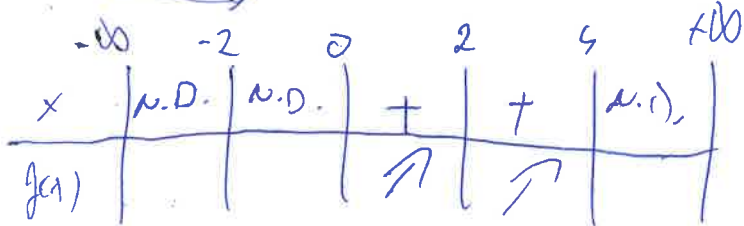
$$\frac{2-x}{\sqrt{4x-x^2}} = 0$$

$$\boxed{y=2}$$

$$2-x=0$$

$$-x=-2$$

$$\boxed{x=2}$$

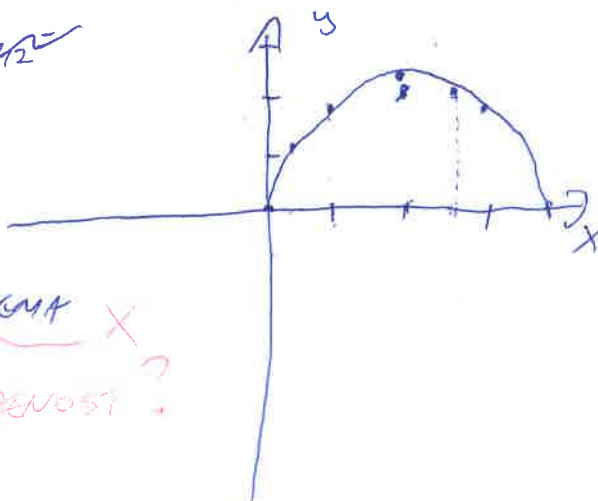


GLOBALNI MAX (2, 2) LOKALNI MAX. NEMA

LOKALNI MIN (0, 0) i (4, 0) GLOBALNI MIN. NEMA

Funkcija je konveksna

ONE DENOSTI?



4. $\sqrt[3]{\frac{4+3i}{2-4i}}$

$\sqrt[3]{z} = \frac{4+3i}{2-4i} \Big| ^3$

~~12/12~~

$4+3i=0$

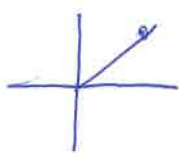
$x=4, y=3$

$r = \sqrt{x^2 + y^2}$

$r = 5$

$\varphi = \arctan \frac{y}{x}$

$\varphi = 0,644$



$z = \frac{(4+3i)^3}{(2-4i)^3}$

$z = \frac{(4+3i)^2 \cdot (4+3i)}{(2-4i)^2 \cdot (2-4i)}$

$z = \frac{(16+24i+9i^2)(4+3i)}{(4-16i+16i^2)(2-4i)}$

$z = \frac{(24i-7)(4+3i)}{(-16i-12)(2-4i)} = \frac{96i+72i^2-28-21i}{-32i+64i^2-24+48i}$

$z = \frac{75i-100}{16i-88} = \frac{25(3i-4)}{4(4i-22)}$

~~$z = \frac{75i-100}{16i-88} = \frac{25(3i-4)}{4(4i-22)}$~~

~~$(4y)(16i-88) = 75i-100$~~

~~$16xi + 88x + 16yi^2 - 88yi = 75i - 100$
 $16xi + 88x - 16y - 88y = -100 + 75i$
 $(16x - 88y)$~~

$k=0$

$w_1 = \sqrt[3]{5} \left[\cos \frac{2k\pi + \varphi}{n} + i \sin \frac{\varphi + 2k\pi}{n} \right] = \sqrt[3]{5} \left[\cos \frac{0,644}{3} + i \sin \frac{0,644}{3} \right]$

$w_1 = 1,6707 + i 0,3671$

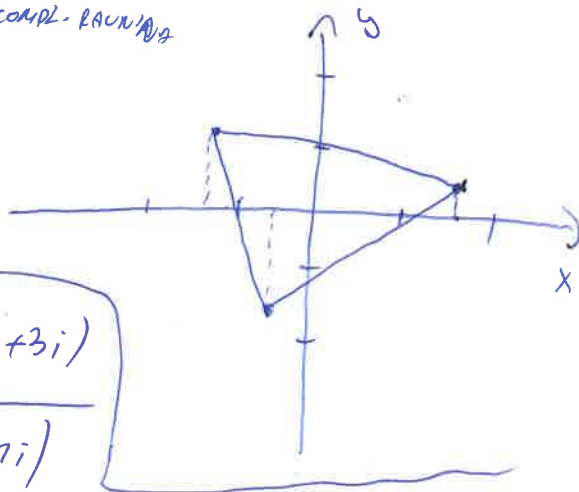
$k=1$

$w_2 = \sqrt[3]{5} \left[\cos \frac{0,644 + 2 \cdot \pi}{3} + i \sin \frac{0,644 + 2 \cdot \pi}{3} \right] = -0,156 + i 1,271$

$k=2$

$w_3 = \sqrt[3]{5} \left[\cos \frac{0,644 + 4 \cdot \pi}{3} + i \sin \frac{0,644 + 4 \cdot \pi}{3} \right] = -0,514 + i 1,631$

KOMPL. RAUNANNA



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: *Luka Grbin*

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): *17-1-0274-2014*

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf. 15 graf
2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf. 15 graf
3. Navesti posebno sve lokalne, a posebno sve globalne ekstreme funkcije $f(x) = \sqrt{4x-x^2}$. Komentirati (8) 8+4 (ne)omeđenost.
4. Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$. Prikaži rješenja u kompleksnoj ravnini! 12+3
5. Ispitati konvergenciju reda: $\sum_{n=1}^{\infty} (\sqrt{n^2-4n}-n)$ 15
6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$. 12+3

Ukupno:

8

1. $f(x) = \sqrt{1+x} - \sqrt{3-x}$

$$1+x \geq 0$$

$$x \geq -1$$

$$x \geq -1$$

$$3-x \geq 0$$

$$-x \geq -3 \quad | \cdot (-1)$$

$$x \leq 3$$



$$D(f(x)) = [-1, 3]$$

N.T.

$$\sqrt{1+x} = 0 \quad |^2$$

$$(\sqrt{1+x})^2 = 0$$

$$1+x=0$$

$$x=-1$$

N.T.

$$(-1, 0)$$

N.T.

$$\sqrt{3-x} = 0 \quad |^2$$

$$3-x=0$$

$$-x = -3$$

$$x=3$$

$$(3, 0)$$

Kosten

Wartung

$$f(x) = \sqrt{1+x} - \sqrt{3-x}$$

$$k = \frac{f(x)}{x} = \frac{\sqrt{1+x} - \sqrt{3-x}}{x} \quad | \cdot x$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{\frac{1}{x^2} + \frac{x}{x^2}} - \sqrt{\frac{3}{x^2} - \frac{x}{x^2}}}{x} = \frac{0 - 0}{1} = 0$$

Keine Kosten asymptote

$$y = \sqrt{1+x} - \sqrt{3-x}$$

$$y' = \frac{1}{2} (1+x)^{-\frac{1}{2}} \cdot (1+x)' - \frac{1}{2} (3-x)^{-\frac{1}{2}} \cdot (3-x)'$$

$$y' = \frac{1}{2} (1+x)^{-\frac{1}{2}} \cdot 1 - \frac{1}{2} (3-x)^{-\frac{1}{2}} \cdot (-1)$$

$$y' = \frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{3-x}}$$

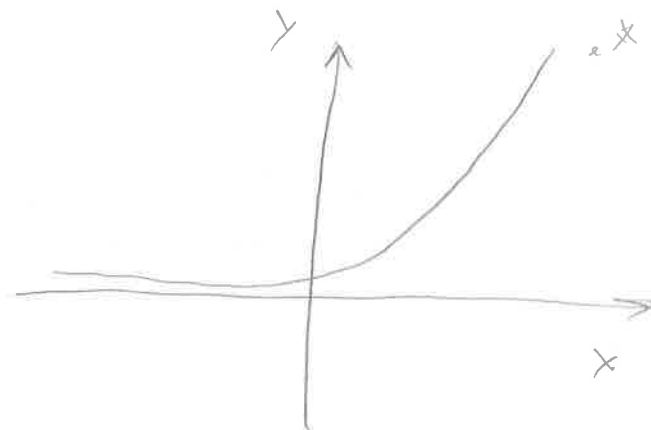
GRAF

$$y' = \frac{2\sqrt{3-x} + 2\sqrt{1+x}}{(2\sqrt{1+x})(2\sqrt{3-x})}$$

2.

$$f(x) = \frac{e^x}{x}$$

$$e^x > 0$$



6.

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \left(\frac{1-1}{\sqrt{1}-1} \right) = \left(\frac{0}{0} \right)$$

$$\lim_{x \rightarrow 1} \frac{1-1}{\sqrt[3]{1}-1} = \frac{0}{0} = 0$$



4.

$$\sqrt[3]{\frac{4+3i}{2-4i}} = 0$$



$$\frac{4+3i}{2-4i} = 0 \Rightarrow 4+3i = 0$$

$$4+3i = 2$$

$$\begin{aligned} b &= 2i \\ 0 &= 2i \\ \frac{2i}{2-4i} &= 2i \\ \frac{2i}{2-4i} &= 2i \times \frac{2+4i}{2+4i} \\ \frac{2i(2+4i)}{2^2 - (4i)^2} &= \frac{4i + 8i^2}{4 - 16i^2} \\ \frac{4i - 8}{4 + 16} &= \frac{4i - 8}{20} \\ \frac{4i - 8}{20} &= \frac{i - 2}{5} \end{aligned}$$

$$2i - \lambda(2i) = 2i(1 - \lambda)$$

Matematika 1

Ime i prezime: Luka Grbin

Matični broj u indeksu: 17-1-0274-2016

3. $f(x) = \sqrt{4x - x^2}$

$$4x - x^2 \geq 0$$

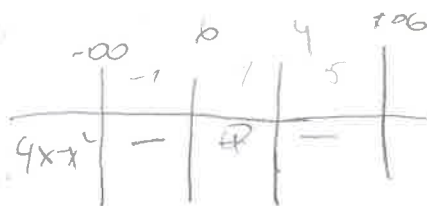
$$-x^2 + 4x = 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4(-1) \cdot 0}}{-2}$$

$$x_{1,2} = \frac{-4 \pm 4}{-2}$$

$$x_1 = 0$$

$$x_2 = 4$$

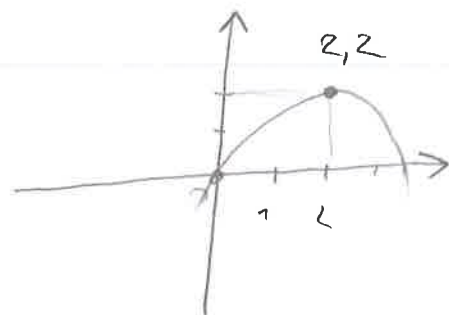


$$D(f(x)) = [0, 4]$$

$$x=0$$

$$\sqrt{4 \cdot 0 - 0^2}$$

$$y=0$$



N.T.

$$\sqrt{4x - x^2} = 0 \quad | \cdot$$

$$(\sqrt{4x - x^2})^2 = 0$$

$$4x - x^2$$

$$-x^2 + 4x = 0$$

N.T.

$$(0, 0) \quad (4, 0)$$

GLOBALNI
(2, 2)

LOKALNI
(0, 0) (4, 0)

ŠTE DENOST?

$$y = \sqrt{4x - x^2}$$

$$y' = \frac{1}{2} (4x - x^2)^{-\frac{1}{2}} \cdot (4x - x^2)'$$

$$y' = \frac{1}{2} (4x - x^2)^{-\frac{1}{2}} \cdot (4 - 2x)$$

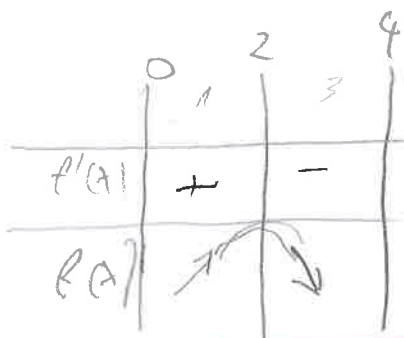
$$y' = \frac{1}{2} (4x - x^2) \cdot 2(2 - x)$$

$$y' = \frac{2 - x}{\sqrt{4x - x^2}}$$

$$2 - x = 0$$

$$-x = -2 \quad | \cdot (-1)$$

S.T. $x = 2$



$(2, 2)$ minimum ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **LOURE RAĐOUČIĆ**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

08:15

17-1-0177-2013

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf. 15 graf
2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf. 15 graf
3. Navesti posebno sve lokalne, a posebno sve globalne ekstreme funkcije $f(x) = \sqrt{4x-x^2}$. Komentirati (ne)omeđenost. 8+8+4
4. Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$. Prikaži rješenja u kompleksnoj ravnini! 12+3
5. Ispitati konvergenciju reda: $\sum_{n=1}^{\infty} (\sqrt{n^2-4n}-n)$ 15
6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$. 12+3

Ukupno:

4

⑥ $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} \rightarrow \lim_{x \rightarrow 1} \frac{1-1}{\sqrt[3]{1}-1} = \frac{0}{0} = \pm \infty = \pm \infty$

③ $f(x) = \sqrt{4x-x^2}$
 $f'(x) = \frac{1}{2\sqrt{4x-x^2}} (4x-x^2)'$
 $= \frac{1}{2\sqrt{4x-x^2}} (4-2x)$
 $= \frac{4-2x}{2\sqrt{4x-x^2}} = \frac{2(2-x)}{2\sqrt{4x-x^2}} = \frac{2-x}{\sqrt{4x-x^2}}$

$f''(x) = \left(\frac{2-x}{\sqrt{4x-x^2}} \right)'$
 $= \frac{(2-x)'(\sqrt{4x-x^2}) - (2-x)(\sqrt{4x-x^2})'}{\sqrt{4x-x^2}^2}$
 $= \frac{-\sqrt{4x-x^2} - (2-x) \cdot \frac{1}{2\sqrt{4x-x^2}}}{4x-x^2}$
 $= \frac{-\sqrt{4x-x^2} - \frac{2-x}{2\sqrt{4x-x^2}}}{4x-x^2}$
 $= \frac{-2\sqrt{4x-x^2} - 2-x}{2\sqrt{4x-x^2}}$
 $= \frac{4x-x^2 - 2\sqrt{4x-x^2} - 2-x}{4x-x^2}$

$\frac{2-x}{\sqrt{4x-x^2}} = 0 / \sqrt{4x-x^2}$

$f(2) = \sqrt{4 \cdot 2 - 2^2} = 2$

$2-x=0$
 $-x=-2$
 $x=2$

	$-\infty$	0	2	3	$+\infty$
$f'(x)$		+	-		
$f(x)$		↗	↘		
		MAX			

$(2, 2)$
MAX FUN

LOKALNI

GLOBALNI ILI LOKALNI

$$f(x) = \sqrt{1+x} - \sqrt{3-x}$$

DOMENA

$$1^\circ 1+x \geq 0 \quad 2^\circ 3-x \geq 0$$

$$x \geq -1 \quad -x \geq -3$$

$$x \leq 3$$

(NE) PARNOST

$$f(-x) = \sqrt{1-x} - \sqrt{3+x}$$

NITI PARNA
NITI NE PARNA

$$D = [-1, 3]$$

ASIMPTOTE

V.A.
 $\lim_{x \rightarrow -1} \sqrt{1-x} - \sqrt{3-x} = -2$

V.A. NE POSTOJI

D.H.A.

$$\lim_{x \rightarrow \infty} \sqrt{1+x} - \sqrt{3-x} \cdot \frac{\sqrt{1+x} + \sqrt{3-x}}{\sqrt{1+x} + \sqrt{3-x}} = \frac{\sqrt{1+x}^2 - \sqrt{3-x}^2}{\sqrt{1+x} + \sqrt{3-x}} = \frac{1+x-3+x}{\sqrt{1+x} + \sqrt{3-x}} = \frac{2x-2}{\sqrt{1+x} + \sqrt{3-x}} \stackrel{0}{=} \frac{2}{\infty} = \infty$$

NEMA D.H.A.

L.H.A.
 $\lim_{x \rightarrow -\infty} \sqrt{1+x} - \sqrt{3-x} \cdot \frac{\sqrt{1+x} + \sqrt{3-x}}{\sqrt{1+x} + \sqrt{3-x}} = \frac{1+x-3+x}{\sqrt{1+x} + \sqrt{3-x}} = \frac{2x-2}{\sqrt{1+x} + \sqrt{3-x}} \stackrel{0}{=} \frac{-2x-2}{\infty} = -\infty$

D.K.A.
 $\lim_{x \rightarrow \infty} \frac{\sqrt{1+x} - \sqrt{3-x}}{x} \stackrel{0}{=} \frac{0}{\infty} = 0$ NEMA D.K.A.

L.K.A.
 $\lim_{x \rightarrow -\infty} \frac{\sqrt{1+x} - \sqrt{3-x}}{x} = \frac{\sqrt{1-x} - \sqrt{3+x}}{-x} \stackrel{0}{=} \frac{0}{-\infty} = -\infty$ NEMA L.K.A.

MULTOČKE

$$\sqrt{1+x} - \sqrt{3-x} = 0 \quad | \quad / 2 \quad (1,0)$$

$$1+x-3+x=0$$

$$2x=3-1$$

$$2x=2$$

$$x=1$$

SPECIŠTE S OSI y

$$f(0) = \sqrt{1+0} - \sqrt{3-0} = -0,73 \quad (0, -0,73)$$

DERIVACIJA

$$f'(x) = (\sqrt{1+x} - \sqrt{3-x})'$$

$$= \frac{1}{2\sqrt{1+x}} (1+x)' - \frac{1}{2\sqrt{3-x}} (3-x)'$$

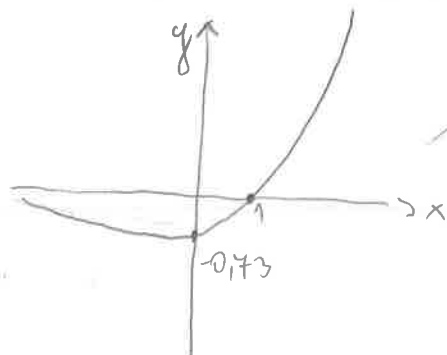
$$= \frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{3-x}} = \frac{2\sqrt{3-x} + 2\sqrt{1+x}}{2\sqrt{1+x} + 2\sqrt{3-x}}$$

KRITIČNE TOČKE

$$3-x+1+x=0$$

$$4=0$$

	$-\infty$	-1	0	1	$+\infty$
$f'(x)$		+		+	
$f(x)$		↗	↘	↗	



Matematika 1

Ime i prezime:

LOVRE RADOVIĆ

Matični broj u indeksu:

17-1-0177-2013

Q. $f(x) = \frac{e^x}{x}$

(NE) PARNOST

$f(-x) = \frac{e^{-x}}{-x}$ NEPARNA

DOMENA

$x \neq 0 \Rightarrow f: \mathbb{R} \setminus \{0\}$

ASIMPTOTE

V.A. $\lim_{x \rightarrow 0} \frac{e^0}{0} = \pm \infty$

V.A. $x=0$

H.A. $\lim_{x \rightarrow \infty} \frac{e^x / |x|}{x / |x|} = \frac{1}{1} = 1$ H.A. $y=1$

K.I.A. $k = \lim_{x \rightarrow \infty} \frac{e^x}{x} = \frac{e^x / |x^2|}{x^2 / |x^2|} = \frac{0}{1} = 0$ NEMA K.I.A.

MULTOČKE

$f(x) = 0$ (oid)
 $\frac{e^x}{x} = 0$
 $e^x = 0$

SPECIŠTE S OSI y
 $f(0) = \frac{e^0}{0} = \infty$

DERIVACIJA

$$f'(x) = \left(\frac{e^x}{x}\right)' = \frac{(e^x)' \cdot x - e^x (x)'}{x^2} = \frac{e^x \cdot x - e^x \cdot 1}{x^2} = \frac{e^x \cdot x - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$$

2. DERIVACIJA

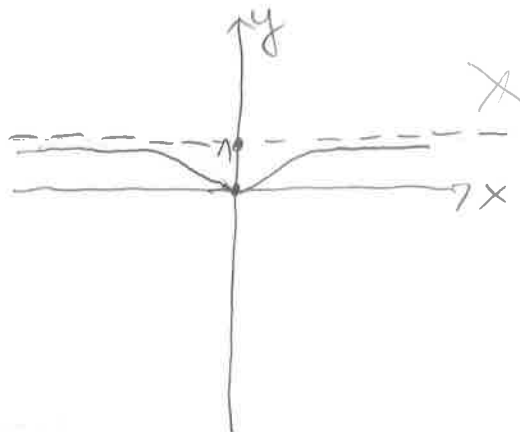
$$f''(x) = \left(\frac{e^x(x-1)}{x^2}\right)' = \frac{(e^x \cdot (x-1) - e^x(x-1) \cdot 2x) \cdot x^2 - (e^x(x-1))^2 \cdot 2x}{x^4}$$

$e^{x^2} - e^x = 0$

$e^x = 0$

$f(0) = \frac{e^0}{0} = \infty$

	$-\infty$	-1	0	1	$+\infty$
$f'(x)$		+		+	
$f(x)$		↗		↗	



$$\omega^+ = \frac{v-v_1 v_2}{v} \sqrt{\frac{v_1^2 - v_2^2}{v^2}} \quad (5)$$

2. DERIVATA

$$f(x) = \frac{2\sqrt{3-x} + 2\sqrt{1+x}}{2\sqrt{1+x} + 2\sqrt{3-x}}$$

$$f'(x) = \frac{\left(\frac{-2}{2\sqrt{3-x}} + \frac{2}{2\sqrt{1+x}} \right) (2\sqrt{1+x} + 2\sqrt{3-x}) - (2\sqrt{3-x} + 2\sqrt{1+x}) \left(\frac{2}{2\sqrt{1+x}} - \frac{2}{2\sqrt{3-x}} \right)}{(2\sqrt{1+x} + 2\sqrt{3-x})^2}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

IME I PREZIME: **KLAUDIO ŠKAPA**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

A1

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf.

15 graf

2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf.

15 graf

3. Navesti posebno sve lokalne, a posebno sve globalne ekstreme funkcije $f(x) = \sqrt{4x-x^2}$. Komentirati (ne)omeđenost.

8+8+4

4. Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$. Prikaži rješenja u kompleksnoj ravnini!

12+3

5. Ispitati konvergenciju reda: $\sum_{n=1}^{\infty} (\sqrt{n^2-4n} - n)$

15

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$

12+3

4.1) Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$

$\sqrt[3]{\frac{4+3i}{2-4i}} = 0 / \sqrt[3]{5}$ X

$\frac{4+3i}{2-4i} = 0 \quad | \cdot (2-4i)$

$4+3i = 0$

$x=4 \quad y=3$

$r = \sqrt{x^2+y^2}$

$r = \sqrt{4^2+3^2}$

$r = 5$

Y aneta $\frac{3}{4} = 0.64$

$\varphi_{\text{pravni}} = 2\pi + 0.64 = 6.92$

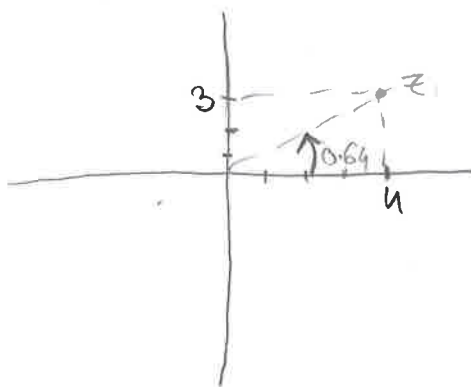
$z_1 = \sqrt[3]{r} \cdot \left(\cos \frac{\varphi + 2 \cdot k \cdot \pi}{n} + i \sin \frac{\varphi + 2 \cdot k \cdot \pi}{n} \right)$

$z_1 = \sqrt[3]{5} \cdot \left(\cos \frac{6.92 + 2 \cdot 0 \cdot \pi}{3} + i \sin \frac{6.92 + 2 \cdot 0 \cdot \pi}{3} \right) =$

$z_1 = 1.71 \cdot (-0.67 + 0.74i) = -1.15 + 1.27i$

$z_2 = 1.71 \cdot (-0.31 - 0.95i) = -0.53 - 1.62i$

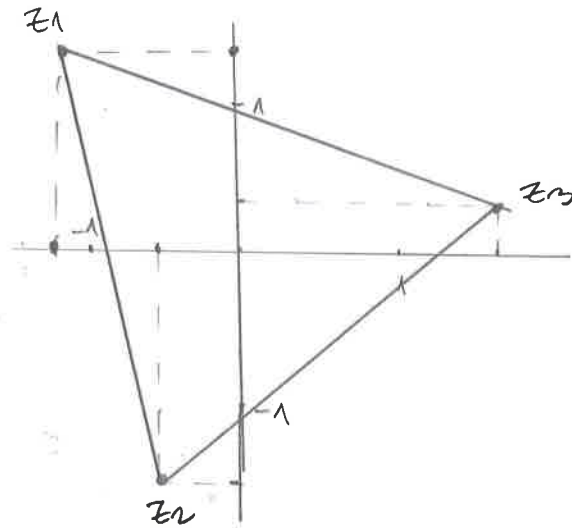
$z_3 = 1.71 \cdot (0.98 + 0.21i) = 1.67 + 0.36i$



$k = 0, 1, 2$

Ukupno:

~~0~~



6. $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} = \frac{1-1}{\sqrt[3]{1}-1} = \frac{0}{0} = \text{NEODREĐENI OBLIK}$

$L^- = \lim_{x \rightarrow 1^-} \frac{0.99999 - 1}{\sqrt[3]{0.99999} - 1} = 2.9$

$L^+ = \lim_{x \rightarrow 1^+} \frac{1.00001 - 1}{\sqrt[3]{1.00001} - 1} = 3$

L^- i $L^+ = \text{ne se poklapaju}$



3. $f(x) = \sqrt{4-x^2}$ I. $4x - x^2 \geq 0$

$b=4 \quad -1a \quad c=0$

$4x - x^2 = 0$

$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot 0}}{-2} = \frac{-4 \pm 4}{-2}$

$x_1 = 0 \quad x_2 = 4$



$D_f \in (-\infty, 4]$

V.A. NE POSTOJI ZATO ŠTO NEMA PREDTOKA FUNKCIJE

H.A.

$\lim_{x \rightarrow \pm\infty} = \frac{\sqrt{4-x^2}}{1} \cdot \frac{\sqrt{4+x^2}}{\sqrt{4+x^2}} = \lim_{x \rightarrow \pm\infty} \frac{|4-x^2|}{\sqrt{4+x^2}}$

$\lim_{x \rightarrow \pm\infty} = \frac{4-x^2/x^2}{\sqrt{4/x^2 + x^2/x^2}} = \frac{4/x^2 - x^2/x^2}{\sqrt{4/x^2 + 1}} = \frac{0-1}{\sqrt{0+1}} = -1$



H.A. Postoji što znači da je funkcija omeđena.

Matematika 1

Ime i prezime: KLAUDIO ŠKALA

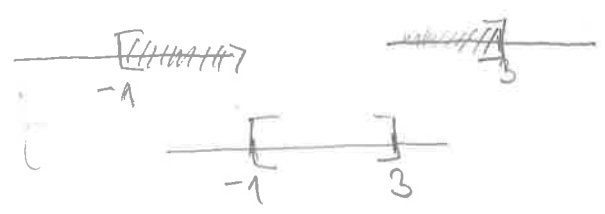
Matični broj u indeksu: 17-1-02 15-20 13

1. $f(x) = \sqrt{1+x} - \sqrt{3-x}$

a) Domena

I. $1+x \geq 0$
 $1+x=0$
 $x=-1$

II. $3-x \geq 0$
 $3-x=0$
 $+x=+3$



$D \in [-1, 3]$

b) asimptote

V. A. - Ne postoji

H. A. $\lim_{x \rightarrow \infty} \frac{\sqrt{1+x} - \sqrt{3-x}}{1} \cdot \frac{\sqrt{1+x} + \sqrt{3-x}}{\sqrt{1+x} + \sqrt{3-x}} = \lim_{x \rightarrow \infty} \frac{(\sqrt{1+x})^2 - (\sqrt{3-x})^2}{\sqrt{1+x} + \sqrt{3-x}} = \lim_{x \rightarrow \infty} \frac{1+x - 3+x}{\sqrt{1+x} + \sqrt{3-x}} = \lim_{x \rightarrow \infty} \frac{1+x - 3+x}{\sqrt{1+x} + \sqrt{3-x}}$

$\lim_{x \rightarrow \infty} = \frac{\frac{1}{x} + \frac{x}{x} - \frac{3}{x} + \frac{x}{x}}{\sqrt{\frac{1}{x} + \frac{x}{x}} + \sqrt{\frac{3}{x} - \frac{x}{x}}} = \lim_{x \rightarrow \infty} \frac{0+1-0+1}{\sqrt{0+1} + \sqrt{0-1}} = \frac{2}{1} = 2$

V. A. - NE POSTOJI ZATO ŠTO IMAMO H. A. H. A. JE PRAVAC $y=2$

c) funkcija je omeđena jer ima H. A.

d) $f(-x) = \sqrt{1-x} - \sqrt{3-x} =$ NIJE PARNA

$f(-x) = -f(x)$

$f(-x) = -\sqrt{1+x} - \sqrt{3-x} =$ NIJE NEPARNA

funkcija je ništa parna ništa neparna

4. SPECIJA S KOORDINATNIM OSIMA

$$f(x) = 0$$

$$f(x) = \sqrt{1+x} - \sqrt{3-x}$$

$$\text{as } y \neq 0 =$$

$$f(0) = \sqrt{1+0} - \sqrt{3-0}$$

$$f(0) = 1 - \sqrt{3}$$

$$f(0) = -0.73$$

$$N(0, -0.73)$$

5. Derivacije

$$f'(x) = (\sqrt{1+x})' - (\sqrt{3-x})'$$

$$f'(x) = \left(-\frac{1}{2\sqrt{1+x}} \cdot (1+x)' \right) - \left(-\frac{1}{2\sqrt{3-x}} \cdot (3-x)' \right)$$

$$f'(x) = \left(-\frac{1}{2\sqrt{1+x}} \cdot 1 \right) - \left(-\frac{1}{2\sqrt{3-x}} \cdot (-1) \right)$$

$$f'(x) = -\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{3-x}}$$

$$f''(x) = \left(-\frac{1}{2\sqrt{1+x}} \right)' - \left(-\frac{1}{2\sqrt{3-x}} \right)'$$

$$f''(x) = \left(\frac{-(-1) \cdot (2\sqrt{1+x}) - (-1) \cdot (2\sqrt{1+x})}{(2\sqrt{1+x})^2} \right) + \left(\frac{-(-1) \cdot (2\sqrt{3-x}) - (-1) \cdot (2\sqrt{3-x})}{(2\sqrt{3-x})^2} \right)$$

$$f''(x) = \frac{1 \cdot (2) + (\sqrt{1+x})'}{(2\sqrt{1+x})^2}$$

GRAF?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **KARLO TERZIĆ**

VRIJEME POČETKA: **08:50**

41

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0361-2014

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf. 15 graf
2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf. 15 graf
3. Navesti posebno sve lokalne, a posebno sve globalne ekstreme funkcije $f(x) = \sqrt{4x-x^2}$. Komentirati (ne)omeđenost. 8+8+4
4. Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$. Prikaži rješenja u kompleksnoj ravnini! 12+3
5. Ispitati konvergenciju reda: $\sum_{n=1}^{\infty} (\sqrt{n^2-4n}-n)$ 15
6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$. 12+3

Ukupno:

4. $\sqrt[3]{\frac{4+3i}{2-4i}}$

$$\frac{4+3i}{2-4i} \cdot \frac{2+4i}{2+4i} = \frac{8+16i+6i+12i^2}{4+8i-8i-16i^2} = \frac{8+22i-12}{4+16}$$

$$= \frac{-4+22i}{20} \quad | \cdot 20$$

$$= -4+22i$$

$$22i = 4 \quad | :22$$

$$i = \frac{4}{22}$$

$$\sqrt[3]{-\frac{4}{20} + \frac{22}{20}i} = ?$$

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod

A1

NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **IVAN KOKIĆ**

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU): **0269086090**

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf. 15 graf
2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf. 15 graf
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Ukupno:

~~0~~

1. $f(x) = \sqrt{1+x} - \sqrt{3-x}$

PARNOST

$f(-x) = \sqrt{1-x} - \sqrt{3+x}$

Neparna

$1+x \geq 0 \quad 3-x \geq 0$
 $x \geq -1 \quad x \leq 3$

$D(f) = \mathbb{R} \quad D(g) = \mathbb{R}$ X

N.T.

$\sqrt{1+x} - \sqrt{3-x} = 0 / \sqrt{\quad}$
 $1+x - 3+x = 0$
 $-2+2x = 0$
 $2x = 2 / :2$
 $x = 1 \quad N_1(1, 0)$

$x = 0$
 $y = \sqrt{1+0} - \sqrt{3-0}$
 $y = \sqrt{1} - \sqrt{3}$
 $y = -0,73$
 $N_2(0, -0,73)$

H.A.

$y = \sqrt{1+x} - \sqrt{3-x} \cdot \frac{\sqrt{1+x} + \sqrt{3-x}}{\sqrt{1+x} + \sqrt{3-x}}$

$y = \frac{(\sqrt{1+x})^2 - (\sqrt{3-x})^2}{\sqrt{1+x} + \sqrt{3-x}}$

$y = \frac{1+x-3+x}{\sqrt{1+x} + \sqrt{3-x}} \quad y = \frac{-2+2x}{\sqrt{1+x} + \sqrt{3-x}} \quad y = \frac{-\frac{2}{x} + \frac{2x}{x}}{\sqrt{\frac{1^0+x^0}{x^2} + \sqrt{\frac{3^0-x^0}{x^2}}}$

$$y = \frac{2}{0} = \infty$$

NEMA
HORIZONTALNE
ASIMPTOTE

K.A.

$$k_1 = \frac{\sqrt{1+x} - \sqrt{3-x}}{x} \quad | :x$$

$$k_1 = \frac{\sqrt{\frac{1^0}{x^2} + \frac{x^0}{x^2}} - \sqrt{\frac{3^0}{x^2} - \frac{x^0}{x^2}}}{\frac{x}{1 \cdot x}}$$

$$k_1 = \frac{0}{1} = 0$$

EKSTREM.

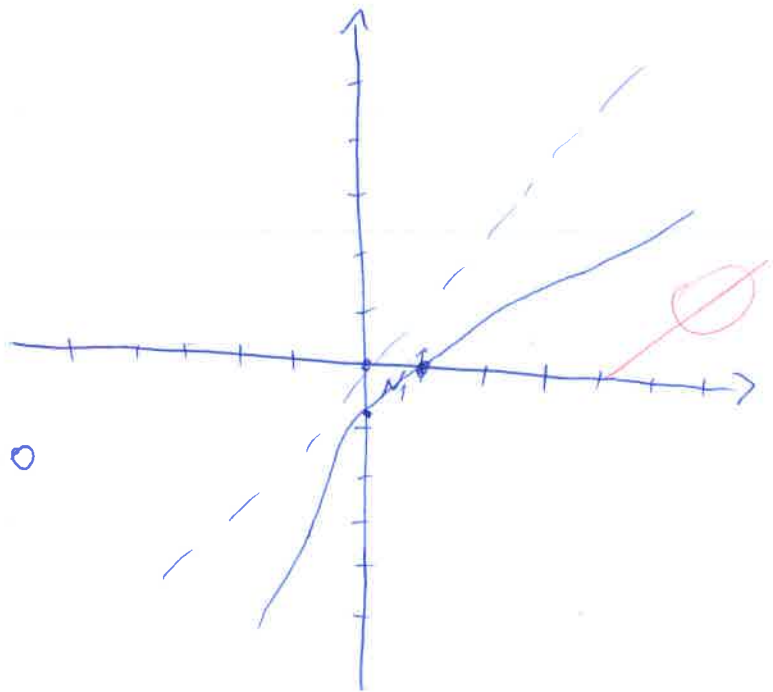
$$f(x) = \sqrt{1+x} - \sqrt{3-x}$$

$$f(x) = (1+x)^{\frac{1}{2}} - (3-x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} + \frac{1}{2}(3-x)^{-\frac{1}{2}}$$

$$\frac{1}{2}(1+x)^{-\frac{1}{2}} + \frac{1}{2}(3-x)^{-\frac{1}{2}} = 0$$

$$x = 0,78$$



	$-\infty$	$-0,78$	0	1	$+\infty$
$f(x)$	$+$	$+$	$+$	$+$	$+$
$f'(x)$	\nearrow	\nearrow	\nearrow	\nearrow	\nearrow

$$2. f(x) = \frac{e^x}{x}$$

$$x = 0$$

$$D(f) = \mathbb{R}$$

N.T.

$$\frac{e^x}{x} = 0 \quad | \cdot x$$

$$e^x = 0$$

$$6. \lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} = \frac{1-1}{\sqrt[3]{1}-1} = \frac{0}{1-1} = 0 \quad \times$$

$$5. \sum_{n=1}^{\infty} (\sqrt{n^2-4n} - n) = \left((\sqrt{1-4} - 1) + (\sqrt{3-1}) \right)$$

$$\begin{aligned} \sqrt{n^2-4n} - n &= 0 \\ n^2-4n-n^2 &= 0 \\ -4n &= 0 \end{aligned}$$

$$\sum_{n=1}^{\infty} (\sqrt{n^2-4n} - n)^2 \quad \times$$

$$\sum_{n=1}^{\infty} (n^2 - 4n - n^2) =$$

$$\sum_{n=1}^{\infty} (-4n) = \sum_{n=1}^{\infty} (-4)$$

$$4. z = \sqrt[3]{\frac{4+3i}{2-4i}} \cdot (2-8i)$$

$$z = \sqrt[3]{4+3i} \quad \times$$

$$x=4 \quad y=3$$

$$\operatorname{tg} \varphi = \frac{y}{x}$$

$$r = \sqrt{16+9}$$

$$\operatorname{tg} \varphi = \frac{3}{4}$$

$$r = \sqrt{25}$$

$$\varphi = 36^\circ 52'$$

$$r=5$$

Matematika 1

Ime i prezime: IVAN KOKIĆ

Matični broj u indeksu: 0269086090

3.

$$f(x) = \sqrt{4x - x^2}$$

$$f(x) = (4x - x^2)^{\frac{1}{2}}$$

$$f(x)' = \frac{1}{2} (4x - x^2)^{-\frac{1}{2}} \cdot (4x - x^2)'$$

$$f(x)' = \frac{1}{2} (4x - x^2)^{-\frac{1}{2}} \cdot (4 - 2x)$$

$$f(x)' = (2 - x)(4x - x^2)^{-\frac{1}{2}}$$

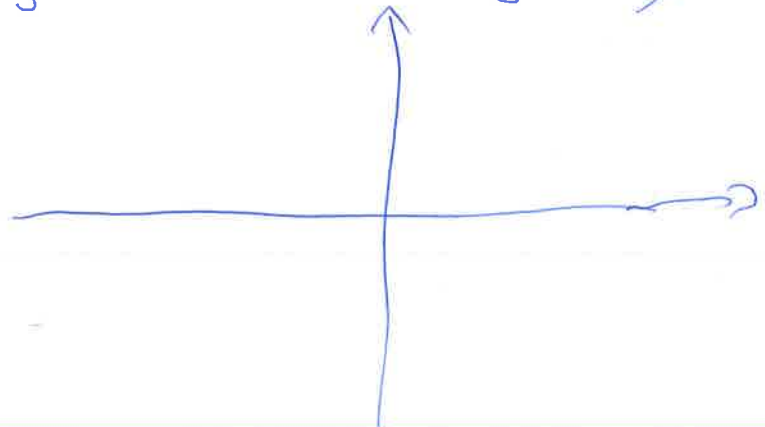
$$(2 - x)(4x - x^2)^{-\frac{1}{2}} = 0$$

4. NASTAVAK

$$k=0 \quad z_1 = \sqrt[3]{5} \left(\cos \frac{36^\circ 52'}{3} + i \sin \frac{36^\circ 52'}{3} \right)$$

$$k=1 \quad z_2 = \sqrt[3]{5} \left(\cos \frac{36^\circ 52' + 2\pi}{3} + i \sin \frac{36^\circ 52' + 2\pi}{3} \right)$$

$$k=2 \quad z_3 = \sqrt[3]{5} \left(\cos \frac{36^\circ 52' + 4\pi}{3} + i \sin \frac{36^\circ 52' + 4\pi}{3} \right)$$



odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

A1

IME I PREZIME: *Petra Delalić*

VRIJEME POČETKA: *9:10*

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

0269080611

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf.

15 graf

2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf.

15 graf

3. Navesti posebno sve lokalne, a posebno sve globalne ekstreme funkcije $f(x) = \sqrt{4x-x^2}$. Komentirati (ne)omeđenost.

8+8+4

4. Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$. Prikaži rješenja u kompleksnoj ravnini!

12+3

5. Ispitati konvergenciju reda: $\sum_{n=1}^{\infty} (\sqrt{n^2-4n}-n)$

15

6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$.

12+3

Ukupno:

0

6) $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} \stackrel{\text{L.H.}}{=} \frac{1}{\frac{1}{\sqrt[3]{x}}} = \frac{1}{\frac{1}{\sqrt[3]{1}}} = \frac{1}{\frac{1}{1}} = \frac{1}{1} = 1$

4) $\sqrt[3]{\frac{4+3i}{2-4i}} \quad z^3 = \frac{4+3i}{2-4i} \cdot \frac{2+4i}{2+4i} = \frac{(4+3i)(2+4i)}{4+8i-8i-16i^2} = \frac{8+16i+6i+12i^2}{4+16}$

$= \frac{8+22i-12}{4+16} = \frac{-4+22i}{20} = -\frac{4}{20} + \frac{22}{20}i = -\frac{1}{5} + \frac{11}{10}i$

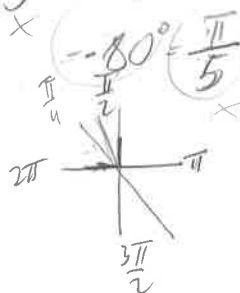
$z^3 = -\frac{1}{5} + \frac{11}{10}i$

$|w| = \sqrt{x^2+y^2}$

$|w| = \sqrt{\frac{1}{25} + \frac{121}{100}} = \frac{\sqrt{5}}{2}$

$\arg z = \frac{y}{x} = \frac{\frac{11}{10}}{-\frac{1}{5}} = -\frac{11}{2}$

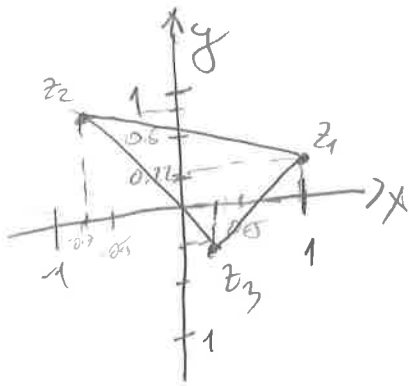
$z = \sqrt[3]{-\frac{1}{5} + \frac{11}{10}i}$



$k=0 \quad z_0 = \sqrt[3]{\frac{\sqrt{5}}{2}} \left(\cos \frac{\frac{\pi}{5} + 2 \cdot 0 \cdot \pi}{3} + i \sin \frac{\frac{\pi}{5} + 2 \cdot 0 \cdot \pi}{3} \right) = 1 + 0.22i$

$k=1 \quad z_1 = \sqrt[3]{\frac{\sqrt{5}}{2}} \left(\cos \frac{\frac{\pi}{5} + 2 \cdot 1 \cdot \pi}{3} + i \sin \frac{\frac{\pi}{5} + 2 \cdot 1 \cdot \pi}{3} \right) = -0.7 + 0.77i$

$k=2 \quad z_2 = \sqrt[3]{\frac{\sqrt{5}}{2}} \left(\cos \frac{\frac{\pi}{5} + 2 \cdot 2 \cdot \pi}{3} + i \sin \frac{\frac{\pi}{5} + 2 \cdot 2 \cdot \pi}{3} \right) = -0.3 - 0.3i$



① $f(x) = \sqrt{1+x} - \sqrt{3-x}$

① DOMENIA

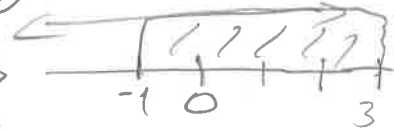
$$1+x \geq 0$$

$$3-x \geq 0$$

$$x \geq -1$$

$$x \leq 3$$

$$D_f \subseteq \mathbb{R} [-1, 3]$$



② $\sqrt{1+x} - \sqrt{3-x} = 0 \quad |^2$

MULTIPLICAZIONE

$$1+x - 3-x = 0$$

~~-2=0~~ NEMA D.T.

③ ASIMPTOTE

v.A.

$$\lim_{x \rightarrow -1^-} \sqrt{1+x} - \sqrt{3-x} = -\infty$$

$$\lim_{x \rightarrow -1^+} \sqrt{1+x} - \sqrt{3-x} = -\infty$$

GRAF?

② $f(x) = \frac{e^x}{x}$

① DOMENA $x \neq 0$ Df. $x \in \mathbb{R} \setminus \{0\}$

PEJKA
DOLAZI

② ~~MULTIPLIK~~ $e^x \neq 0$ LEMA

③ ASIMPTOTE

V.A.

$\lim_{x \rightarrow 0^-} \frac{e^x}{x} = -\infty$ $\lim_{x \rightarrow 0^+} \frac{e^x}{x} = +\infty$ $x=0$ s. lije i desno

H.A.

$\lim_{x \rightarrow +\infty} \frac{e^x}{x} \left[\frac{\infty}{\infty} \right] \stackrel{L.H.}{=} \frac{(e^x)'}{x'} = e^x = e^\infty = \infty$

LEMA H.A.

$\lim_{x \rightarrow -\infty} \frac{e^x}{x} \left[\frac{\infty}{\infty} \right] \stackrel{L.H.}{=} \frac{(e^x)'}{x'} = e^x = e^{-\infty} = -\infty$

KOSA

$g = kx + l$ $k = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \frac{\frac{e^x}{x}}{\frac{x}{1}} = \frac{e^x}{x^2} \stackrel{L.H.}{=} \frac{e^x}{2x} \stackrel{L.H.}{=} \frac{e^x}{2}$

$\lim_{x \rightarrow +\infty} \frac{e^\infty}{2} = +\infty$

$\lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \frac{e^{-\infty}}{2} = -\infty$

nema kose

④ I DEN. (SITAC. TOČKE)

$f(x) = \frac{e^x}{x}$ $f'(x) = \frac{(e^x)' \cdot x - e^x \cdot x'}{x^2} = \frac{e^x \cdot x - e^x}{x^2}$

$e^x \cdot x - e^x = 0$
 $e^x \cdot x = e^x / : e^x$
 $x = 1$

	-∞	0	1	+∞
f'	-	-	+	
f	↘	↘	↗	

$m(1, 2.7)$

$g(1) = \frac{e^1}{1} = 2.7$

⑤ II DER. (TOOKA INF.)

$$f'(x) = \frac{e^x \cdot x - e^x}{x^2} = \frac{(e^x \cdot x - e^x)' \cdot x^2 - (e^x \cdot x - e^x) \cdot (x^2)'}{x^4}$$

$$\frac{(e^x \cdot x - e^x)' \cdot x^2 - (e^x \cdot x - e^x) \cdot 2x}{x^4} = \frac{-2x(e^x \cdot x - e^x)}{x^4} = \frac{-2x^2 e^x + 2x e^x}{x^4}$$

$$\frac{2x e^x (-x + 1)}{x^4}$$

$$2x e^x (-x + 1) = 0 \quad -x + 1 = 0$$

$$2x e^x = 0 / :2$$

$$-x = -1$$

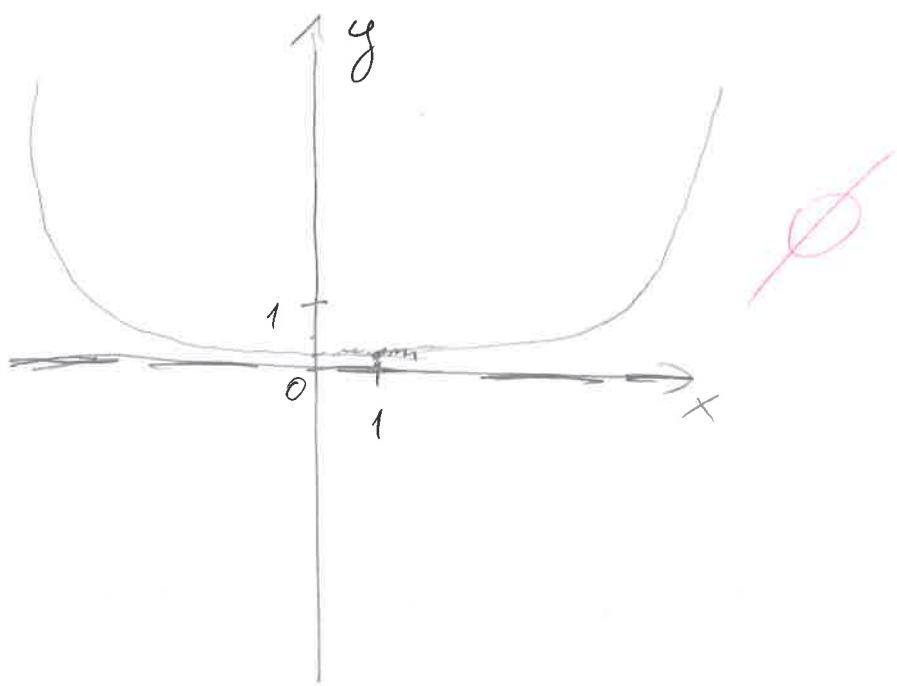
$$x e^x = 0 / e^x$$

$$\boxed{x = 1}$$

$$\boxed{x = 0}$$

x	-2	0	1	+∞
f''	+	+	+	
f'	U	U	U	

KEWA TOOKI INF.



PETRA DEZARIC

$$\textcircled{3} \quad f(x) = \sqrt{4x-x^2}$$

$$f'(x) = \frac{1}{\sqrt{4x-x^2}} \cdot (4x-x^2)' = \frac{1}{\sqrt{4x-x^2}} \cdot (4-2x) = \frac{4-2x}{\sqrt{4x-x^2}}$$

$$4-2x=0$$

$$-2x = -4$$

$$\boxed{x=2}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ↓↓

A1

IME I PREZIME: DORA BUŽONJA

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA (IZNAD SLIKE U INDEKSU):

17-2-0307-2013.

1. Odrediti tok funkcije $f(x) = \sqrt{1+x} - \sqrt{3-x}$ i skicirati graf. 15 graf
2. Ispitati tijek funkcije $f(x) = \frac{e^x}{x}$ i skicirati njen graf. 15 graf
3. Navesti posebno sve lokalne, a posebno sve globalne ekstreme funkcije $f(x) = \sqrt{4x-x^2}$. Komentirati (ne)omeđenost. 8+8+4
4. Među kompleksnim brojevima odrediti $\sqrt[3]{\frac{4+3i}{2-4i}}$. Prikaži rješenja u kompleksnoj ravnini! 12+3
5. Ispitati konvergenciju reda: $\sum_{n=1}^{\infty} (\sqrt{n^2-4n}-n)$ 15
6. Ispitati i na neki način provjeriti $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$. 12+3

Ukupno:

3) $f(x) = \sqrt{4x-x^2}$

DOMENA: $4x-x^2 \geq 0$

$x(4-x) \geq 0$

$x \geq 0$

$4-x \geq 0$

$-x \geq -4$

$x \leq 4$



Preklapaju se
Df. [4, 0]

F-JA JE OMEĐENA
SPOD I NEOMEĐENA
PREMA V (KREĆE SE
LILJE PREMA GORE)

$$f'(x) = \frac{1}{2\sqrt{4x-x^2}} \cdot (4x-x^2)' = \frac{4-2x}{2\sqrt{4x-x^2}} = \frac{2(2-x)}{2\sqrt{4x-x^2}} = \frac{2-x}{\sqrt{4x-x^2}}$$

$f'(x) = 0$

$2-x = 0$

$-x = -2$

$x = 2$

$f(2) = \sqrt{8-4} = 2$

T(2, 2)

$$f''(x) = \frac{(2-x)' \cdot (\sqrt{4x-x^2}) - (2-x) \cdot (\sqrt{4x-x^2})'}{(4x-x^2)^2} = \frac{-1 \cdot (\sqrt{4x-x^2}) - (2-x) \cdot \left(\frac{1}{2\sqrt{4x-x^2}} \cdot (4x-x^2)'\right)}{(4x-x^2)^2}$$

$$= \frac{-\sqrt{4x-x^2} - (2-x) \cdot \frac{4-2x}{2\sqrt{4x-x^2}}}{4x^2-x^4} = \frac{-\sqrt{4x-x^2} - (2-x) \cdot \frac{2(2-x)}{2\sqrt{4x-x^2}}}{x(4-x)^2}$$

$$= \frac{-\sqrt{4x-x^2} - \frac{4-2x-x^2+x^2}{\sqrt{4x-x^2}}}{x(4-x)^2} = \frac{-(4x-x^2) - (4-4x+x^2)}{x(4-x)^2 \sqrt{4x-x^2}} = \frac{-4x+x^2-4+4x-x^2}{x(4-x)^2 \sqrt{4x-x^2}} = \frac{-4}{x(4-x)^2 \sqrt{4x-x^2}}$$

$$= \frac{\frac{-4}{\sqrt{4-x^2}}}{\frac{x(3-x)}{1}} = \frac{-4}{(4x-x^2) \cdot \sqrt{4-x^2}} \cdot \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}} = \frac{-4\sqrt{4-x^2}}{(4x-x^2)(4-x^2)} = \frac{-4\sqrt{4-x^2}}{16x-4x^3-4x^2+x^4}$$

$$f'' = 0$$

$$-4\sqrt{4-x^2} = 0 / : (-4)$$

$$\sqrt{4-x^2} = 0 / ^2$$

$$4-x^2 = 0$$

$$-x^2 = -4$$

$$x^2 = 4 / \sqrt{}$$

$$x = \pm 2$$

	$-\infty$	-2	2	$+\infty$
$f''(x)$		-	-	-
$f(x)$		\cap	\cap	\cap

$$f(2) = 2 > 0 \Rightarrow \text{m.}$$

$$f(-2) = \text{nema}$$

	$-\infty$	0	2	$+\infty$
$f'(x)$			+	-
$f(x)$			\cap	

MINIMUM
MAKSIMUM

2) NASTAVAK

	$-\infty$	-1	$+\infty$
$f''(x)$			+
$f(x)$			\cup

6) a) V.A.

$$\lim_{x \rightarrow 1} \left(\sqrt{1+x} - \sqrt{3-x} \cdot \frac{\sqrt{1+x} + \sqrt{3-x}}{\sqrt{1+x} + \sqrt{3-x}} \right) = \lim_{x \rightarrow 1} \frac{(\sqrt{1+x})^2 - (\sqrt{3-x})^2}{\sqrt{1+x} + \sqrt{3-x}} = \lim_{x \rightarrow 1} \frac{1+x-3+x}{\sqrt{1+x} + \sqrt{3-x}} = \lim_{x \rightarrow 1} \frac{2x-2}{\sqrt{1+x} + \sqrt{3-x}} = \infty \Rightarrow \text{NEMA V.A.}$$

b) K.A.

$$y = kx + l$$

$$k = \lim_{x \rightarrow \pm\infty} \frac{\sqrt{1+x} - \sqrt{3-x}}{x} = \lim_{x \rightarrow \pm\infty} \frac{\frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{3-x}}}{1} = \lim_{x \rightarrow \pm\infty} \left(\frac{1}{2\sqrt{1+x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} \right) + \left(\frac{1}{2\sqrt{3-x}} \cdot \frac{\sqrt{3-x}}{\sqrt{3-x}} \right) =$$

$$= \lim_{x \rightarrow \pm\infty} \frac{\sqrt{1+x}}{2(1+x)} + \frac{\sqrt{3-x}}{2(3-x)} = \lim_{x \rightarrow \pm\infty} \frac{\sqrt{1+x} \cdot \frac{1}{\sqrt{1+x}}}{2 \cdot \frac{1+x}{1+x}} + \frac{\sqrt{3-x} \cdot \frac{1}{\sqrt{3-x}}}{2 \cdot \frac{3-x}{3-x}} = \frac{0}{2} + \frac{0}{2} = 0$$

$$l = \lim_{x \rightarrow \pm\infty} \left(\sqrt{1+x} - \sqrt{3-x} - kx \right) = \lim_{x \rightarrow \pm\infty} \left(\sqrt{1+x} - \sqrt{3-x} \cdot \frac{\sqrt{1+x} + \sqrt{3-x}}{\sqrt{1+x} + \sqrt{3-x}} \right) = \lim_{x \rightarrow \pm\infty} \frac{1+x-3+x}{\sqrt{1+x} + \sqrt{3-x}} = \pm\infty$$

$$y = 0 \Rightarrow \text{H.A.}$$

6. $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} = \lim_{x \rightarrow 1} \frac{1-1}{\sqrt[3]{1}-1} = 0 \cdot \infty$

$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1} = \lim_{x \rightarrow 1} \frac{1}{\frac{1}{\sqrt[3]{x}} \cdot x'} = \lim_{x \rightarrow 1} \frac{1}{\frac{1}{3\sqrt{x}}} = \frac{3\sqrt{x}}{1} = 3\sqrt{x} = 3 \cdot \sqrt{1} = 3$

$\lim_{x \rightarrow 1} \frac{0-1}{\sqrt[3]{0}-1} = \frac{+1}{-1} = -1$

1. $f(x) = \sqrt{1+x} - \sqrt{3-x}$

1) DOMENA:

I. $1+x \geq 0$
 $x \geq -1$

II. $3-x \geq 0$
 $-x \geq -3 \cdot (-1)$
 $x \geq 3$



$D_f: [-1, 3] \cup [3, +\infty)$

2) NULTOČKE

$\sqrt{1+x} - \sqrt{3-x} = 0 / ^2$

$1+x - (3-x) = 0$

$1+x - 3+x = 0$

$2x - 2 = 0$

$2x = 2$

$x = 1$

$A(1, 0)$



3) PARNOST

$f(-x) = \sqrt{1-x} - \sqrt{3+x} \Rightarrow$ F-JA NIJE NI PARNI NI NEDARNA

4) $f'(x) = \frac{1}{2\sqrt{1+x}} \cdot (1+x)' - \frac{1}{2\sqrt{3-x}} \cdot (3-x)' = \frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{3-x}}$

$f'(x) = 0$ F-JA NEMA LOKALNIH EKSTREMA

$1+1=0$

$2=0 \Rightarrow$ nemoguće

5) $f''(x) = \frac{-1(2 \cdot \sqrt{1+x} + 2 \cdot \frac{1}{2\sqrt{1+x}})}{(2\sqrt{1+x})^2} = \frac{-\left(\frac{2}{\sqrt{1+x}}\right)}{(2\sqrt{1+x})^2} = \frac{-1}{4(1+x)} = \frac{-1}{4+4x}$

$f''(x) = 0$
 $\sqrt{1+x} = 0 / ^2$
 $1+x = 0$
 $x = -1$

$= \frac{-1}{(4+4x)(\sqrt{1+x})} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{-1}{(4+4x)(1+x)}$

4.

$$\sqrt[3]{\frac{4+3i}{2-4i}} = \sqrt[3]{0} = 0$$

DORA BUZONA $x = \frac{-1-4}{20} = -\frac{1}{5}$

$$r = \sqrt{\left(-\frac{1}{5}\right)^2 + \left(\frac{11}{10}\right)^2} = \frac{\sqrt{5}}{2}$$

$$y = \frac{11}{20} = \frac{11}{10}$$

$$\operatorname{tg} \varphi = \frac{y}{x} =$$

$$\frac{4+3i}{2-4i} = \frac{2+4i}{2+4i} = \frac{(4+3i)(2+4i)}{(2-4i)(2+4i)} = \frac{8+16i+6i+12i^2}{4+8i-8i-16i^2} = \frac{8+22i-12}{4+16} = \frac{-4+22i}{20}$$

$\varphi = -10^\circ 23'$?

$$z = \sqrt[3]{5} \left(\cos \frac{-10^\circ 23' + 2k\pi}{3} + i \sin \frac{-10^\circ 23' + 2k\pi}{3} \right)$$

$k = 0, 1, 2$

1) $k=0$

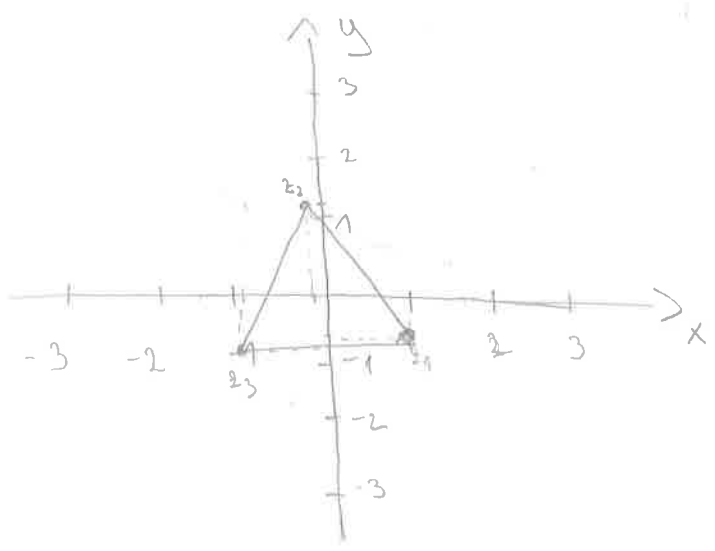
$$z = \frac{\sqrt{5}}{2} \left(\cos \frac{-10^\circ 23'}{3} + i \sin \frac{-10^\circ 23'}{3} \right) = 1 - 0.5i$$

2) $k=1$

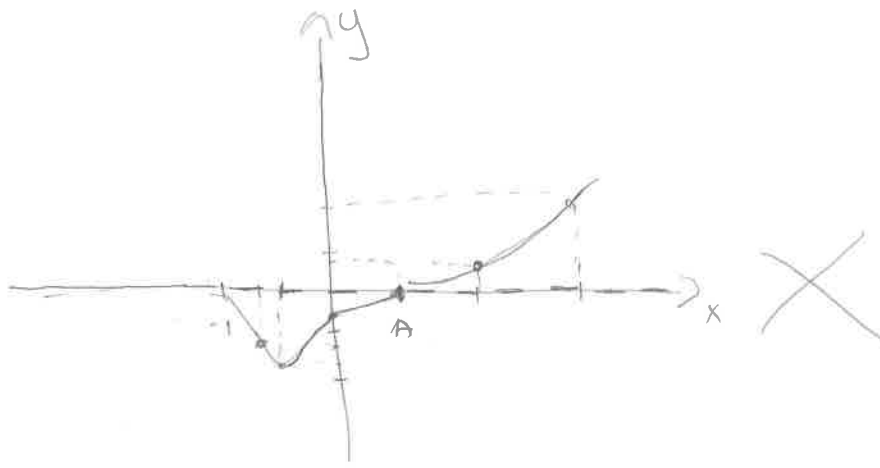
$$z = \frac{\sqrt{5}}{2} \left(\cos \frac{-10^\circ 23' + 2\pi}{3} + i \sin \frac{-10^\circ 23' + 2\pi}{3} \right) = -0.1 + 1.1i$$

3) $k=2$

$$z = \frac{\sqrt{5}}{2} \left(\cos \frac{-10^\circ 23' + 4\pi}{3} + i \sin \frac{-10^\circ 23' + 4\pi}{3} \right) = -0.9 - 0.6i$$



2.



$$f(-0,5) = -1,7 \quad f(-0,7) = -1,3$$

$$f(0) = -0,7 \quad f(3) = 2$$

$$f(1) = 0 \quad f(2) = 0,7$$

$$(5.) \sum_{n=1} (\sqrt{n^2 - 4n} - n) = \sum_{n=1} (\sqrt{-3} - 1) \quad \times$$

