MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C3

IME I PREZIME: 17-2-0223-2012 VRIJEME POČETKA:

MATIČNI BROJ STUDENTA: Ines Kurfürst Vukušić USTMENI ISPIT KOD NASTAVNIKA: Oglešić

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) + y'(t) + y(t) = 0, \quad y(0) = 2,\ y'(0) = 4,\ y''(0) = 4\,.$$

Pomoć: $x^3 + x^2 + x + 1 = (x+1)\left(x^2+1\right)$.

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 2^2$ i ravninama $z = y + 1$ i $z = -2$. 20

3. Neka je K krug radijusa A s centrom u ishodištu. Po rubu kruga postavljena je žica u obliku kružnice ∂K. Izračunaj elektormotornu silu $\int_{\partial K} \vec{E} \cdot ds$ ako je električno polje $\vec{E}$ statičko i zadano izrazom $\vec{E} = \frac{1}{x^2+y^2}\left(x\vec{\mathrm{i}} + y\vec{\mathrm{j}}\right)$. Podrazumijeva se pozitivna orjentacija krivulje. 20

4. Izračunati $\iint_S (x^2+y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2+y^2}$ i $0 \le z \le 4$. 20

5. Izračunati 20

$$\int_{(0,\pi)}^{(2,2\pi)} 2x\sin y\,\mathrm{d}x + \left(x^2+1\right)\cos y\,\mathrm{d}y$$

Ukupno: 60

5. $\int_{(0,\pi)}^{(2,2\pi)} 2x\sin y\,\partial x + (x^2+1)\cos y\,\partial y =$

$\partial_x F = 2x\sin y \quad / \int \partial x$

$\partial_y F\ (x^2+1)\cos y$

$F = x^2\sin y + \varphi(y) / \partial y$

$\partial_y F = x^2\cos y + \varphi'(y) = (x^2+1)\cos y$

$x^2\cancel{\cos y} + \varphi'(y) = x^2\cancel{\cos y} + \cos y$

$\varphi'(y) = \cos y / \int$

$\varphi(y) = \sin y$

$\Rightarrow F(x,y) = x^2\sin y + \sin y$

potencijalno polje

$F(2,2\pi) - F(0,\pi) = 0$

4.) $\iint_S (x^2+y^2)$

S stožac $z=\sqrt{x^2+y^2}$, $0 \leq z \leq 4$

parametrizacija $r(x,y) = x\vec{i} + y\vec{j} + \sqrt{x^2+y^2}\,\vec{k}$

$f(x,y) = x^2+y^2$ područje D je krug radijusa

$f\circ r = x^2+y^2$

$$\iint_S f\,dS = \iint_D \left((f\circ r)\,\|\vec{n}\|\right)(x,y)\,dx\,dy$$

$$\partial_x r = \begin{pmatrix} 1 \\ 0 \\ \frac{x}{\sqrt{x^2+y^2}} \end{pmatrix}$$

$f(x,y) = x^2+y^2$ područje D je krug radijusa 4

$f\circ r = x^2+y^2$

$$\iint_S f\,dS = \iint_D \left((f\circ r)\,\|\vec{n}\|\right)(x,y)\,dx\,dy =$$

$$\partial_x r = \begin{pmatrix} 1 \\ 0 \\ \frac{x}{\sqrt{x^2+y^2}} \end{pmatrix}$$

$$\partial_y r = \begin{pmatrix} 0 \\ 1 \\ \frac{y}{\sqrt{x^2+y^2}} \end{pmatrix}$$

$$\vec{n} = \partial_x r \times \partial_y r = \begin{vmatrix} i & j & k \\ 1 & 0 & \frac{\cancel{x}}{\sqrt{x^2+y^2}} \end{vmatrix} = \begin{pmatrix} \frac{-x}{\sqrt{x^2+y^2}} \\ 4 \end{pmatrix}$$

$f = r = x^2+y^2$

Matematika 3

Ime i prezime: Ines Kurtišić Valušić **Matični broj u indeksu:**

$$\iint_S f\,dS = \iint_D \left((f \circ r)\, \|\vec{n}\| \right)(x,y)\,dx\,dy =$$

$$\partial_x r = \begin{pmatrix} 1 \\ 0 \\ \frac{x}{\sqrt{x^2+y^2}} \end{pmatrix}$$

$$\partial_y r = \begin{pmatrix} 0 \\ 1 \\ \frac{y}{\sqrt{x^2+y^2}} \end{pmatrix}$$

$$\vec{n} = \partial_x r \times \partial_y r = \begin{vmatrix} i & j & k \\ 1 & 0 & \frac{x}{\sqrt{x^2+y^2}} \\ 0 & 1 & \frac{y}{\sqrt{x^2+y^2}} \end{vmatrix} = \begin{pmatrix} \frac{-x}{\sqrt{x^2+y^2}} \\ -\frac{y}{\sqrt{x^2+y^2}} \\ 1 \end{pmatrix}$$

$$\|\vec{n}\| = \sqrt{\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2+y^2}}\right)^2 + 1} = \sqrt{2}$$

$$* = \sqrt{2} \iint_D (x^2+y^2)\,dx\,dy =$$

$x = r\cos\varphi \qquad r \in [0,4]$

$y = r\sin\varphi \qquad \varphi \in [0,2\pi]$

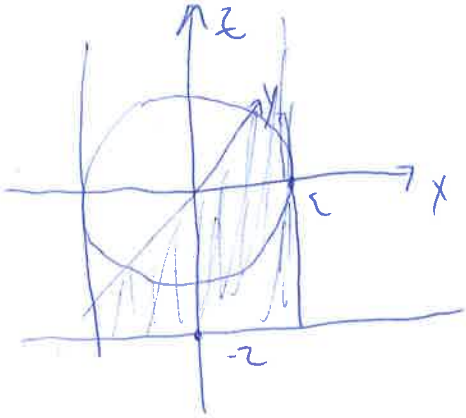
$$= \sqrt{2} \int_0^{2\pi}\int_0^4 (r^2\cos^2\varphi + r^2\sin^2\varphi)\cdot r\,dr\,d\varphi =$$

$$= \sqrt{2} \int_0^{2\pi} 64\,d\varphi = \sqrt{2}\, 64 \cdot 2\pi = 128\sqrt{2}\,\pi$$

2) valjak $x^2+y^2=2^2$

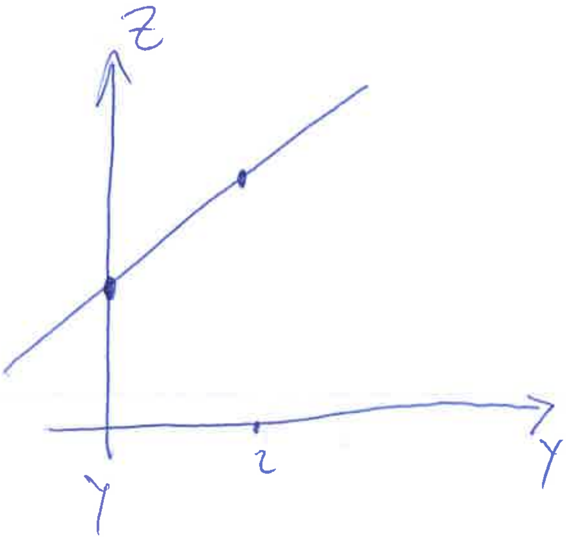
ravnina $z=y+1$

$z=-2$



$r^2=2^2 \quad r\in[0,2]$

$\varphi\in[0,2\pi]$



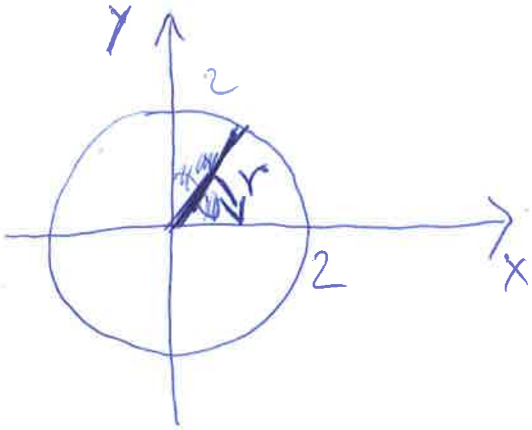
cilindrična koordinata

$x=r\cos\varphi$

$y=r\sin\varphi$

$z=z$

$z\,[-2,\ r\sin\varphi-1]$



$$V=\int_0^{2\pi}\int_0^{2}\int_{-2}^{r\sin\varphi+1} 1\cdot r\,dz\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\int_0^{2}(r^2\sin\varphi+3r)\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\left(\frac{r^3}{3}\sin\varphi+\frac{3r^2}{2}\right)\Bigg|_0^2 d\varphi\int_0^{2\pi}\left(\frac{8}{3}\sin\varphi+6\right)d\varphi$$

$$=\left(-\frac{8}{3}\cos\varphi+6\varphi\right)\Bigg|_0^{2\pi}=-\frac{8}{3}+12\pi+\frac{8}{3}=12\pi$$

1. $y'''(t)+y''(t)+y'(t)+y(t)=0$ $\quad y(0)=2$ $\quad y'(0)=4$ $\quad y''(0)=4$

Ines Kurfürst Vukušić

$$s^3Y(s)-s^2y(0)-sy'(0)-y''+$$

$$+s^2Y(s)-sy(0)-y'(0)+sY(s)-y(0)$$

$$+Y(s)=0$$

$$Y(s)(s^3+s^2+s+1)=2s^2+4s+4+2s+4+4$$

$$Y(s)(s^3+s^2+s+1)=2s^2+6s+12$$

$$Y(s)=\frac{2s+6s+12}{(s+1)(s^2+1)}=\frac{A}{s+1}+\frac{Bs+C}{s^2+1}\Big/(s+1)(s^2+1)$$

$$2s^2+6s+12=A(s^2+1)+(Bs+C)(s+1)$$

$$2s^2+6s+12=As^2+A+Bs^2+Bs+Cs+C$$

$$\left.\begin{array}{l}A+B=2\\ B+C=6\end{array}\right\}- \quad = \quad \left.\begin{array}{l}A-C=-4\\ A+C=12\end{array}\right\}+$$

$$A+C=12$$

$$2A=8$$

$$\boxed{A=4}$$

$$\boxed{B=-2}$$

$$\boxed{C=8}$$

$$Y(s)=\frac{4}{s^2+1}=\frac{-2s+8}{s^2+1}$$

$$Y(s)=4\cdot\frac{1}{s+1}-2\frac{s}{s^2+1}+\frac{8}{s^2+1}\Big/\mathcal{L}^{-1}$$

$y(t) = 4 \cdot e^{-t} - 2\cos t + 5\sin t$

$y(0) = 4-2=2$

$y'(t) = -4e^{-t}$

$y'(0) = -4+8=4$

$y''(t) = 4e^{-t} + 2\cos t - 5\sin t$

$y''(0) = 4$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $C3$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: IVAN VUKAŠINA **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: 1720182-12 **USTMENI ISPIT KOD NASTAVNIKA:**	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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5. Izračunati 20

$$\int_{(0,\pi)}^{(2,2\pi)} 2x\sin y\,\mathrm{d}x + (x^2+1)\cos y\,\mathrm{d}y$$

Ukupno: 60

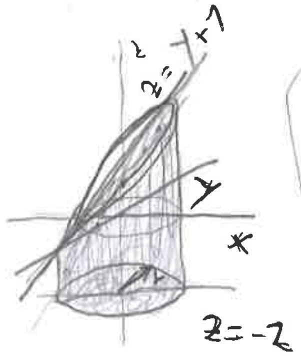
2.

$x^2 + y^2 = 2^2$

$r = 2$

$z = y + 1$

$z = -2$



y	0	1	2
z	1	2	3

Pol. koor.

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$$\int_0^{2\pi}\int_0^2\int_{-2}^{r\cos\varphi+1} 1\cdot r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^2 r\cdot z\Big|_{-2}^{r\cos\varphi+1} dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^2 \left(r\cdot(r\cos\varphi+1) - (-2r)\right) dr\,d\varphi = \int_0^{2\pi}\int_0^2 r^2\cos\varphi + \underbrace{r + 2r}_{3r}\,dr\,d\varphi$$

$$\int_0^{2\pi} \frac{r^3}{3}\cos\varphi + 3\cdot\frac{r^2}{2}\Big|_0^2 d\varphi = \int_0^{2\pi} \frac{2^3}{3}\cos\varphi + 3\cdot\frac{2^2}{2} - 0 = \int_0^{2\pi}\frac{8}{3}\cos\varphi + 6 = \frac{8}{3}\sin\varphi + 6\varphi\Big|_0^{2\pi}$$

$$\frac{8}{3}\sin 2\pi + 6\cdot 2\pi - \left(\frac{8}{3}\sin 0 + 6\cdot 0\right)$$

$$= 12\pi$$

(3.)

$$\oint_{\partial E} \vec{E} \cdot ds \qquad E = \frac{1}{x^2+y^2} \qquad r(t)\begin{pmatrix} x \\ y \end{pmatrix} \qquad r = A$$

$$\vec{E} = \frac{1}{x^2+y^2}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$r(t) = \begin{pmatrix} A\cos t \\ A\sin t \end{pmatrix}$$

$$\|r'\| = \sqrt{((A\cos t)')^2 + ((A\sin t)')^2}$$

$$\|r'\| = \sqrt{(-A\sin t)^2 + (A\cos t)^2}$$

$$\|r'\| = \sqrt{A^2\sin^2 t + A^2\cos^2 t}$$

$$\|r'\| = \sqrt{A^2(\underbrace{\sin^2 t + \cos^2 t}_{1})} = \sqrt{A^2} = A$$

$$\int_0^{2\pi} \frac{1}{(A\cos t)^2 + (A\sin t)^2} \cdot A \, dt$$

$$\int_0^{2\pi} \frac{A}{A^2\cos^2 t + A^2\sin^2 t} dt = \int_0^{2\pi} \frac{A}{A^2(\underbrace{\sin^2 t + \cos^2 t}_{1})} dt = \int_0^{2\pi} \frac{\cancel{A}}{A^{\cancel{2}}} dt$$

$$= \int_0^{2\pi} \frac{1}{A} dt = \frac{1}{A} 2\pi - \frac{1}{A} \cdot 0 = \frac{2}{A}\pi$$

$$[(0 \cdot 1) - 0] - [(-4 \cdot 0) - 0] = 0$$

(4.)

(4.)

$$\int_{A\,(0,\pi)}^{B\,(2,2\pi)} 2x \sin y \, dx + (x^2+1)\cos y \, dy$$

$$+(x^2\cos y + \cos y)$$

projekcija

$f = -\mathrm{grad}\, f$ ako je $F(A) - F(B)$

$$= -\cancel{2}\frac{x^2}{2}\sin y + C(y)$$

$$= -x^2 \sin y + c(y) / \cdot f'(y)$$

$$-x^2\cos y - \cos y = \cancel{-x^2\cos y} + c'(y)$$

$$-\cos y = c'(y) / \int dy$$

$$-\sin y = c(y)$$

$$-x^2\sin y - \sin y = -\mathrm{grad}\, f$$

$$x' = -2x\sin y$$

$$y' = -x^2\cos y - \cos y = (-x^2-1)\cos y$$

$$\int_{0,\pi}^{2,2\pi} 2x\sin y \, dx + (x^2+1)\cos y \, dy = F(A) - F(B)$$

$$[-(0^2) \cdot \sin\pi - \sin\pi] - [-(2)^2\sin 2\pi - \sin 2\pi] = 0$$

Matematika 3

Ime i prezime: IVAN VUKAŠINA **Matični broj u indeksu:** 1720182-12

$$y'''(t)+y''(t)+y'(t)+y(t)=0 \quad y(0)=2 \quad y'(0)=4 \quad y''(0)=4$$

$$s^3Y(s)-s^2\overset{2}{Y(0)}-s\overset{4}{Y'(0)}-\overset{4}{Y''(0)}+s^2Y(s)-s\overset{2}{Y(0)}-\overset{4}{Y'(0)}+sY(s)-\overset{2}{Y(0)}+Y(s)=0$$

$$s^3Y(s)-2s^2-4s-4+s^2Y(s)-2s-4+sY(s)-2+Y(s)=0$$

$$Y(s)\left(s^3+s^2+s+1\right)-2s^2-4s-4-2s-4-2=0$$

$$Y(s)(s^3+s^2+s+1)=2s^2+4s+4+2s+4+2$$

$$Y(s)(s^3+s^2+s+1)=2s^2+6s+10 \quad /(s^3+s^2+s+1=(s+1)(s^2+1)$$

$$Y(s)=\frac{2s^2+6s+10}{(s+1)(s^2+1)}=\frac{A}{s+1}+\frac{Bs+C}{s^2+1} \quad /\cdot(s+1)(s^2+1)$$

$$2s^2+6s+10=A(s^2+1)+(Bs+C)(s+1)$$

$$2s^2+6s+10=As^2+A+Bs^2+Bs+Cs+C$$

$$2s^2+6s+10=s^2(A+B)+s(B+C)+(A+C)$$

$A+B=2$

$B+C=6 \Rightarrow C=6-B$

$A+C=10$

$A=2-B$

$2-B+6-B=10$ $\quad A=3$ // $\quad C=7$ //

$-2B=2$

$B=-1$ //

$$=\frac{3}{s+1}+\frac{-s+7}{s^2+1}$$

$$=3\frac{1}{s+1}-\frac{s}{s^2+1}+7\frac{1}{s^2+1}$$

$$=3e^{-t}-\cos t+7\sin t$$ ✓

Provjera:

$$\overset{3}{3e^0} - \overset{-1}{\cos 0} + 7\sin 0 = 2 \;//\qquad y(0)=2$$

$$-\overset{-3}{3e^0} + \sin 0 + 7\overset{1}{\cos 0} = 4 \;//\qquad y'(0)=4$$

$$\overset{3}{3e^0} + \overset{1}{\cos 0} + 7\overset{0}{\sin 0} = 4 \;//$$

$$3e^{-t} - \cos t + 7\sin t \;// = f'(x)$$

$$3e^{-t}\cdot(-t)' + \sin t + 7\cos t$$

$$-3e^{-t} + \sin t + 7\cos t \;// = f''(x)$$

$$-3e^{-t}\cdot(-t)' + \cos t - 7\sin t$$

$$3e^{-t} + \cos t - 7\sin t \;//$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{C}3$	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ANAMARIA JOZIĆ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: 17-2-0104-2011 **USTMENI ISPIT KOD NASTAVNIKA:** Prof. Uglešić	

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5. Izračunati 20

$$\int_{(0,\pi)}^{(2,2\pi)} 2x\sin y\,\mathrm{d}x + \left(x^2+1\right)\cos y\,\mathrm{d}y$$

Ukupno: 55

1. $y'''(t) + y''(t) + y'(t) + y(t) = 0, \quad y(0) = 2,\ y'(0) = 4,\ y''(0) = 4$

$s^3F(s) - s^2 f(0) - sf'(0) - f''(0) + s^2F(s) - sf(0) - f'(0) + sF(s) - f(0) + F(s) = 0$

$F(s)\left[s^3 + s^2 + s + 1\right] - 2s^2 - 4s - 4 - 2s - 4 - 2 = 0$

$F(s)(s+1)(s^2+1) = 2s^2 + 6s + 10$

$F(s) = \frac{2s^2+6s+10}{(s+1)(s^2+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+1} \quad \Big/ \cdot (s+1)(s^2+1)$

$2s^2 + 6s + 10 = As^2 + A + Bs^2 + Bs + Cs + C$

(1) $A + B = 2 \Rightarrow B = 2 - A \Rightarrow \underline{B = -1}$

(2) $B + C = 6$

(3) $A + C = 10 \Rightarrow C = 10 - A \Rightarrow \underline{C = 7}$

(2) $2 - A + 10 - A = 6$

$2A = 6$

$\underline{A = 3}$

$F(s) = \frac{3}{s+1} + \frac{-s+7}{s^2+1} = \frac{3}{s+1} - \frac{s}{s^2+1} + \frac{7}{s^2+1}$

$y(t) = 3e^{-t} - \cos t + 7\sin t$ ✓

$y(0) = 3 - 1 = 2$

② $x^2+y^2=2^2, \quad z=y+1 \;;\; z=-2$

$-2=y+1 \Rightarrow y=-3$

$x^2+y^2=r^2 \qquad dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$r=2$

$r\in[0,2]$

$\varphi\in[0,2\pi]$

$z\in[-3,-2]$

$$\int_0^{2\pi}\int_0^{2}\int_{-3}^{-2} r\,dr\,d\varphi\,dz = \int_0^{2\pi}\int_0^{2} r\cdot z\Big|_{-3}^{-2} dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^{2} r\,(-2-(-3))\,dr\,d\varphi = \int_0^{2\pi}\int_0^{2} r\,dr\,d\varphi$$

$$= \int_0^{2\pi} \frac{1}{2}r^2\Big|_0^2 d\varphi = \int_0^{2\pi} 2\,d\varphi = 2\varphi\Big|_0^{2\pi} = 4\pi$$

Matematika 3

Ime i prezime: ANAMARIA JOZIĆ **Matični broj u indeksu:** 17-2-0104-2011

(4.) $\iint_S (x^2+y^2)\,dS$, $z=\sqrt{x^2+y^2}$; $0 \le z \le 4$

$dS = \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy = \sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}\,dx\,dy$

$= \sqrt{\frac{2(x^2+y)^2}{x^2+y^2}}\,dx\,dy = \sqrt{2}\,dx\,dy$

$\iint_{S_{xy}} (x^2+y^2)\sqrt{2}\,dx\,dy$

POLARNE KORDINATE $x = r\cos\varphi$ $\quad r\Big|_0^4$

$y = r\sin\varphi$ $\quad \varphi\Big|_0^{2\pi}$

$dx\,dy = r\,dr\,d\varphi$

$\int_0^{2\pi} d\varphi \int_0^4 (r^2\cos^2\varphi + r^2\sin^2\varphi)\sqrt{2}\,r\,dr = 2\pi\cdot\sqrt{2}\int_0^4 r^3(\cos^2\varphi+\sin^2\varphi)\,dr$

$= 2\sqrt{2}\pi\,\frac{r^4}{4}\Big|_0^4 = 8\sqrt{2}\pi$

15

(5.) $\int_{(0,\pi)}^{(2,2\pi)} 2x\sin y\,dx + (x^2+1)\cos y\,dy$

$f = 2x\sin y$

$f = (x^2+1)\cos y$

$\text{grad} = \begin{bmatrix} -\frac{\partial f}{\partial x} \\ -\frac{\partial f}{\partial y} \end{bmatrix}$

$\frac{\partial \varphi}{\partial x} = -2x\sin y \quad \Big| \int dx$

$\varphi = -x^2\sin y + c(y) \quad \Big| \frac{\partial}{\partial y}$

$\frac{\partial \varphi}{\partial y} = -x^2\cancel{\cos y} + c'(y) = -x^2\cancel{\cos y} - \cos y$

$c'(y) = -\cos y \;/ \int dy$

$c(y) = -\sin y$

$F(x,y) = -x^2\sin y - \sin y$

$F(0,\pi) \quad F(2,2\pi) = 0\cdot(-400) = 0$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C3	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: JURE DUNDOVIĆ **VRIJEME POČETKA:**	
MATIČNI BROJ STUDENTA: **USTMENI ISPIT KOD NASTAVNIKA:**	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) + y'(t) + y(t) = 0, \quad y(0) = 2,\, y'(0) = 4,\, y''(0) = 4\,.$$

Pomoć: $x^3 + x^2 + x + 1 = (x+1)(x^2+1)$.

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 2^2$ i ravninama $z = y + 1$ i $z = -2$. 20

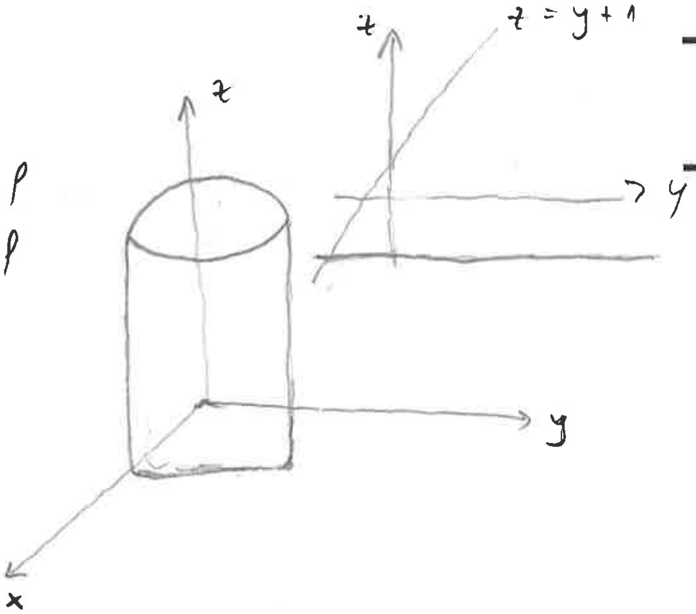
3. Neka je K krug radijusa A s centrom u ishodištu. Po rubu kruga postavljena je žica u obliku kružnice ∂K. Izračunaj elektormotornu silu $\int_{\partial K} \vec{E} \cdot \mathrm{d}s$ ako je električno polje $\vec{E}$ statičko i zadano izrazom $\vec{E} = \frac{1}{x^2+y^2}\left(x\,\vec{\mathbf{i}} + y\,\vec{\mathbf{j}}\right)$. Podrazumijeva se pozitivna orijentacija krivulje. 20

4. Izračunati $\iint_S (x^2+y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2+y^2}$ i $0 \le z \le 4$. ~~20~~ 15

5. Izračunati 20

$$\int_{(0,\pi)}^{(2,2\pi)} 2x \sin y \, \mathrm{d}x + (x^2+1)\cos y \, \mathrm{d}y$$

Ukupno: 55



2) $x^2 + y^2 = 2^2$

$z = y + 1$

$z = -2$

$x = r\cos\varphi$

$y = r\sin\varphi$

$r \in [0, 2]$

$\varphi \in [0, 2\pi]$

$z \in [-2, r\sin\varphi + 1]$

$$V = \int_0^{2\pi}\int_0^2\int_{-2}^{r\sin\varphi+1} r \, dz\, dr\, d\varphi = \int_0^{2\pi}\int_0^2 r z \Big|_{-2}^{r\sin\varphi+1} dr\, d\varphi = \int_0^{2\pi}\int_0^2 (r^2\sin\varphi + 3r)\, dr\, d\varphi$$

$$= \int_0^{2\pi}\left(\frac{r^3}{3}\sin\varphi + \frac{3r^2}{2}\right)\Big|_0^2 d\varphi = \int_0^{2\pi}\left(\frac{8}{3}\sin\varphi + 6\right) d\varphi = \left. -\frac{8}{3}\cos\varphi + 6\varphi \right|_0^{2\pi} =$$

$$= -\cancel{\frac{8}{3}} + 12\pi + \cancel{\frac{8}{3}} = 12\pi$$

4) $\iint_S (x^2+y^2)\,dS$

$z = \sqrt{x^2+y^2}$

$0 \leq z \leq 4$

$$\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1 + \frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}} =$$

$$= \sqrt{2}$$

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$

$x^2+y^2 = r^2$

$z = \sqrt{x^2+y^2} = \sqrt{r^2} = r \Rightarrow r \in [0,4]$

$$\iint_S (x^2+y^2)\,dS = \int_0^{2\pi}\int_0^4 r^2 \cdot \sqrt{2} \cdot r \, dr\, d\varphi = \int_0^{2\pi} \sqrt{2}\, \frac{r^4}{4}\Big|_0^{4} d\varphi =$$

$$= \int_0^{2\pi} 4\sqrt{2}\, d\varphi = 4\sqrt{2}\,\varphi \Big|_0^{2\pi} = 8\sqrt{2}\,\pi$$

15

Matematika 3

Ime i prezime: JURE DUNDOVIĆ **Matični broj u indeksu:**

1) $y'''(t) + y''(t) + y'(t) + y(t) = 0 \quad / \cdot \mathcal{L}$

$s^3 y(s) - s^2 y(0) - s y'(0) - y''(0) + s^2 y(s) - s y(0) -$

$- y'(0) + s y(s) - y(0) + y(s) = 0$

$s^3 y(s) + s^2 y(s) + s y(s) + y(s) = 2s^2 + 6s + 10$

$$y(s) = \frac{2(s^2+3s+5)}{s^3+s^2+s+1} = \frac{2(s^2+3s+5)}{(s+1)(s^2+1)}$$

RASTAV NA PARCIJALNE RAZLOMKE

$$\frac{2(s^2+3s+5)}{(s+1)(s^2+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+1} = \frac{A(s^2+1)+(Bs+C)(s+1)}{(s+1)(s^2+1)} =$$

$$= \frac{s^2(A+B) + s(B+C) + (A+C)}{(s+1)(s^2+1)}$$

$A + B = 2$ $\quad$ $A = 3$

$B + C = 6$ $\Rightarrow$ $B = -1$

$A + C = 10$ $\quad$ $C = 7$

$$y(s) = \frac{3}{s+1} + \frac{-s+7}{s^2+1} \quad / \mathcal{L}^{-1}$$

$$y(t) = 3e^{-t} - \cos t + 7\sin t$$

$$y(s) = \frac{3}{s+1} + \frac{-s+7}{s^2+1} \quad / \cdot \mathcal{L}^{-1}$$

$$y(t) = 3e^{-t} - \cos t + 7\sin t$$ ✓

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ C3

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: MARTIN BEDNAR

VRIJEME POČETKA:

MATIČNI BROJ STUDENTA:

USTMENI ISPIT KOD NASTAVNIKA:

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) + y'(t) + y(t) = 0, \quad y(0) = 2,\ y'(0) = 4,\ y''(0) = 4\,.$$

Pomoć: $x^3 + x^2 + x + 1 = (x+1)(x^2+1)$.

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 2^2$ i ravninama $z = y + 1$ i $z = -2$. ~~20~~ 15

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4. Izračunati $\iint_S (x^2 + y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2 + y^2}$ i $0 \le z \le 4$. 20

5. Izračunati 20

$$\int_{(0,\pi)}^{(2,2\pi)} 2x \sin y \,\mathrm{d}x + \left(x^2 + 1\right)\cos y \,\mathrm{d}y$$

Ukupno: 55

$$\int 2x \sin y \,dx = x^2 \sin y$$

$$\int (x^2+1) \cos y \,dy = (x^2+1)\sin y$$

SADRŽI OBA RJEŠENJA

$$(x^2+1)\sin y \Big|_{(x,y)=(0,\pi)}^{(2,2\pi)}$$

$$5 \cdot 0 - 1 \cdot 0 = 0$$ ✓

4. $\iint_S (x^2+y^2)$ $\quad z=\sqrt{x^2+y^2}$ $\quad 0\le z\le 4$

$$\iint_S (x^2+y^2)\,dS = \sqrt{2}\iint_D x^2+y^2\,dx\,dy \qquad |\vec{m}| = \sqrt{1+\frac{x^2+y^2}{\sqrt{x^2+y^2}}} = \sqrt{2}$$

$D \downarrow$ KRUG S1OBĆA

$$= \sqrt{2}\int_0^{2\pi}\int_0^{4}\left[(r\cos\varphi)^2+(r\sin\varphi)^2\right]\cdot r\,dr\,d\varphi$$

$$= \sqrt{2}\int_0^{2\pi}\int_0^{4} r^3\,dr\,d\varphi = \sqrt{2}\cdot 2\pi\cdot\frac{4^4}{4} =$$

$= 568,6890161$

$=$

Matematika **3**

Ime i prezime: MARTIN SEDMAK **Matični broj u indeksu:**

2. $x^2+y^2=2^2$ $\qquad x=r\cos\varphi$

$x^2+y^2=r^2$ $\qquad y=r\sin\varphi$

$r=2$ $\qquad dx\,dy\,dz=r\,dr\,d\varphi\,dz$

$r\in[0,2]$ $\varphi\in[0,2\pi]$

$z\in[-2,y+1]=[-2,r\sin\varphi+1]$

$$\int_0^{2\pi}\int_0^2\int_{-2}^{r\sin\varphi+1} r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^2 r\cdot z\Big|_{2}^{r\sin\varphi+1} dr\,d\varphi = \int_0^{2\pi}\int_0^2 r\,(r\sin\varphi+1+2)\,dr\,d\varphi$$

15

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C3	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: JELENA MALEŠ VRIJEME POČETKA: 08:30	
MATIČNI BROJ STUDENTA: 17-2-0103-2011 USTMENI ISPIT KOD NASTAVNIKA: Prof. Uglešić	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) + y'(t) + y(t) = 0, \quad y(0) = 2,\ y'(0) = 4,\ y''(0) = 4.$$

Pomoć: $x^3 + x^2 + x + 1 = (x+1)(x^2+1)$.

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 2^2$ i ravninama $z = y + 1$ i $z = -2$. 20

3. Neka je K krug radijusa A s centrom u ishodištu. Po rubu kruga postavljena je žica u obliku kružnice ∂K. Izračunaj elektormotornu silu $\int_{\partial K} \vec{E} \cdot ds$ ako je električno polje $\vec{E}$ statičko i zadano izrazom $\vec{E} = \frac{1}{x^2+y^2}\left(x\vec{i} + y\vec{j}\right)$. Podrazumijeva se pozitivna orijentacija krivulje. 20

4. Izračunati $\iint_S (x^2+y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2+y^2}$ i $0 \le z \le 4$. ~~20~~

5. Izračunati 20

$$\int_{(0,\pi)}^{(2,2\pi)} 2x \sin y \, dx + (x^2+1)\cos y \, dy$$

Ukupno: 20

1. $y'''(t) + y''(t) + y'(t) + y(t) = 0 \quad y(0)=2 \quad y'(0)=4 \quad y''(0)=4$

$y'''(t) = s^3 y(s) - s^2 \underbrace{y(0)}_{2} - s \underbrace{y'(0)}_{4} - \underbrace{y''(0)}_{4} = s^3 y(s) - 2s^2 - 8$

$y''(t) = s^2 y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{4} = s^2 y(s) - 2s - 4$

$y'(t) = s y(s) - \underbrace{y(0)}_{2} = s y(s) - 2$

$y(t) = y(s)$

$s^3 y(s) - 2s^2 - 8 + s^2 y(s) - 2s - 4 + s y(s) - 2 + s y(s) = 0$

$s^3 y(s) + s^2 y(s) + s y(s) + y(s) = 2s^2 + 8 + 2s + 4 + 2$

$y(s)\underbrace{(s^3 + s^2 + s + 1)}_{(s+1)(s^2+1)} = 2s^2 + 2s + 14 \quad /:(s+1)(s^2+1)$

$y(s) = \frac{2s^2+2s+14}{(s+1)(s^2+1)}$

$\frac{2s^2+2s+14}{(s+1)(s^2+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+1} \quad / (s+1)(s^2+1)$

$2s^2 + 2s + 14 = A(s^2+1) + (Bs+c)(s+1)$

$2s^2 + 2s + 14 = As^2 + A + Bs^2 + Bs + Cs + C$

$2s^2 + 2s + 14 = s^2(A+B) + s(B+C) + A + C$

$A + B = 2 \quad B + C = 2 \quad A + C = 14$

Matematika 3

Ime i prezime: JELENA MALEŠ **Matični broj u indeksu:** 17-2-0103-2011

$A + B = 2$

$B + C = 2$

$A + C = 14$

$B + C = 2 \quad / \cdot (-1)$

$A + C = 14$

$-B - C = -2$

$A + C = 14$

$A - B = 12$

$A + B = 2$

$2A = 14$

$\boxed{A = 7}$

$A + C = 14$

$7 + C = 14$

$C = 14 - 7$

$\boxed{C = 7}$

$A + B = 2$

$7 + B = 2$

$B = 2 - 7$

$\boxed{B = -5}$

$$f(s) = \frac{7}{s+1} + \frac{-5s + 7}{s^2 + 1}$$

$$f(s) = 7 \cdot \frac{1}{s+1} - 5 \cdot \frac{s}{s^2 + 1^2} + 7 \cdot \frac{1}{s^2 + 1^2}$$

$$\boxed{f(t) = 7 \cdot e^{-t} - 5 \cdot \cos t + 7 \cdot \sin t}$$

PROVJERA ?

2. $\underbrace{x^2+y^2}_{r}=2^2$ $\qquad z=y+1$ pravac $\qquad z=-2$

$r^2=2^2$

$r=2$

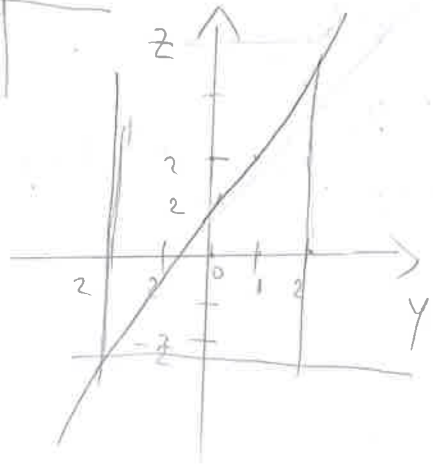
y	$y+1$
1	2
2	3
3	4

$x=r\cos\varphi$

$y=r\sin\varphi$

$z=z$

$dx\,dy\,dz=r\,dr\,d\varphi\,dz$



$$V=\int_0^{2\pi}\int_0^{2}\int_{-2}^{\boxed{y+1}\ r\sin\varphi+1} r\,dz\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\int_0^{2} rz\Big|_{-2}^{r\sin\varphi+1} dr\,d\varphi=\int_0^{2\pi}\int_0^{2}\Big((r\cdot(r\sin\varphi+1))-(r\cdot(-2))\Big)\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\int_0^{2} r^2\sin\varphi+r+2r\,dr\,d\varphi=\int_0^{2\pi}\int_0^{2}\left(r^2\sin\varphi+3r\right)dr\,d\varphi$$

$$=\int_0^{2\pi}\frac{1}{3}r^3\sin\varphi+\frac{3}{2}r^2\Big|_0^2\,d\varphi=\int_0^{2\pi}\left(\frac{1}{3}2^3\sin\varphi+\frac{3}{2}2^2\right)-0\,d\varphi$$

$$=\int_0^{2\pi}\frac{8}{3}\sin\varphi+\frac{12}{2}=6\,d\varphi=$$

$$=\frac{8}{3}\cos\varphi+6\varphi\Big|_0^{2\pi}=\frac{8}{3}\cos 2\pi+6\cdot 2\pi=\boxed{12\pi}$$

4. $\iint_S (x^2+y^2)\,dS \quad dx\,dy$

$z = \sqrt{x^2+y^2} \qquad 0 \le z \le 4$

$z = \sqrt{r^2}$

$\boxed{z = r}$

$z \in [0,4]$

$\boxed{r = 4} \in (0,4)$

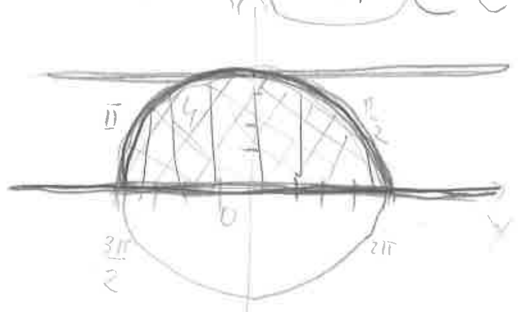
$x = r\cos\varphi$

$y = r\sin\varphi$

$x^2 = r\cos^2\varphi$

$y^2 = r\sin^2\varphi$

$\iint_{\varphi\, r} r\,dr\,d\varphi$



$\int_0^{\pi}\int_0^{4} (r\cos^2\varphi + r\sin^2\varphi)\, r\,dr\,d\varphi =$

$= \int_0^{\pi}\int_0^{4} r^2\cos^2\varphi + r^2\sin^2\varphi\, dr\,d\varphi =$

$= \int_0^{\pi} \frac{r^3}{3}\cos^2\varphi + \frac{r^3}{3}\sin^2\varphi \Big|_0^4 d\varphi =$

$= \int_0^{\pi} \frac{4^3}{3}\cos^2\varphi + \frac{4^3}{3}\sin^2\varphi - (0\cos^2\varphi + 0\sin^2\varphi)$

$= -\frac{64}{3}\sin^2\varphi + \frac{64}{3}\cos^2\varphi \Big|_0^{\pi}$

$= -\frac{64}{3}(\sin^2\pi + \frac{64}{3}(\cos^2\pi$

$= 0$

JELENA MALEŠ 17-2-0103-2011

3. $\int_{\partial K} \vec{E} \cdot ds \rightarrow dt$

$r(t) \begin{pmatrix} x = r\cos t \\ y = r\sin t \end{pmatrix}$

$r'(t) = \begin{pmatrix} -r\sin t \\ \\ \end{pmatrix}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C3

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: JOSIP FEŠTINI **VRIJEME POČETKA:**

MATIČNI BROJ STUDENTA: 26. 02. 2015. **USTMENI ISPIT KOD NASTAVNIKA:**

17-2-0233-2012 N. Uglešić

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) + y'(t) + y(t) = 0, \quad y(0) = 2,\ y'(0) = 4,\ y''(0) = 4.$$

Pomoć: $x^3 + x^2 + x + 1 = (x+1)(x^2+1)$.

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5. Izračunati 20

$$\int_{(0,\pi)}^{(2,2\pi)} 2x \sin y\, \mathrm{d}x + (x^2+1)\cos y\, \mathrm{d}y$$

Ukupno: 20

(1) $y'''(t) + y''(t) + y'(t) + y(t) = 0 \qquad y(0) = 2,\ y'(0) = 4 \qquad y''(0) = 4$

$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + s^2 Y(s) - s y(0) - y'(0) + s Y(s) - y(0) + Y(s) = 0$

$s^3 Y(s) - 2s^2 - 4s - 4 + s^2 Y(s) - 2s - 4 + s Y(s) - 2 + Y(s) = 0$

$s + s^3 + s^2 + 1 = 2s^2 + 4s + 4 + 2s + 4 + 2$

$s^3 + s^2 + s + 1 = 2s^2 + 4s + 4 + 2s + 4 + 2$

$s^3 + s^2 + s + 1 = 2s^2 + 6s + 10$

$Y(s) = \dfrac{2s^2 + 6s + 10}{s^3 + s^2 + s + 1} = \dfrac{A}{(s+1)} + \dfrac{Bs + C}{(s^2+1)}$

$Y(s)\ \dfrac{2s^2 + 6s + 10}{(s+1)(s^2+1)} = \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+1} \quad \Big/ (s+1)(s^2+1)$

$2s^2 + 6s + 10 = A(s^2+1) + (Bs+C)(s+1)$

$2s^2 + 6s + 10 = As^2 + A + Bs^2 + Bs + Cs + C$

$2 = A + B$

$6 = B + C$

$10 = A + C$

$\Rightarrow$ druga strana

① $2 = A + B$

$6 = B + C \Rightarrow -C = -6 + B$

$10 = A + C \qquad C = 6 - B$

$$\left|\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 6 \\ 1 & 0 & 1 & 10 \end{array}\right| = \quad \text{III} - \text{I}$$

$$= \left|\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 6 \\ 0 & -1 & 1 & 8 \end{array}\right| \begin{array}{l} \text{I} - \text{II} \\ \\ \text{III} + \text{II} \end{array} = \left|\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 2 & 14 \end{array}\right| /:2 = \left|\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & 7 \end{array}\right| \begin{array}{l} \text{I} + \text{III} \\ \text{II} - \text{III} \\ \\ \end{array}$$

$$= \left|\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 7 \end{array}\right|$$

$A = 3$

$B = -1$

$C = 7$

Provjera:

$$\frac{2+6+10}{4} = \frac{3}{2} + \frac{-1+7}{2}$$

$$\frac{18}{4} = \frac{3}{2} + \frac{6}{2}$$

$$\frac{9}{2} = \frac{9}{2}$$

$$y(s) = \frac{3}{s+1} + \frac{-1s + 7}{s^2+1}$$

$y(t) =$

$$y(t) = \mathcal{L}^{-1}\left[\frac{3}{s+1}\right] + \mathcal{L}^{-1}\left[\frac{-s+7}{s^2+1}\right]$$

$$y(t) = 3\mathcal{L}^{-1}\left[\frac{1}{s+1}\right] - \mathcal{L}^{-1}\left[\frac{+s}{s^2+1}\right] + 7\mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right]$$

$$y(t) = 3e^{-t} - \cancel{\cos}\ t + 7\sin(t)$$

$$y(t) = 3e^{-t} - \cos(t) + 7\sin(t)$$

(5)

$$\int_{(0,\pi)}^{(2,2\pi)} 2x \sin y \, dx + (x^2+1)\cos y \, dy = \left| \frac{2x^2}{2} \cdot \sin y \, x + (x^2+1)\, y \cdot \sin y \, x y \right|_{0,\pi}^{2,2\pi}$$

$$= \cancel{\ \ } + \left| \frac{2 \cdot 2^2}{2} \sin 2\pi \cdot 2 + (2^2+1) \cdot 2\pi \cdot \sin 2\pi \cdot 2 \cdot 2\pi \right.$$

$$= + \left[4 \sin 2\pi \cdot 2 + 5 \cdot 2\pi \cdot \sin 2\pi \cdot 2 \cdot 2\pi \right\}$$

$$= 0.875540855 + 43.20620959$$

$$= 44.0817$$

Matematika 3

Ime i prezime: JOSIP FEŠTINI

26.02.2015.

Matični broj u indeksu: 17-2-0233-2012

(2) valjak $x^2+y^2=2^2$

ravnine $z=y+1$

$z=-2$

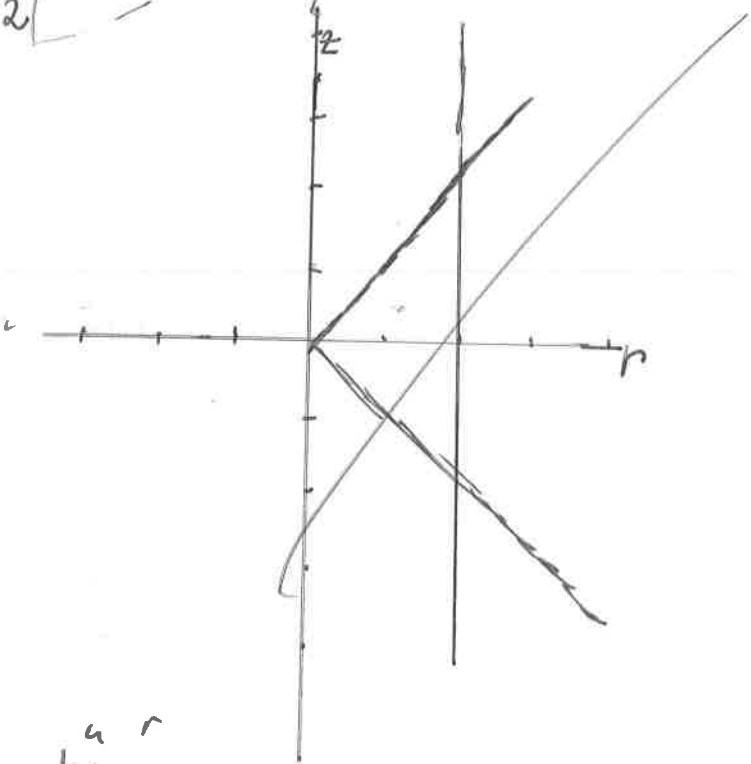
$r^2=z^2 \Rightarrow$ ~~$r^2=\ldots$~~ $z^2=r^2$

$r=|z| \Rightarrow r=4$

$r=-4$

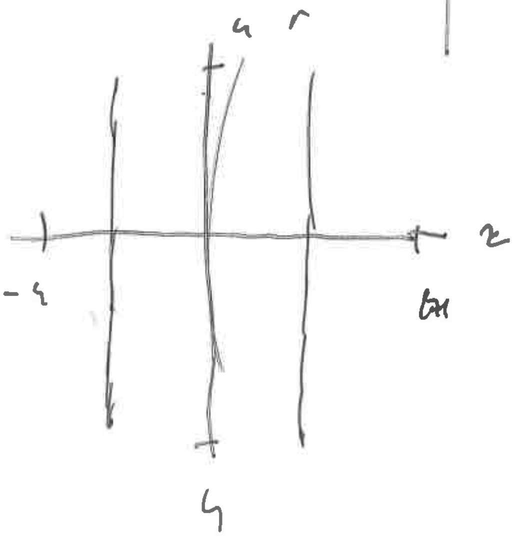
$z^2=r^2$

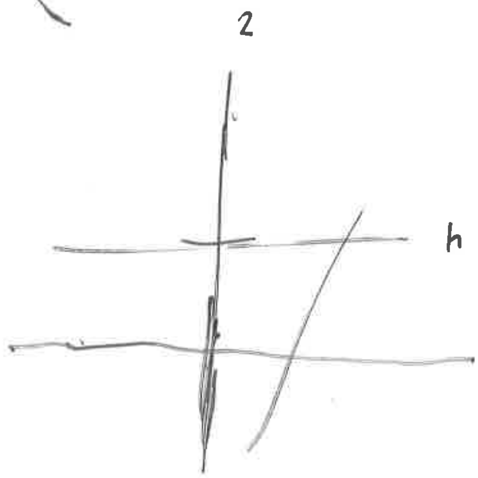
$z=r$



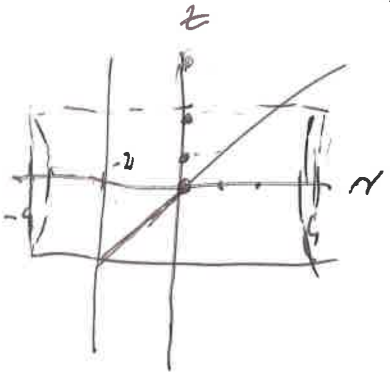
$z=y+1$

$$\frac{2}{4}\left|\frac{4}{0}\right|\frac{2}{1}\left|\frac{\cdot}{\cdot}\right|$$



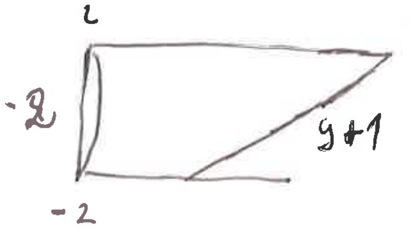


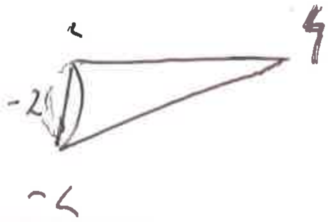
JOSIP FESTINI



$z = y + 1$

y	0	1	-1	3
z	1	2	0	4





$$V = \int_0^{2\pi} \int_z^{2} \int_{-2}^{4} 1 \; r \; dr \, dz \, d\varphi$$

$$= \int_0^{2\pi} \int_z^{2} \frac{r^2}{2} \Big|_{-2}^{4} = \int_0^{2\pi} \int_z^{2}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊ C3

Ime i prezime: MARIN MATEK **Vrijeme početka:**

Matični broj studenta: 17-1-0111-12 **Ustmeni ispit kod nastavnika:**

Popunjava nastavnik Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + y''(t) + y'(t) + y(t) = 0, \quad y(0) = 2,\ y'(0) = 4,\ y''(0) = 4\,.$$

Pomoć: $x^3 + x^2 + x + 1 = (x+1)(x^2+1)$.

2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 2^2$ i ravninama $z = y + 1$ i $z = -2$. 20

3. Neka je K krug radijusa A s centrom u ishodištu. Po rubu kruga postavljena je žica u obliku kružnice ∂K. Izračunaj elektormotornu silu $\int\limits_{\partial K} \vec{E}\cdot ds$ ako je električno polje $\vec{E}$ statičko i zadano izrazom $\vec{E} = \frac{1}{x^2+y^2}\left(x\,\vec{\mathbf{i}} + y\,\vec{\mathbf{j}}\right)$. Podrazumijeva se pozitivna orijentacija krivulje. 20

4. Izračunati $\iint_S (x^2+y^2)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z = \sqrt{x^2+y^2}$ i $0 \le z \le 4$. 20

5. Izračunati 20

$$\int\limits_{(0,\pi)}^{(2,2\pi)} 2x\sin y\,\mathrm{d}x + (x^2+1)\cos y\,\mathrm{d}y$$

Ukupno: 20

$y(0)=2$

$y'(0)=4$

$y''(0)=4$

1. $y'''(t) + y''(t) + y'(t) + y(t) = 0$

$$s^3Y(s) - s^2\underbrace{y(0)}_{2} - s\underbrace{y'(0)}_{4} - \underbrace{y''(0)}_{4} + s^2Y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{4} + sY(s) - \underbrace{y(0)}_{2} + Y(s) = 0$$

$$s^3Y(s) - 2s^2 - 4s - 4 + s^2Y(s) - 2s - 4 + sY(s) - 2 + Y(s) = 0$$

$$s^3Y(s) + s^2Y(s) + sY(s) + Y(s) = 2s^2 + 4s + 4 + 2s + 4 + 2 = 0$$

$$Y(s)(s^3 + s^2 + s + 1) = 2s^2 + 6s + 10$$

$$Y_s = \frac{2s^2+6s+10}{s^3+s^2+s+1} = \frac{2s^2+6s+10}{(s+1)(s^2+1)}$$

$$\frac{2s^2+6s+10}{(s+1)(s^2+1)} = \frac{A}{s+1} + \frac{Bs+C}{(s^2+1)} \Big/ (s+1)(s^2+1)$$

$$2s^2+6s+10 = A(s^2+1) + (Bs+C)(s+1)$$

$$= As^2 + A + Bs^2 + Bs + Cs + C$$

$$= s^2(A+B) + s(B+C) + A + C$$

→

$A+B=2 \quad \Rightarrow B = 2-A \qquad 2-A+C=6$

$B+C=6 \qquad B=-1 \qquad C=4+A \Rightarrow C=7$

$A+C=10$

$A+4+A=10$

$2A=6$

$A=3$

$= \frac{3}{s+1} + \frac{-1s+7}{s^2+1} = 3 \cdot \frac{1}{s+1} - \frac{s}{s^2+1} + 7\frac{1}{s^2+1}$

$= 3e^{-t} - \cos(t) + 7\sin(t)$ //

PROVJERA

$y(0) = 3 \cdot e^{-0} - \cos(0) = 3 \cdot 1 - 1 = 2$ ✓

$y'(0) = -3e^{-0} + \sin(0) + 7\cos(0) = 4$ ✓

2. $x^2+y^2=2^2 \qquad z=y+1 \quad z=-2 \qquad \varphi \in [0, 2\pi]$

$r=2 \qquad x = r\cos\varphi \qquad r \in [0,2]$

$y = r\sin\varphi \qquad z \in [-2, y+1]$

$dx\,dy = r\,dr\,d\varphi$

$\emptyset$

Matematika **3**

Ime i prezime: MARIN MATEK **Matični broj u indeksu:** 17-1-0111-12

4. $\iint_S (x^2+y^2)\,ds$ $\quad r=\sqrt{x^2+y^2}$ $\quad 0\leq R\leq 4$

$d\varphi \in [0, 2\pi]$

$dr \in [r, \sqrt{x^2+y^2}]$

$\varnothing$

5.

$\iint$

$\int_0^2\int_\pi^{2\pi} 2x \sin y\, dx + (x^2+1)\cos y\, dy$

$= \int_0^2 2\left(\frac{x^2}{2}\right)\sin y + \frac{x^3}{3}\cos y + y\cos y \Big|_\pi^{2\pi}$

$\varnothing$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C3

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: LUKA MILIN VRIJEME POČETKA:

MATIČNI BROJ STUDENTA: 0269066346 USTMENI ISPIT KOD NASTAVNIKA: prof. UGLEŠIĆ

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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5. Izračunati 20

$$\int_{(0,\pi)}^{(2,2\pi)} 2x \sin y \,\mathrm{d}x + (x^2+1)\cos y \,\mathrm{d}y$$

Ukupno: 0

(1) $y'''(t) + y''(t) + y'(t) + y(t) = 0 \qquad y(0)=2,\ y'(0)=4,\ y''(0)=4$

$y'(t) = sF(s) - f(0)$

$y''(t) = s^2F(s) - sf(0) - f(0)$

$y'''(t) = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

$x^3 + x^2 + x + 1 = (x+1)(x^2+1)$

$s^3y(s) - s^2y(0) - sy'(0) - y''(0) + s^2y(s) - sy(0) - y'(0) - sy(s) - y(0) + y(s) = 0$

$s^3y(s) - s^2\cdot 2 - s\cdot 4 - 4 + s^2y(s) - s\cdot 2 - 4 - sy(s) - 2 + y(s) = 0$

$s^3y(s) - 2s^2 - 4s - 4 + s^2y(s) - 2s - 4 - sy(s) - 2 + y(s) = 0$

$s^3y(s) + s^2y(s) - sy(s) + y(s) - 2s^2 - 6s - 10 = 0$

$s^3y(s) + s^2y(s) - sy(s) + y(s) = 2s^2 + 6s + 10$

$y(s)(s^3 + s^2 - s + 1) = 2s^2 + 6s + 10 \ /: (s^3 + s^2 - s)$

$y(s) = \dfrac{2s^2 + 6s + 10}{s^3 + s^2 - s + 1} = \dfrac{2s^2 + 6s + 10}{(s+1)(s^2-1)}$

$y(s) = \dfrac{A}{s+1} + \dfrac{Bx + C}{s^2 - 1}$

$s^3 - s + s^2 - 1$

$(s^3 + s^2 - s + 1)$

$(s+1)(s^2+1)$

$s^3 + s + s^2 + 1$

$s^3 + s^2 + s + 1$

$s^3 - s + s^2 + 1$

$s^3 + s^2 - s + 1$

(2) $x^2+y^2=2^2$ $\quad$ $z=y+1$, $z=-2$