MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A3

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: VEDRAN ČIZMIN **VRIJEME POČETKA:** 08:00

MATIČNI BROJ STUDENTA: 17-2-0089-2011 **USTMENI ISPIT KOD NASTAVNIKA:** prof. Nikica Uglešić

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: 20

$$f'''(t) + 4f'(t) = t, \quad f(0) = 3,\ f'(0) = 5,\ f''(0) = 2.$$

2. Izračunati integral funkcije $f(x,y,z) = z$ u dijelu prostora omeđenog plohama $x = z^2$, $y = 0$ i $y = 2 - x$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 3t$, $t \in [0,2]$. Izračunaj $\int_C f\,ds$, kada je $f(x,y,z) = z\left(x^2+y^2\right)$. 20

4. Neka je S parabolična ploha $z = x^2$ u dijelu prostora $x \in [-1,1]$ i $y \in [-1,1]$ s orijentacijom koja najbolje odgovara vektoru $\vec{\mathbf{k}}$. Odrediti $\iint_S \vec{\mathbf{k}} \cdot dS$. 20

5. Neka je C cilindar zadan sa $C = \{(x,y,z) : x^2+z^2 \le 3,\ 1 \le y \le 2\}$. Izračunati plošni integral 20

$$\iint_{\partial C} z\,dydz + y\,dxdz + x\,dxdy$$

Ukupno: 40

3. $x = \cos t \quad y = \sin t \quad z = 3t \qquad t \in [0,2]$

$$r(t) = \begin{pmatrix} \cos t \\ \sin t \\ 3t \end{pmatrix} \quad r'(t) = \begin{pmatrix} -\sin t \\ \cos t \\ 3 \end{pmatrix}$$

$$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (3)^2} = \sqrt{\underbrace{\sin^2 t + \cos^2 t}_{1} + 9} = \sqrt{10}$$

$$\int_0^2 3t\left((\cos t)^2 + (\sin t)^2\right)\cdot\sqrt{10}\,dt = \int_0^2 3t\cdot\sqrt{10}\,dt = 3\sqrt{10}\,\frac{t^2}{2}\Big|_0^2$$

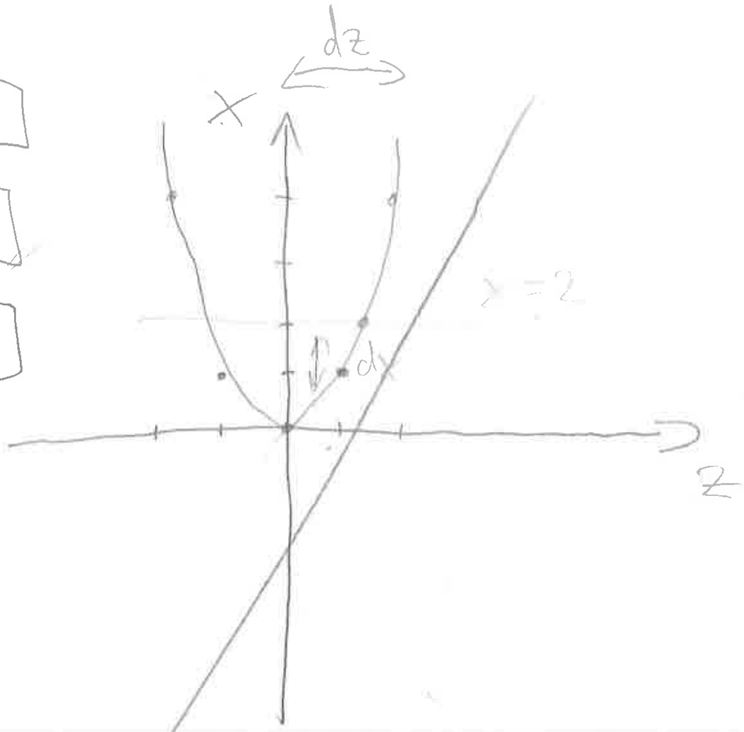
$$= 3\sqrt{10}\,\frac{2^2}{2} = 3\sqrt{10}\cdot 2 = 6\sqrt{10}$$ ✓

2. $f(x,y,z)=z \quad x=z^2 \quad y=0 \quad y=2-x \qquad 0=2-x \quad x=2$

$y\in[0,2-x]$

$x\in[z^2,2]$

$z\in[0,\sqrt{2}]$

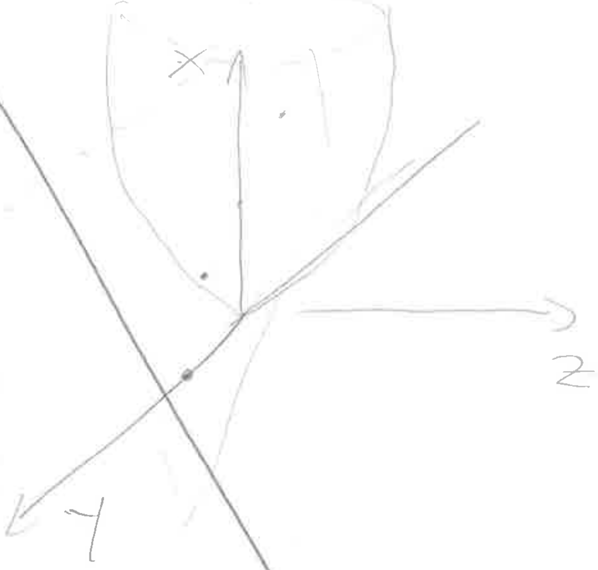


$$\int_0^{\sqrt{2}}\int_{z^2}^{2}\int_0^{2-x} z \, dy\, dx\, dz =$$

$$=\int_0^{\sqrt{2}}\int_{z^2}^{2} zy \Big|_0^{2-x} dx\, dz$$

z	-1	0	1	2	-2
$x=z^2$	1	0	1	4	4

$$\int_0^{\sqrt{2}}\int_{z^2}^{2} z(2-x)\, dx\, dz$$



$$\int_0^{\sqrt{2}}\int_{z^2}^{2} 2z-zx \, dx\, dz$$

$$\int_0^{\sqrt{2}} 2zx - z\frac{x^2}{2}\Big|_{z^2}^{2} dz$$

x	0	1	-1	2
$y=2-x$	2	1	3	0

$$\int_0^{\sqrt{2}} (4z-2z)-\left(2z^3-z\frac{z^4}{2}\right) dz$$

$$\int_0^{\sqrt{2}} 2z-2z^3+\frac{z^5}{2}\, dz = 2\frac{z^2}{2}-2\frac{z^4}{4}+\frac{1}{2}\frac{z^6}{6}\Big|_0^{\sqrt{2}} = (\sqrt{2})^2-2+\frac{4}{3}$$

$$=\frac{4}{3}$$

② VEDRAN ČIŽMIN

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{z^2}^{2} \int_{0}^{2-x} z \, dy \, dx \, dz = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{z^2}^{2} zy \Big|_{0}^{2-x} dx \, dz$$

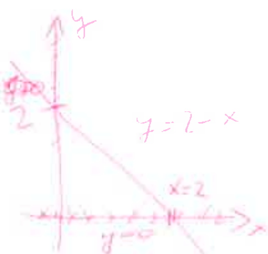
$$= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{z^2}^{2} z(2-x) \, dx \, dz = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{z^2}^{2} 2z - xz \, dx \, dz$$

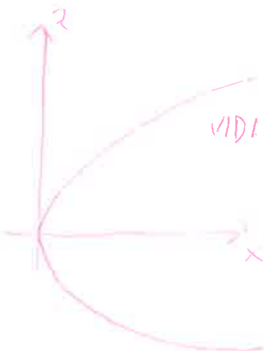
$$= \int_{-\sqrt{2}}^{\sqrt{2}} 2zx - \frac{x^2}{2} z \Big|_{z^2}^{2} dz = \int_{-\sqrt{2}}^{\sqrt{2}} (4z - 2z) - \left(2z^3 - \frac{z^5}{2}\right) dz$$

$$\int_{-\sqrt{2}}^{\sqrt{2}} 2z - 2z^3 + \frac{1}{2} z^5 \, dz = 2\frac{z^2}{2} - 2\frac{z^4}{4} + \frac{1}{2}\frac{z^6}{6} \Big|_{-\sqrt{2}}^{\sqrt{2}}$$

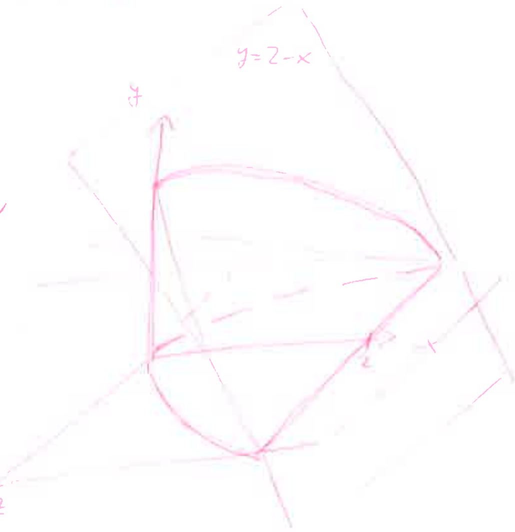
$$= \left(2\frac{(\sqrt{2})^2}{2} - 2\frac{(\sqrt{2})^4}{4} + \frac{1}{2}\frac{(\sqrt{2})^6}{6}\right) - \left(2\frac{(-\sqrt{2})^2}{2} - 2\frac{(-\sqrt{2})^4}{4} + \frac{1}{2}\frac{(-\sqrt{2})^6}{6}\right)$$

$$= \cancel{2} - \cancel{2} + \cancel{\frac{4}{3}} - \cancel{2} + \cancel{2} - \cancel{\frac{4}{3}} = 0$$





VIDI: x POZITIVAN



$\int_0^{\sqrt{2}}\int_0^{2-x}\int_{z^2}^{2}$ $\qquad \int^{\sqrt{2}}\int^{2-x} zx \Big|_{z^2}^{2} = \int_0^{\sqrt{2}}\int_0^{2-x} 2z - z^3 \, dy\, dz$

$\int_0^{\sqrt{2}} \qquad \int_0^{2-x}$

$\frac{1}{12} \qquad \frac{1}{2}$

$\frac{4}{3}$

$\frac{1}{2}\frac{(\sqrt{2})}{6} \qquad \frac{1}{2}\,\frac{2}{6}$

$2\frac{(\sqrt{2})^4}{4} = 2\,\frac{2^2}{4} = 2$

$$\frac{\frac{55}{16}}{s} + \frac{\frac{1}{4}}{s^3} + \frac{-\frac{7}{16}s + 5}{s^2+4}$$

$$\frac{55}{16}\cdot\frac{1}{s} + \frac{1}{4}\cdot\frac{1}{s^3} + \frac{-\frac{7}{16}s}{s^2+4} + \frac{5}{s^2+4} = \frac{55}{16} + \frac{1}{4}t - \frac{7}{16}\frac{s}{s^2+4} + \frac{5}{s^2+4}$$

$$\frac{1}{s} \downarrow 1$$

$$\frac{1}{s^3} \downarrow \frac{1}{s}\cdot\frac{1}{s^2} \downarrow 1\cdot t$$

$$\frac{s}{s^2+4} \downarrow \frac{s}{s^2+2^2} \downarrow \cos(2t)$$

$$\frac{5}{s^2+4} \downarrow 5\cdot\frac{1}{s^2+2^2} \downarrow \frac{5}{2}\cdot\frac{2}{s^2+2^2} \downarrow \frac{5}{2}\sin(2t)$$

$$\frac{55}{16} + \frac{1}{4}t - \frac{7}{16}\cos 2t + \frac{5}{2}\sin 2t$$

PROVJERA

$$f(0) = \frac{55}{16} + \frac{-7}{16} = \cancel{\frac{48}{16}} = 3$$

$$f'(t) = \frac{1}{4} + \frac{7}{8}\sin 2t + 5\cos 2t \qquad f'(0) = \frac{1}{4} + 5 \neq 5$$

Matematika 3

Ime i prezime: VEDRAN ČRNIN **Matični broj u indeksu:** 17-2-0089-2011

(1.) $f'''(t) + 4f'(t) = t \quad f(0)=3 \quad f'(0)=5 \quad f''(0)=2$

$$s^3F(s) - s^2 3 - s5 - 2 + 4(sF(s) - 3) = \frac{1}{s^2}$$

$$s^3F(s) - 3s^2 - 5s - 2 + 4sF(s) - 12 = \frac{1}{s^2}$$

$$s^3F(s) + 4sF(s) = \frac{1}{s^2} + 3s^2 + 5s + 2 + 12$$

$$F(s)(s^3 + 4s) = \frac{1}{s^2} + 3s^2 + 5s + 14$$

$$F(s) = \frac{1 + 3s^4 + 5s^3 + 14s^2}{s^2(s^3 + 4s)} \qquad \frac{1 + 3s^4 + 5s^3 + 14s^2}{s^2}$$

$$F(s) = \frac{1 + 3s^4 + 5s^3 + 14s^2}{s^3(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4} =$$

$$As^4 + 4As^2 + Bs^3 + 4Bs + Cs^2 + 4C + Ds^4 + Es^3$$

$A\, s^2(s^2+4)$

$As^4 + 4As^2$

$Bs(s^2+4)$

$Bs^3 + 4Bs$

$C(s^2+4)$

$Cs^2 + 4C$

$(Ds+E)(s^3)$

$Ds^4 + Es^3$

$A + D = 3$

$B + E = 5$

$4A + C = 14$

$4B = 0$

$4C = 1$

$\boxed{C = \frac{1}{4}}$

$\boxed{B = 0}$

$\boxed{E = 5}$

$4A + C = 14$

$4A = 14 - \frac{1}{4}$

$4A = \frac{55}{4}$

$\boxed{A = \frac{55}{16}}$

$\frac{55}{16} + D = 3$

$D = 3 - \frac{55}{16}$

$D = \frac{48}{16} - \frac{55}{16} \qquad \boxed{D = -\frac{7}{16}}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}3$ **IME I PREZIME:** MATE MAROVIĆ **VRIJEME POČETKA:** **MATIČNI BROJ STUDENTA:** 1710145 2012 **USTMENI ISPIT KOD NASTAVNIKA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: 20

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3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 3t$, $t \in [0, 2]$. Izračunaj $\int_C f\,ds$, kada je $f(x, y, z) = z\,(x^2 + y^2)$. ~~20~~ 15

4. Neka je S parabolična ploha $z = x^2$ u dijelu prostora $x \in [-1, 1]$ i $y \in [-1, 1]$ s orijentacijom koja najbolje odgovara vektoru $\vec{\mathbf{k}}$. Odrediti $\iint_S \vec{\mathbf{k}} \cdot dS$. 20

5. Neka je C cilindar zadan sa $C = \{(x, y, z) : x^2 + z^2 \le 3,\ 1 \le y \le 2\}$. Izračunati plošni integral ~~20~~

$$\iint_{\partial C} z\,dydz + y\,dxdz + x\,dxdy$$

Ukupno:

30

1.

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 4\big(sF(s) - f(0)\big) = \frac{1}{s^2}$$

$$s^3F(s) - 3s^2 - 5s - 2 + 4sF(s) - 12 = \frac{1}{s^2}$$

$$F(s)\left[s^3 + 4s\right] = \frac{1}{s^2} + 3s^2 + 5s + 14$$

$$s^3 + 4s = s(s^2+4)$$

$$F(s) = \frac{\frac{1}{s^2} + 3s^2 + 5s + 14}{s^3 + 4s} = \frac{1}{s^2 \cdot s(s^2+4)} + \frac{3s^2 + 5s + 14}{s(s^2+4)}$$

$$\frac{1}{s^2 \cdot s(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s} + \frac{Ds+E}{s^2+4} = \frac{As^4 + 4As^2 + Bs^3 + 4Bs + Cs^4 + 4Cs^2 + Ds^4 + Es^3}{s^2 \cdot s(s^2+4)}$$

$A + C + D = 0$

$B + E = 0 \Rightarrow E = 0$

$4A + 4C = 0$

$4B = 0 \Rightarrow B = 0$

$A + C + D = 0 \quad / \cdot (-4)$

$4A + 4C + D = 0$

$D = 0$

$A = -D - C$

$A = -D - 4A$

$5A = -D$

$D = -5A$

$$\frac{1}{s^2 \cdot s(s^2+4)} = 0$$

$$\frac{3s^2 + 5s + 14}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4} = \frac{As^2 + 4A + Bs^2 + Cs}{s(s^2+4)}$$

$A + B = 3$

$4A + C = 5$

$0 = 14$

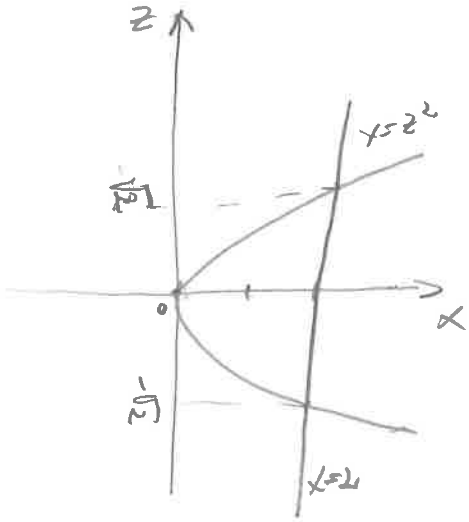
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 4 & 0 & 1 & 5 \\ 0 & 0 & 0 & 14 \end{array}\right]$$

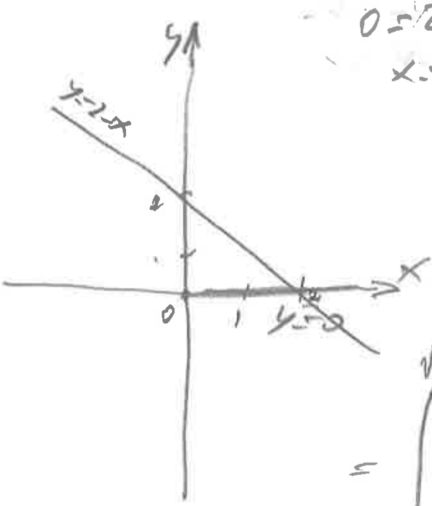
② $f(x,y,z) = z$; $x = z^2$, $y = 0$ i $y = 2 - x$

y	0	2
x	2	0

$0 = 2 - x$

$x = 2$





$$\iiint_X 1 \, dx\,dy\,dz = \iiint z \, d$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{z^2}^{2} \int_{0}^{2-x} z \, dy\,dx\,dz \checkmark$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{z^2}^{2} z(2-x)\,dx\,dz = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{z^2}^{2} 2z - zx \, dx\,dz = \int_{-\sqrt{2}}^{\sqrt{2}} \left[2zx - z\frac{x^2}{2}\right]_{z^2}^{2} dz$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} 4z - 2z - 2z^3 + \frac{z^5}{2} \, dz = \left[z^2 - \frac{1}{2}z^4 + \frac{z^6}{12}\right]_{-\sqrt{2}}^{\sqrt{2}} = 0$$

$$= 2 - 2 + \frac{2}{3} - 2 + 2 + \frac{2}{3} = 1.33$$

Matematika **3**

Ime i prezime: MATE MITROVIĆ **Matični broj u indeksu:** 171 0145 2012

(3.) $r(t) = (\cos t, \sin t, 3t)$

$f \circ r(t) = f(\varphi(t), \psi(t), \chi(t)) = 3t\,(\underbrace{\cos^2 t + \sin^2 t}_{1})$

$\|r'(t)\| = \sqrt{(\varphi'(t))^2 + (\psi'(t))^2 + \chi'(t)^2}$

$r'(t) = \sqrt{(-\sin t)^2 + (\cos t)^2 + (3)^2} = \sqrt{10}$

$\int_c f\,ds = \int f \circ r(t) \|r'\|(t)\,dt = \int_0^2 3t\,(\underbrace{\cos^2 t + \sin^2 t}_{1})\sqrt{10}\,dt$ ✓

$= \sqrt{10}\left(3\int_0^2 t\,dt \int_0^2 1\,t\,dt\right) = \sqrt{10}\left(\left[\frac{3}{2}t^2\right]_0^2 \cdot \left[\frac{t^2}{2}\right]_0^2\right) = \sqrt{10}\cdot 6\cdot 2$

$= 12\sqrt{10}$

(5.) $\oint_{\partial S} P\,dx + Q\,dy + R\,dz = \iint_{S} z\,dy\,dz + y\,dx\,dz + x\,dx\,dy$

$\iint_S \left(\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\cos\alpha + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\cos\beta + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\cos\gamma\right) dS$

$= \iint 0 + \cancel{1+1} + 0 = 0$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}3$	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: BRANIMIR PISIACA **Vrijeme početka:**	
Matični broj studenta: 17-2-0086-2011 **Ustmeni ispit kod nastavnika:**	

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$$\iint_{\partial \widehat{C}} z\,\mathrm{d}y\mathrm{d}z + y\,\mathrm{d}x\mathrm{d}z + x\,\mathrm{d}x\mathrm{d}y$$

Ukupno: 20

$$f(xyz) = z(x^2+y^2)$$

$$r(t) = \begin{pmatrix} \cos t \\ \sin t \\ 3t \end{pmatrix} \to r'(t) = \begin{pmatrix} -\sin t \\ \cos t \\ 3 \end{pmatrix}$$

$$= 3t(\cos^2 t + \sin^2 t)$$
$$= 3t$$

$$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (3^2)} = \sqrt{\sin^2 t + \cos^2 t + 9} = \sqrt{1+9} = \sqrt{10}$$

$$\int_0^2 \sqrt{10}\cdot(3t)\,dt \Rightarrow \sqrt{10}\cdot\left(3\frac{t^2}{2}\right)\Big|_0^2$$

$$\sqrt{10}\cdot\left[3\cdot\frac{2^2}{2} - 3\cdot\frac{0^2}{2}\right] = \sqrt{10}\cdot 6 = 6\sqrt{10}$$

$f(xyz) = z$ $\qquad x = z^2 \quad y = 0 \quad y = 2 - x$

$\downarrow$

$x = 2 - y$

$y \in [0, 2-x]$

$x \in [z^2, 2-y]$

$z \in [\sqrt{x}, \sqrt{2-y}]$

$$\int_0^{2-x}\int_{\sqrt{x}}^{\sqrt{2-y}}\int_{z^2}^{2-y} dx\, dz\, dy$$

$$\int_0^{2-x}\int_{\sqrt{x}}^{\sqrt{2-y}} x \Big|_{z^2}^{2-y} = \int_0^{2-x}\int_{\sqrt{x}}^{\sqrt{2-y}} (2-y) - z^2\, dz\, dy$$

$$\int_0^{2-x} z(2-y) - z^3 \Big|_{\sqrt{x}}^{\sqrt{2-y}} dy \Rightarrow \int_0^{2-x} \sqrt{2-y}(2-y) - (\sqrt{2-y})^3 - [\sqrt{x}(2-y) - \sqrt{x}^3]$$

$$\int_0^{2-x} 2\sqrt{2} - 2y - \sqrt{2}y + y^2 + (\sqrt{2})^3 + y^3 - [2\sqrt{x} - \sqrt{x}y - \sqrt{x}^3]\ dy$$

$$\int_0^{2-x} 2\sqrt{2} - 2y - \sqrt{2}y + y^2 + (\sqrt{2})^3 + y^3 - 2\sqrt{x} + \sqrt{x}y + \sqrt{x}^3$$

Matematika 3

Ime i prezime: Branimir Pijaca **Matični broj u indeksu:** 17-2-0086-2011

$f(xyz) = z$ u dijelu prostora omeđenog plohama

$x = z^2,\ y = 0 \qquad y = 2 - x$

$$\int_{-\sqrt{x}}^{\sqrt{x}} \int_{0}^{2-x} \int_{z^2}^{-2} z \, dx\, dy\, dz$$

$0 = 2 - x$

$x = -2$

$z^2 = x$

$z = \sqrt{x}$

$y \in [0, 2-x]$

$x \in [z^2, -2]$

$z \in [-\sqrt{x}, \sqrt{x}]$

$$\int_{-\sqrt{x}}^{\sqrt{x}} \int_{0}^{2-x} z \cdot x \Big|_{z^2}^{-2} dy\, dz = \int_{-\sqrt{x}}^{\sqrt{x}} \int_{0}^{2-x} z\cdot(-2) - z\cdot(z^2)\, dy\, dz$$

$$\int_{-\sqrt{x}}^{\sqrt{x}} \int_{0}^{2-x} -2z - z^3 \, dy\, dz = \int_{-\sqrt{x}}^{\sqrt{x}} -2zy - z^3 y \Big|_{0}^{2-x} dz$$

$$\int_{-\sqrt{x}}^{\sqrt{x}} -2z(2-x) - z^3(2-x) - 0 \, dz \Rightarrow \int_{-\sqrt{x}}^{\sqrt{x}} -4z + 2zx - 2z^3 + z^3 x \, dz$$

$$-4z \cdot z + 2zx \cdot z - 2z^3 \cdot z + z^3 x \cdot z \Big|_{-\sqrt{x}}^{\sqrt{x}}$$

$$= -4z^2 + 2z^2 x - 2z^4 + z^4 x \Big|_{-\sqrt{x}}^{\sqrt{x}}$$

$$= -4\cdot(\sqrt{x})^2 + 2\cdot\sqrt{x}^2\cdot x - 2\cdot\sqrt{x}^4 + \sqrt{x}^4 x - [-4$$

$f(x,y,z) = z$ $\quad x = z^2 \quad y = 0 \quad y = 2 - x$

$y \in [0, 2-x]$ $\quad 0 = 2 - x$

$x = 2$

$x \in [z^2, 2]$

$z \in [\sqrt{x}, \sqrt{2}]$

$$\int_0^{2-x}\int_{\sqrt{x}}^{\sqrt{2}}\int_{z^2}^{2} z \, dx \, dz \, dy$$

$$\int_0^{2-x}\int_{\sqrt{x}}^{\sqrt{2}} z \cdot x \Big|_{z^2}^{2} dz\,dy = \int_0^{2-x}\int_{\sqrt{x}}^{\sqrt{2}} z \cdot 2 - [z \cdot z^2] \, dz\,dy = \int_0^{2-x}\int_{\sqrt{x}}^{\sqrt{2}} 2z - z^3 \, dz\,dy$$

$$\int_0^{2-x} 2z^2 - z^4 \Big|_{\sqrt{x}}^{\sqrt{2}} dy \Rightarrow \int_0^{2-x} 2 \cdot (\sqrt{2})^2 - (\sqrt{2})^4 - [2 \cdot \sqrt{x}^2 - \sqrt{x}^4] \, dy$$

$$\int_0^{2-x} \cancel{2 \cdot 2} - \cancel{(2)^2} - [2x - x^2] \, dy = \int_0^{2-x} (-2x + x^2) \, dy$$

$$-2xy + x^2 y \Big|_0^{2-x} = -2x(2-x) + x^2(2-x) - 0$$

$$-4x - \cancel{2x^2} + \cancel{2x^2} - x^3$$

$$-4x - x^3$$

?????

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A3

IME I PREZIME: ALEN BURA

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA: 17-2-0095-2011

USTMENI ISPIT KOD NASTAVNIKA: prof. UGLEŠIĆ

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: 20

$$f'''(t) + 4f'(t) = t, \quad f(0) = 3,\ f'(0) = 5,\ f''(0) = 2.$$

2. Izračunati integral funkcije $f(x,y,z) = z$ u dijelu prostora omeđenog plohama $x = z^2$, $y = 0$ i $y = 2 - x$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 3t$, $t \in [0,2]$. Izračunaj $\int_C f\,ds$, kada je $f(x,y,z) = z\,(x^2+y^2)$. 20

4. Neka je S parabolična ploha $z = x^2$ u dijelu prostora $x \in [-1,1]$ i $y \in [-1,1]$ s orijentacijom koja najbolje odgovara vektoru $\vec{\mathbf{k}}$. Odrediti $\iint_S \vec{\mathbf{k}} \cdot dS$. 20

5. Neka je C cilindar zadan sa $C = \{(x,y,z) : x^2 + z^2 \le 3,\ 1 \le y \le 2\}$. Izračunati plošni integral 20

$$\iint_{\partial C} z\,dydz + y\,dxdz + x\,dxdy$$

Ukupno: 20

$x \in [0,2]$

$y \in [0,2]$

$z \in [0, \sqrt{x}]$

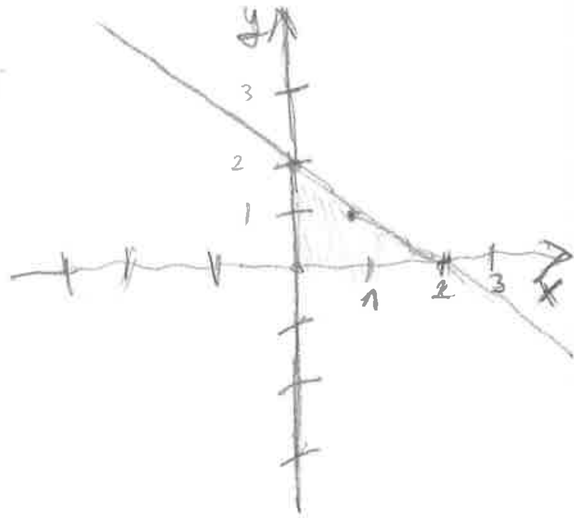
$(y \geq 0,\ z \in [0, \sqrt{x}]$

2. $f(x,y,z) = z$

$x = z^2$

$y = 0$

$y = 2 - x$



$$\int_0^2 \int_0^2 \int_0^{\sqrt{x}} z\,dx\,dy\,dz = \int_0^2 dx \int_0^2 dy \int \frac{z^2}{2}\Big|_0^{\sqrt{x}}$$

$$= \int_0^2 dx \int_0^2 \frac{x}{2}\,dy = \frac{1}{2}\int_0^2 x\,dx \cdot y\Big|_0^2$$

$$= \frac{x^2}{2}\Big|_0^2 = 2$$

3. $r(t) \begin{pmatrix} \cos t \\ \sin t \\ 3t \end{pmatrix} \begin{matrix} x \\ y \\ z \end{matrix} \quad r'(t) = \begin{pmatrix} \sin t \\ -\cos t \\ 3 \end{pmatrix}$

$$\|r'(t)\| = \sqrt{(\sin t)^2 + (-\cos t)^2 + 3^2}$$

$$= \sqrt{\sin^2 t + \cos^2 t + 9}$$

$$= \sqrt{1+9} = \sqrt{10}$$

$$f(x,y,z) = z(x^2+y^2)$$

$$= 3t(\cos^2 t + \sin^2 t)$$

$$= 3t \cdot 1$$

$$= 3t$$

$$\int f\,ds = \int_0^2 3t \cdot \sqrt{10}\,dt = \int_0^2 3\sqrt{10}\,t = 6\sqrt{10} \,//\; ✓$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{A}3$

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: MARIO IVANAC

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA: 17-1-0096-2011

USTMENI ISPIT KOD NASTAVNIKA: NIKICA UGLEŠIĆ

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: ~~20~~

$$f'''(t) + 4f'(t) = t, \quad f(0) = 3,\ f'(0) = 5,\ f''(0) = 2.$$

2. Izračunati integral funkcije $f(x,y,z) = z$ u dijelu prostora omeđenog plohama $x = z^2$, $y = 0$ i $y = 2 - x$. ~~20~~ 15

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 3t$, $t \in [0,2]$. Izračunaj $\int_C f \, ds$, kada je $f(x,y,z) = z\,(x^2 + y^2)$. 20

4. Neka je S parabolična ploha $z = x^2$ u dijelu prostora $x \in [-1,1]$ i $y \in [-1,1]$ s orijentacijom koja najbolje odgovara vektoru $\vec{\mathbf{k}}$. Odrediti $\iint_S \vec{\mathbf{k}} \cdot d\vec{S}$. 20

5. Neka je C cilindar zadan sa $C = \{(x,y,z) : x^2 + z^2 \le 3,\ 1 \le y \le 2\}$. Izračunati plošni integral 20

$$\iint_{\partial C} z\,dydz + y\,dxdz + x\,dxdy$$

Ukupno: 15

2) $f(x,y,z) = z$

$x = z^2$

$y = 0$

$y = 2-x$

$y \in [0, 2-x]$

$x \in [-2, z^2]$

$z \in [-\sqrt{2}, \sqrt{2}]$

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-2}^{z^2} \int_{0}^{2-x} z \, dy\, dx\, dz$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-2}^{z^2} z \cdot y \Big|_0^{2-x} dx\, dz = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-2}^{z^2} z[2-x] \, dx\, dz$$

$\left.\begin{array}{l} y = 0 \\ y = 2-x \end{array}\right\}$ $0 = 2-x$, $-x = 2$, $x = -2$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} 2z - zx \, dx\, dz = \int_{-\sqrt{2}}^{\sqrt{2}} 2z \cdot x - z \cdot \frac{x^2}{2} \Big|_{-2}^{z^2}$$

$\left.\begin{array}{l} x = z^2 \\ x = -2 \end{array}\right\}$ $z^2 = -2$, $z = \pm\sqrt{2}$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \left(2z \cdot z^2 - z \cdot \frac{z^4}{2}\right) - \left(2z \cdot (-2) - z \cdot \frac{(-2)^2}{2}\right) dz$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} 2z^3 - \frac{z^5}{2} - (-4z - 2z)\, dz = \int_{-\sqrt{2}}^{\sqrt{2}} 2z^3 - \frac{1}{2} z^5 + 4z + 2z \, dz$$

$$= \cancel{2} \frac{z^4}{\cancel{4}_2} - \frac{1}{2} \frac{z^6}{6} + \frac{\cancel{4}^2 z^2}{\cancel{2}} + \frac{\cancel{2} z^2}{\cancel{2}} \Big|_{-\sqrt{2}}^{\sqrt{2}} = \frac{z^4}{2} - \frac{z^6}{12} + 2z^2 + z^2 \Big|_{-\sqrt{2}}^{\sqrt{2}} = \frac{z^4}{2} - \frac{z^6}{12} + 3z^2 \Big|_{-\sqrt{2}}^{\sqrt{2}}$$

$$= \left(\frac{(\sqrt{2})^4}{2} - \frac{(\sqrt{2})^6}{12} + 3(\sqrt{2})^2\right) - \left(\frac{(-\sqrt{2})^4}{2} - \frac{(-\sqrt{2})^6}{12} + 3 \cdot (-\sqrt{2})^2\right) = \left(\frac{2^2}{2} - \frac{2^3}{12} + 3 \cdot 2\right) - \left(\frac{2^2}{2} - \frac{2^3}{12} + 3 \cdot (2)\right)$$

NASTAVAK NA DRUGOM LISTU

1) $f'''(t) + 4f'(t) = t \quad , \; f(0) = 3 \; , \; f'(0) = 5 \; , \; f''(0) = 2$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 4\left(sF(s) - f(0)\right) = \frac{1}{s^2}$$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 4sF(s) - 4f(0) = \frac{1}{s^2}$$

$$s^3F(s) - \left(s^2 \cdot 3\right) - \left(s \cdot 5\right) - 2 + 4sF(s) - \left(4 \cdot 3\right) = \frac{1}{s^2}$$

$$s^3F(s) - 3s^2 - 5s - 2 + 4sF(s) - 12 = \frac{1}{s^2}$$

$$F(s)\left(s^3 + 4s\right) = \frac{1}{s^2} + 3s^2 + 5s + 2 + 12$$

$$F(s)\, s\left(s^2+4\right) = \frac{1}{s^2} + 3s^2 + 5s + 14$$

$$F(s)\, s\left(s^2+4\right) = \frac{1 + 3s^4 + 5s^3 + 14s^2}{s^2} \quad / : s\left(s^2-4\right)$$

$$F(s) = \frac{3s^4 + 5s^3 + 14s^2 + 1}{s^2 \cdot s\left(s^2+4\right)} = \frac{3s^4+5s^3+14s^2+1}{s^3\left(s^2+4\right)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4} \quad / \cdot s^3\left(s^2+4\right)$$

$$3s^4+5s^3+14s^2+1 = A\left(s^2\left(s^2+4\right)\right) + B\left(s\left(s^2+4\right)\right) + C\left(s^2+4\right) + \left(Ds+E\right)s^3$$

$$3s^4+5s^3+14s^2+1 = A\left(s^4+4s^2\right) + B\left(s^3+4s\right) + Cs^2 + 4C + Ds^4 + Es^3$$

$$3s^4+5s^3+14s^2+1 = As^4 + 4As^2 + Bs^3 + 4Bs + Cs^2 + 4C + Ds^4 + Es^3$$

$3 = A + D$

$5 = B + E$

$14 = 4A + C$

$1 = 4C$

$4C = 1$

$\boxed{C = \frac{1}{4}}$

$14 = 4A + C$

$4A + C = 14$

$4A + \frac{1}{4} = 14$

$4A = 14 - \frac{1}{4}$

$4A = \frac{56-1}{4}$

$4A = \frac{55}{4} \quad / : 4$

$\boxed{A = \frac{55}{16}}$

$3 = A + D$

$A + D = 3$

$\frac{55}{16} + D = 3$

$D = 3 - \frac{55}{16}$

$D = \frac{48-55}{16}$

$\boxed{D = -\frac{7}{16}}$

$4B = 0$

$\boxed{B = 0}$

$5 = B + E$

$B + E = 5$

$0 + E = 5$

$\boxed{E = 5}$

RJEŠENJE ?

Matematika **3**

Ime i prezime: MARIO IVANAC **Matični broj u indeksu:** 17-1-0096-2011

NASTAVAK 1 ZADATKA

$$F(s) = \frac{\frac{55}{16}}{s} + \frac{\cancel{0}}{\cancel{s^2}} + \frac{\frac{1}{4}}{s^3} + \frac{-\frac{7}{16}s+5}{s^2+4}$$

$$= \frac{55}{16} + \frac{1}{4}t - \frac{7}{16} \cdot \frac{s}{s^2+(\sqrt{4})^2} + \frac{5}{s^2+(\sqrt{4})^2}$$

$$= \frac{55}{16} + \frac{1}{4}t - \frac{7}{16} \cdot \cos(\sqrt{4}t) + \sin(\sqrt{4}t) \;//$$

NASTAVAK 2. ZADATKA

$$= \frac{4}{2} - \frac{16}{12} + 6 - \left(\frac{4}{2} - \frac{16}{12} + 6\right)$$

$$= \cancel{2} - \cancel{\frac{16}{12}} + 6 - \cancel{2} + \cancel{\frac{16}{12}} + 6$$

$$= 6 + 6$$

$$= 12 \;//$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A3

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: ANTE ŠIŠAK

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA: 17-2-0247-2012

USTMENI ISPIT KOD NASTAVNIKA:

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: 20

$$f'''(t) + 4f'(t) = t, \quad f(0) = 3,\ f'(0) = 5,\ f''(0) = 2.$$

2. Izračunati integral funkcije $f(x,y,z) = z$ u dijelu prostora omeđenog plohama $x = z^2$, $y = 0$ i $y = 2 - x$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 3t$, $t \in [0,2]$. Izračunaj $\int_C f ds$, kada je $f(x,y,z) = z(x^2+y^2)$. 20

4. Neka je S parabolična ploha $z = x^2$ u dijelu prostora $x \in [-1,1]$ i $y \in [-1,1]$ s orijentacijom koja najbolje odgovara vektoru $\vec{k}$. Odrediti $\iint_S \vec{k}\cdot d\vec{S}$. 20

5. Neka je C cilindar zadan sa $C = \{(x,y,z) : x^2+z^2 \leq 3,\ 1 \leq y \leq 2\}$. Izračunati plošni integral 20

$$\iint_{\partial C} z\,dydz + y\,dxdz + x\,dxdy$$

Ukupno:

1. $f'''(t) + 4f'(t) = t, \quad f(0)=3, \quad f'(0)=5, \quad f''(0)=2$

$s^3(f)(s) - s^2 f(0) - s(f)'(0) - f''(0) + 4s(f)(s) - 4(f(0)) = \frac{1}{s^2}$

$s^3(f)(s) - 3s^2 - 5s - 2 + 4s(f)(s) - 12 = \frac{1}{s^2}$

$f(s)(s^3+4s) = \frac{1}{s^2} + 3s^2 + 5s + 2 + 12$

$f(s)(s^3+4s) = \frac{1+3s^4+5s^3+2s^2+12s^2}{s^2} = \frac{1+3s^4+5s^3+14s^2}{s^2}$

$f(s) = \frac{1+3s^4+5s^3+14s^2}{s^2(s^3+4s)} = \frac{1+3s^4+5s^3+14s^2}{s^3(s^2+4)}$

$\frac{1+3s^4+5s^3+14s^2}{s^3(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4}$

$1+3s^4+5s^3+14s^2 = A(s^2(s^2+4)) + B(s(s^2+4)) + C(s^2+4) + (Ds+E)(s^3)$

$1+3s^4+5s^3+14s^2 = As^4+4A + Bs^3+4B + 4C + Cs^2 + (Ds+E)(s^3)$

$\boxed{A=3}$ $\boxed{C=14}$ $1 = 4A+4B+4C$ $-68 = 4B$

$5 = B+E$ $1 = 12+4B+56$ $\boxed{B = \frac{-68}{4} = -17}$

$\boxed{D=0}$ $1 = 68+4B$

$5 = -17 + E$

$\vdots$

$+5 = -17 + E$

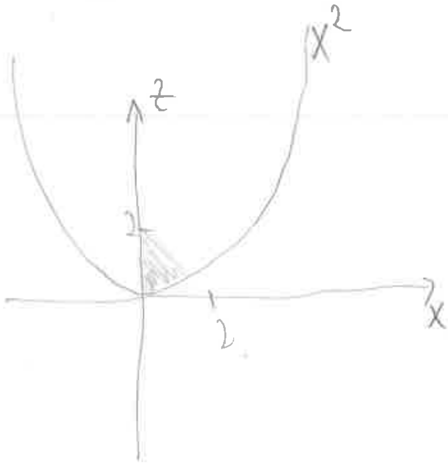
$-E = +17 - 5 \quad \boxed{E = 22}$

$f(s) = 3\frac{1}{s} - 17\frac{1}{s^2} + \frac{5}{s}\frac{1}{s^2} + 22 \cdot \frac{1}{s^2+4}$

$\underbrace{f(s) = 3\frac{1}{s} - 17\frac{1}{s^2} + \frac{5}{s}\frac{1}{s^2} + 22 \cdot \frac{1}{s^2+a^2}}$

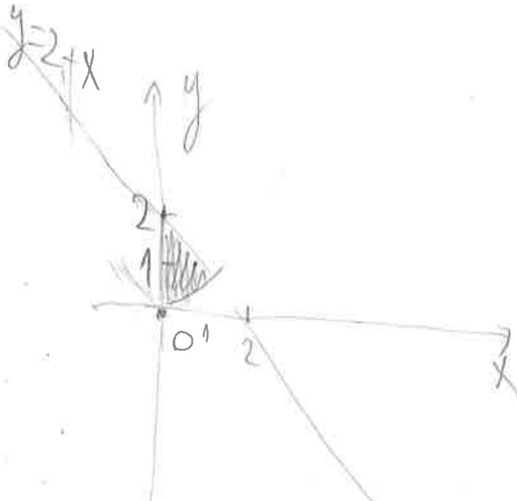
$\underbrace{f(t) = 3 - 17t + \frac{5}{s}t + 22\sin(22t)}$

2. $f(x,y,z) = z \qquad x = z^2 \qquad y = 0 \quad y = 2-x$



$\int_0^{2-x}\int_0^{x^2}\int_0^2 z \, dz\, dy\, dx =$ ✗

$= \int_0^{2-x}\int_0^{x^2} \frac{z^2}{2}\Big|_0^2 \, dy\, dx = \int_0^{2-x}\int_0^{x^2} 2 \, dy\, dx =$



$= 2\int_0^{2-x} dx \cdot x\Big|_0^{x^2} = 2\int_0^{2-x} dx \cdot x^2 = 2 \cdot \frac{x^3}{3}\Big|_0^{2-x} =$

$= 2\,\frac{(2-x)^3}{3} + C = \frac{2}{3}(2-x^3) + C$

4. $\iint_S \vec{k} \cdot dS = \int_{-x^2}^{x^2}\int_0^1 1\, dy\, dx$ ✗

5. $\iint z\,dy\,dz + y\,dx\,dz + x\,dx\,dy$ ✗

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A3	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Duje Mitrović VRIJEME POČETKA:	
MATIČNI BROJ STUDENTA: 017-2-0205-2012 USTMENI ISPIT KOD NASTAVNIKA: prof. Uglešić	

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: 20

$$f'''(t) + 4\,f'(t) = t, \quad f(0) = 3,\ f'(0) = 5,\ f''(0) = 2.$$

2. Izračunati integral funkcije $f(x,y,z) = z$ u dijelu prostora omeđenog plohama $x = z^2$, $y = 0$ i $y = 2 - x$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 3t$, $t \in [0,2]$. Izračunaj $\int_C f\,ds$, kada je $f(x,y,z) = z\left(x^2 + y^2\right)$. 20

4. Neka je S parabolična ploha $z = x^2$ u dijelu prostora $x \in [-1,1]$ i $y \in [-1,1]$ s orijentacijom koja najbolje odgovara vektoru $\vec{\mathbf{k}}$. Odrediti $\iint_S \vec{\mathbf{k}} \cdot \mathrm{d}S$. 20

5. Neka je C cilindar zadan sa $C = \left\{(x,y,z) \,:\, x^2 + z^2 \le 3,\ 1 \le y \le 2\right\}$. Izračunati plošni integral 20

$$\iint_{\partial C} z\,\mathrm{d}y\mathrm{d}z + y\,\mathrm{d}x\mathrm{d}z + x\,\mathrm{d}x\mathrm{d}y$$

Ukupno: 0

$As^2+4A+Bs^2+Cs=3s^2+5s+14$

$A+B=3$ $\qquad$ $B=3-A$

$4A=14 \Rightarrow \underline{A=\frac{7}{2}}$ $\qquad$ $B=3-\frac{7}{2}$

$\underline{B=-\frac{1}{2}}$

$\underline{C=5}$

$=\frac{7}{2}\cdot\frac{1}{s}-\frac{1}{2}\frac{s+5}{s^2+4}=\frac{7}{2}\cdot\frac{1}{s}-\frac{1}{2}\left[\frac{s}{s^2+4}+\frac{2}{s^2+4}+\frac{2}{s^2+4}+\frac{1}{s^2+4}\right]$

$=\frac{7}{2}-\frac{1}{2}\cos(2t)-\underbrace{\frac{1}{2}\sin(2t)-\frac{1}{2}\sin 2t-\frac{1}{4}\sin(2t)}_{-(5/4)\sin t}$

$f(0)=3$

$f'(t)=\sin(2t)-\frac{5}{4}\cos t$

$f'(0)+3$

$\mathcal{L}^{-1}[F(s)]=-\frac{1}{16}+\frac{1}{8}t^2+\frac{1}{16}\cos(2t)+\frac{7}{2}-\frac{1}{2}\cos(2t)-\frac{1}{2}\sin(2t)-\frac{1}{2}\sin(2t)-\frac{1}{4}\sin(2t)$

---

(2) $x=z^2 \quad y=0 \quad f(x,y,z)=z$

$y=2-x$

$z\in[-\sqrt{x},\sqrt{x}]$

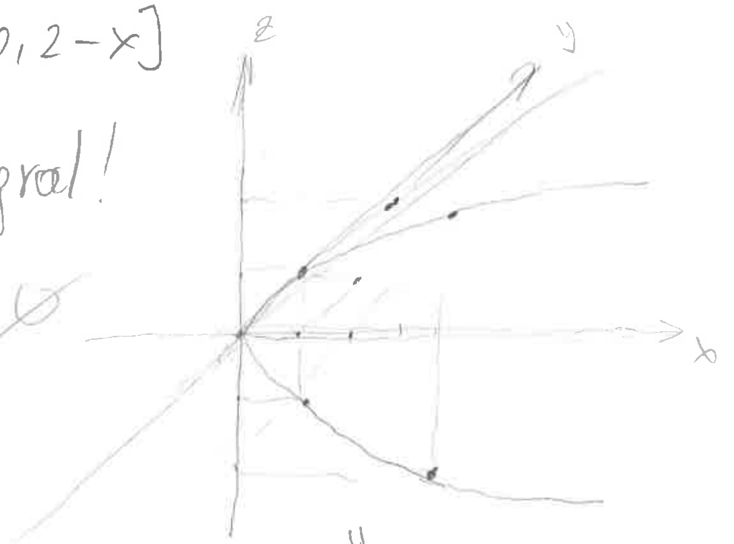
$y\in[0,2-x]$

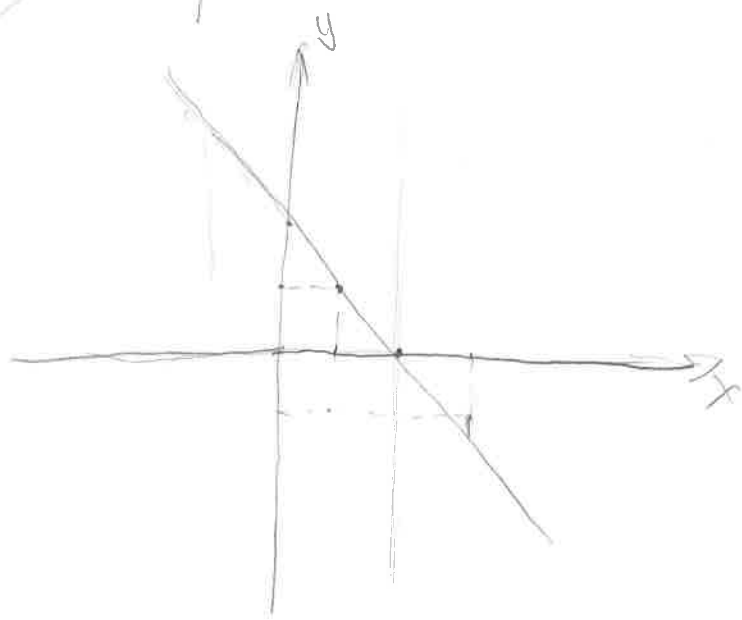
Nije moguće izračunati integral!

$\emptyset$

~~$x=2$~~

VIDI ČIŽMIN, MITROVIĆ





Matematika **3**

Ime i prezime: Ilije Mitrović **Matični broj u indeksu:** 17-2-0205-2012

① $f'''(t) + 4f'(t) = t$ $\quad f(0)=3,\ f'(0)=5 \quad f''(0)=2$

$\mathcal{L}[f'''(t)] + 4\mathcal{L}[f'(t)] = \mathcal{L}[t]$

$s^3F(s) - s^2\underbrace{f(0)}_{3} - s\underbrace{f'(0)}_{5} - \underbrace{f''(0)}_{2} + 4[sF(s) - \underbrace{f(0)}_{3}] = \frac{1}{s^2}$

$s^3F(s) - 3s^2 - 5s - 2 + 4sF(s) - 12 = \frac{1}{s^2}$

$s^3F(s) + 4sF(s) = \frac{1}{s^2} + 3s^2 + 5s + 14$

$F(s) = \frac{\frac{1}{s^2} + 3s^2 + 5s + 14}{s^3 + 4s} = \underbrace{\frac{1}{s^2(s^3+4s)}}_{I} + \underbrace{\frac{3s^2+5s+14}{s^3+4s}}_{II}$

$I = \frac{1}{s^3(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+4} = 1$

$A[s^2(s^2+4)] + B[s(s^2+4)] + C[s^2+4] + [Ds+E]s^3 = 1$

$As^4 + 4As^2 + Bs^3 + 4Bs + Cs^2 + 4C + Ds^4 + Es^3 = 1$

$C = \frac{1}{4}$

$A + D = 0 \Rightarrow D = \frac{1}{16}$

$B + E = 0 \Rightarrow E = 0$

$4A + C = 0 \Rightarrow A = -\frac{1}{16}$

$4B = 0 \Rightarrow B = 0$

$= -\frac{1}{16}\cdot\frac{1}{s} + \frac{1}{4}\cdot\frac{1}{s^3} + \frac{1}{16}\cdot\frac{s}{s^2+4} = \mathcal{L}^{-1}[I] = -\frac{1}{16} + \frac{1}{8}t^2 + \frac{1}{16}\cdot\cos(2t)$

$II = \frac{3s^2+5s+14}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4} = 3s^2+5s+14$

$A[s^2+4] + [Bs+C]s = 3s^2+5s+14$

$As^2 + 4A + Bs^2 + Cs = 3s^2+5s+14$

$\longrightarrow$

Dojc Mitrović 17-2-0205-2012

(3) $x = \cos t$ $\quad t \in [0, 2\pi]$

$y = \sin t$

$z = 3t$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ A3

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: ANTONIO SEKULA **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA: A-2-0025-2010 **USTMENI ISPIT KOD NASTAVNIKA:**

1. Koristeći Laplaceovu transformaciju nađi realnu funkciju f koja zadovoljava sljedeće uvjete: 20

$$f'''(t) + 4f'(t) = t, \quad f(0) = 3,\ f'(0) = 5,\ f''(0) = 2.$$

2. Izračunati integral funkcije $f(x, y, z) = z$ u dijelu prostora omeđenog plohama $x = z^2$, $y = 0$ i $y = 2 - x$. 20

3. Neka je C kružna uzvojnica (spirala) s jednadžbama $x = \cos t$, $y = \sin t$ i $z = 3t$, $t \in [0, 2]$. Izračunaj $\int_C f\,ds$, kada je $f(x, y, z) = z\left(x^2 + y^2\right)$. 20

4. Neka je S parabolična ploha $z = x^2$ u dijelu prostora $x \in [-1, 1]$ i $y \in [-1, 1]$ s orijentacijom koja najbolje odgovara vektoru $\vec{\mathbf{k}}$. Odrediti $\iint_S \vec{\mathbf{k}} \cdot d\vec{S}$. 20

5. Neka je C cilindar zadan sa $C = \{(x, y, z) : x^2 + z^2 \leq 3,\ 1 \leq y \leq 2\}$. Izračunati plošni integral 20

$$\iint_{\widehat{\partial C}} z\,dydz + y\,dxdz + x\,dxdy$$

Ukupno: 0

(1.) $f'''(t) + 4f'(t) = t$

$f(0) = 3$

$f'(0) = 5$

$f''(0) = 2$

$$f'''(t) \to s^3F(s) - s^2\underbrace{f(0)}_{3} - s\underbrace{f'(0)}_{5} - \underbrace{f''(0)}_{2}$$

$$\to s^3F(s) - 3s^2 - 5s - 2$$

$$4f'(t) \to sF(s) - \underbrace{f(0)}_{3}$$

$$\to 4sF(s) - 12$$

$$t \to \frac{1}{s^2}$$

$$s^3F(s) - 3s^2 - 5s - 2 + 4sF(s) - 12 = \frac{1}{s^2}$$

$$F(s)\left(s^3 + 4s\right) = \frac{1}{s^2} + 3s^2 + 5s + 14$$

$$s\left(s^2 + 4\right) =$$

(2.)

$x = z^2$

$y = 0$

$y = 2 - x$, $y =$

$x = 2 - y$

$x = 2$

$y \in [0, 2]$

$x \in [2, 2]$

$x \in [\sqrt{2}, \sqrt{2}]$