26.02.2015,

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: Rikardo Radovčić

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA: 17-2-0228-2012

USTMENI ISPIT KOD NASTAVNIKA:

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 2,\ f'(0) = 5,\ f''(0) = 3\,.$$

Pomoć: $x^3 + 2x^2 + x + 2 = (x+2)(x^2+1)$.

2. Izračunati volumen tijela omeđenog plohama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 3 + x^2$, $z = -y^2$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 2t$, $t \in [0,3]$. Izračunaj $\int_C f \cdot \mathrm{d}s$, kada je $f(x,y,z) = x\vec{\mathbf{i}} + y\vec{\mathbf{j}} + \vec{\mathbf{k}}$. 20

4. Izračunati integral funkcije $f(x,y,z) = x + y + z$ na dijelu ravnine $x + y + z = 1$ koji se nalazi u dijelu prostora određenom nejednadžbama $x \geq 0$, $y \geq 0$ i $z \geq 0$. 20

5. Neka je točkama $A(1,0)$, $B(1,4)$, $C(-2,2)$ $D(-2,0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 20

$$\oint_C (x+y)^2 \mathrm{d}y$$

Ukupno: 40

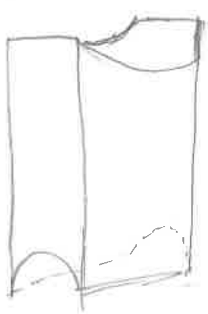
(2)

$x = 1 \qquad x = -1$

$y = 1 \qquad y = -1$

$z = 3 + x^2 \qquad z = -y^2$

SKICA



$$V = \int_{-1}^{1}\int_{-1}^{1}\int_{-y^2}^{3+x^2} dz\,dy\,dx$$

$$= \int_{-1}^{1}\int_{-1}^{1} (3 + x^2 + y^2)\,dy\,dx$$

$$= \int_{-1}^{1} \left(3y + x^2 y + \frac{y^3}{3}\right)\Big|_{-1}^{1} dx$$

$$= \int_{-1}^{1} \left(\left(3 + x^2 + \frac{1}{3}\right) - \left(-3 - x^2 - \frac{1}{3}\right)\right) dx$$

⇒ okreni

$$= \int_{-1}^{1} \left(\frac{20}{3} + 2x^2 \right) dx = \left(\frac{20x}{3} + \frac{2x^3}{3} \right) \Big|_{-1}^{1}$$

$$= \left(\frac{20}{3} + \frac{2}{3} \right) + \left(\frac{-20}{3} - \frac{2}{3} \right) = \frac{22}{3} + \frac{22}{3} = \frac{44}{3}$$ ✓

---

③ $x = \cos t$ $\quad t \in [0, 3]$

$y = \sin t$ $\quad f(x,y,z) = x\vec{i} + y\vec{i} + \vec{k}$

$z = 2t$

$(\cos t)' = -\sin t$

$(\sin t)' = \cos t$

$(2t)' = 2$

$$\int_C t\, ds = \int_0^3 (\cos t\, \vec{i} + \sin t\, \vec{j} + \vec{k}) (-\sin t\, \vec{n} + \cos t\, \vec{g} + 2\vec{k})\, dt$$ ✓ ✓ 20

$$= \int_0^3 (-\sin t \cos t + \sin t \cos t + 2)\, dt = 2t \Big|_0^3 = 6$$

26.02.2015.

Matematika 3

Ime i prezime: Rikardo Radovčić

Matični broj u indeksu:

(5) $f(x,y,z) = x+y+z$

$x+y+z=1$

$x \geq 0$

$y \geq 0$

$z \geq 0$

$z = 1-x-y$

$$\boxed{\int_0^1 \int_0^1 \int_0^{1-x-y} (x+y+z)\, dz\, dy\, dx}$$

$$= \int_0^1 \int_0^1 \left(xz + yz + \frac{z^2}{2} \right) \Big|_0^{1-x-y} dy\, dx$$

$$= \int_0^1 \int_0^1 \left(x - x^2 - xy + y - xy - y^2 + \frac{1-2x+x^2-2y+2xy+y^2}{2} \right) dy\, dx$$

$$= \frac{1}{2} \int_0^1 \int_0^1 \left(2x - 2x^2 - 2xy + 2y - 2xy - 2y^2 + 1 - 2x + x^2 - 2y + 2xy + y^2 \right) dy\, dx$$

$$= \frac{1}{2} \int_0^1 \int_0^1 \left(-x^2 - y^2 - 2xy + 1 \right) dy\, dx$$

$$= \frac{1}{2} \int_0^1 \left(-x^2 y - \frac{y^3}{3} - xy^2 + y \right) \Big|_0^1 dx =$$

$$= \frac{1}{2} \int_0^1 \left(-x^2 - \frac{1}{3} - x + 1 \right) dx$$

$$= \frac{1}{2} \int_0^1 \left(-x^2 - x + \frac{2}{3} \right) dx = \frac{1}{2} \left(-\frac{x^3}{3} - \frac{x^2}{2} + \frac{2}{3}x \right) \Big|_0^1 = \frac{1}{2} \left(-\frac{1}{3} - \frac{1}{2} + \frac{2}{3} \right)$$

$$= -\frac{1}{2}$$

① $f'''(t)+2f''(t)+f'(t)+2f(t)=t$ $\qquad f(0)=2,\quad f''(0)=3$

$f'(0)=5,$

$f(t)=f(s)$

$f'(t)=sf(s)-2$

$f''(t)=s^2 f(s)-2s-5$

$f'''(t)=s^3 f(s)-2s^2-5s-3$

$x^3+2x^2+x+2=(x+2)(x^2+1)$

$s^3 f(s)-2s^2-5s-3+2s^2f(s)-4s-10+sf(s)-2+2f(s)=\frac{1}{s^2}$

$f(s)(s^3+2s^2+s+2)=\frac{1}{s^2}+9s+15$

$f(s)=\dfrac{9s^3+15s^2+1}{s^2(s+2)(s^2+1)}=-\frac{1}{4}\cdot\frac{1}{s}+\frac{1}{2}\cdot\frac{1}{s^2}-\frac{11}{20}\cdot\frac{1}{s+2}+\frac{4}{5}\cdot\frac{s}{s^2+1}+\frac{37}{5}\cdot\frac{1}{s^2+1}$

PROVJERA: $f(0)=-\frac{1}{4}-\frac{11}{20}+\frac{4}{5}=\frac{-5-11+16}{20}$

$f(0)=0$ ✗

$$\boxed{f(t)=-\frac{1}{4}+\frac{1t}{2}-\frac{11}{20}e^{-2t}+\frac{4}{5}\cos t+\frac{37}{5}\sin t}$$

$\dfrac{9s^2+15s^2+1}{s^2(s+2)(s^2+1)}=\dfrac{A}{s}+\dfrac{B}{s^2}+\dfrac{C}{s+1}+\dfrac{Ds+E}{s^2+1}\ \Big/\cdot s^2(s+2)(s^2+1)$

$9s^3+15s^2+1=As(s^3+2s+s+2)+B(s^3+2s^2+s+2)+Cs^2(s^2+1)+$
$+(Ds^3+Es^2)(s+2)$

$9s^3+15s^2+1=As^4+2As^3+As^2+2As+Bs^3+2Bs^2+Bs+2B+$
$+Cs^4+Cs^2+Ds^4+2Ds^3+Es^3+2Es^2$

$0=A+C+D$

$9=2A+B+E+2D$

$15=A+2B+C+2E$

$0=2A+B$

$1=2B\Rightarrow \boxed{B=\frac{1}{2}}$

$A=-\frac{1}{4}$

$C+D=\frac{1}{4}$

$E+2D=9+\frac{1}{2}-\frac{1}{2}\Rightarrow \boxed{E=9-2D}$

$C+2E=15-1+\frac{1}{4}$

$C=-18+4D+14+\frac{1}{4}$

$C=-\frac{15}{4}+4D$

$-\frac{15}{4}+4D+D=\frac{1}{4}$

$5D=4$

$\boxed{D=\frac{4}{5}}$

$C=\frac{-15}{4}+\frac{16}{5}=$
$=\frac{-75+64}{20}$

$\boxed{C=-\frac{11}{20}}$

$E=9-\frac{8}{5}\Rightarrow \boxed{E=\frac{37}{5}}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: FILIP BAČINIĆ **VRIJEME POČETKA:** 8:30

MATIČNI BROJ STUDENTA: 17-2-0168-2012 **USTMENI ISPIT KOD NASTAVNIKA:** UGLEŠIĆA

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 2,\ f'(0) = 5,\ f''(0) = 3.$$

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2. Izračunati volumen tijela omeđenog plohama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 3 + x^2$, $z = -y^2$. ~~20~~ 15

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 2t$, $t \in [0, 3]$. Izračunaj $\int_C f \cdot \mathrm{d}s$, kada je $f(x, y, z) = x\,\vec{\mathbf{i}} + y\,\vec{\mathbf{j}} + \vec{\mathbf{k}}$. ~~20~~

4. Izračunati integral funkcije $f(x, y, z) = x + y + z$ na dijelu ravnine $x + y + z = 1$ koji se nalazi u dijelu prostora određenom nejednadžbama $x \geq 0$, $y \geq 0$ i $z \geq 0$. 20

5. Neka je točkama $A(1, 0)$, $B(1, 4)$, $C(-2, 2)$ $D(-2, 0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 20

$$\oint_C (x + y)^2 \mathrm{d}y$$

Ukupno: 35

① $f'''(t) + 2f''(t) + f'(t) + 2f(t) = t \qquad f(0) = 2 \quad f'(0) = 5$

$f''(0) = 3$

$\mathcal{L}[f'''(t) + 2f''(t) + f'(t) + 2f(t)] = \mathcal{L}[t]$

$[s^3F(s) - s^2\overset{=2}{f(0)} - s\overset{=5}{f'(0)} - \overset{=3}{f''(0)} + 2(s^2F(s) - s\overset{2}{f(0)} - \overset{5}{f'(0)}) +$

$+ sF(s) - \overset{=2}{f(0)} + 2F(s)] = \dfrac{1}{s^2}$

$s^3F(s) - 2s^2 - 5s - 3 + 2s^2F(s) - 4s - 10 + sF(s) - 2 + 2F(s) = \dfrac{1}{s^2}$

$s^3F(s) + 2s^2F(s) + sF(s) + 2F(s) = \dfrac{1}{s^2} + 2s^2 + 5s + 3 + 4s + 10 + 2$

$F(s)(s^3 + 2s^2 + s + 2) = \dfrac{1}{s^2} + 2s^2 + 9s + 15$

$F(s)(s+2)(s^2+1) = \dfrac{1 + s^2(2s^2 + 9s + 15)}{s^2}$

$x_{1,2} = \dfrac{-9 \pm \sqrt{81 - 80}}{2}$

$x_{1,2} = \dfrac{-9 \pm 1}{2}$

$x_1 = -4 \qquad x_2 = -5$

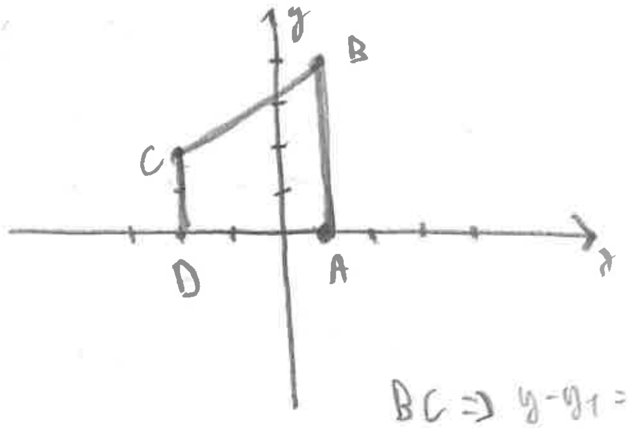
4 $f(x,y,z) = x+y+z$

$x+y+z=1$ $\quad$ $x \geq 0$

$y \geq 0$

$z \geq 0$

(5) $A(1,0)$
$B(1,4)$
$C(-2,2)$
$D(-2,0)$



$\oint (x+y)^2\,dy = ?$

$x \in [-2, 1]$

$y \in [0, \frac{10}{3} + \frac{2x}{3}]$

$BC \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$BC \Rightarrow y - 4 = \frac{2-4}{-2-1}(x-1) \Rightarrow y = \frac{-2}{-3}(x-1)+4$

$\Rightarrow y = \frac{-2x+2}{-3} + 4 \Rightarrow y = \frac{2x}{3} - \frac{2}{3} + \frac{12}{3} \Rightarrow y = \frac{10}{3} + \frac{2x}{3}$

$\oint (x+y)^2\,dy \Rightarrow P = 0$

$\oint (x^2+2xy+y^2)\,dy \rightarrow Q = x^2+2xy+y^2$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dy\,dx = \int_{-2}^{1}\int_{0}^{\frac{10}{3}+\frac{2x}{3}} \left(\frac{\partial (x^2+2xy+y^2)}{\partial x} - \frac{\partial 0}{\partial y}\right) dy\,dx$$

$$= \int_{-2}^{1}\int_{0}^{\frac{10}{3}+\frac{2x}{3}} 2x+2y\;dy\,dx \quad ✓$$

1. $\int_{0}^{\frac{2x}{3}+\frac{10}{3}} (2x+2y)\,dy = 2xy + \frac{2y^2}{2}\Big|_{0}^{\frac{2x}{3}+\frac{10}{3}} = \frac{4}{9}(4x^2+25x+25)$

2. $\int_{-2}^{1} \left(\frac{16}{9}x^2 + \frac{100}{9}x + \frac{100}{9}\right) dx = \frac{16}{9}\frac{x^3}{3} + \frac{100}{9}\cdot\frac{x^2}{2} + \frac{100}{9}x\Big|_{-2}^{1}$

$= \left(\frac{16}{27} + \frac{100}{18} + \frac{100}{9}\right) - \left(\frac{128}{27} + \frac{400}{18} - \frac{200}{9}\right) =$

$= \underline{\underline{22}}$ ✓

Matematika 3

Ime i prezime: FILIP BAČINIĆ **Matični broj u indeksu:** 17-2-0168-2012

(2.)

$x \in [-1, 1]$

$y \in [-1, 1]$

$z \in [-y^2, 3 + x^2]$

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-y^2}^{3+x^2} 1 \, dz \, dy \, dx$$

15

(3)

$x = \cos t$ $\quad t \in [0, 3]$

$y = \sin t$

$z = 2t$ $\quad \int_C f \cdot ds$ $\quad$ KAKO JE $f(x,y,z) = x\vec{i} + y\vec{j} + \vec{k}$

$$r(t) = \begin{bmatrix} \cos t \\ \sin t \\ t \end{bmatrix} \quad r'(t) = \begin{bmatrix} -\sin t \\ \cos t \\ 1 \end{bmatrix}$$

$$\|r'(t)\| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$

$$\sqrt{2}\int_0^3 (\cos t + \sin t + 1)\,dt = \sqrt{2}\,(\sin t - \cos t + t)\Big|_0^3$$

$$= \sqrt{2}\left[(\sin 3 - \cos 3 + 3) - (\underbrace{\sin 0}_{0} - \cos 0 + 0)\right]$$

$$= \sqrt{2}\left[\sin 3 - \cos 3 + 3 + 1\right] = \underline{\underline{5{,}1311 \cdot \sqrt{2}}}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3

IME I PREZIME: STIPE BRKLJAČA **VRIJEME POČETKA:** 08:27

MATIČNI BROJ STUDENTA: **USTMENI ISPIT KOD NASTAVNIKA:**

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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$$\oint_C (x+y)^2 dy$$

Ukupno: 20

$$s^3F(s) - s^2\overbrace{f(0)}^{2} - s\overbrace{f'(0)}^{5} - \overbrace{f''(0)}^{3} + 2s^2F(s) - 2s\overbrace{f(0)}^{2} - 2\overbrace{f'(0)}^{5}$$

$$+ sF(s) - \overbrace{f(0)}^{2} + 2F(s) = \frac{1}{s^2}$$

$$F(s)(s^3 + 2s^2 + s + 2) - 2s^2 - 5s - 3 - 4s - 10 - 2 = \frac{1}{s^2}$$

$$F(s)(s^3 + 2s^2 + s + 2) = \frac{1}{s^2} + 9s + 15 + 2s^2$$

$$F(s)(s^3 + 2s^2 + s + 2) = \frac{1 + 9s^3 + 15s^2 + 2s^4}{s^2} \quad / (s+2)(s^2+1)$$

$$F(s) = \frac{\dfrac{1 + 9s^3 + 15s^2 + 2s^4}{s^2}}{\dfrac{(s+2)(s^2+1)}{1}}$$

$$F(s) = \frac{9s^3 + 15s^2 + 2s^4 + 1}{s^2(s+2)(s^2+1)}$$

$$F(s) = \frac{9s^3 + 15s^2 + 2s + 1}{s^2(s+2)(s^2+1)} =$$

$$F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+2)} + \frac{Ds+E}{(s^2+1)} \quad / \; s^2(s+2)(s^2+1)$$

$Bs + 2B$ $\quad Ds^3 + Es^2$

$$F(s) = As(s+2)(s^2+1) + B(s+2)(s^2+1) + Cs^2(s^2+1) + (Ds+E)(s^2)(s+2)$$

$$F(s) = \underline{As^4} + \boxed{As^2} + \underline{2As^3} + 2As + \underline{Bs^3} + Bs + \boxed{2Bs^2} + \boxed{2B} + \underline{Cs^4} + \boxed{Cs^2} + \underline{Ds^4} + \underline{2Ds^3} + \underline{Es^3} + \boxed{2Es^2}$$

$$F(s) = s^4(A + C + D) + s^3(2A + B + 2D + E) + s^2(A + 2B + C + 2E)$$
$$+ s(2A + B) + 2B$$

$0 = A + C + D \rightarrow A = 0 - C - D \rightarrow D = 0 - C - A$

$C = 0 - A - D$ $\quad 2E - 12 = 2 -$

$9 = 2A + B + 2D + E$

$15 = A + 2B + C + 2E \rightarrow C = 15 - A - 2B - 2E$

$1 = 2B$

$\boxed{B = \frac{1}{2}}$

$\rightarrow 15 = \cancel{A} + 1 + 2 - \cancel{A} - D + 2E$

$15 = 3 - D + 2E$

$12 = -D + 2E$

$9 = 2A$ $\quad \frac{D}{2} + 6 \quad D = 2E - 12$

$2E = D + 12 \;/ :2$

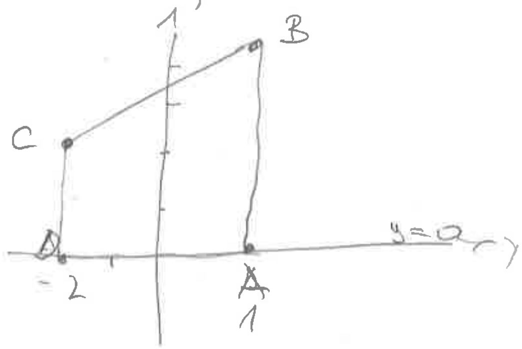
$E = \frac{D}{2} + 6$

-15

Matematika **3**

Ime i prezime: STIPE BRKLJAČA **Matični broj u indeksu:**

A(1,0)
B(1,4)
C(-2,2)
D(-2,0)



$\overline{DD} \ldots y=0$

$\overline{BC}$

$\overline{CB}$ C(-2,2) B(1,4)

$y-2 = \frac{4-2}{1+2}(x+2)$

$y-2 = \frac{2}{3}x + \frac{4}{3}$

$y = \frac{2}{3}x + \frac{10}{3}$

$\oint_c \overbrace{(x+y)^2}^{Q} dy$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$

$\frac{x^2+2xy+y^2}{\partial x} - \frac{\partial P}{\partial y} = 2x+2y$

$\int_{-2}^{1}\int_{0}^{\frac{2}{3}x+\frac{10}{3}} 2x+2 \, dy\, dy$

$\int_{-2}^{1} 2xy + 2y \Big|_{0}^{\frac{2}{3}x+\frac{10}{3}} dx$

$\int_{-2}^{1} 2x\left(\frac{2}{3}x+\frac{10}{3}\right) + 2\left(\frac{2}{3}x+\frac{10}{3}\right)$

$\int_{-2}^{1} \frac{4}{3}x^2 + \frac{20}{3} + \frac{4}{3}x + \frac{20}{3} \, dx$

$\int_{-2}^{1} \frac{4}{3}x^2 + \frac{4}{3}x + \frac{40}{3} \, dx$

$= \frac{4}{3}\frac{x^3}{3} + \frac{4}{3}\frac{x^2}{2} + \frac{40}{3}x \Big|_{-2}^{1} = \left(\frac{4}{3}\cdot\frac{1}{3} + \frac{4}{3}\cdot\frac{1}{2} + \frac{40}{3}\right) - \left(\frac{4}{3}\frac{(-2)^3}{3} + \frac{4}{3}\cdot\frac{(-2)^2}{2} + \frac{40}{3}(-2\right.$

$= \frac{130}{9} - \left(-\frac{248}{9}\right) = \underline{\underline{42}}$

STIPE BEKLJANOV

2.

$x = 1$ $\quad$ $x \in [-1, 1]$

$y = 1$ $\quad$ $y \in [-1, 1]$

$y = -1$ $\quad$ $z \in [-y^2, 3+x^2]$

$z = 3+x^2$

$z = -y^2$

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-y^2}^{3+x^2} 1 \, dz\, dx\, dy =$$

$$\int_{-1}^{1}\int_{-1}^{1} 3+x^2+y^2 \, dx\, dy = \int_{-1}^{1} 3x + \frac{x^3}{3} + x\cdot y^2 \Big|_{-1}^{1} dy =$$

$$= \int_{-1}^{1} \left(3 + \frac{1}{3} + y^2\right) - \left(-3 + \frac{(-1)^3}{3} - y^2\right) dy$$

$$= \int_{-1}^{1} \frac{10}{3} + y^2 - \left(-\frac{10}{3} - y^2\right) dy = \int_{-1}^{1} \frac{10}{3} + y^2 + \frac{10}{3} + y^2 \, dy =$$

$$= \int_{-1}^{1} \frac{20}{3} + 2y^2 \, dy = \frac{20}{3} y + \frac{2}{3} y^3 \Big|_{-1}^{1}$$

$$= \left(\frac{20}{3} + \frac{2}{3}\right) - \left(\frac{20}{3}(-1) + \frac{2}{3}\cdot(-1)\right) =$$

$$= \frac{22}{3} + \left(-\frac{22}{3}\right) = \frac{22}{3} + \frac{22}{3} = \frac{44}{3} \checkmark$$

STIPE BRKIĆ

3. $f\begin{pmatrix} \cos t \\ \sin t \\ 2t \end{pmatrix} \quad r'(t) = \begin{pmatrix} -\sin t \\ \cos t \\ 2 \end{pmatrix} \qquad t \in (0, 3)$

$$\int_0^3 f \cdot ds = \cancel{2} \cdot (\cos t + \sin t + 2t)\,dt =$$

$$r = \sqrt{\sin t^2 + \cos^2 t + 4}$$

$$r = 2$$

$$= \int_0^3 2\cos t + 2\sin t + 4t \, dt$$

$$= -2\sin t + 2\cos t + \frac{4t^2}{2}$$

$$= (-2\sin(3) + 2\cos(3) + 9) - (-2\cancel{\sin}(0) + 2\cos(0)) =$$

$$= -2 + 9 - (2) = -2 + 9 - 2 =$$

$$= 5$$

4. $f(x, y, z) = x + y + z \quad x \geq 0 \quad y \geq 0 \quad z \geq 0$

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} x + y + z \; dz\, dy\, dx$$

$x + y + z = 1$

$y = 1 - x$

$z = 1 - x - y$

$$\begin{pmatrix} \cos\varphi \\ \sin\varphi \\ z \end{pmatrix} = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} = \sqrt{\sin^2\varphi + \cos^2\varphi} = 1$$

$$\int_0^1 \int_0^{1-\cos\varphi} \int_0^{1-\cos\varphi-\sin\varphi} \cos\varphi + \sin\varphi + z$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: DONATO PREDOVAN **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: 0036461512 **USTMENI ISPIT KOD NASTAVNIKA:** prof. Uglešić	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 2,\ f'(0) = 5,\ f''(0) = 3\,.$$

Pomoć: $x^3 + 2x^2 + x + 2 = (x+2)(x^2+1)$.

2. Izračunati volumen tijela omeđenog plohama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 3 + x^2$, $z = -y^2$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 2t$, $t \in [0, 3]$. Izračunaj $\int_C f \cdot \mathrm{d}s$, kada je $f(x, y, z) = x\,\vec{\mathbf{i}} + y\,\vec{\mathbf{j}} + \vec{\mathbf{k}}$. 20

4. Izračunati integral funkcije $f(x, y, z) = x + y + z$ na dijelu ravnine $x + y + z = 1$ koji se nalazi u dijelu prostora određenom nejednadžbama $x \geq 0$, $y \geq 0$ i $z \geq 0$. 20

5. Neka je točkama $A(1, 0)$, $B(1, 4)$, $C(-2, 2)$ $D(-2, 0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 20

$$\oint_C (x + y)^2 \mathrm{d}y$$

Ukupno: 20

$\boxed{1.}$ $f'''(t) + 2f''(t) + f'(t) + 2f(t) = t$ $\quad f(0) = 2,\ f'(0) = 5,\ f''(0) = 3$

$$f'''(t) \to s^3F(s) - s^2 f(0) - sf'(0) - f''(0)$$
$$= s^3F(s) - s^2\cdot 2 - s\cdot 5 - 3$$
$$= s^3F(s) - 2s^2 - 5s - 3$$

$$f''(t) \to s^2F(s) - sf(0) - f'(0)$$
$$= s^2F(s) - s\cdot 2 - 5$$

$$f'(t) \to sF(s) - f(0)$$
$$= sF(s) - 2$$

$$f(t) \to F(s)$$
$$t \to \frac{1}{s^2}$$

$$s^3F(s) - 2s^2 - 5s - 3 + 2(s^2F(s) - 2s - 5) + s\cdot F(s) - 2 + 2\cdot F(s) = \frac{1}{s^2}$$

$$s^3\cdot F(s) - 2s^2 - 5s - 3 + 2\cdot s^2\cdot F(s) - 4s - 10 + s\cdot F(s) - 2 + 2\cdot F(s) - \frac{1}{s^2} = 0 \,/\cdot s^2$$

$$s^6 F(s) - 2s^4 - 5s^3 - 3s^2 + 2\cdot s^4\cdot F(s) - 4s^3 - 10s^2 + s^3\cdot F(s) - 2s^2 + 2\cdot F(s)\cdot s^2 - 1 = 0$$

$$s^6 \cdot F(s) - 2s^4 - 5s^3 - 3s^2 + 2 \cdot s^4 \cdot F(s) - 4s^3 - 10s^2 + s^3 \cdot F(s) - 2s^2 + 2 \cdot F(s) \cdot s^2 - 1 = 0$$

$$F(s)\left(s^6 + 2s^4 + s^3 + 2s^2\right) - 2s^4 - 5s^3 - 3s^2 - 4s^3 - 10s^2 - 2s^2 - 1 = 0$$

$$F(s)\left(s^6 + 2s^4 + s^3 + 2s^2\right) - 2s^4 - 9s^3 - 15s^2 - 1 = 0$$

$$F(s)\left(s^6 + 2s^4 + s^3 + 2s^2\right) = 1 + 2s^4 + 9s^3 + 15s^2$$

$$F(s) = \frac{1 + 2s^4 + 9s^3 + 15s^2}{s^6 + 2s^4 + s^3 + 2s^2}$$

$$F(s)\left(s^2\left(s^3 + 2s^2 + s + 2\right)\right) = 1 + s^2\left(15 + 9s + 2s^2\right)$$

$$F(s)\left(s^2(s+2)(s^2+1)\right) = 1 + s^2\left(15 + 9s + 2s^2\right) \Big/ : s^2(s+2)(s^2+1)$$

$$F(s) = \frac{1 + 2s^4 + 9s^3 + 15s^2}{s^2(s+2)(s^2+1)}$$

RASTAV

$$\frac{1 + 2s^4 + 9s^3 + 15s^2}{s^2(s+2)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{Ds + E}{s^2+1} \Big/ \cdot s^2(s+2)(s^2+1)$$

$$1 + 2s^4 + 9s^3 + 15s^2 = A \cdot s(s+2)(s^2+1) + B(s+2)(s^2+1) + C\left(s^2(s^2+1)\right) + (Ds+E)(s^2(s+2))$$

$$1 + 2s^4 + 9s^3 + 15s^2 = A\left((s^2+2s)(s^2+1)\right) + B\left(s^3 + s + 2s^2 + 2\right) + C\left(s^4 + s^2\right) + (Ds+E)(s^3 + 2s^2)$$

$$1 + 2s^4 + 9s^3 + 15s^2 = A\left(s^4 + s^2 + 2s^3 + 2s\right) + Bs^3 + Bs + 2Bs^2 + 2B + Cs^4 + Cs^2 + Ds^4 + 2Ds^3 + Es^3 + 2Es^2$$

$$1 + 2s^4 + 9s^3 + 15s^2 = As^4 + As^2 + 2As^3 + 2As + Bs^3 + Bs + 2Bs^2 + 2B + Cs^4 + Cs^2 + Ds^4 + 2Ds^3 + Es^3 + 2Es^2$$

$$2s^4 + 9s^3 + 15s^2 + 1 = s^4(A + C + D) + s^3(2A + B + 2D + E) + s^2(A + 2B + C + 2E) + s(2A+B) + 2B$$

$s^4 \ldots \quad 2 = A + C + D$

$s^3 \ldots \quad 9 = 2A + B + 2D + E$

$s^2 \ldots \quad 15 = A + 2B + C + 2E$

$s \ldots \quad 0 = 2A + B$

$1 = 2B$

$\boxed{B = \frac{1}{2}}$

$2A = -B$

$2A = -\frac{1}{2}$

$\boxed{A = -\frac{1}{4}}$

$C = 2 - A - D$

$2A + B + 2D + E - 9 = 0 \,/\cdot(-2)$

$A + 2B + C + 2E - 15 = 0$

$-4A - 2B - 4D - 2E + 18 = 0$

$A + 2B + C + 2E - 15 = 0$

$-3A - 4D + C + 3 = 0$

$-3A - 4D + 2 - A - D + 3 = 0$

$-4A - 5D + 5 = 0$

$-4 \cdot \left(-\frac{1}{4}\right) - 5D + 5 = 0$

$-5D = -6$

$\boxed{D = \frac{6}{5}}$

$9 = -\frac{1}{2} + \frac{1}{2} + \frac{12}{5} + E$

$E = 9 - \frac{12}{5}$

$E = \frac{45 - 12}{5}$

$\boxed{E = \frac{33}{5}}$

$C = \frac{2}{1} + \frac{1}{4} - \frac{6}{5}$

$C = \frac{40 + 5 - 24}{20} = \frac{21}{20}$

$\boxed{C = \frac{21}{20}}$

DONATO PREDOVAN

NASTAVAK ZAD. 1.

$$\frac{6}{5}s+\frac{33}{5} = \frac{\frac{6s+33}{5}}{\frac{s^2+1}{1}} \qquad \frac{21}{20(s+2)} \qquad \frac{6s+33}{5(s^2+1)}$$

$$F(s) = \frac{-\frac{1}{4}}{\frac{s}{1}} + \frac{\frac{1}{2}}{\frac{s^2}{1}} + \frac{\frac{21}{20}}{\frac{s+2}{1}} + \frac{\frac{6}{5}s+\frac{33}{5}}{s^2+1}$$

$$= -\frac{1}{4}\cdot\frac{1}{s} + \frac{1}{2}\cdot\frac{1}{s^2} + \frac{21}{20}\cdot\frac{1}{s+2} + \frac{6}{5}\cdot\frac{s}{s^2+1} + \frac{33}{5}\cdot\frac{1}{s^2+1}$$

$$F(t) = -\frac{1}{4}\cdot 1 + \frac{1}{2}\cdot t + \frac{21}{20}\cdot e^{-2t} + \frac{6}{5}\cdot\cos t + \frac{33}{5}\cdot\sin t$$

$$F(t) = -\frac{1}{4} + \frac{1}{2}t + \frac{21}{20}e^{-2t} + \frac{6}{5}\cos t + \frac{33}{5}\sin t$$

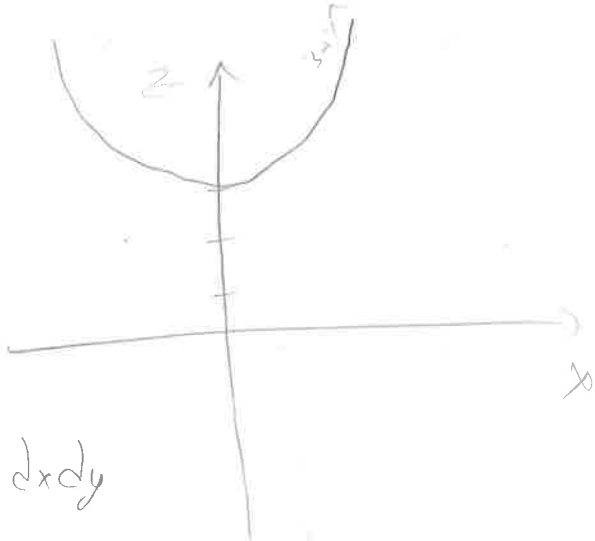
$f(0) = -\frac{1}{4} + \frac{21}{20} + \frac{6}{5} = \frac{-5+21+24}{20} = \frac{40}{20} = 2$

$f'(t) = \frac{1}{2} - \frac{21}{10}e^{-2t} - \frac{6}{5}\sin t + \frac{33}{5}\cos t$, $f'(0) = \frac{1}{2} - \frac{21}{10} + \frac{33}{5} = \frac{10-21+66}{10} = \frac{55}{10} \neq 5$

2.

$x = 1 \quad y = 1 \quad z = 3 + x^2$

$x = -1 \quad y = -1 \quad z = -y^2$



$$\int_{-1}^{1}\int_{-1}^{1}\int_{-y^2}^{3+x^2} 1\, dz\, dx\, dy$$ ✓

$$\int_{-1}^{1}\int_{-1}^{1} z \Big|_{-y^2}^{3+x^2} dx\, dy = \int_{-1}^{1}\int_{-1}^{1} (3+x^2+y^2)\, dx\, dy$$

$$= \int_{-1}^{1} \left(3x + \frac{x^3}{3} + y^2x\right)\Big|_{-1}^{1} dy$$

$$= \int_{-1}^{1} \left(3+\frac{1}{3}+y^2\right) - \left(-3-\frac{1}{3}-y^2\right)$$

$$= \int_{-1}^{1} \left(3+\frac{1}{3}+y^2+3+\frac{1}{3}+y^2\right) dy = \int_{-1}^{1} \frac{20}{3} + 2y^2\, dy$$

$$= \frac{20}{3}y + \frac{2y^3}{3}\Big|_{-1}^{1} = \frac{20}{3} + 2\cdot\frac{1}{3} - \left(-\frac{20}{3} - 2\cdot\frac{1}{3}\right) =$$

$$= \frac{20}{3} + \frac{2}{3} + \frac{20}{3} + \frac{2}{3} = \frac{20+2+20+2}{3} = \frac{44}{3} \approx 14.66666$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{B}3$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: JASMIN NEKIĆ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: 17-1-0058-2011 **USTMENI ISPIT KOD NASTAVNIKA:** N. Mogleščić	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 2,\ f'(0) = 5,\ f''(0) = 3\,.$$

Pomoć: $x^3 + 2x^2 + x + 2 = (x+2)\,(x^2+1)$.

2. Izračunati volumen tijela omeđenog plohama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 3 + x^2$, $z = -y^2$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 2t$, $t \in [0, 3]$. Izračunaj $\int_C f \cdot \mathrm{d}s$, kada je $f(x,y,z) = x\,\vec{\mathbf{i}} + y\,\vec{\mathbf{j}} + \vec{\mathbf{k}}$. 20

4. Izračunati integral funkcije $f(x,y,z) = x + y + z$ na dijelu ravnine $x + y + z = 1$ koji se nalazi u dijelu prostora određenom nejednadžbama $x \geq 0$, $y \geq 0$ i $z \geq 0$. 20

5. Neka je točkama $A(1,0)$, $B(1,4)$, $C(-2,2)$ $D(-2,0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 20

$$\oint_C (x+y)^2 \mathrm{d}y$$

Ukupno: 20

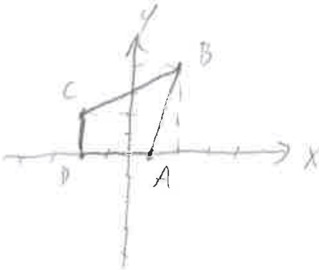
Matematika 3

Ime i prezime: JASMIN NEKIĆ

Matični broj u indeksu: 17-1-0058-2014

(5) A (1,0)
B (1,4)
C (-2,2)
D (-2,0)

$$\oint_C (x+y)^2 \, dy$$



$\overline{DA} \Rightarrow y = 0$
$\overline{CB} \Rightarrow y = x + 2$
$\overline{CD} \Rightarrow x = -2$
$\overline{AB} \Rightarrow y = x + 3$

$$\oint_C \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 \qquad P = 0 \, dx \quad Q = (x+y)^2 \, dy \qquad \frac{\partial P}{\partial y} = 0 \qquad \frac{\partial Q}{\partial x} \Rightarrow x^2 + 2xy + y^2 = 2x + 2y$$

0

(2) $x = 1$
$x = -1$
$y = 1$
$y = -1$
$z = 3 + x^2$
$z = -y^2$

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-y^2}^{3+x^2} 1 \, dz \, dy \, dx = \int_{-1}^{1}\int_{-1}^{1} (3 + x^2 + y^2) \, dy \, dx =$$

$$= \int_{-1}^{1} \left[3y + x^2 y + \frac{y^3}{3}\right]_{-1}^{1} dx = \int_{-1}^{1} \left(3 + x^2 + \frac{1}{3}\right) - \left(-3 - x^2 - \frac{1}{3}\right) dx =$$

$$= \int_{-1}^{1} \left(6 + 2x^2 + \frac{2}{3}\right) dx = \left[6x + 2\frac{x^3}{3} + \frac{2}{3}x\right]_{-1}^{1} = \left(6 + \frac{2}{3} + \frac{2}{3}\right) - \left(-6 - \frac{2}{3} - \frac{2}{3}\right) =$$

$$= 12 + \frac{8}{3} = \frac{44}{3}$$

(1) $s^3X(s) - s^2\underset{2}{f(0)} - s\underset{5}{f'(0)} - \underset{3}{f''(0)} + 2\left(s^2X(s) - s\underset{2}{f(0)} - \underset{5}{f'(0)}\right) + sX(s) - \underset{2}{f(0)} + 2X(s) = \frac{1}{s^2}$

$s^3X(s) - 2s^2 - 5s - 3 + 2s^2X(s) - 4s - 10 + sX(s) - 2 + 2X(s) = \frac{1}{s^2}$

$s^3X(s) + 2s^2X(s) + sX(s) + 2X(s) = \frac{1}{s^2} + 15 + 2s^2 + 9s$

$X(s)\left(s^3 + 2s^2 + s + 2\right) = \frac{1}{s^2} + 2s^2 + 9s + 15 \quad \Big| : (s+2)(s^2+1)$

$X(s) = \frac{1}{s^2(s+2)(s^2+1)} + \frac{2s^2+9s+15}{(s+2)(s^2+1)}$

$(s^2+2s)(s^2+1) = s^4+s^2+2s^3+2s$

s^3+2s^2

s^3+2s^2+s+2

$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+1} = As(s+2)(s^2+1) + B(s+2)(s^2+1) + Cs^2(s^2+1) + (Ds+E)s^2(s+2) =$

$= As^4 + 3As^3 + As^2 + \cancel{2As} + Bs^3 + 2Bs^2 + \cancel{Bs} + 2B + Cs^3 + \cancel{2Cs} + Ds^4 + 2Ds^3 + Es^3 + 2Es^2$

$A + D = 0$

$3A + B + C + 2D + E = 0$

$A + 2B + 2E = 0$

$2A + B + 2C = 0$

$2B = 1 \Rightarrow \boxed{B = \frac{1}{2}}$

$D = -A$

$\boxed{D = \frac{1}{2}}$

$2E = -A - 1$

$E = -\frac{1}{2}A - \frac{1}{2}$

$2C = -2A - \frac{1}{2}$

$C = -A - \frac{1}{4}$

$3A + \frac{1}{2} - A - \frac{1}{4} - 2A - \frac{1}{2}A - \frac{1}{2} = 0$

$-\frac{1}{2}A = \frac{1}{4}$

$\boxed{A = -\frac{1}{2}}$ $\boxed{E = -\frac{1}{4}}$ $\boxed{C = \frac{1}{4}}$

$\frac{G}{s+2} + \frac{Hs+I}{s^2+1} = Gs^2 + G + Hs^2 + 2Hs + Is + 2I$

$G + H = 2 \Rightarrow H = 2 - G$

$2H - I = 9 \Rightarrow I = 2H - 9$

$G + 2I = 15 \Rightarrow I = \frac{15}{2} - \frac{1}{2}G$

$I = -5 - 2G$

$-5 - 2G = \frac{15}{2} - \frac{1}{2}G$

$2G - \frac{1}{2}G = -5 - \frac{15}{2}$

$\frac{3}{2}G = -\frac{25}{2}$

$G = -\frac{25}{3}$

$H = \frac{31}{3}$

$I = \frac{15}{2} + \frac{25}{6} = \frac{70}{6} = \frac{35}{3}$

Matematika **3**

Ime i prezime: JASMIN MEHIĆ **Matični broj u indeksu:**

... (1)

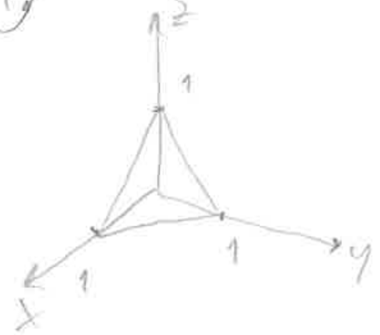
$$X(s) = -\frac{1}{2}\cdot\frac{1}{s} + \frac{1}{2}\cdot\frac{1}{s^2} + \frac{1}{4}\cdot\frac{1}{s+2} + \frac{\frac{1}{2}s - \frac{1}{4}}{s^2+1} - \frac{25}{3}\cdot\frac{1}{s+2} + \frac{\frac{31}{3}s + \frac{35}{3}}{s^2+1}$$

$$x(t) = -\frac{1}{2} + \frac{1}{2}t + \frac{1}{4}e^{-2t} + \frac{1}{2}\cos(t) - \frac{1}{4}\sin(t) - \frac{25}{3}e^{-2t} + \frac{31}{3}\cos(t) + \frac{35}{3}\sin(t)$$

$$x(0) = -\frac{1}{2} + \frac{1}{4} + \frac{1}{2} - \frac{25}{3} + \frac{31}{3} = 2 + \frac{1}{4} \neq 2$$

×

(4.)



$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} (x+y+z)\, dz\, dy\, dx$$

$$(1-x-y)^2 =$$

$$= \int_0^1 \int_0^{1-x} \left[xz + yz + \frac{z^2}{2}\right]_0^{1-x-y} dy\, dx =$$

$$1 + x^2 + y^2 - 2y - 2x + 2xy$$

$$= \int_0^1 \int_0^{1-x} \left(x - x^2 - xy + \cancel{y} - xy - y^2 + \frac{1}{2} + \frac{x^2}{2} + \frac{y^2}{2} - \cancel{y} - \cancel{x} + \cancel{xy}\right) dy\, dx =$$

$$= \int_0^1 \int_0^{1-x} \left(-\frac{1}{2}x^2 - \frac{1}{2}y^2 - xy - \frac{1}{2}\right) dy\, dx = \int_0^1 \left[-\frac{1}{2}x^2 y - \frac{1}{6}y^3 - \frac{1}{2}xy^2 - \frac{1}{2}y\right]_0^{1-x} dx =$$

$$= \int_0^1 \left(-\frac{x^2}{2} + \frac{x^3}{2} - \frac{1}{6} + \frac{x}{6} - \frac{x^2}{6} + \frac{x^3}{6} - \frac{x}{2} + x^2 - \frac{x^3}{2} - \frac{1}{2} + \frac{x}{2}\right) dx =$$

$$= \int_0^1 \left(\frac{x^3}{3} + \frac{x^2}{3} + \frac{x}{6} - \frac{2}{3}\right) dx = \left[\frac{1}{12}x^4 + \frac{1}{9}x^3 + \frac{1}{12}x^2 - \frac{2}{3}x\right]_0^1 =$$

$$= \frac{3 + 4 + 3 - 24}{36} = -\frac{14}{36} = -\frac{7}{18}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Antonijo Knežević VRIJEME POČETKA:	
MATIČNI BROJ STUDENTA: 57672 USTMENI ISPIT KOD NASTAVNIKA:	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 2,\ f'(0) = 5,\ f''(0) = 3.$$

Pomoć: $x^3 + 2x^2 + x + 2 = (x+2)(x^2+1)$.

2. Izračunati volumen tijela omeđenog plohama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 3 + x^2$, $z = -y^2$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 2t$, $t \in [0, 3]$. Izračunaj $\int_C f \cdot \mathrm{d}s$, kada je $f(x, y, z) = x\vec{\mathbf{i}} + y\vec{\mathbf{j}} + \vec{\mathbf{k}}$. 20

4. Izračunati integral funkcije $f(x, y, z) = x + y + z$ na dijelu ravnine $x + y + z = 1$ koji se nalazi u dijelu prostora određenom nejednadžbama $x \geq 0, y \geq 0$ i $z \geq 0$. 20

5. Neka je točkama $A(1,0)$, $B(1,4)$, $C(-2,2)$ $D(-2,0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 20

$$\oint_C (x+y)^2 \mathrm{d}y$$

Ukupno: 20

$\boxed{1.}$ $f'''(t) + 2f''(t) + f'(t) + 2f(t) = t$

$f(0) = 2$

$f'(0) = 5$

$f''(0) = 3$

$$s^3F(s) - \underbrace{s^2f(0)}_{2} - \underbrace{sf'(0)}_{5} - \underbrace{f''(0)}_{3} + 2\left(s^2F(s) - s\underbrace{f(0)}_{2} - \underbrace{f'(0)}_{5}\right)$$
$$+ sF(s) - \underbrace{f(0)}_{2} + 2F(s) = \frac{1}{s-1}$$

$$\underline{s^3F(s)} - 2s - 5s - 3 + \underline{2s^2F(s)} - 4s - 10 + \underline{sF(s)} - 2 + \underline{2F(s)} = \frac{1}{s-1}$$

$$F(s)(s^3 - 2s^2 + s + 2) - 2s^2 - 5s - 4s - 15 = \frac{1}{s-1}$$

$$F(s)(s^3 - 2s^2 + s + 2) = \frac{1}{s-1} + 2s^2 + 9s + 15$$

$$F(s)(s+2)(s^2+1) = \frac{2s^3 + 7s^2 + 6s - 15}{s-1} \quad /:(s+2)(s^2+1)$$

$$F(s) = \frac{2s^3 + 7s^2 + 6s - 15}{(s-1)(s+2)(s^2+1)}$$

→

$$\frac{2s^3+7s^2+6s-15}{(s-1)(s+2)(s^2+1)} = \frac{A}{s-1} + \frac{B}{s+2} + \frac{Cs+D}{s^2+1} \quad \Big/ (s-1)(s+2)(s^2+1)$$

$$2s^3+7s^2+6s-15 = A(s+2)(s^2+1) + B(s-1)(s^2+1) + (Cs+D)(s-1)(s+2)$$

$$2s^3+7s^2+6s-14 = A(s^3+2s^2+s+2) + B(s^3-s^2+s-1) + (Cs+D)(s^2+s-2)$$

$$2s^3+7s^2+6s-14 = As^3 + 2As^2 + As + 2A + Bs^3 - Bs^2 + Bs - B + Cs^3 + Cs^2 + Ds^2 + Ds - 2Cs - 2D$$

$A+B+C=2$

$2A-B+C+D=7$

$A+B+D-2C=6$

$2A-B-2D=-14$

$D-3C=4$

$D=4+3C$

$2A-B+C+4+3C=7$

$$\begin{cases} 2A-B+4C=3 \\ 2A-B-6C-8=-14 \end{cases}$$

$4C+6C=3+6$

$10C=9$

$C=\frac{9}{10}$

$D=4+3\cdot\frac{9}{10}$

$D=4+\frac{27}{10}=\frac{67}{10}=D$

$A+B=2-\frac{9}{10} \to A+B\ \frac{11}{10}$

$2A-B-2\frac{67}{10}=-14$

$2A-B-\frac{67}{5}=-14$

$2A-B=-14+\frac{67}{5}=-\frac{3}{5}$

$3A=\frac{5}{10}=\frac{1}{2}$

$\boxed{A=\frac{1}{6}}$

$B=\frac{11}{10}-\frac{1}{6}=\frac{56}{60}$

$\boxed{B=\frac{14}{15}}$

Matematika 3

Ime i prezime: Antonijo Knežević **Matični broj u indeksu:** 57672

1.

$$F(s) = \frac{1}{6}\cdot\underset{e^{t}}{\frac{1}{s-1}} + \frac{14}{15}\cdot\underset{e^{-2t}}{\frac{1}{s+2}} + \frac{9}{10}\cdot\underset{\cos t}{\frac{s}{s^2+1}} + \frac{67}{10}\cdot\underset{\sin t}{\frac{1}{s^2+1}}$$

$$F(t) = \frac{1}{6}e^{t} + \frac{14}{15}e^{-2t} + \frac{9}{10}\cos t + \frac{13}{3}\sin t$$

PROVJERA

$$F(0) = \frac{1}{6} + \frac{14}{15} + \frac{9}{10} = \frac{5+28+27}{30} = \frac{60}{30} = 2$$

0

$$f'(t) = \frac{1}{6}e^{t} - \frac{28}{15}e^{-2t} - \frac{9}{10}\sin t + \frac{13}{3}\cos t,\quad f'(0) = \frac{1}{6} - \frac{28}{15} + \frac{13}{3} = \frac{5-56+130}{30} = \frac{79}{30} \neq 5$$

2.

$x = 1 \qquad y = 1 \qquad z = 3 + x^2$

$x = -1 \qquad y = -1 \qquad z = -y^2$

$$x \in [-1, 1] \quad y \in [-1, 1] \quad z \in [-y^2, 3 + x^2]$$

$$V = \int_{-y^2}^{3+x^2} dz\,dx\,dy = \int_{-1}^{1}\int_{-1}^{1} z\Big|_{-y^2}^{3+x^2} dx\,dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} (3 + x^2 + y^2)\,dx\,dy = \int_{-1}^{1} 3x + \frac{1}{3}x^3 + y^2 x \Big|_{-1}^{1} dy$$

$$= \int_{-1}^{1} \left(\left(3 + \frac{1}{3} + y^2\right) - \left(-3 - \frac{1}{3} + y^2\right) - \left(-3 - \frac{1}{3} - y^2\right)\right) dy$$

$$= \int_{-1}^{1} \left(6 + \frac{2}{3} + 2y^2\right) dy = \int_{-1}^{1} \left(\frac{20}{3} + 2y^2\right) dy$$

$$= \frac{20}{3}y + \frac{2}{3}y^3 \Big|_{-1}^{1} = \frac{20}{3} + \frac{2}{3} - \left(-\frac{20}{3} - \frac{2}{3}\right)$$

$$= \frac{22}{3} - \left(-\frac{22}{3}\right) = \frac{44}{3}$$

$\boxed{4}$ $f(x,y,z) = x+y+z$

$x+y+z=1 \qquad x \geq 0, \; y \geq 0, \; z \geq 0$

$x \in [0,1], \; y \in [0,1] \quad z \in [0,1]$

$$\int_0^1\int_0^1\int_0^1 (x+y+z)\,dx\,dy\,dz =$$

$$\int_0^1\int_0^1 \left.\frac{1}{2}x^2 + yx + zx \right|_0^1 dy\,dz =$$

$$= \int_0^1\int_0^1 \left(\frac{1}{2} + y + z\right) dy\,dz$$

$$= \int_0^1 \left.\left(\frac{1}{2}y + \frac{1}{2}y^2 + zy\right)\right|_0^1 dz$$

$$= \int_0^1 \left(\frac{1}{2} + \frac{1}{2} + z\right) dz = \int_0^1 1 + z\,dz$$

$$= \left. z + \frac{1}{2}z^2 \right|_0^1$$

$$= 1 + \frac{1}{2} = \frac{3}{2}$$

3.

$x = \cos t$

$y = \sin t$

$z = 2t \quad t \in [0,3]$

Antonijo Knežević
57672

$\int_C f \cdot ds = ? \qquad f(x,y,z) = x\vec{i} + y\vec{j} + \vec{k}$

$r = \begin{bmatrix} \cos t \\ \sin t \\ 2t \end{bmatrix} \qquad r(t) = \begin{bmatrix} -\sin t \\ \cos t \\ 2 \end{bmatrix} \quad t \in [0,3]$

$\| r(t) \| = \sqrt{(-\sin t)^2 + (\cos^2 t) + 2^2}$

$= \sqrt{\sin^2 t + \cos^2 t + 4}$

$= \sqrt{1+4} = \sqrt{5}$

$f(x,y,z) \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$\int_0^3 \sqrt{5} \cdot (\cos t + \sin t + 1)\, dt$

$= \sqrt{5} \int_0^3 (\cos t + \sin t + 1)\, dt$

$= \sqrt{5}\, (\sin t - \cos t + t) \Big|_0^3$

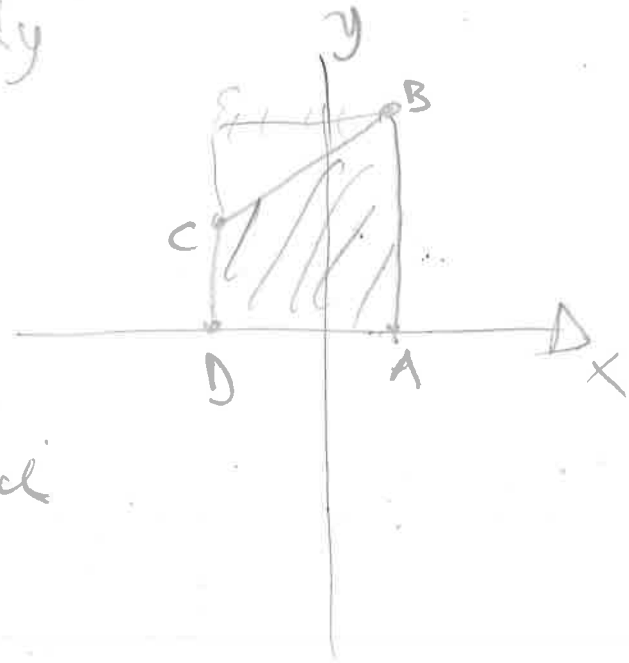
$= \sqrt{5}\,(\sin 3 - \cos 3 + 3) - \sqrt{5}\,(\underbrace{\sin 0}_{0} - \cos 0)$

$= \sqrt{5}\,(\sin 3 - \cos 3 + 3) + \sqrt{5}$

$= \sqrt{5}\,(\sin 3 - \cos 3 + 4)$

5) $A(1,0)$, $B(1,4)$, $C(-2,2)$, $D(-2,0)$

$$\oint_C (x+y)^2 \, dy$$



ABCD četverokut

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{B}3$	Popunjava nastavnik
Ime i prezime: GABRIJELA JORDAN **Vrijeme početka:**	Broj ↓ bodova
Matični broj studenta: 17-2-0118-2011 **Ustmeni ispit kod nastavnika:** N. UGLEŠIĆ	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 2,\ f'(0) = 5,\ f''(0) = 3\,.$$

Pomoć: $x^3 + 2x^2 + x + 2 = (x+2)(x^2+1)$.

2. Izračunati volumen tijela omeđenog plohama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 3 + x^2$, $z = -y^2$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 2t$, $t \in [0, 3]$. Izračunaj $\int_C f \cdot \mathrm{d}s$, kada je $f(x, y, z) = x\vec{\mathbf{i}} + y\vec{\mathbf{j}} + \vec{\mathbf{k}}$. 20

4. Izračunati integral funkcije $f(x, y, z) = x + y + z$ na dijelu ravnine $x + y + z = 1$ koji se nalazi u dijelu prostora određenom nejednadžbama $x \geq 0$, $y \geq 0$ i $z \geq 0$. 20

5. Neka je točkama $A(1, 0)$, $B(1, 4)$, $C(-2, 2)$ $D(-2, 0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 20

$$\oint_C (x+y)^2 \mathrm{d}y$$

Ukupno: 20

$f'''(t)+2f''(t)+f'(t)+2f(t)=t \quad f(0)=2 \quad f'(0)=5 \quad f''(0)=3$

10 $s^3F(s)-s^2f(s)-sf(0)-f(0)+2(s^2F(s)-sf(0))+sF(s)-f(0)+2F(s)$

$s^3F(s)-s^2\cdot 2-5s-3+2s^2F(s)-2s\cdot 2-2\cdot 5+s(s)-2+2F(s)=\frac{1}{s^2}$

$F(s)[s^3+2s^2+s+2]=\frac{1}{s^2}+2s^2+5s+3+4s+10+2$

$F(s)[s(s^2+1)+2(s^2+1)]=\frac{1}{s^2}+2s^2+9s+15$

$[(s^2+1)(s+2)=\frac{2s^4+9s^3+15s^2+1}{s^2}$

$F(s)=\frac{2s^4+9s^3+15s^2+1}{s^2(s^2+1)(s+2)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+2}+\frac{Ds+E}{s^2+1}$

$A(s(s+2)(s^2+1)]=B[(s+2)(s^2+1)]+C[(s^2)\cdot(s^2+1)]+$
$Ds+E[s^2(s+2)]^2=$

$s^4: A+C+D=2$
$s^3: 2A+B+2D+E=9$
$s^2: A+2B+C+2E=15 \quad A=-\frac{1}{4}$
$s\ \ 2B=1 \quad B=\frac{1}{2}$

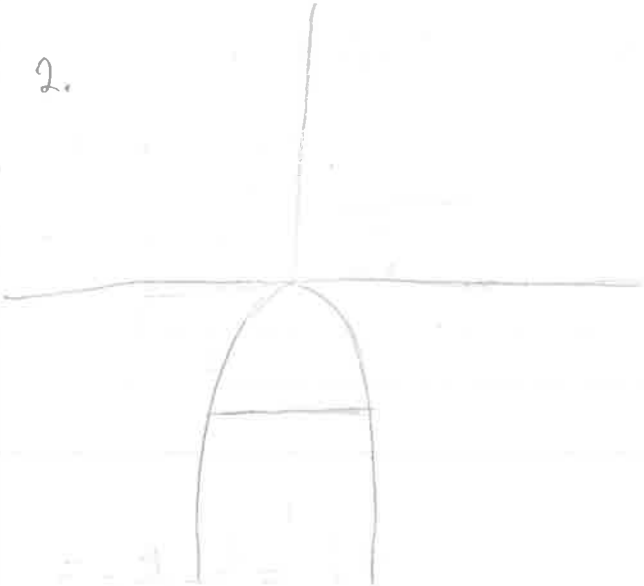
$C+D=\frac{9}{4} \quad C=\frac{9}{4}-D$
$2D+E=9 \quad E=9-2D$
$-\frac{1}{4}+1+C+2E=15$
$\frac{9}{4}-D+18-4D=\frac{63}{4}$
$-5D=\frac{54}{4}-18$
$-5D=-\frac{9}{2}$
$D=\frac{9}{10}$

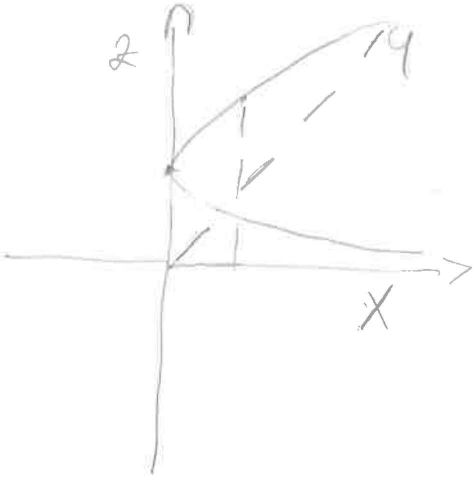
$$E = 9 - \frac{9}{5} = \frac{36}{5} \qquad C = \frac{9}{4} - \frac{9}{10} = \frac{45-18}{20} = \frac{27}{20}$$

$$F(s) = \frac{1}{4}\cdot\frac{1}{s} + \frac{1}{2}\cdot\frac{1}{s^2} + \frac{27}{10}\cdot\frac{1}{s+2} + \frac{\frac{9}{10}s + \frac{36}{8}}{s^2+1}$$

$$= -\frac{1}{4} + \frac{1}{2}t + \frac{27}{20}\cdot e^{-2t} + \frac{9}{10}\cos(t) + \frac{36}{8}\sin(t) \quad \times$$

2.





$$\int_{-1}^{1} dx \int_{-1}^{1} dy \int_{y^2}^{3+x^2} dz = \int_{-1}^{1}\int_{-1}^{1} 3+x^2-y^2 = \int_{-1}^{1} (3+x^2)\cdot y \Big|_{-1}^{1} - \frac{y^3}{3}\Big|_{-1}^{1} = \int (3+x^2)\cdot 2 - \left[\frac{1}{3}+\frac{1}{3}\right]$$

$$= 2\int_{-1}^{1} 3+x^2+\frac{1}{3} = 2\int_{-1}^{1} \frac{10}{3}+x^2 = 2\left[\frac{10}{3}x\Big|_{-1}^{1} \; \frac{x^3}{3}\Big|_{-1}^{1}\right]$$

$$= 2\left[\frac{10}{3}\cdot 2 + \frac{2}{3}\right] = 2\cdot\frac{22}{3} = \frac{44}{3} \quad \checkmark$$

PROVJERA: $f(0) = -\frac{1}{4} + \frac{27}{20} + \frac{9}{10} = \frac{-5+27+18}{20} = \frac{40}{20} = 2 \checkmark$

$f'(t) = \frac{1}{2} - \frac{27}{10}e^{-2t} - \frac{9}{10}\sin t + \frac{36}{8}\cos t, \quad f'(0) = \frac{1}{2} - \frac{27}{10} + \frac{\cancel{36}^{9}}{\cancel{8}_{2}} = \frac{5-27+45}{10} = \frac{23}{10} \neq 5$

Matematika **3**

Ime i prezime: GABRIJELA JORDAN **Matični broj u indeksu:** 17-2-0118-2011

3. $x=\cos t$
$y=\sin t$
$z=2t$
$t\in(0,3)$

$$f(x,y,z)=x\vec{i}+y\vec{j}+\vec{k}$$

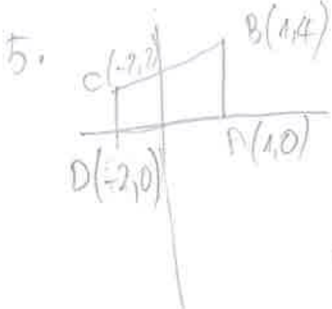
$$r(t)=\begin{pmatrix}\cos t\\ \sin t\\ 2t\end{pmatrix} \qquad r'(t)\begin{pmatrix}-\sin t\\ \cos t\\ 2\end{pmatrix}$$

$$|r'(t)|=\sqrt{\sin^2+\cos^2+4}=\sqrt{5}$$

$$\int_c f\,ds=\sqrt{5}\int_0^3 \cos t\,\vec{i}+\sin t\,\vec{j}+2t\,\vec{k}\,dt$$

$$=\sqrt{5}\left(-\sin t+\cos t+t^2\right)\Big|_0^3$$

$$=\sqrt{5}\left(-\sin 3+\cos 3-1+9\right)=\sqrt{5}\cdot\left(8-\sin 3+\cos 3\right)$$

5. 

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y=\frac{2}{3}x+\frac{10}{3}$$

φ

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B3

IME I PREZIME: AUGUSTIN PTIČAR **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA: **USTMENI ISPIT KOD NASTAVNIKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2\,f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 2,\; f'(0) = 5,\; f''(0) = 3\,.$$

Pomoć: $x^3 + 2x^2 + x + 2 = (x+2)\,(x^2+1)$.

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3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 2t$, $t \in [0,3]$. Izračunaj $\int_C f \cdot \mathrm{d}s$, kada je $f(x,y,z) = x\,\vec{\mathbf{i}} + y\,\vec{\mathbf{j}} + \vec{\mathbf{k}}$. 20

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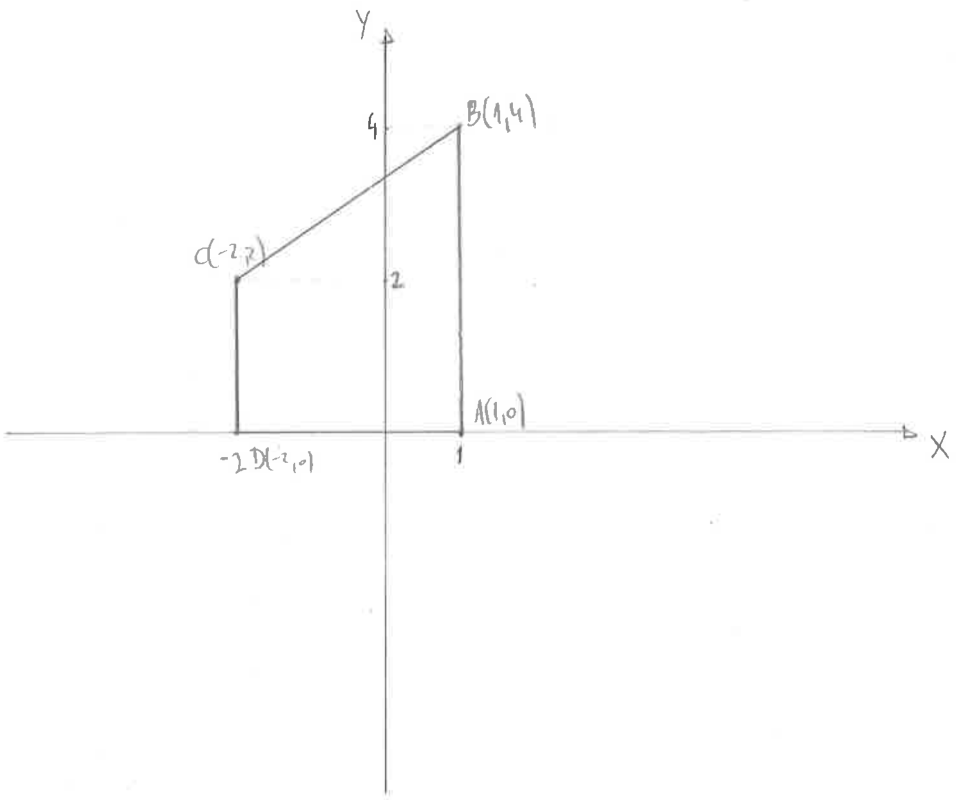
5. Neka je točkama $A(1,0)$, $B(1,4)$, $C(-2,2)$ $D(-2,0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 20

$$\oint_C (x+y)^2 \mathrm{d}y$$

Ukupno: 15

5.

A(1,0)
B(1,4)
C(-2,2)
D(-2,0)



$$\oint_C (x+y)^2 dy \rightarrow P=0 \quad , \quad Q=(x+y)^2 dy$$

$$\frac{\partial Q}{\partial x} ; \frac{\partial P}{\partial y}$$

$$\frac{\partial Q}{\partial x} = ((x+y)^2)' = 2(x+y)$$

$$\frac{\partial P}{\partial y} = 0' = 0$$

$$\iint_D 2(x+y)\,dy$$

$x \in [-2, 1]$

$C(\overset{x_2}{-2}, \overset{y_2}{2})$

$B(\overset{x_1}{1}, \overset{y_1}{4})$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2 = \frac{2}{3}(x+1)$$

$$y - 2 = \frac{2}{3}x + \frac{2}{3}$$

$$y = \frac{2}{3}x + \frac{2}{3} - 2$$

$$y = \frac{2}{3}x - \frac{4}{3}$$

$$\int_{-2}^{1}\int_{0}^{\frac{2}{3}x-\frac{4}{3}} 2(x+y)\,dy = \int_{-2}^{1} 2x \int_{0}^{\frac{2}{3}x-\frac{4}{3}} y\,dy = \int_{-2}^{1} 2x \left.\frac{y^2}{2}\right|_{0}^{\frac{2}{3}x-\frac{4}{3}}$$

$$= \int_{-2}^{1} 2x\,\frac{1}{2}\left(\frac{2}{3}x - \frac{4}{3}\right)^2 = \int_{-2}^{1} 2x\,\frac{1}{2}\left(\frac{4}{9}x^2 - \frac{4}{3}\cdot\frac{4}{3} + \frac{16}{9}\right)$$

$$= \int_{-2}^{1} 2x\,\frac{1}{2}\left(\frac{4}{9}x^2 - \cancel{\frac{16}{9}} + \cancel{\frac{16}{9}}\right) = \int_{-2}^{1} 2x\,\frac{2}{9}x^2 = \int_{-2}^{1}\frac{4}{9}x^3 = \frac{4}{9}\left.\frac{x^4}{4}\right|_{-2}^{1}$$

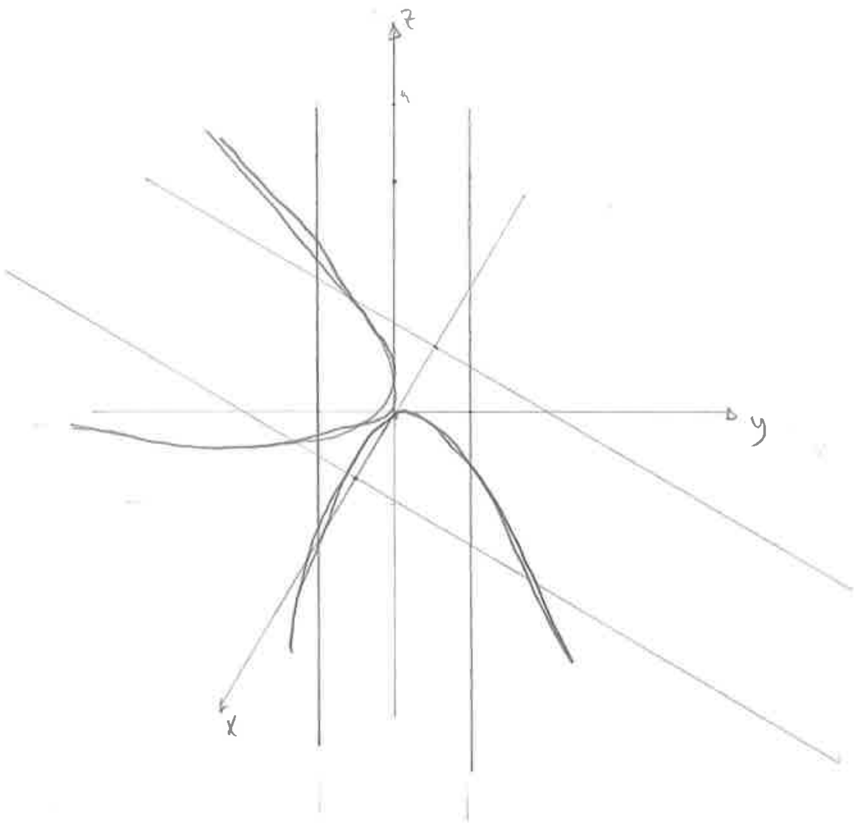
$$= \frac{4}{9}\cdot\frac{1}{4}\left(1-(-2)^2\right) = \frac{1}{9}(1-4) = \frac{1}{9}\cdot(-3) = \frac{1}{3}$$

Matematika 3

Ime i prezime: AUGUSTIN PTIČAR **Matični broj u indeksu:**

x	0	1	2	-1	-2
$z=3+x^2$	3	4	7	4	7

2.



$$\int_{-1}^{1}\int_{-1}^{1}\int_{-y^2}^{3+x^2} 1\, dz\, dy\, dx = \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{-y^2}^{3+x^2} 1\, dz$$

$$= \int_{-1}^{1} dx \int_{-1}^{1} dy\; z \Big|_{-y^2}^{3+x^2} = \int_{-1}^{1} dx \int_{-1}^{1} dy\; (3+x^2+y^2)$$

$$= \int_{-1}^{1} x^2 dx \int_{-1}^{1} (3+y^2)\, dy = \int_{-1}^{1} x^2 dx \int_{-1}^{1} 3\, dy + \int_{-1}^{1} y^2 dy$$

$$= \int_{-1}^{1} x^2 dx\; 3y\Big|_{-1}^{1} + \frac{y^3}{3}\Big|_{-1}^{1} = \int_{-1}^{1} x^2 dx\; 3(2) + \frac{1}{3}\left(1^3-(-1)^3\right)$$

$$= \int_{-1}^{1} x^2 dx \, 6 + \frac{1}{3}(1+1) = \int_{-1}^{1} x^2 dx \left(6 + \frac{2}{3}\right)$$

$$= \int_{-1}^{1} x^2 dx \, \frac{20}{3} = \frac{20}{3} \left. \frac{x^3}{3} \right|_{-1}^{1} = \frac{20}{9} (1+1) = \frac{40}{9}$$

X

1. $f'''(t)+2f''(t)+f'(t)+2f(t)=t$ PTICAR

$f(0)=2,\ f'(0)=5,\ f''(0)=3$

$$s^3F(s)-s^2\underbrace{f(0)}_{=2}-s\underbrace{f'(0)}_{=5}-\underbrace{f''(0)}_{=3}+2\left(s^2F(s)-s\underbrace{f(0)}_{=2}-\underbrace{f'(0)}_{=5}\right)+sF(s)-\underbrace{f'(0)}_{=5}+2F(s)=\frac{1}{s}$$

$$s^3F(s)-2s^2-5s-3+2\left(s^2F(s)-2s-5\right)+sF(s)-5+2F(s)=\frac{1}{s}$$

$$s^3F(s)-2s^2-5s-3+2s^2F(s)-4s-10+sF(s)-5+2F(s)=\frac{1}{s}$$

$$s^3F(s)+2s^2F(s)+sF(s)+2F(s)=\frac{1}{s}+2s^2+5s+18+4s$$

$$F(s)\left(s^3+2s^2+s+2\right)=\frac{1}{s}+2s^2+9s+18$$

$$F(s)\left(s^3+2s^2+s+2\right)=\frac{1+2s^3+9s^2+18s}{s}\quad\Big/\cdot\frac{1}{(s^3+2s^2+s+2)}$$

$$F(s)=\frac{1+2s^3+9s^2+18s}{s(s^3+2s^2+s+2)}$$

$s_1=0$

$s^3+2s^2+s+2=(s+2)(s^2+1)$

$$\frac{1+2s^3+9s^2+18s}{s(s+2)(s^2+1)}=\frac{A}{s}+\frac{B}{(s+2)}+\frac{C}{(s^2+1)}\quad\Big/\cdot s(s+2)(s^2+1)$$

$$\frac{1+2s^3+9s^2+18s}{s(s+2)(s^2+1)} = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s^2+1)} \quad / \cdot s(s+2)(s^2+1)$$

$$1+2s^3+9s^2+18s = A(s+2)(s^2+1) + B(s(s^2+1)) + C(s(s+2))$$

$$1+2s^3+9s^2+18s = A(s^3+s+2s^2+2) + B(s^3+s) + C(s^2+2s)$$

$$1+2s^3+9s^2+18s = As^3+As+2As^2+2A + Bs^3+Bs + Cs^2+2Cs$$

$s^3 \Rightarrow 2 = A+B$

$s^2 \Rightarrow 9 = 2A+C$

$s \Rightarrow 18 = A+2C$

$(s=0) \quad 1 = 2A$

$A = \frac{1}{2}$

$2 = \frac{1}{2} + B$

$B = 2 - \frac{1}{2}$

$B = \frac{4-1}{2} = \frac{3}{2}$

$9 = 2A + C$

$9 = 1 + C$

$C = 8$

PROVJERA:

$f(0) = \frac{1}{2} + \frac{3}{2} = 2$ ✓

$f'(t) = -3 + 8\cos t \Rightarrow f'(0) = 5$ ✓

$f''(t) = -8\sin t \Rightarrow f''(0) = 0$ ✗

$A = \frac{1}{2}$

$B = \frac{3}{2}$

$C = 8$

$$F(s) = \frac{\frac{1}{2}}{s} + \frac{\frac{3}{2}}{(s+2)} + \frac{8}{(s^2+1)} = \frac{1}{2s} + \frac{3}{2(s+2)} + \frac{8}{(s^2+1)}$$

0

$$F(t) = \frac{1}{2} + \frac{3}{2}e^{-2t} + 8\sin(t)$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{B}3$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: VESNA ŠARIĆ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: **USTMENI ISPIT KOD NASTAVNIKA:** prof. UGLEŠIĆ	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 2,\ f'(0) = 5,\ f''(0) = 3.$$

Pomoć: $x^3 + 2x^2 + x + 2 = (x + 2)(x^2 + 1)$.

2. Izračunati volumen tijela omeđenog plohama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 3 + x^2$, $z = -y^2$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 2t$, $t \in [0, 3]$. Izračunaj $\int_C f \cdot ds$, kada je $f(x, y, z) = x\vec{i} + y\vec{j} + \vec{k}$. 20

4. Izračunati integral funkcije $f(x, y, z) = x + y + z$ na dijelu ravnine $x + y + z = 1$ koji se nalazi u dijelu prostora određenom nejednadžbama $x \geq 0$, $y \geq 0$ i $z \geq 0$. 20

5. Neka je točkama $A(1, 0)$, $B(1, 4)$, $C(-2, 2)$ $D(-2, 0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 20

$$\oint_C (x + y)^2 dy$$

Ukupno: $\emptyset$

1 $f'''(t)+2f''(t)+f'(t)+2f(t)=t$ $\quad f(0)=2,\ f'(0)=2,\ f'(0)=5,\ f''(0)=3$

$$\left(s^3F(s)-s^2f(0)-sf'(0)-f''(0)\right)+2\left(s^2F(s)-sf(0)-f'(0)\right)+\left(sF(s)-f(0)\right)+2f(t)=\frac{1}{s^2}$$

$$\left(s^3F(s)-s^2\cdot 2-s\cdot 5-3\right)+2\left(s^2F(s)-s\cdot 2-5\right)+\left(sF(s)-2\right)+2f(t)=\frac{1}{s^2}$$

$$\left(s^3F(s)-s^2 2-s5-3\right)+2\left(s^2F(s)-s\cdot 2-5\right)+\left(sF(s)-2\right)+2f(t)=\frac{1}{s^2}$$

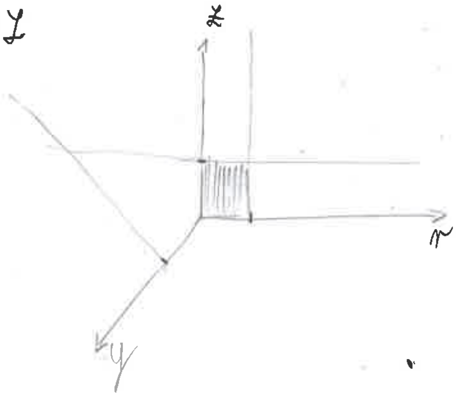
$$\underline{s^3F(s)}-s^2 2-s5-3+\underline{2s^2F(s)}-4s-10+\underline{sF(s)}-2+2s=\frac{1}{s^2}$$

$$s^3-s^2 2+2s^2+s=5s-\cancel{3}-4s-\cancel{10}-\cancel{2}+2s+\frac{1}{s^2}$$

$\emptyset$

③ $x+y+z=0$ $\quad x \geqslant 0,\ y \geqslant 0$ i $z \geqslant 0$ $\quad x+y+z=1$

$f(x)=x+y+z$



③ $\begin{bmatrix} x=\cos t \\ x=-\sin t \end{bmatrix}$ $\quad T=(0,3)$ $\quad \int_C f \cdot ds$ $\quad xi+yj+k$

$\begin{bmatrix} y=\sin t \\ x=\cos t \end{bmatrix}$

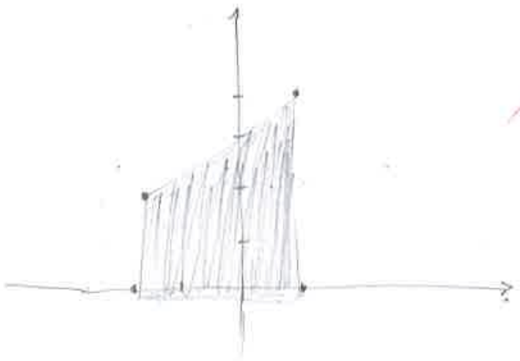
$\begin{bmatrix} z=2t \\ z=t \end{bmatrix}$

⑤ $A(1,0)$

$B(1,4)$

$C(-2,2)$

$D(-2,0)$



x	0	1	2
$z=3+x^2$	3	4	7

y	0	1	2
$z=-y^2$	0	1	4

② $x=1$ $\quad z=3+x^2$

$x=-1$ $\quad z=-y^2$

$y=1$

$y=-1$

Matematika **3**

Ime i prezime: VESNA ŠARIĆ **Matični broj u indeksu:**

① $f'''(t)+2f''(t)+f'(t)+2f(t)=t$ $f(0)=2,\ f'(0)=5,\ f''(0)=3$

$$\left(s^3F(s)-s^2f(0)-sf'(0)-f''(0)\right)+2\left(s^2F(s)-sf(0)-f'(0)\right)+\left(sF(s)-f(0)\right)+2f(t)=\frac{1}{s^2}$$

$$\left(s^3F(s)-s^2 2-s\cdot 5-3\right)+2\left(s^2F(s)-s\cdot 2-5\right)+\left(sF(s)-2\right)+2f(t)=\frac{1}{s^2}$$

$$F(s)=\frac{s^3+}{\quad}$$